Torsional cooling of a nanodiamond via the interaction with the electron spin of the embedded nitrogen-vacancy center

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We propose a way to cool the torsional motion of a nanodiamond, which contains a nitrogen-vacancy (NV) center and is levitated by an optical tweezer. Following the rotation of the particle, the NV-center electron spin experiences varying external fields and so leads to spin-torsion coupling. By optically pumping the electrons from a higher energy level to a lower level, the torsional energy is dissipated. We give the analytical result for the damping torque exerted on the nanodiamond and evaluate the final cooling temperature by the fluctuation-dissipation theorem. It is shown that the quantum regime of the torsion can be reached with our scheme.

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I. INTRODUCTION

Since the seminal experiments by Ashkin in 1970 [1], the techniques of optical trapping and manipulation have developed rapidly over the past decades and stimulated remarkable advances in various fields of physics [4]. In atomic physics, these techniques have greatly enhanced the ability to manipulate single atoms, leading to the experimental discovery of Bose-Einstein condensation [2,3], the implementation of atom interferometry [5], and quantum simulations of condensed-matter systems with cold atoms [6]. More recently, optical manipulation has also been applied to larger objects such as micromirrors, cantilevers, and dielectric nanoparticles to control the mechanical degrees of freedom [7–14], with the purposes of quantum information processing [15–17], ultrasensitive sensing [18–21], studying quantum-classical boundaries [22,23], etc. Theories regarding the cooling of center-of-mass (c.m.) motion were proposed [24–27], and the quantum ground-state cooling of a mechanical oscillator was realized experimentally [28,29]. Besides, the interaction between the rotation of a nanobody and the light has also been investigated [30–35]. It is suggested that angular trapping and cooling of a dielectric can be achieved using multiple Laguerre-Gaussian cavity modes. The frequency of torsional vibration can be one order of magnitude higher than the c.m. frequency [35], which is promising for ground-state cooling. The quantum master equation for the coupled particle-cavity system is also derived [32], and it is demonstrated that cooling into the deep quantum regime is achievable. Aside from being used for fundamental purposes, optically levitated nanoparticles can also serve as an ultrasensitive torque balance [36,37].

Recently, the coupling between the motion of a nanodiamond and the nitrogen-vacancy (NV)-center electron spin has attracted much research interest [38–42]. The NV-center spin experiences varying external fields following the motion of the nanodiamond, thus induces interaction between the spin and the mechanical motion, either translational or torsional. In this paper we propose a cooling scheme based on the spin-torsion coupling, the mechanism of which is similar to that of atomic laser-cooling [43,45]. A nanodiamond that contains an NV center is levitated by an optical tweezer, hence its motion is confined. By applying external fields, the energy levels of the NV center are altered and left with an effective two-level system. In the course of rotation, the electrons in the higher level are optically pumped to the lower level, resulting in the dissipation of torsional energy of the nanodiamond, and thus achieves the effect of cooling. This paper is organized as folllowings: in Sect. I we outline the setup of the system and give a qualitative explanation of the cooling mechanism; Sec. II contains the calculation of the electronic state of the NV center and the torque exerted on the nanodiamond; based on these results, the cooling effect is analyzed in Sec. III; and, finally, we draw conclusions in Sec. IV.

A. Setup of the system

We want to cool the rotation of a diamond nanoparticle that contains an NV center. First, the center of mass motion and rotation of the particle should be confined, which can be achieved with an optical tweezer. We consider a spheroid-shaped nanoparticle with semimajor axis a and semiminor axis b placed in a linearly polarized optical tweezer. If the size of the spheroid is much smaller than the wavelength of the laser, the electrostatic approximation can be used to describe the light-matter interaction. In a laser field with Z polarization, the potential energy of the nanoparticle is

$$U = -\frac{1}{2}\sum_i \alpha_i E_i^2 = -\frac{1}{2}\alpha_x E^2 - \frac{1}{2}(\alpha_z - \alpha_x)E^2 \cos^2\beta, \quad (1)$$

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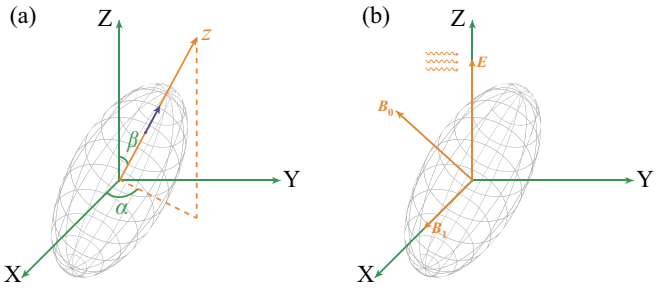

