Localized optical micromotors based on twin laser cavity solitons

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It has recently been shown that vertical-cavity surface-emitting lasers with an intracavity saturable absorber are capable of forming binary localized structures called *twin* laser cavity solitons (LCSs). Apart from their asymmetric intensity distribution, they can spontaneously rotate about their center of mass with a frequency that is controllable via a bifurcation parameter representing the ratio of carrier lifetimes in the amplifier and in the absorber. Moreover, circles of maximum phase are found to form around the center of the binary structure which start rotation once the twin LCSs begin to do so. In this paper, we propose to use the fact that these phase circles continually shrink on the center of mass to trap microparticles at the center and subsequently rotate them through the dipole force provided by the asymmetric intensity distribution. This finding can be regarded as a first step in the realization of a localized microdimensional optical motor potentially useful in the manipulation of microparticles and nanofluids.

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I. INTRODUCTION

The formation of regular and localized structures in spatially extended systems far from thermodynamical equilibrium has been the subject of a vast amount of research in the past two decades [1–5]. For optical systems, spatiotemporal phenomena arise in the structure of the electromagnetic field on the plane orthogonal to the direction of propagation as a result of the nonlinear response of the materials to intense laser beams and the spatial coupling provided by diffraction. Dissipation and driving and/or feedback are the features that can bring about lots of intriguing and novel properties once introduced to the schemes. Among these, localized bright spots or "cavity solitons (CSs)" have received a great deal of attention because of their experimental realizability in semiconductor microresonators and potential applications in information processing [6–10]. This is because CSs are attractors, i.e., stable solutions towards which the system evolves spontaneously from a wide set of initial conditions. This entails that, at the difference from their conservative analogs, CSs do not rely on a proper seeding of the initial conditions. This makes them extremely interesting for applications and, in particular, for information processing in which CSs are used as bit units. For example, CS-based all-optical buffers [11,12], logic gates [13], ultra-low-energy switches [14], and many others have been shown to be feasible candidates [15–17]. Among the different configurations reported for the realization of CSs, systems of no external injection have been studied extensively as they are free from the limitation that requires a controlled detuning to the resonance of the microcavity. Called laser cavity solitons (LCSs), they possess phase symmetry and freedom of choosing their polarization and frequency. More particularly, LCSs are realized in a single vertical-cavity surface-emitting laser (VCSEL) containing two media: an amplifier and an absorber [18]. The presence of two media with different timescales has put forward many possibilities for realizing LCSs of different dynamical behaviors owing to a translational [19] or Andronov-Hopf [20] instability.

Moreover, in certain switching conditions where the injection pulse energy exceeds a threshold twin LCSs are created from a single spot and trapped by each other's attractive force exchanging their amplitudes such that the sum of the intensities remains constant. In typical situations where a single LCS is present, the translational instability leads to a drifting LCS; however, in this regime it makes the twin LCSs rotate about the center of the line connecting their intensity maxima as a result of their mutual interaction. As the details of such binary localized structures can be found in Ref. [21], in this paper we propose to make use of their optical force in the rotating regime for designing localized optical micromotors.

In Sec. II, we present the details of the dynamical equations and the relevant physical parameters, Sec. III is devoted to discussions on the existence and properties of twin LCSs, followed by Sec. IV where their rotation feature and the idea of a localized micromotor are presented. Finally, conclusions are drawn in Sec. V.

II. THE MODEL

The dynamics of such a VCSEL-based CS laser containing a saturable absorber is described by the following set of equations for an intracavity field and populations in the two media [18,22]:

$$\begin{aligned}\partial_t F &= [(1-i\alpha)D + (1-i\beta)d - 1 + i\nabla^2]F,\\ \partial_t D &= b_1[\mu - D(1+|F|^2) - BD^2],\\ \partial_t d &= b_2[-\gamma - d(1+s|F|^2) - Bd^2],\end{aligned} \tag{1}$$

