Diffraction of an atom laser in the Raman-Nath regime

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An atom interferometer is a ubiquitous tool for measuring fundamental constants and inertial sensing. While it has been extremely useful in measuring inertial rotations, the fine-structure constant, gravity gradients, and local gravity, the measurement process lacks the ability to probe continuously due to its single-shot nature. In this work, we experimentally demonstrate the diffraction of an atom laser in the Raman-Nath regime, a key step towards the development of an atom-laser-based interferometer. The diffraction orders can be precisely controlled, and momenta up to $\pm 18\hbar k$ can be imparted to the atom laser. We form the "atom laser" by outcoupling a quasicontinuous beam of coherent atoms from a reservoir of ^{87}Rb Bose-Einstein condensate lasting up to 400 ms. This atom laser then interacts with a grating formed by a standing wave of far-detuned laser light. By controlling the interaction time, the strength of diffraction into various orders can be controlled. Such diffraction would allow for the construction of an atom-interferometer to probe changes in physical environments continuously up to a few hundred milliseconds.

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I. INTRODUCTION

Atom interferometry is a matter-wave-based interferometer realized through coherent manipulation of translational and internal degrees of freedom of atoms or molecules [1]. Atoms and molecules possess several physical properties (e.g., magnetic moment, mass, a high collision cross section, polarizability, etc.) which enable them to interact with various environments (e.g., magnetic fields, gravity, electric fields, etc.), making the interferometer extremely sensitive to the quantity of interest as opposed to photon-based interferometers. Atom interferometry has gained impetus in studying fundamental quantum science, inertial sensing [2–4], and precision metrology [5,6] and for next-generation quantum technologies [7]. Atom interferometry has been used to measure rotations [3,8–10], the fine-structure constant α [11–13], local gravity [14–18], gravity gradients [19,20], atomic polarizability [21], etc., with unprecedented accuracy and precision and has been proposed as a potential candidate for gravitational-wave detection [22,23]. In addition, precision metrology based on atom interferometry has also provided worldwide frequency standards [24–26]. The most popular atom interferometers are based on thermal or cold atomic samples. A thermal beam of atoms can measure a phase change with a resolution of $\sim 10^{-3}$ rad [1]. However, an ultracold cloud of Bose-Einstein condensate (BEC) can further improve the readout from the interferometer.

In spite of the advantages mentioned above, these interferometers cannot be used for monitoring a slowly varying physical parameter, and hence, the measurement needs to be repeated many times. This measurement process gets severely limited by the duty cycle of the production of the atomic wave packets and hence is limited in bandwidth and suffers from the aliasing effect [27], also known as the Dick effect [28]. Although there have been considerable efforts to reduce the wave-packet preparation time [29] and construct a zero-dead-time interferometer [30,31] using light-pulse atom interferometry, another possible solution to improve the data rate is to pursue continuous interrogation of the atomic wave packets by operating the optical grating continuously. A suitable form of matter wave needed to serve this purpose (a) should be a continuous source of the interfering wave packets (similar to a laser beam) and (b) should maintain long coherence times. These properties can be achieved with a continuously outcoupled stream of ultracold atoms from a reservoir. One such source is the outcoupling of a coherent beam of ultracold atoms from a reservoir formed by a BEC. Such an outcoupled beam of atoms, possessing a long coherence length, high collimation, and high brightness (limited by the flux), is known as an atom laser [32]. Because of better spatial-mode properties and fewer wave-front aberrations [33] atom lasers show promise for precision measurements of inertial effects. There has been a significant amount of work over the last few decades in the generation and characterization of atom lasers [32,34–46]. In Ref. [47], interference due to the overlap of many matter waves leaking from many sites of an optical lattice has been demonstrated. In Ref. [48] the authors demonstrated a pulsed atom laser formed due to the interference of many Airy functions from a tilted optical lattice. However, to the best of our knowledge, no attempt has been made so far to set up an interferometer with an atom laser. The first step towards such atom interferometry based on an atom laser would be to coherently split the atom laser and cause the different continuous wave packets to interfere.