FIG. 1. (a), An ellipsoidal nanodiamond with a NV-center(blue arrow) in it. The quantization axis of the NV-center is in the z direction, i.e. the major axis of the ellipsoid. α and β are angle coordinates of z direction relative to the laboratory frame XYZ. (b), The external fields applied to the nanodiamond. B_0 is a static magnetic field that lies in the XZ plane and has an angle 45° to the X axis, B_1 is an oscillating magnetic field in the X direction and E is the elelctric field of a laser.

where α_x and α_z are the polarizibilities along the principal axes, E is the electric field strength of the laser, and β is the nutation angle of the nanoparticle (Fig. 1). The electric field at the laser waist is determined by $\varepsilon_0 E^2 = \frac{2P}{\pi c w^2}$, where P is the laser power, c is the light speed, and w is the waist. To give an estimation of the potential, we take $P = 100$ mW, $\pi w^2 = 2\ \mu\text{m}^2$, $a = 40$ nm, $b = 20$ nm, $\varepsilon_r = 5.7$ (the dielectric constant of diamond); then the depth of the trap is

$$U\left(\beta = \frac{\pi}{2}\right) - U(\beta = 0) \simeq 5.6 \times 10^{-21}\ \text{J} \equiv k_B T_0 \qquad (2)$$

with $T_0 \simeq 400$ K. At a temperature T, the nutation of the nanoparticle is confined in the range $[0, \beta_m]$, where β_m is roughly determined by $k_B T_0(1 - \cos^2 \beta_m) = \frac{1}{2} k_B T$, e.g., for $T = 5$ K, $\beta_m \simeq 4.5°$.

Now that the trap is produced, cooling shall be achieved through interactions between the NV center and external fields. The electronic states of the NV center are illustrated in Fig. 2, where GS is the ground-state configuration and ES is

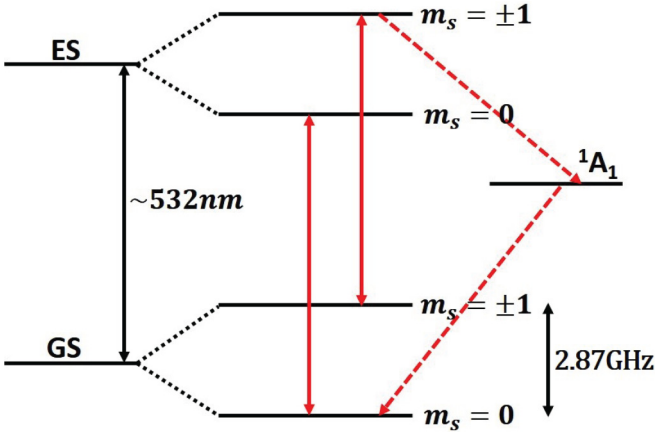

FIG. 2. Electronic structure of the NV-center. GS is the ground state configuration and ES is the excited state configuration, each contains three sublevels $m_s = 0, \pm 1$. A $532nm$ laser beam pumps the electrons in the $m_s = \pm 1$ states in the GS to $m_s = 0$ state.

the excited-state configuration, each contains three spin sublevels with magnetic quantum number $m_s = 0, \pm 1$. Sublevels in the ground state have a zero-field splitting $D = 2.87$ GHz between state $|0\rangle$ and state $|\pm 1\rangle$. Our cooling scheme borrows the idea of atomic cooling proposed by Cohen-Tannoudji *et al.* [43]. First, a static magnetic field $\mathbf{B}_0$ (Fig. 1) is applied to modulate the energies of states $|\pm 1\rangle$ when the quantization axis of the NV center (which is the z axis) rotates. This is analogous to the modulation of the light-shifted energies for a moving atom. Second, another laser beam of 532-nm wavelength is required to pump the electrons from $|\pm\rangle 1$ to $|0\rangle$ (Fig. 2). The pumping process includes spontaneous emissions to an intermediate state and then to the $|0\rangle$ state, which causes dissipation of energy. Finally, we need a microwave field $\mathbf{B}_1$ to induce transitions between $|0\rangle$ and $|\pm 1\rangle$, since otherwise all the electrons will be pumped to $|0\rangle$, which is insensitive to external fields.