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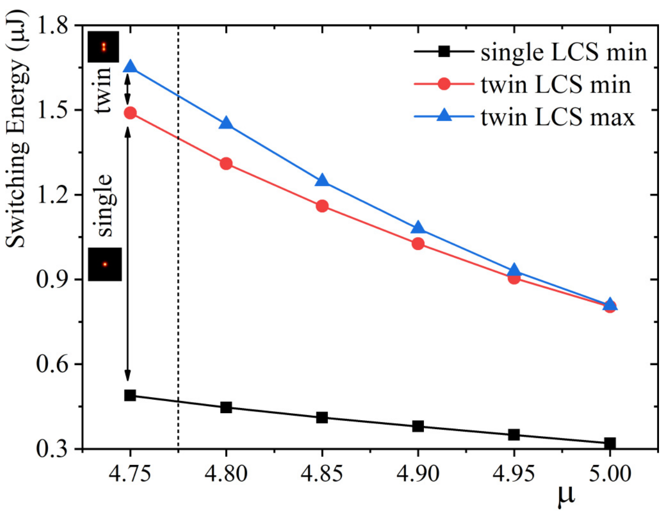

FIG. 1. Energy bands available for switching single and twin LCSs versus pump current μ. r is fixed at 0.85. The vertical dashed line marks the boundary which separates only oscillating twin LCSs (to the right) from those of oscillating and rotating (to the left) LCSs.

in which F is the slowly varying amplitude of the electric field and $D,\ d$ are the population variables defined as

$$D = \eta_1(N_1/N_{1,0} - 1), \quad d = \eta_2(N_2/N_{2,0} - 1), \tag{2}$$

where N_1 and N_2 are the carrier densities in the active and passive materials, respectively, $N_{1,0}$ and $N_{2,0}$ are their transparency values, and η_1 and η_2 are dimensionless coefficients related to gain and absorption, respectively. The parameters α and b_1 (β and b_2) are the linewidth enhancement factor and the ratio of the photon lifetime for the carrier lifetime in the active (passive) material, μ is the pump parameter of the active material, γ is the absorption parameter of the passive material, s is the saturation parameter, and B is the coefficient of radiative recombination. Time is scaled to the photon lifetime, and space is scaled to the diffraction length. Typically, a time unit is a few picoseconds and a space unit is $\sim$4 μm. The integration of the dynamical equations is performed by a split-step method separating the time and space derivatives on a 128×128 spatial grid with space step 0.5 implying the physical distance of 2 μm between two consecutive grid points. It uses the fast Fourier transform for the Laplacian and Runge-Kutta for time derivatives along with the periodic boundary condition. We have used the following parameter values throughout the paper unless stated otherwise: $s = 1,\ B = 0.1,\ \alpha = 2,\ \beta = 1,\ b_1 = 0.01$, and $\gamma = 2$. The bifurcation parameter which is responsible for passing from stable stationary to the moving LCS region in the translational instability is defined as $r = b_2/b_1$.

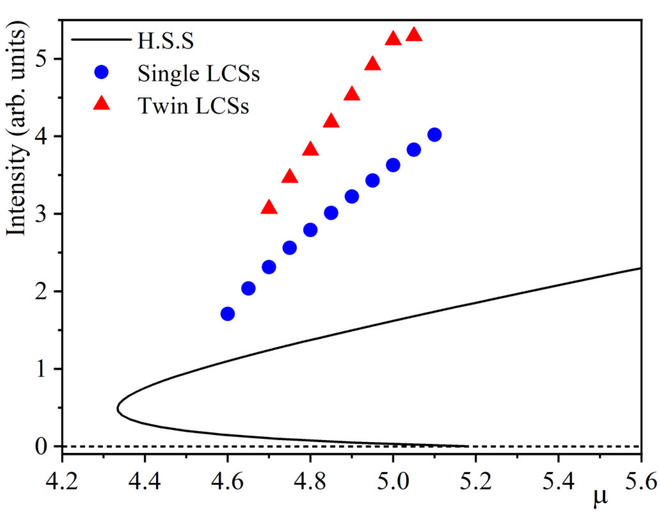

FIG. 2. The homogenous steady-state (H.S.S) solution and the single-twin LCS branches. $r = 1$.

