Electrodynamics with a preferred frame

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(Received 27 June 2018; published 4 October 2018)

We introduce a variant of quantum and classical electrodynamics formulated on the grounds of a hypothesis of existence of a preferred frame of reference—a formalism complementary to that regarding the structure of the space of photonic states, presented by us recently [Phys. Rev. A **97**, 062106 (2018)]. The present approach makes a unique test theory to search for a preferred frame for photons in experiments, one of which is suggested.

DOI: 10.1103/PhysRevA.98.042107

I. INTRODUCTION

One of the possible consequences of contemporary approaches to extensions of the Standard Model and quantization of gravity [1,2] is the existence of a preferred frame (PF) of reference. Such possibility might exhibit a preferred foliation of the space-time at its most fundamental level. Let us mention particular Lorentz-violating extensions of the Standard Model [3–6], approaches to classical and quantum gravity [7–9], and the so-called doubly special relativity theories [10] characterized by modified dispersion relations, common for Lorentz violating models. In almost all of the above models certain specific effects are predicted, usually suppressed by a power of the Planck scale, like, e.g., vacuum birefringence. This is a consequence of asymmetry of the modified, helicity dependent, dispersion relations for the photon which result in a rotation of the polarization plane, of magnitude depending on the distance between the source and the detector [11]. References to the notion of PF in the context of quantum theory are owed to several authors. Referring to the concept of aether Dirac pointed out that the ideas regarding symmetries in the classical theory could turn out much different at the quantum level [12,13]. The historical notion of aether was superseded by that of preferred frame, as, e.g., in de Broglie–Bohm formulation of quantum mechanics [14,15]. Bell suggested that it would have been helpful to consider a PF at the fundamental level for resolving certain incompatibilities between special relativity theory and nonlocality of quantum mechanics [16], an opinion shared by other authors [17,18]. This brief outlook demonstrates that the concept of PF has been frequently referred to in the context of Lorentz symmetry violation within numerous contemporary theories and ideas. A discussion of the structure of photonic states in the context of the PF hypothesis has recently been presented by us [19] comprising suggestions for possible experiments, one of which involves two observers in relative motion. In the present paper we construct quantum and classical electrodynamics founded on the hypothesis of existence of a preferred frame in nature. The pertinent Lorentz-covariant formalism constitutes a unique test theory for addressing the issue of existence of a PF for photons.

II. PRELIMINARIES

In quantum electrodynamics, the Hilbert space of photonic states is the carrier space of the unitary irreducible representation of the inhomogeneous Lorentz group. Action of the Lorentz group on the states is achieved by means of the Wigner-Mackey induction procedure [20] implemented on eigenvectors of the four-momentum operator and extended by linearity to the entire space. In our recent paper we exploited the Wigner-Mackey induction procedure to obtain the one-particle space of photonic states under a working hypothesis of existence of a preferred frame of reference [19] with preserved Lorentz covariance. This approach is equivalent to assuming the description of photonic states be frame dependent by way of the four-velocity of the preferred frame as seen by a given observer.

Let $u^\mu = (u^0; \boldsymbol{u})$ be the timelike four-velocity of the PF as seen by an inertial observer (${u^0}^2 - \boldsymbol{u}^2 = 1$); in particular $u \equiv u_{\rm PF} = (1; 0, 0, 0)$ for an observer at rest with respect to this frame (below we assume natural units $\hbar = c = 1$). Let $k^\mu = (k^0; \boldsymbol{k})$ denote the photon four-momentum (${k^0}^2 - \boldsymbol{k}^2 = 0$). Our working hypothesis can be nontrivially realized only if monochromatic photonic states are frame dependent, i.e., are parametrized not only by k^μ but also by u^μ. Such one-photonic four-momentum eigenvectors, denoted as $|k, u, \lambda\rangle$, where $\lambda = \pm 1$ is the photon helicity, are identified with monochromatic, circularly polarized states of the photons. The starting point of the Wigner-Mackey induction procedure is the determination of the little group $\mathrm{O}(2) \sim \mathrm{E}(2) \bigcap \mathrm{O}(3)$ of a pair of four-vectors (k, u). The pair (k, u) can be obtained from the "standard" one $(q, u_{\rm PF})$, where $q = \kappa\,(1; 0, 0, 1)$ and $\kappa > 0$, by the sequence of Lorentz transformations $L_u R_{\boldsymbol{n}}$, where L_u is the Lorentz boost transforming $u_{\rm PF}$ into u and $R_{\boldsymbol{n}}$ is the rotation of q into the four-vector $\kappa(1; \boldsymbol{n})$, provided