Now one may have a qualitative understanding of the cooling mechanism. If the frequency of the microwave field is ω, the effective energies of states $|\pm 1\rangle$ are $\delta_\pm = D - \omega \mp \gamma \mathbf{B}_0 \cdot \hat{z}$, where $\hat{z}$ is the unit vector along the z direction. Since the rotation of the spheroid is confined by the optical trap, a suitable choice of ω and $\mathbf{B}_0$ means that $\delta_+ \ll \delta_-$ and the ground state reduces to a two-level system with $m_s = 0, 1$. In the course of rotation, the electrons in level $|1\rangle$ are pumped to $|0\rangle$, and since the state of this system doe not follow the rotation adiabatically, as long as $\delta_+ > 0$ the energy is continuously dissipated and a friction force is produced; otherwise, the system absorbs more energy from the 532-nm laser than it loses in the spontaneous emissions. So to make sure the force is always frictional, the torsion angle must be confined to a range in which $\delta_+ > 0$. In the following we give the details of the calculation, which confirm the above analysis.

B. Calculation of the torque

We assume that the quantization axis of the NV center is in the z direction, i.e., the major axis of the ellipsoid. Then in the laboratory frame, the Hamiltonian describing the ground-state configuration of the NV center is

$$H = DS_z^2 - \gamma(\mathbf{B}_0 + \mathbf{B}_1 \cos \omega t) \cdot \mathbf{S}, \qquad (3)$$

where S_z is the z component of the spin-1 operator. It relates to the spin operators in the XYZ frame as: $S_z = \mathbf{S} \cdot \hat{z} = S_X \sin\beta\cos\alpha + S_Y \sin\beta \sin\alpha + S_Z \cos\beta$. For a rotating nanoparticle, α, β, and also S_z are time dependent in the laboratory frame, making the calculations difficult. So it is better to move to the rotating frame, in which

$$\begin{aligned} H' &= R^\dagger H R - i R^\dagger \frac{\partial R}{\partial t} = DS_Z^2 - \gamma(\mathbf{B}_0 + \mathbf{B}_1 \cos\omega t) \cdot R^\dagger \mathbf{S} R \\ &= DS_Z^2 - \gamma(\mathbf{B}_0 + \mathbf{B}_1 \cos\omega t) \cdot R^\dagger \mathbf{S} R \\ &\quad - \dot{\alpha}(S_Z \cos\beta - S_X \sin\beta) - \dot{\beta} S_Y, \end{aligned} \qquad (4)$$

where $R = e^{-i\alpha S_Z} e^{-i\beta S_Y}$ is the rotating operator. Physically, this transformation is analogous to the unitary transformation made in the rotating-wave approximation. In our setup, $\mathbf{B}_0$ is in the XZ plane at an angle 45° to the X axis ($\mathbf{B}_0$ cannot be in the Z direction. The reason for this is reported later.), and

$\mathbf{B}_1$ points in the X direction. This Hamiltonian is still time dependent due to the existence of microwave field $\mathbf{B_1}$, so we apply a second unitary transformation, $U = e^{-i\omega t S_z^2}$, and make the rotating-wave approximation; then it becomes

$$\begin{aligned} H_e = (D-\omega)S_Z^2 - \frac{1}{\sqrt{2}}\gamma B_0(\cos\alpha\sin\beta+\cos\beta)S_Z \\ -\dot\alpha\cos\beta S_Z - \frac{1}{2}\gamma B_1\cos\alpha\cos\beta S_X \\ +\frac{1}{2}\gamma B_1\sin\alpha S_Y. \end{aligned} \tag{5}$$

This Hamiltonian sets up the basis of our following calculations. The terms depending on S_Z give the unperturbed energies of $|0\rangle$ and $|\pm1\rangle$, which are $E_0 = 0$ and $E_\pm = D-\omega \mp [\frac{1}{\sqrt{2}}\gamma B_0(\cos\alpha\sin\beta+\cos\beta)+\dot\alpha\cos\beta]$, respectively. As discussed before, by suitably choosing ω and B_0 we can make $E_+ \ll E_-$ so that this system becomes effectively two-level. This Hamiltonian explicitly depends on the angles α and β, so the nanoparticle experiences a torque,