II. THEORETICAL BACKGROUND

In this work, we demonstrate the Kapitza-Dirac [49] diffraction using an atom laser in the Raman-Nath [50] regime, which will open up a new direction towards building such an interferometer. This configuration can be easily modified to set up a standard three-grating interferometer [51] using an atom laser.

Among all the known techniques to outcouple an atom laser from a reservoir in cw or pulsed form, we choose the "spilling" method to generate a cw atom laser for several reasons: (i) a well-collimated atom laser can be achieved even without guiding, (ii) the flux rate can be controlled precisely, and (iii) the acceleration in the direction of gravity can be reduced by applying a magnetic field gradient. To study the diffraction of an atom laser, a pulsed light grating is placed in the path of the falling atom laser. We use square-pulse switching of the light grating with different pulse widths to study the coherent splitting of the atom laser. Below, we briefly summarize the relevant theory to understand the phenomenon. The light grating is made of a one-dimensional (1D) optical lattice whose electric field is given by

$$E(x,t) = 2E_0 \cos(kx)\cos(\omega t). \tag{1}$$

The strength of the interaction can be defined using the traveling-wave Rabi frequency $\Omega = \mu E/\hbar$, where μ is the dipole matrix element between the two atomic states coupled by light. Assuming that the atom laser has negligible momentum in the direction of the lattice, we can understand the effect of the grating as a diffraction of the atoms into several momentum eigenstates. The momentum of each eigenstate can be labeled as a multiple of $2\hbar k$, such that atoms in the nth diffracted order will possess $2n\hbar k$ momenta. Here $k = 2\pi/\lambda$ represents the wave number of the lattice laser. The population P_n in the nth diffraction order can be expressed as

$$P_n = J_n^2(\theta), \quad n = 0, \pm1, \pm2, \ldots, \tag{2}$$

where J_n is the nth-order Bessel function of the first kind and θ is the pulse area for the corresponding interaction time τ. θ can be expressed in terms of peak Rabi frequency Ω_0 and τ as

$$\theta = \frac{\Omega_0^2 \tau}{2\delta}, \tag{3}$$

where $\delta = (\omega_l - \omega_a)$ is the detuning of the laser, with ω_l being the laser frequency and ω_a beng the atomic resonance frequency. The condition for Kapitza-Dirac diffraction comes from the Raman-Nath approximation. The basic assumption underlying the theory is that the interaction time should be such that the distance traveled by the atoms during the interaction time is much less than the wavelength of the lattice. Thus, the limiting pulse width is given by $\tau \ll 1/\omega_{\rm rec}$, where $\omega_{\rm rec} = \hbar k^2/2m$ is the recoil frequency. Following the argument given in Ref. [52], a more precise condition for Kapitza-Dirac scattering is given by $\tau < \tau_{\rm osc}/4$, where

$$\tau_{\rm osc} = \frac{\pi}{\Omega_0}\left(\frac{|\delta|}{\omega_{\rm rec}}\right)^{1/2}. \tag{4}$$

Here $\tau_{\rm osc}$ is the period of oscillation of the atoms due to the curvature of the optical potential at the center of each lattice site.

III. EXPERIMENT AND RESULTS

The basic experimental setup to realize the system is similar to that given in Ref. [53]. We load a standard magneto-optical trap (MOT) with $\sim 10^7$ atoms of ^{87}Rb. With laser cooling we reach a temperature up to $\sim 30\ \mu$K [54]. The atoms are then transferred into a crossed optical dipole trap (wavelength $\lambda = 1064$ nm). Further evaporative cooling is performed by reducing the intensity of the lasers of the dipole trap to reach the quantum degeneracy level. We produce a BEC with $\sim 7\times10^4$ atoms. Starting from the loading of the MOT, the quadrupolar magnetic field is kept at a constant axial gradient of $\sim$24.5 G/cm.