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the unit vector $\boldsymbol{n}$ is equal to [19]

$$\boldsymbol{n} = \boldsymbol{n}(k,u) = \frac{1}{uk}\left(\boldsymbol{k} - \frac{|\boldsymbol{k}| + uk}{1+u^0}\boldsymbol{u}\right),$$

where $uk = u^\mu k_\mu = \kappa$ (see Appendix A). Applying the Wigner-Mackey procedure to the base vectors $|k,u,\lambda\rangle$ leads to the unitary action of the Lorentz group of the form

$$U(\Lambda)|k,u,\lambda\rangle = e^{i\lambda\phi(\Lambda,k,u)}|\Lambda k,\Lambda u,\lambda\rangle, \tag{1}$$

where $e^{i\lambda\phi(\Lambda,k,u)}$ is the phase factor related to the subgroup SO(2) and the little group O(2), corresponding to the Wigner rotation

$$W(\Lambda,k,u) = (L_{\Lambda u}R_{\boldsymbol{n}(\Lambda k,\Lambda u)})^{-1}\Lambda L_u R_{\boldsymbol{n}(k,u)} \tag{2}$$

(see Appendix B). As follows from (2), the Wigner phase $\phi(\Lambda,k,u)$ is in general different from that calculated in the standard theory, $\phi_S(\Lambda,k)$ [21]. Linearly polarized monochromatic states are given as usual by the following superposition of circularly polarized states:

$$|\theta,k,u\rangle := \frac{1}{\sqrt{2}}(e^{i\theta}|k,u,1\rangle + e^{-i\theta}|k,u,-1\rangle). \tag{3}$$

Thus, by means of (1), these states transform unitarily under the Lorentz group action, according to the transformation law

$$U(\Lambda)|\theta,k,u\rangle = |\theta + \phi(\Lambda,k,u),\Lambda k,\Lambda u\rangle.$$

III. QUANTUM ELECTRODYNAMICS AND PREFERRED FRAME

We construct the free quantum electrodynamics assuming the existence of a preferred frame (PF QED) in close analogy to the standard formalism. In this case the base vectors are obtained by action of creation operators $a_\lambda^\dagger(k,u)$ on the vacuum vector $|0\rangle$, normalized to unity, defined by the condition $a_\lambda(k,u)|0\rangle = 0$, namely

$$|k,u,\lambda\rangle = a_\lambda^\dagger(k,u)|0\rangle.$$

Creation and annihilation operators fulfill the standard canonical commutation relations. The only nonzero commutators satisfy

$$[a_\lambda(k,u), a_\sigma^\dagger(p,u)] = 2k^0\delta_{\lambda\sigma}\delta^3(\boldsymbol{k}-\boldsymbol{p}) \tag{4}$$

so the scalar product of the base vectors reads

$$\langle k,u,\lambda|p,u,\sigma\rangle = 2k^0\delta_{\lambda\sigma}\delta^3(\boldsymbol{k}-\boldsymbol{p}).$$

In order to reproduce (1), creation and annihilation operators should transform under the Lorentz group action according to the rule

$$U(\Lambda)a_\lambda^\dagger(k,u)U^\dagger(\Lambda) = e^{i\lambda\phi(\Lambda,k,u)}a_\lambda^\dagger(\Lambda k,\Lambda u). \tag{5}$$