III. TWIN LASER CAVITY SOLITONS: EXISTENCE AND PROPERTIES

Single LCSs in VCSELs with saturable absorbers are excited by typical address pulses which are Gaussian in shape and rectangular in time and it has been shown that there is always a band of possible energies for the address pulse above

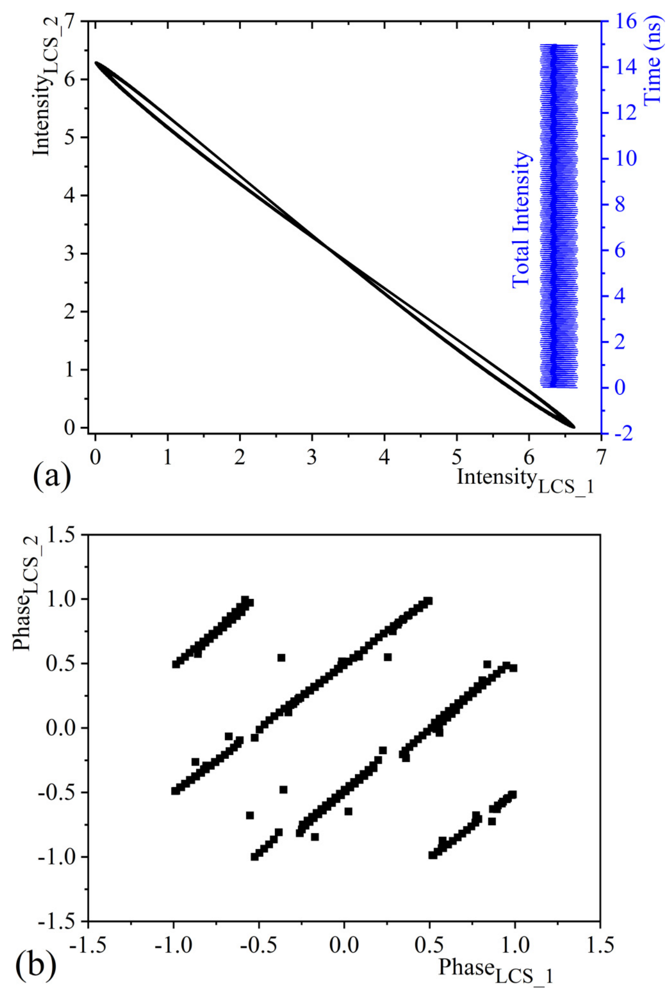

FIG. 3. Entanglement of the two LCSs in the twin structure in terms of (a) intensity and (b) phase. The values on the vertical and horizontal axes in (b) are divided by π. Parameter values are as follows: $\mu = 5.05,\ r = 1$.