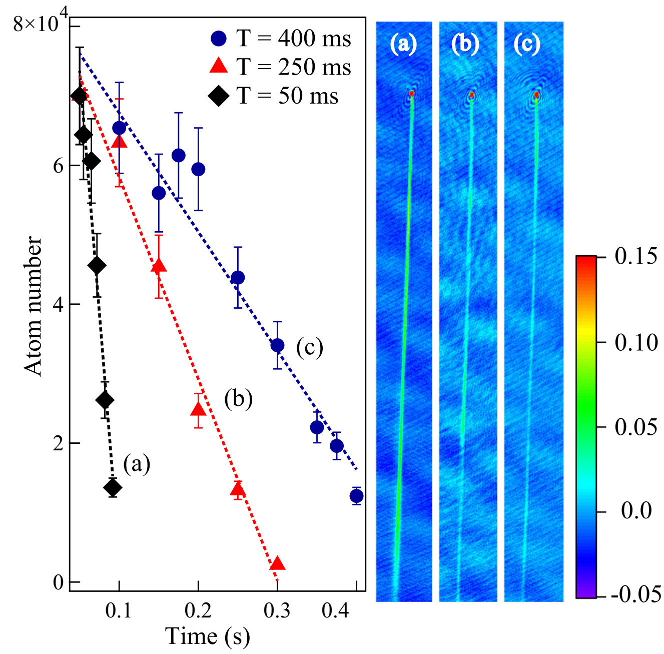

FIG. 1. Left: Number of atoms in the reservoir as a function of time for three different outcoupling rates. Atomic fluxes for the three cases are (a) 1.39×10^6, (b) 2.91×10^5, and (c) 1.71×10^5 atoms/s. Right: Images of the atom laser corresponding to the three flux rates in the left panel. These images are taken at (a) 40, (b) 200, and (c) 300 ms after the start of outcoupling. The color scale represents optical density. Although in the experiment the atom laser falls vertically along the axis of gravity, due to a relative tilt between the planes of the final imaging lens and the camera sensor, the absorption images of our atom laser look slanted towards left.

To outcouple the atoms from the reservoir of the BEC, the trap depth of the crossed dipole trap is further reduced adiabatically while keeping the magnetic field gradient constant. The flux of the atom laser is controlled by the rate of lowering of the depth of the optical dipole trap. However, the axial field gradient helps us to reduce the net acceleration felt by the falling atoms. For the value of the field gradient used in our experiment, we can slow the atom laser down to an acceleration of $\sim g/5$, where g is the acceleration due to gravity. In our experiment, keeping the magnetic field gradient constant, we have been able to outcouple an atom laser with varying flux rates. Three different rates of outcoupling are represented in Fig. 1. The absorption images shown in Fig. 1 are taken at (a) 40, (b) 200, and (c) 300 ms after the start of outcoupling. To increase the signal-to-noise ratio, each of these images has been averaged over 15 realizations for the same set of experimental parameters. The three images correspond to three different outcoupling rates: (a) 1.39×10^6,