The electromagnetic four-potential operator, $\hat{A}^\mu(x,u)$, is defined as the Fourier transform

$$\hat{A}^\mu(x,u) = \frac{1}{(2\pi)^{3/2}}\int\frac{d^3\boldsymbol{k}}{2k^0}[e^{ikx}e^{\mu\lambda}(k,u)a_\lambda^\dagger(k,u) + e^{-ikx}e^{*\mu\lambda}(k,u)a_\lambda(k,u)] \tag{6}$$

obeying the Lorentz covariant transformation rule

$$U(\Lambda)\hat{A}^\mu(x,u)U^\dagger(\Lambda) = \Lambda^{-1\mu}{}_\nu\hat{A}^\nu(\Lambda x,\Lambda u). \tag{7}$$

We note that, in contrast to the standard case, no inhomogeneous gauge term appears on the right-hand side of the above equation, owing to the natural reduction of the Wigner little group to the compact O(2) group. Next, we obtain the Weinberg consistency condition [22] of (5) and (7) in the form

$$e^{\mu\lambda}(\Lambda k,\Lambda u) = \Lambda^\mu{}_\nu e^{\nu\lambda}(k,u)e^{i\lambda\phi(\Lambda,k,u)}. \tag{8}$$

Replacing the pair (k,u) by $(q,u_{\rm PF})$ in (8) and Λ by rotation about the z axis, $R_z(\varphi)$, as well as taking into account that $\phi(R_z(\varphi),q,u_{\rm PF}) = \varphi$ follows from (2), we obtain (up to a nonzero factor)

$$e^{0\lambda}(q,u_{\rm PF}) = e^{3\lambda}(q,u_{\rm PF}) = 0;\quad e^{1\lambda}(q,u_{\rm PF}) = 1/\sqrt{2};$$
$$e^{2\lambda}(q,u_{\rm PF}) = -i\lambda/\sqrt{2}. \tag{9}$$

In view of the vanishing phase $\phi(L_{u_{\rm PF}}R_{\boldsymbol{n}(q,u_{\rm PF})},k,u) = 0$ we conclude that

$$e^{\mu\lambda}(k,u) = (L_u R_{\boldsymbol{n}(k,u)})^\mu{}_\nu e^{\nu\lambda}(q,u_{\rm PF}). \tag{10}$$

Now, it is a matter of simple calculations that (9) and (10) imply the following covariant transversality and normalization relations:

$$k_\mu e^{\mu\lambda}(k,u) = 0;\quad u_\mu e^{\mu\lambda}(k,u) = 0;$$
$$e^{*\mu\lambda}(k,u)e^\sigma_\mu(k,u) = -\delta^{\lambda\sigma};$$
$$\sum_\lambda e^{*\mu\lambda}(k,u)e^{\nu\lambda}(k,u) = -\left(\eta^{\mu\nu} - \frac{1}{uk}(u^\mu k^\nu + k^\mu u^\nu) + \frac{1}{(uk)^2}k^\mu k^\nu\right). \tag{11}$$

Therefore considering the definition of the four-potential operator (6) we see that the following covariant relations hold for $\hat{A}^\mu(x,u)$ in PF QED:

$$\text{Massless Klein-Gordon equation: } \partial^2\hat{A}^\mu(x,u) = 0, \tag{12}$$

$$\text{Lorenz condition: } \partial_\mu\hat{A}^\mu(x,u) = 0, \tag{13}$$

$$\text{PF transversality condition: } u_\mu\hat{A}^\mu(x,u) = 0. \tag{14}$$

We can now define the electromagnetic tensor operator $\hat{F}^{\mu\nu}$ by means of $\hat{A}^\mu$,

$$\hat{F}^{\mu\nu}(x,u) = \partial^\mu\hat{A}^\nu(x,u) - \partial^\nu\hat{A}^\mu(x,u), \tag{15}$$

with the following identification of the electric and magnetic-field operators:

$$\hat{E}^l(x,u) = \hat{F}^{0l}(x,u);\qquad \hat{B}^l(x,u) = \frac{1}{2}\epsilon^{lij}\hat{F}^{ij}(x,u). \tag{16}$$

The Maxwell equations are obtained from the definition of the four-potential (6), the electromagnetic tensor (15), as well as by means of the transversality condition $k_\mu e^{\mu\lambda} = 0$ (11) and the spectral condition $k^2 = 0$,

$$\partial_\mu\hat{F}^{\mu\nu}(x,u) = 0,$$
$$\partial_\mu\hat{\tilde{F}}^{\mu\nu}(x,u) = 0.$$

Here $\hat{\tilde{F}}$ is dual to $\hat{F}$.