$$\begin{aligned} \langle M_\alpha\rangle &= \left\langle -\frac{\partial H_e}{\partial\alpha}\right\rangle = -\frac{1}{2}\gamma B_1\sin\alpha\cos\beta\langle S_x\rangle \\ &\quad -\frac{1}{2}\gamma B_1\cos\alpha\langle S_y\rangle, \\ \langle M_\beta\rangle &= \left\langle -\frac{\partial H_e}{\partial\beta}\right\rangle = \frac{1}{\sqrt{2}}\gamma B_0(\cos\alpha\cos\beta-\sin\beta)\langle S_z\rangle \\ &\quad -\frac{1}{2}\gamma B_1\cos\alpha\sin\beta\langle S_x\rangle, \end{aligned} \tag{6}$$

where $\langle\dots\rangle$ means $\mathrm{Tr}(\rho\dots)$ and ρ is the density matrix describing the state of the NV center. In our setup, the microwave field B_1 is much smaller than the static field B_0, so in the following we focus on M_β.

The evolution of ρ is governed by the master equation $\dot\rho = -i[H_e,\rho]+\mathcal{D}(\rho)$, or, in the component form,

$$\begin{aligned} \dot\rho_{11} &= -\frac{1}{\sqrt{2}}g_2(\rho_{01}+\rho_{10}) + \frac{i}{\sqrt{2}}g_1(\rho_{01}-\rho_{10}) - \Gamma\rho_{11}, \\ \dot\rho_{01} &= i(\delta_+ - \dot\alpha\cos\beta)\rho_{01} + \frac{i}{\sqrt{2}}g_1(\rho_{11}-\rho_{00}) \\ &\quad +\frac{1}{\sqrt{2}}g_2(\rho_{11}-\rho_{00}) - \Gamma_1\rho_{01}, \end{aligned} \tag{7}$$

where we have introduced the notations $g_1 = \frac{1}{2}\gamma B_1\cos\alpha\cos\beta$, $g_2 = \frac{1}{2}\gamma B_1\sin\alpha$, $\delta_+ = D-\omega-\frac{1}{\sqrt{2}}\gamma B_0(\cos\alpha\sin\beta+\cos\beta)$. Γ and Γ_1 are the decay rates for ρ_{11} and ρ_{01}, respectively.

If the nanoparticle is at rest, i.e., $\dot\alpha = \dot\beta = 0$, the steady state of the NV center is obtained by putting $\dot\rho_{11} = \dot\rho_{01} = 0$. We get the steady values $\rho_{11}^s = \frac{1}{2+f_0}$, $\rho_{01}^s = \frac{(ig_1+g_2)f_0}{\sqrt{2}(2+f_0)(\Gamma_1-i\delta_+)}$ with $f_0 = \frac{\Gamma(\Gamma_1^2+\delta_+^2)}{\Gamma_1(g_1^2+g_2^2)}$. For a rotating nanoparticle, the steady state satisfies $\dot\rho = \dot\alpha\frac{\partial\rho}{\partial\alpha}+\dot\beta\frac{\partial\rho}{\partial\beta}$, and Eq. (7) can be solved order by order. Note that $\Gamma_1 \gg \Gamma$ [44], so ρ_{01} follows almost adiabatically with the evolution of ρ_{11} [45]: $\rho_{01} = \frac{(ig_1+g_2)(\rho_{11}-\rho_{00})}{\sqrt{2}(\Gamma_1-i\delta_+)}$.

Inserting this formula to the first equation (7) we get

$$\dot\alpha\frac{\partial\rho_{11}}{\partial\alpha}+\dot\beta\frac{\partial\rho_{11}}{\partial\beta} = -\frac{(2+f)\Gamma}{f}(\rho_{11}-p_s), \tag{8}$$

where $f = \frac{\Gamma[\Gamma_1^2+(\delta_+-\dot\alpha\cos\beta)^2]}{\Gamma_1(g_1^2+g_2^2)}$, $p_s = \frac{1}{2+f_0}$. Expanding $\rho_{11} = p_s + p^{(1)} + \dots$ and $\frac{(2+f)\Gamma}{f} = \frac{(2+f_0)\Gamma}{f_0} + \dot\alpha A_1 + \dots$, then to the first order of $\dot\alpha$, $\dot\beta$,

$$\dot\alpha\left(\frac{\partial p_s}{\partial\alpha} - A_1 p_s\right) + \dot\beta\frac{\partial p_s}{\partial\beta} = -\frac{(2+f_0)\Gamma}{f_0}p^{(1)} \tag{9}$$