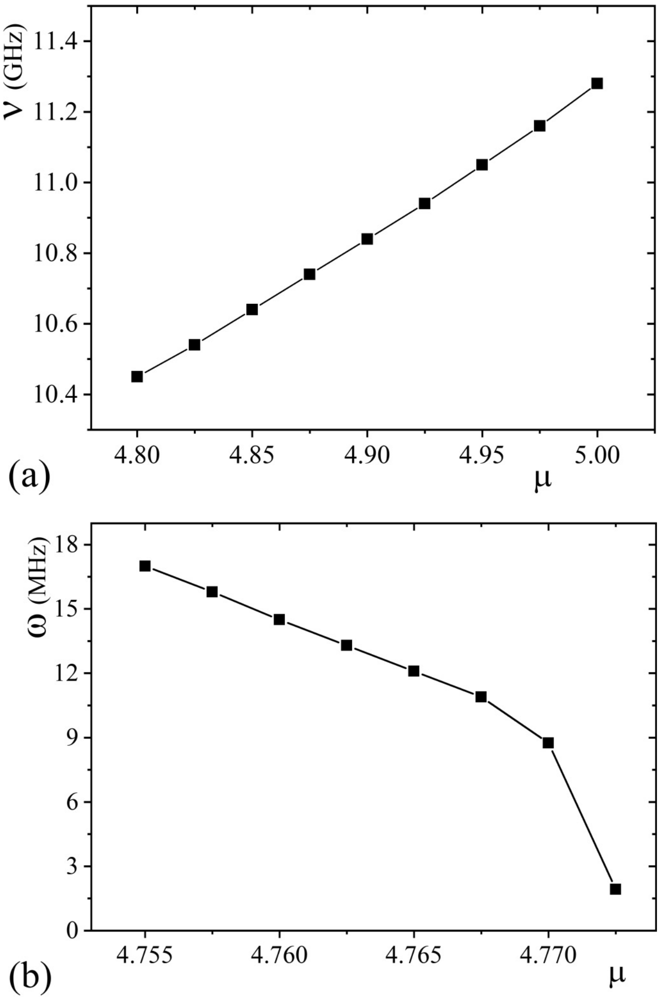

FIG. 4. Frequency of (a) intensity oscillations ν and that of (b) rotation about the center of mass (in cycles per second) as a function of the pump current μ. In (a) the frequencies are calculated for the regime of only oscillating LCSs. $r = 0.85$.

and below which the switching process fails [23]. Also, it has been reported that double-peak (twin) LCSs can be excited if the switching pulse energy slightly exceeds the maximum for exciting a single LCS [21]. In Fig. 1, the available switching energy bands for single and twin LCSs are shown with respect to the pump value. The creation of twin LCSs shows itself first through the widening of the initial bright spot and then through its breakup into two attached LCSs trapped by each other's attractive force exchanging their amplitudes such that the sum of the intensities remains constant at a value roughly equal to two times the intensity of a single regular LCS in the same regime. Single and twin LCS branches are shown in Fig. 2 along with the homogeneous steady-state solution. The term *twin* translates to the fact that the two LCSs are born from a single carrier dip and behave in a very much entangled way in their intensities and phases. In Fig. 3(a), it is seen that the exchange of intensities between the two elements in the pair happens very tightly. This is further verified by the figure shown on the right side of Fig. 3(a) where the sum of intensities exhibit minor fluctuations manifesting a very efficient energy exchange. This behavior is accompanied by some sort of phase boundedness between the LCSs in the twin structure such that they only take certain phase values and their relative phase always remains a multiple of $\pi/2$ as shown in Fig. 3(b).

Although twin LCSs are shown to exhibit periodic intensity oscillations, rotating and traveling features can also be excited by changing the bifurcation parameter r, associated with the carrier lifetimes' ratio in the amplifier and absorber materials. This bifurcation parameter affects the dynamics of the twin LCSs such that, for $\mu = 5$, in the range of $r \geqslant 0.75$ they only oscillate periodically, in the range of $0.70 < r \leqslant 0.74$ they oscillate and rotate about each other, and finally in the range of $r \leqslant 0.70$ a traveling behavior is added to the other two. It should also be mentioned that for r values lower than 0.675 the motion is so fast that the LCS pair cannot survive and ends up in a transversely turbulent structure. The parameter r also controls the frequency of intensity oscillations, frequency of the rotation about the center of mass, and the velocity of motion across the transverse plane. Figure 4(a) depicts the frequency ν of intensity oscillations at the peak of one of the LCSs as a function of μ, which has a growing trend, and that of rotation around the center of mass ω is shown in Fig. 4(b),