IV. POLARIZED STATES AND POLARIZATION OPERATORS

Linearly polarized states (3) are generated from the vacuum state by the action of creation operators,

$$|\theta, k, u\rangle = a_\theta^\dagger(k,u)|0\rangle,$$

where

$$a_\theta^\dagger(k,u) := \frac{1}{\sqrt{2}}(e^{i\theta}a_{+1}^\dagger(k,u) + e^{-i\theta}a_{-1}^\dagger(k,u)),$$

and analogously for orthogonally polarized states,

$$|\theta_\perp, k, u\rangle = a_{\theta_\perp}^\dagger(k,u)|0\rangle,$$

where $\theta_\perp = \theta + \frac{\pi}{2}$. Evidently $\langle\theta, k, u|\theta', p, u\rangle = 2k^0\cos(\theta - \theta')\delta^3(\boldsymbol{k}-\boldsymbol{p})$. The operators $a_\lambda^\dagger$ can be expressed in terms of $a_\theta^\dagger$ and $a_{\theta_\perp}^\dagger$ as follows:

$$a_\lambda^\dagger(k,u) = \frac{e^{-i\lambda\theta}}{\sqrt{2}}(a_\theta^\dagger(k,u) - i\lambda a_{\theta_\perp}^\dagger(k,u)).$$

The four-potential operator (6) can be expressed in terms of the operators $a_\theta^\dagger$ and $a_{\theta_\perp}^\dagger$ and their Hermitian conjugates,

$$\begin{aligned}\hat{A}^\mu(x,u) = \frac{1}{(2\pi)^{3/2}}\int\frac{d^3\boldsymbol{k}}{2k^0}\big[e^{ikx}\big(e_\theta^\mu(k,u)a_\theta^\dagger(k,u)\\ + e_{\theta_\perp}^\mu(k,u)a_{\theta_\perp}^\dagger(k,u)\big) + e^{-ikx}\big(e_\theta^{*\mu}(k,u)a_\theta(k,u)\\ + e_{\theta_\perp}^{*\mu}(k,u)a_{\theta_\perp}(k,u)\big)\big],\end{aligned} \tag{17}$$

where

$$e_\theta^\mu(k,u) = \sum_\lambda\left(e^{\mu\lambda}(k,u)\frac{e^{-i\lambda\theta}}{\sqrt{2}}\right) = (L_u R_{\boldsymbol{n}(k,u)})^\mu{}_\nu e_\theta^\nu(q,u_{\rm PF}), \tag{18}$$

are obtained using (9) and (10). Here the coefficients $e_\theta^\nu(q,u_{\rm PF})$ are as follows:

$$\begin{aligned}&e_\theta^0(q,u_{\rm PF}) = 0;\quad e_\theta^1(q,u_{\rm PF}) = \cos\theta;\\ &e_\theta^2(q,u_{\rm PF}) = -\sin\theta;\quad e_\theta^3(q,u_{\rm PF}) = 0;\\ &e_{\theta_\perp}^0(q,u_{\rm PF}) = 0; e_{\theta_\perp}^1(q,u_{\rm PF}) = -\sin\theta;\\ &e_{\theta_\perp}^2(q,u_{\rm PF}) = -\cos\theta;\quad e_{\theta_\perp}^3(q,u_{\rm PF}) = 0.\end{aligned} \tag{19}$$