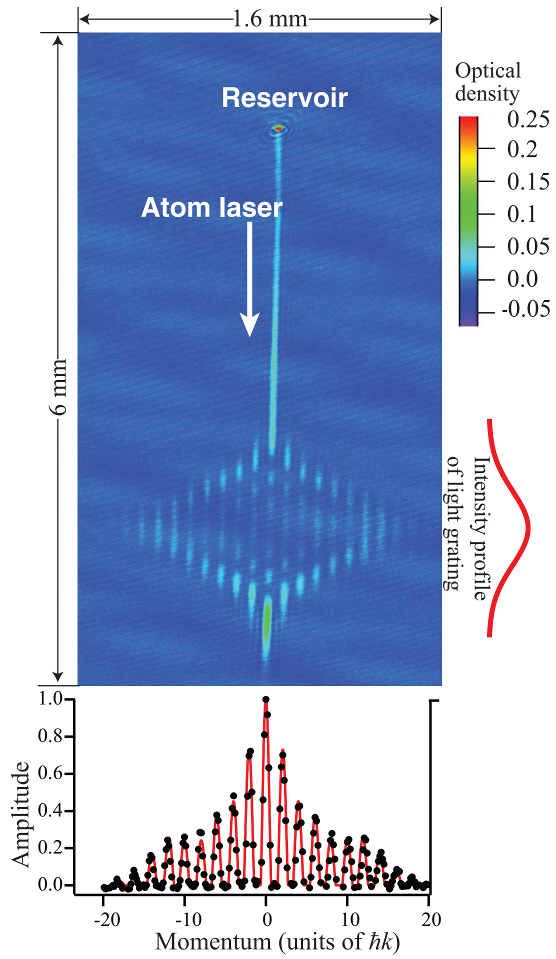

FIG. 2. The image in the top panel shows a typical time-of-flight absorption image of the atom-laser diffraction. The red curve drawn beside the image is the Gaussian intensity profile of the standing wave of light that forms the diffraction grating with which the atom laser interacts. The bottom panel shows the normalized population at different diffraction orders (up to $\pm 18\hbar k$). Circles represent experimental data, whereas the solid line shows the numerical simulation.

(b) 2.91×10^5, and (c) 1.71×10^5 atoms/s. The rate of outcoupling is measured by tracking the number of atoms left in the reservoir (see the left panel in Fig. 1). For the three cases of outcoupling shown in Fig. 1, we see the atom laser for a duration of (a) 50, (b) 250, and (c) 400 ms, respectively, before the reservoir is emptied.

To realize the Kapitza-Dirac effect with the above atom laser, a far-detuned 1D optical lattice is kept ∼3 mm below the reservoir. The lattice laser is −6.8 GHz detuned from the $5S_{1/2}F = 1 \rightarrow 5P_{3/2}F = 2$ transition of ^{87}Rb. The lattice is made of a collimated laser beam (waist $\omega_d = 720\,\mu$m). Once the outcoupled atom laser falls low enough to get close to the position of the optical lattice, the lattice is turned on for a short time τ. After the interaction, a free evolution time (time of flight) is given to resolve all the diffracted orders, and an absorption image is taken. Figure 2 shows a typical time-of-flight image of the diffraction of an atom laser for $\tau = 0.5\,\mu$s. A red Gaussian envelope is shown in Fig. 2 to schematically represent the region where the atom laser overlaps with the lattice beam. Due to the Gaussian profile of the optical lattice beam, different sections of the atom laser see a different intensity in the vertical direction. So the strength of the diffraction is maximum at the center of the lattice beam and falls as we move away from the center of the laser beam. The bottom panel in Fig. 2 shows the normalized population in different momentum eigenstates due to the diffraction of an atom laser. To find the population in the diffraction orders, we choose the section of the image where the effect of the grating can be observed. The size of the interaction region was found to be $\sim 2\omega_d$, which includes ∼95% of the total power of the lattice beam. To find the normalized population from the selected region we do a column integration of the optical density and divide it by the maximum population. In the bottom panel of Fig. 2, the black circles represent the experimental data of the normalized population for the diffraction pattern shown in the top panel. The solid line displays the normalized population calculated numerically.

A broader picture of the interaction of the atom laser with the light grating is displayed in Fig. 3. In this figure, we represent diffraction of an atom laser by varying the interaction time. For our experimental parameters, the recoil frequency is found to be $\omega_{\rm rec} = 23.68$ KHz. Therefore, to satisfy the Raman-Nath condition for a thin grating, the interaction time has to be less than 42 μs. Also, the condition for the transition to the classical oscillation regime can be found using Eq. (4). For the 1D lattice used in our experiment, the peak value of the traveling wave Rabi frequency is ∼470 MHz. Hence, the upper limit of the interaction time to observe Kapitza-Dirac diffraction is given by $\tau_{\rm osc}/4 \sim 0.9\ \mu$s. The values of the interaction times chosen in our experiment are $\tau = 0.15\,\mu$s, $\tau = 0.3\,\mu$s, $\tau = 0.4\,\mu$s, and $\tau = 0.5\,\mu$s, shown in Figs. 3(a)–3(d), respectively. Figure 3 displays how the distribution of the