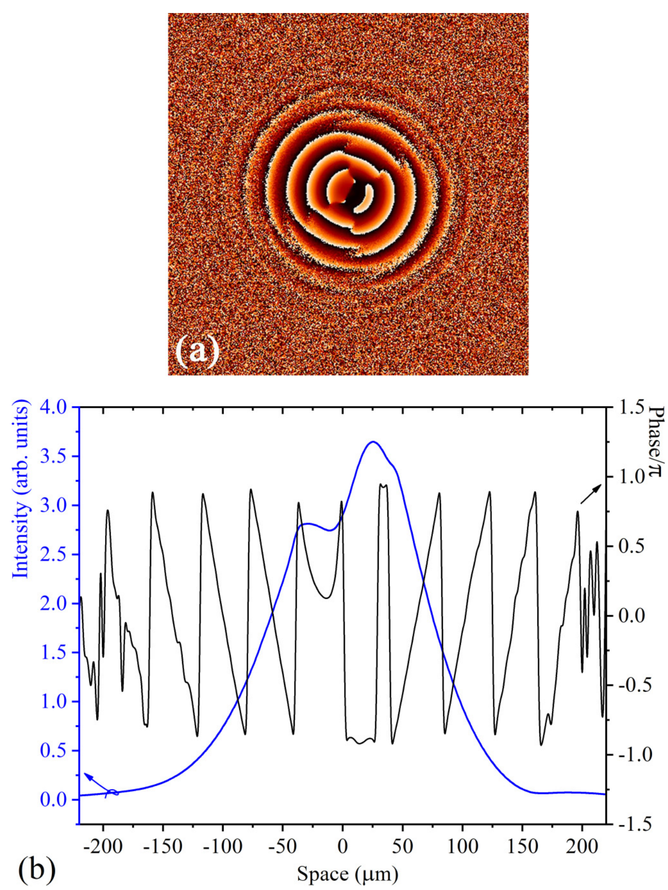

FIG. 5. (a) Phase structure of the transverse field taken from simulations and (b) the phase-field profiles along the line connecting the centers of the two peaks. A zero phase value is found almost at the intensity minimum between the two LCSs. $\mu = 5.0,\ r = 0.85$. See the Supplemental Material for a multimedia file (simulation movie) corresponding to this complex motion of the circles of the maximum phase [24].

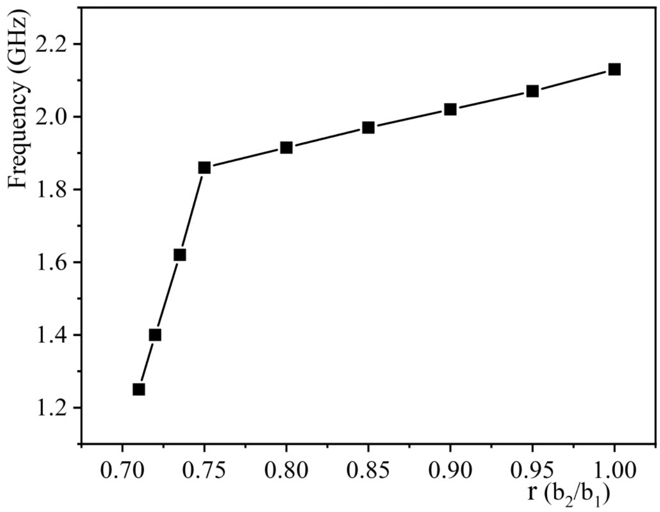

FIG. 6. The frequency at which circles of the maximum phase value shrink toward the center of mass of the binary structure versus the bifurcation parameter r. $\mu = 5$.

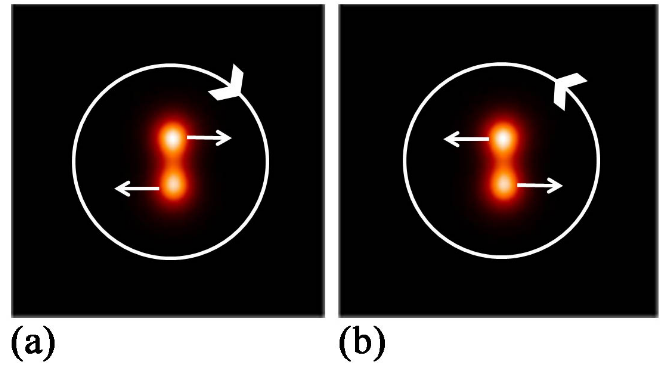

FIG. 7. Simultaneous injection of rotatory pulses (low in amplitude and short in time) in the vicinity of the peaks causing (a) clockwise and (b) counterclockwise rotation of the twin LCSs. The arrows are pointed at the locations where the rotatory pulses are injected. See the Supplemental Material for a multimedia file (simulation movie) corresponding to the counterclockwise rotation [24].

which decreases as the pump current increases. We note that in experiments one can equally use the pump current μ instead of r as a control parameter.