The form of $e_{\theta_\perp}^\mu(k,u)$ follows from (18) by replacing θ with $\theta + \frac{\pi}{2}$. The terms e_θ^μ and $e_{\theta_\perp}^\mu$ are real and obey the following transversality, normalization, and Minkowski orthogonality relations:

$$\begin{aligned}&k_\mu e_\theta^\mu(k,u) = 0;\quad u_\mu e_\theta^\mu(k,u) = 0;\\ &e_{\theta\mu}e_\theta^\mu = e_{\theta_\perp\mu}e_{\theta_\perp}^\mu = -1;\quad e_{\theta\mu}e_{\theta_\perp}^\mu = 0.\end{aligned}$$

The Lorentz group transformation of e_θ^μ (and analogously for $e_{\theta_\perp}^\mu$), by means of the Weinberg consistency condition (8), can be written as

$$e_{\theta+\phi(\Lambda,k,u)}^\mu(\Lambda k, \Lambda u) = \Lambda^\mu{}_\nu e_\theta^\nu(k,u). \tag{20}$$

It should be stressed that the rule (20), together with (2) defining the phase $\phi(\Lambda,k,u)$, can also be obtained from (18) by means of an analogon of the Wigner procedure.

Now, the electromagnetic field operator (15) takes the form

$$\begin{aligned}\hat{F}^{\mu\nu}(x,u) = \int\frac{d^3\boldsymbol{k}}{2k^0}\big[ie^{ikx}\big(f_\theta^{\mu\nu}(k,u)a_\theta^\dagger(k,u)\\ + f_{\theta_\perp}^{\mu\nu}(k,u)a_{\theta_\perp}^\dagger(k,u)\big) + \text{H.c.}\big],\end{aligned} \tag{21}$$

where the real polarization tensor $f_\theta^{\mu\nu}$ reads

$$f_\theta^{\mu\nu}(k,u) = k^\mu e_\theta^\nu(k,u) - k^\nu e_\theta^\mu(k,u). \tag{22}$$

The transformation rule (20) and the definition (21) imply the following transformation law for the polarization tensor:

$$f_{\theta+\phi(\Lambda,k,u)}^{\mu\nu}(\Lambda k, \Lambda u) = \Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta f_\theta^{\alpha\beta}(k,u). \tag{23}$$

Furthermore, by means of (21) and (22), equations for the electric- and magnetic-field operators (16) become

$$\begin{aligned}\hat{E}^l(x,u) = \frac{1}{2(2\pi)^{3/2}}\int d^3\boldsymbol{k}\big[ie^{ikx}\big(f_\theta^l(k,u)a_\theta^\dagger(k,u)\\ + f_{\theta_\perp}^l(k,u)a_{\theta_\perp}^\dagger(k,u)\big) + \text{H.c.}\big],\end{aligned} \tag{24}$$

$$\begin{aligned}\hat{B}^l(x,u) = \frac{1}{2(2\pi)^{3/2}}\int d^3\boldsymbol{k}\big[ie^{ikx}\big(g_\theta^l(k,u)a_\theta^\dagger(k,u)\\ + g_{\theta_\perp}^l(k,u)a_{\theta_\perp}^\dagger(k,u)\big) + \text{H.c.}\big],\end{aligned} \tag{25}$$

where

$$f_\theta^l(k,u) = \frac{1}{k^0}f_\theta^{0l}(k,u);\qquad g_\theta^l(k,u) = \frac{1}{2k^0}\epsilon^{lij}f_\theta^{ij}(k,u).$$

After some calculations one obtains the following form of the electric and magnetic polarization vectors $\boldsymbol{f}_\theta$ and $\boldsymbol{g}_\theta$ by means of which the classical electromagnetic field can be conveniently expressed:

$$\boldsymbol{f}_\theta = \boldsymbol{f}_0\cos\theta + \boldsymbol{f}_0\times\hat{\boldsymbol{k}}\sin\theta;\qquad \boldsymbol{g}_\theta = -\boldsymbol{f}_{\theta_\perp}, \tag{26}$$

where $\boldsymbol{f}_0 = \boldsymbol{f}_{\theta=0}$. Moreover, the following conditions are fulfilled for arbitrary θ:

$$\boldsymbol{f}_\theta^2 = \boldsymbol{g}_\theta^2 = 1;\qquad \boldsymbol{k}\boldsymbol{f}_\theta = \boldsymbol{k}\boldsymbol{g}_\theta = 0;\qquad \boldsymbol{f}_\theta\boldsymbol{g}_\theta = 0.$$