Hence

$$p^{(1)} = -\frac{f_0}{\Gamma(2+f_0)}\left(\frac{\partial p_s}{\partial\alpha}\dot\alpha - A_1 p_s\dot\alpha + \frac{\partial p_s}{\partial\beta}\dot\beta\right). \tag{10}$$

And from Eq. (6),

$$\begin{aligned} \langle M_\beta\rangle &\approx \frac{1}{\sqrt{2}}\gamma B_0(\cos\alpha\cos\beta-\sin\beta) \\ &\quad\times\left(p_s - \frac{f_0}{\Gamma(2+f_0)}\frac{\partial p_s}{\partial\alpha}\dot\alpha - \frac{f_0}{\Gamma(2+f_0)}\frac{\partial p_s}{\partial\beta}\dot\beta\right) \\ &\equiv M_\beta^0 - \kappa_{\alpha\beta}\dot\alpha - \kappa_\beta\dot\beta, \end{aligned} \tag{11}$$

where M_β^0 is the conservative force and

$$\begin{aligned} \kappa_{\alpha\beta} &= \frac{1}{\sqrt{2}}\gamma B_0(\cos\alpha\cos\beta-\sin\beta)\frac{f_0}{\Gamma(2+f_0)}\frac{\partial p_s}{\partial\alpha} \\ &\simeq -\frac{(\gamma B_0)^2\delta_+ f_0}{\Gamma_1(g_1^2+g_2^2)(2+f_0)^3} \\ &\quad\times\sin\alpha\sin\beta(\cos\alpha\cos\beta-\sin\beta), \\ \kappa_\beta &= \frac{1}{\sqrt{2}}\gamma B_0(\cos\alpha\cos\beta-\sin\beta)\frac{f_0}{\Gamma(2+f_0)}\frac{\partial p_s}{\partial\beta} \\ &\simeq [\gamma B_0(\cos\alpha\cos\beta-\sin\beta)]^2\frac{\delta_+ f_0}{\Gamma_1(g_1^2+g_2^2)(2+f_0)^3}. \end{aligned} \tag{12}$$

The effect of $\kappa_{\alpha\beta}\dot\alpha$ is discussed later and we focus on $\kappa_\beta\dot\beta$ now, which is a friction force as long as $\kappa_\beta > 0$. From the above equation it is seen that the sign of κ_β is determined by δ_+, which confirms the qualitative analysis in the previous section. As mentioned before, β is confined by the optical trap in the range $[0,\beta_m]$; thus, by suitably choosing the frequency ω such that at a certain angle $\beta_0 > \beta_m$ (but less than 45°), $D-\omega-\gamma B_0\cos(45^\circ-\beta_0) = 0$, one can make sure that δ_+ is always positive in the course of rotation (Fig. 3).

The reason $\mathbf{B}_0$ cannot lie in the Z axis is now clear. It is seen that κ_β contains a factor $[B_0(\cos\alpha\cos\beta-\sin\beta)]^2$ which is proportional to $[\frac{d}{d\beta}(\mathbf{B}_0\cdot\hat z)]^2$; thus if $\mathbf{B}_0$ is on the Z axis, this will be $(B_0\sin\beta)^2$, making the friction force negligible for small β.

C. Analysis of the cooling effect

We now estimate the time scale over which the torsion is damped and the final cooling temperature. For small β, the Hamiltonian for the nanoparticle, which consists of the

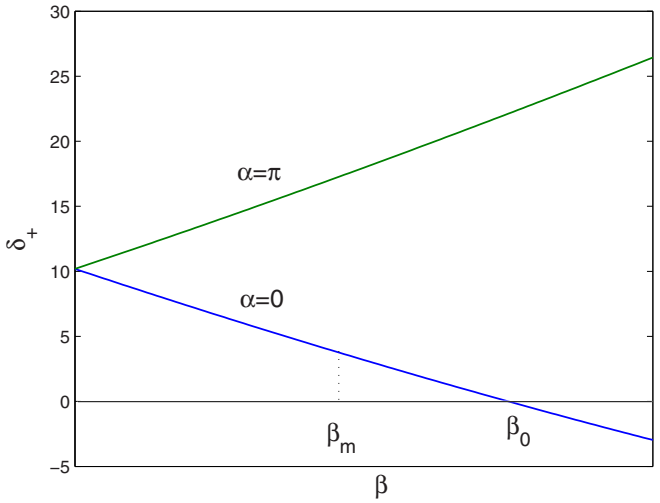

FIG. 3. A sketch of δ_+ as a function of β. The upper curve represents $\alpha = \pi$ and the lower curve $\alpha = 0$; the value of δ_+ for arbitrary α is between these two curves. At $\beta = \beta_0$ the lower curve crosses 0, as long as $\beta \leqslant \beta_m < \beta_0$, δ_+ is always positive.