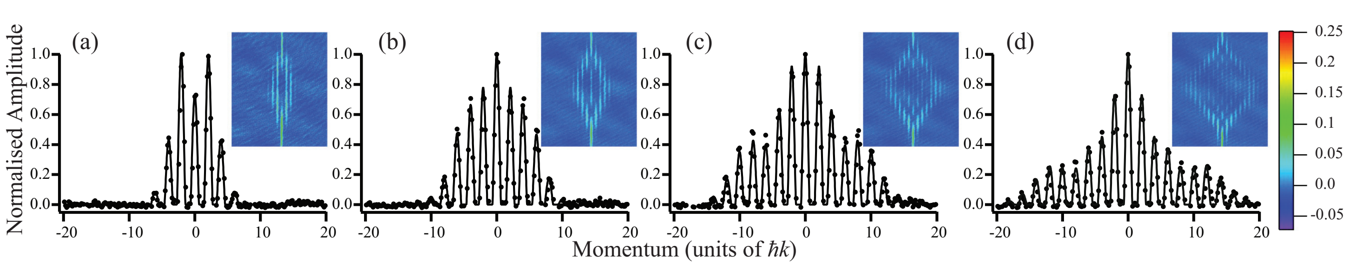

FIG. 3. Normalized population at different diffraction orders with varying interaction time. (a) $\tau = 0.15\,\mu$s, (b) $\tau = 0.3\,\mu$s, (c) $\tau = 0.4\,\mu$s, and (d) $\tau = 0.5\ \mu$s, keeping the peak Rabi frequency and detuning constant. Black circles represent experimental data. Solid lines represent numerical simulation for (a) $\theta = 2.64$, (b) $\theta = 4.29$, (c) $\theta = 6.62$, and (d) $\theta = 8.23$. Insets show corresponding time-of-flight absorption images of the diffracted atom laser. The dimension of each image frame is $\sim 1.85\times1.54$ mm^2. The color scale represents the optical density.

population in different momentum eigenstates evolves with an increasing value of the interaction time. For the maximum value of the interaction time ($\tau = 0.5\,\mu$s) chosen in this experiment, we were able to transfer maximum momenta of $\pm 18\hbar k$ to the atoms. In Fig. 3, the black circles represent the experimental data for the normalized population in different diffraction orders. The solid lines represent the numerically simulated population in diffracted orders for the values of θ approximately equal to that used in the experiment. The population in various diffraction orders is calculated numerically by convoluting the Thomas-Fermi distribution with Eq. (2). The average Thomas-Fermi radius of the atom laser used to demonstrate diffraction is found to be $\sim 23\ \mu$m. The values of θ for which we found the best fit to the diffracted population agree with the values of θ used in the experiment within $\pm 10\%$ uncertainty.

IV. CONCLUSIONS

In conclusion, we have demonstrated diffraction of a slowed-down cw atom laser in the Kapitza-Dirac limit. The atom laser can be outcoupled with a well-controlled flux rate. In our experiment, the atom laser could be extracted up to 400 ms for the slowest outcoupling rate used. Since, for a constant outcoupling rate, the time for which one can extract the atom laser is proportional to the initial atom number in the reservoir, improving the number of atoms in the BEC will essentially allow us to use an atom laser with a high flux for a longer time. We have also numerically simulated the population distribution in the diffracted orders. Our experimental results are in good agreement with the numerical simulations which include the effect of the Gaussian intensity profile of the grating lattice. An extension of this work is to realize a three-grating interferometer using an atom laser which can be used for quasicontinuous probing of physical quantities with high bandwidth and a high data rate. Since the population in the different momentum states can be controlled precisely, such diffraction can also be used to construct an interferometer by selectively using two wave packets with large and opposite momenta. To use the advantage of high flux and long operation time simultaneously, we propose to produce a BEC on an atom chip with a fast production rate similar to that shown in Refs. [29,55,56] and to have multiple reservoirs on a chip that operate in a synchronized manner to produce an effective perpetual source of atom lasers that can be integrated with this diffraction effect to realize a continuously operable interferometer.

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