Another feature is the dynamical phase structure appearing as concentric circles of maximum phase values around the center of mass of the twin LCSs. As shown in Fig. 5, the phase matrix of the transverse field exhibits periodic phase circles which shrink toward the center and disappear one after another. These phase circles start to rotate as the twin LCSs do so when the bifurcation parameter r is put in the $0.70 < r \leqslant 0.74$ interval. Once more, the frequency (speed) of creation and annihilation of the phase circles by shrinking on the center of mass is controlled by the bifurcation parameter r as shown in Fig. 6.

IV. ROTATING TWIN LASER CAVITY SOLITONS: LOCALIZED MICROMOTORS

Many scenarios have been suggested so far for trapping and rotating microdimensional objects starting with the pioneering work of Ashkin *et al.* where the reorientation of the objects affected by the symmetry of the beam caused the desired rotation of the rod-shaped bacteria and their rearrangement in the direction of the beam axis [25]. It is naturally preferred for an elongated object to adjust its longer axis in the direction of the beam propagation. If the light is asymmetrically focused, then nonspherical objects will find themselves co-oriented with the beam. It is therefore concluded that the rotation of an asymmetric beam leads to a subsequent rotation of an object.

Unlike this first mechanism where the rotation forces and torques are directly originated from the beam asymmetry, imposing rotation to the objects is also possible by beams of circular symmetry in intensity distribution and through the transfer of angular momentum. It is well known that light can have both spin angular momentum (from its polarization state) and orbital angular momentum (from its phase structure). Depending on the right or left handedness of the circular polarization of the light, spin angular momentum can take the values of $\pm\hbar$; whereas orbital angular momentum is described by $\phi = \exp(il\theta)$ for a beam having helical phase fronts which can have an azimuthal component in the Poynting vector giving orbital angular momentum $l\hbar$ per photon. A detailed description of the subject of inducing rotation by optical gradients or transfer of angular momentum can be found in Ref. [26].

Trapping and rotating an object will be more efficient if both the gradient forces of an asymmetric beam and its phase are used together. For instance, in Ref. [27] Roichman *et al.* have shown that phase gradients in a light field can be used to create a new category of optical traps complementary to the more familiar intensity-gradient traps known as optical tweezers.

We propose in this paper that twin LCSs can realize such a combination as the binary structure is nonsymmetric in intensity distribution and possesses phase circles of maximum values which repeatedly move towards the center of mass. Our explanation of the proposal includes two sections: (i) the gradient force of the asymmetric intensity distribution whose direction depends on the detuning between the atomic resonance frequency and that of the optical field, and (ii) the trapping feature of the shrinking phase circles for which the resultant force lines are always towards the center.

As mentioned earlier, spontaneous rotation of the asymmetric intensity distribution of twin LCSs is excited by adjusting the bifurcation parameter r which, in turn, activates the so-called Goldstone modes close to the translational instability threshold. As the activity of the Goldstone modes creates random directions of a gradient with closer or farther distances from the peaks of the LCSs in the twin structure, the rotation could start clockwise or counterclockwise with no preference. To control the direction of rotation and its angular frequency we propose to use ultrashort pulses perpendicular to the axis of the twin structure according to Fig. 7 when the value of r is at the threshold of the spontaneous rotation regime for which a fixed position of peaks is expected. Working close to the boundary of the rotating regime gives us a further possibility for maintaining lower rotation frequencies on the order of kilohertz. For example, injecting rotatory pulses