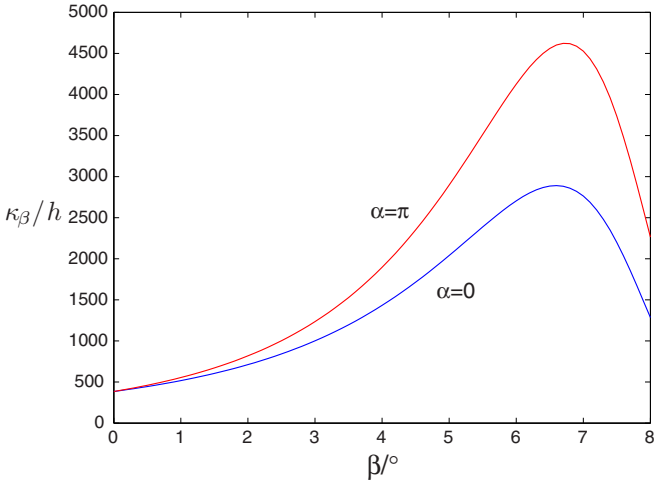

FIG. 4. The β dependence of κ_β (in units of the Planck constant h) with the following parameters: $\gamma B_0 = 2\pi \times 100$ MHz, $\gamma B_1 = 2\pi$ MHz, $\Gamma = 0.4$ MHz, $\Gamma_1 = 5$ MHz, and $\beta_0 = 9^\circ$. Upper curve, $\alpha = \pi$; lower curve, $\alpha = 0$.

rotating energy and the potential of the optical trap, is

$$H_p = \frac{I_1}{2}(\dot\beta^2 + \dot\alpha^2 \sin^2\beta) + \frac{I_3}{2}(\dot\alpha\cos\beta + \dot\gamma)^2 + U \simeq \frac{I_1}{2}\dot\beta^2 + \frac{I_3}{2}(\dot\alpha + \dot\gamma)^2 + \frac{1}{2}(\alpha_z - \alpha_x)E^2\beta^2 \tag{13}$$

where a constant energy is omitted, and I_1 and I_3 are moments of inertia about the x and z axes, respectively. The torque exerted on the particle is produced by the optical trap as well as M_β, so the motion equation of β is

$$I_1\ddot\beta = -\frac{\partial U}{\partial\beta} + M_\beta = -\frac{\partial U}{\partial\beta} + M_\beta^0 - \kappa_{\alpha\beta}\dot\alpha - \kappa_\beta\dot\beta + M'_\beta, \tag{14}$$

where $M'_\beta = M_\beta - \langle M_\beta\rangle$ is the fluctuation of the torque. The time scale for damping is $t = \frac{I_1}{\kappa_\beta}$. In our case, $I_1 = \frac{4\pi}{15}\rho ab^2(a^2 + b^2)$, where $\rho = 3.5\,\mathrm{g/cm^3}$ is the density of diamond the other parameters are $\Gamma = 0.4\ \mu\mathrm{s}^{-1}/2\pi$, $\Gamma_1 = 5\ \mu s^{-1}/2\pi$, and $\beta_0 = 9^\circ$. The dependence of κ_β on β is shown in Fig. 4, where we take $\alpha = 0$ and π as examples.

To give a rough estimation, taking $\kappa_\beta = 500$ h as the average, then $t \simeq 2.8 \times 10^{-4}$ s.

The damping of $\dot\alpha$ is much slower than that of $\dot\beta$ since from Eqs. (6) and (10), $M_\alpha \propto \gamma B_1$ and is negligible compared to M_β for $B_1 \ll B_0$. So after some cooling time, $\dot\alpha \gg \dot\beta$, and in a period τ during which α changes from 0 to 2π, β can be taken as a constant. Then the impulse of $\kappa_{\alpha\beta}\dot\alpha$ in such a period is

$$\int_0^\tau \kappa_{\alpha\beta}\dot\alpha dt = \int_0^{2\pi} \kappa_{\alpha\beta} d\alpha = 0, \tag{15}$$

where the last equality results from $\kappa_{\alpha\beta}(\alpha) = -\kappa_{\alpha\beta}(2\pi - \alpha)$.

To estimate the final cooling temperature, we need to calculate the correlation function of the friction force. Let $G(t) = \langle S_z(t)S_z(0)\rangle - \langle S_z\rangle^2$; since $\langle S_z\rangle = p_s$, we have from the quantum regression theorem [46] that $\frac{dG}{dt} = -\frac{(2+f_0)\Gamma}{f_0}G$ with initial condition $G(0) = p_s - p_s^2$. Then $G = G(0)\exp(-\frac{(2+f_0)\Gamma}{f_0}t)$, and from (6),

$$\langle M'_\beta(t)M'_\beta(0)\rangle = \frac{1}{2}[\gamma B_0(\cos\alpha\cos\beta - \sin\beta)]^2 \times \left(p_s - p_s^2\right)\exp\left(-\frac{(2+f_0)\Gamma}{f}t\right). \tag{16}$$

Hence the momentum diffusion coefficient is

$$D_p = \frac{1}{2}\int_{-\infty}^{\infty}\langle M'_\beta(t)M'_\beta(0)\rangle dt = \frac{1}{2}[\gamma B_0(\cos\alpha\cos\beta - \sin\beta)]^2\left(p_s - p_s^2\right)\frac{f_0}{(2+f_0)\Gamma}, \tag{17}$$

where we have used the fact that for $t < 0$, $\langle S_z(t)S_z(0)\rangle = \langle S_z(-t)S_z(0)\rangle$. So according to the fluctuation-dissipation theorem [46], the temperature is

$$k_B T_f = \frac{D_p}{\kappa_\beta} = \frac{(2+f_0)^2\left(p_s - p_s^2\right)\left(g_1^2 + g_2^2\right)\Gamma_1}{\delta_+\Gamma} = \frac{(1+f_0)(g_1^2 + g_2^2)\Gamma_1}{\delta_+\Gamma}. \tag{18}$$

The angle β will be about 0 after a period of cooling, so we take $\beta = 0$ in the above equation. Then $\delta_+ = \gamma B_0[\cos(45^\circ - \beta_0) - \cos 45^\circ]$ and $g_1^2 + g_2^2 = (\frac{1}{2}\gamma B_1)^2$. Let us consider the case $f_0 = \frac{\Gamma(\Gamma_1^2 + \delta_+^2)}{\Gamma_1(g_1^2 + g_2^2)} \gg 1$, which can be fulfilled for $\gamma B_0 \gg \gamma B_1$; then Eq. (19) is simplified as

$$k_B T_f \simeq \frac{f_0\left(g_1^2 + g_2^2\right)\Gamma_1}{\delta_+\Gamma} = \frac{\Gamma_1^2 + \delta_+^2}{\delta_+}. \tag{19}$$

So the temperature reaches its minimum $2\Gamma_1/k_B$ at $\delta_+ = \Gamma_1$. For $\Gamma_1 = 5$ MHz, the lowest temperature is $T_f \simeq 0.6 \times 10^{-4}$ K. We want to see whether this temperature is low enough to reach the quantum regime. The Hamiltonian H_p describes a harmonic oscillator with frequency $\omega_0 = \sqrt{(\alpha_z - \alpha_x)E^2/I_1}$, so the temperature for the quantum regime is $\hbar\omega_0/k_B \sim 10^{-4}$ K, which is of the same order as T_f.

II. CONCLUSION

To conclude, we study the torsional cooling of a nanodiamond which contains an NV center. Through the coupling between its torsion and the NV-center electron spin, the torsional energy is dissipated, which is similar to atomic laser-cooling. By suitably choosing the parameters of the system setup, the quantum regime can be reached. In our theory, the motion of nanoparticles is treated classically. However, when the temperature is low enough, the quantum effect must be taken into account. A full quantum mechanical approach is of our future interest.

Above we have assumed that there is only one NV center in the nanodiamond. If the number of NV centers is n, and if these NV centers are uncorrelated, the damping coefficient κ_β should be n times the present one. However, the final cooling temperature is unchanged since D_p is also multiplied by a factor n.

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