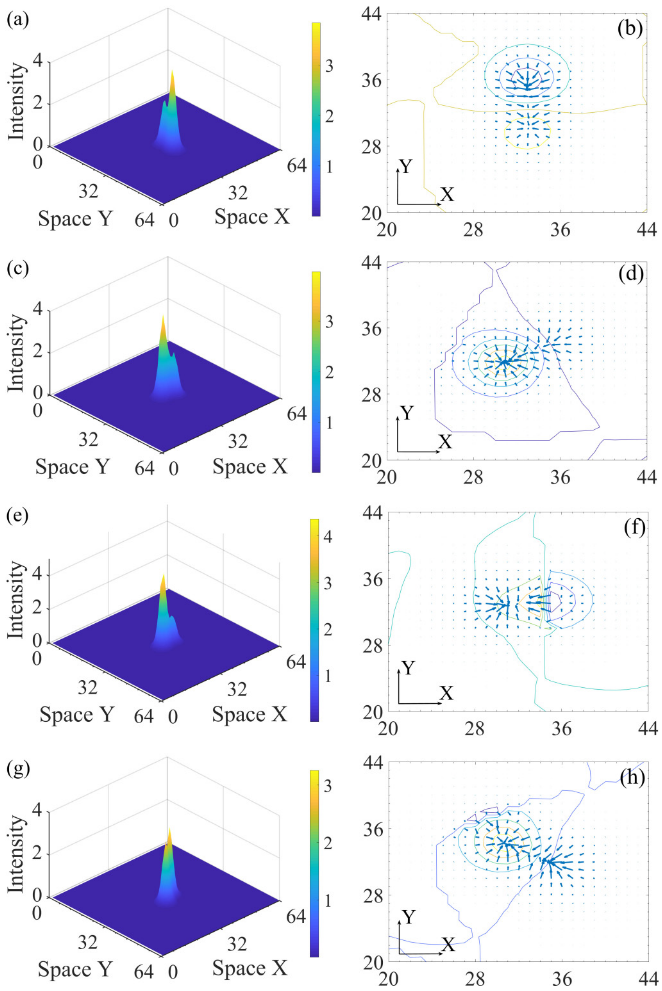

FIG. 8. Force field experienced by the particle as the twin LCSs are in clockwise rotation. Intensity distributions (a), (c), (e), (g) and their respective force fields (b), (d), (f), and (h). The values are as follows: $\Gamma = 0.0621$, $\Omega = -0.05$ GHz, $\mu = 5$, $r = 0.72$. The time between consecutive snapshots is $\sim 90\ \mu$s.

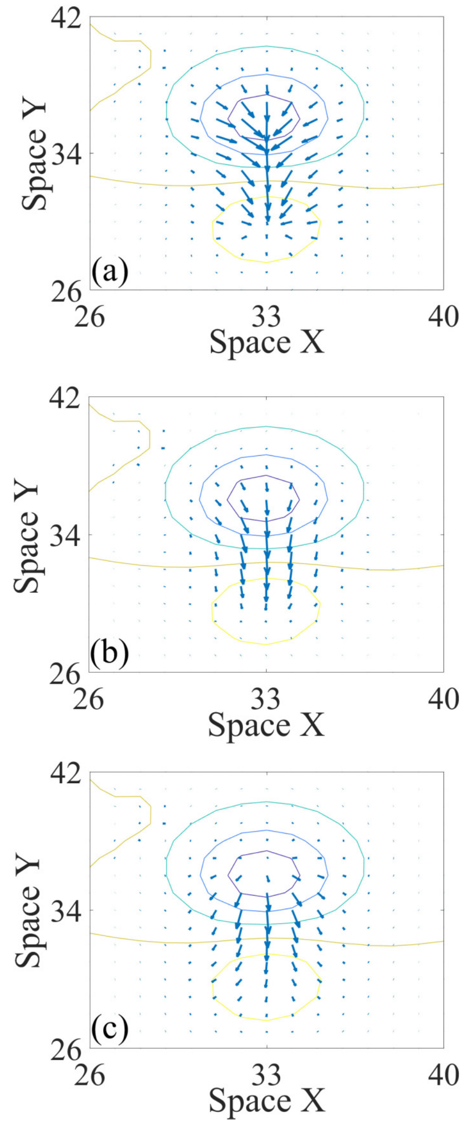

FIG. 9. Change in the direction of the force lines for different signs of the detuning: (a) for $\Omega = -0.015$ GHz the forces point to the center of the intensity maxima, (b) for $\Omega = 0$ the forces point to the center of the less intense LCS, and (c) for $\Omega = 0.015$ GHz the forces point outwards. Note that for $+0.010 < \Omega < +0.015$ GHz the gradients from the field and phase almost cancel out. The values are the same as in Fig. 8.

with an energy equal to 0.05 nJ for a twin LCS at $r = 0.755$ results in a rotation with the steady-state frequency of 40 kHz. Although, on average, the particle would experience cylindrically symmetric intensity distribution, the dipole force proportional to the intensity gradient with a sign depending on the detuning between the field and the atomic resonance will be responsible for the rotation of the particle.

Microparticles tend to move toward the phase maximum when they are exposed to a light beam of an inhomogeneous phase pattern, see Eq. (3) below. In our proposed scheme here since the phase increases by moving away from the center, the particle would experience a force in the same direction which would move it away if these phase circles were only stationary. However, we should note that the circles of the phase maximum repeatedly shrink on the center of mass and are replaced by new circles of larger radius by the frequencies shown in Fig. 6. We recall from Fig. 6 that the frequency of the process is determined by the value of r and considering the threshold case where rotatory pulses put the twin structure into rotation at $r = 0.755$, shrinking of the phase circles is actually much slower than the values shown in Fig. 6. This is the mechanism that tends to push the particles towards the center. To study the joint effect of the asymmetric intensity distribution and the shrinking phase circles, we added another dynamical equation to those of Eq. (2) which calculates the dynamical optical force $F_{\rm opt}$ exerted on a test particle by employing the amplitude F and phase Φ of the electric field,

$$F_{\rm opt} = |F|^2\left[-\Omega\nabla(\ln|F|) + \frac{\Gamma}{2}\nabla\Phi\right], \tag{3}$$

where Ω is the detuning of the atomic resonance frequency from that of the optical field and Γ represents the natural radiative decay rate of the atom for which we use the one for sodium given by 0.0621 GHz. The details of such calculations for optical force and motion of atoms in a trap can be found in the seminal paper of Gordon and Ashkin [28].

The sign of the detuning Ω in Eq. (3) is crucial since the force is proportional to the sum of two gradients, field modulus, and its phase. Clearly, the dipole force points toward the intensity maxima only if the detuning is negative. Instead, the component proportional to the gradient of the phase should always point toward the center. In Figs. 8(a), 8(c) 8(e), and 8(g) which show the intensity and orientation of the binary structure along with Figs. 8(b), 8(d) 8(f), and 8(h) which depict the corresponding contour plot of the optical force, we have used a negative value for Ω on the order of Γ. These figures illustrate that the net result of the gradients from the intensity and phase dynamics is always toward the two LCSs regardless of their intensity with respect to each other; and as the twin LCSs rotate around the center, the direction of the net force lines also follow thus dragging the particle around. We also show in Figs. 9(a)–9(c) that, by increasing the detuning value from negative to positive, the net gradient directions start to diverge from the intensity maxima and there is a critical positive detuning value for which the two forces more or less compensate each other.

V. CONCLUSION

We numerically studied the switching and dynamical behavior of rotating *twin* laser cavity solitons in a broad-area semiconductor laser with a saturable absorber. It is observed that such binary structures form circles of maximum phase around their center of mass which continually rotate and shrink on the center. We used the fact that these rotating structures have an asymmetric intensity distribution and shrinking maximum phase circles to propose an optical micromotor. By adding an equation to calculate field and phase gradients required for obtaining the force exerted upon a two-level atom, we showed that rotating *twin* laser cavity solitons are capable of trapping and rotating microparticles since the force field experienced by the particle is found to be always directed towards the LCSs if the detuning between the atomic resonance frequency and that of the light field is negative. We believe that this kind of study involving the calculation of optical force in a dynamical system is noteworthy and the proposed scheme can be implemented in real experimental conditions to realize localized optical micromotors.

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