V. CLASSICAL ELECTROMAGNETIC FIELD

Let us define a general normalized monochromatic state as

$$|\varepsilon, k, u\rangle = \varepsilon_\mu(k,u)\sum_\lambda e^{\mu\lambda}(k,u)|k,u,\lambda\rangle, \tag{27}$$

where ε_μ denotes an arbitrary covariant four-vector. The complex forms of the electric and the magnetic fields are identified in the usual way as follows:

$$\mathfrak{E} \equiv \mathcal{E}e^{-ikx} = \langle 0|\hat{\boldsymbol{E}}(x,u)|\varepsilon,k,u\rangle, \tag{28}$$

$$\mathfrak{B} \equiv \mathcal{B}e^{-ikx} = \langle 0|\hat{\boldsymbol{B}}(x,u)|\varepsilon,k,u\rangle. \tag{29}$$

Using (24), (25), and (27), making use of the reality of the polarization tensor components f_θ^j, as well as the commutation relations (4), one obtains for the complex electric field $\mathcal{E}$

$$\begin{aligned}\mathcal{E}(k,u) &= \frac{-ik^0}{(2\pi)^{3/2}}(\boldsymbol{\varepsilon}(k,u) - \hat{\boldsymbol{k}}\varepsilon^0(k,u))\\ &= \boldsymbol{f}_\theta(k,u)\mathcal{E}_\theta + \boldsymbol{f}_{\theta_\perp}(k,u)\mathcal{E}_{\theta_\perp},\end{aligned} \tag{30}$$

where

$$\mathcal{E}_\theta = \frac{-ik^0}{(2\pi)^{3/2}}\big(\varepsilon_\mu(k,u)e^\mu_\theta(k,u)\big);$$
$$\mathcal{E}_{\theta_\perp} = \frac{-ik^0}{(2\pi)^{3/2}}\big(\varepsilon_\mu(k,u)e^\mu_{\theta_\perp}(k,u)\big) \tag{31}$$

and in analogy for the complex magnetic field $\mathcal{B}$

$$\mathcal{B}(k,u)=\frac{-ik^0}{(2\pi)^{3/2}}\hat{\boldsymbol{k}}\times\boldsymbol{\varepsilon}(k,u)=\boldsymbol{f}_\theta(k,u)\mathcal{E}_{\theta_\perp}-\boldsymbol{f}_{\theta_\perp}(k,u)\mathcal{E}_\theta. \tag{32}$$

According to the standard description of polarization the classical electromagnetic field can be expressed as $\varepsilon(k,u)\equiv(\boldsymbol{b}_1+i\boldsymbol{b}_2)e^{i\alpha}$ under the conditions $\boldsymbol{b}_1\boldsymbol{b}_2=0$ and $\boldsymbol{b}_1^2-\boldsymbol{b}_2^2\geqslant 0$, where α stands for the phase. By comparing with (30)–(32), we can rewrite (28) and (29) in the following form:

$$\mathfrak{E}=(\boldsymbol{f}_\theta(k,u)|\mathcal{E}_\theta|+i\boldsymbol{f}_{\theta_\perp}(k,u)|\mathcal{E}_{\theta_\perp}|)e^{-i(kx-\alpha)},$$
$$\mathfrak{B}=(i\boldsymbol{f}_\theta(k,u)|\mathcal{E}_{\theta_\perp}|-\boldsymbol{f}_{\theta_\perp}(k,u)|\mathcal{E}_\theta|)e^{-i(kx-\alpha)}.$$

The physical electric and magnetic fields are defined as real parts of $\mathfrak{E}$ and $\mathfrak{B}$, respectively. Thus

$$\begin{aligned}\boldsymbol{E} &= \mathrm{Re}\mathfrak{E} = \boldsymbol{f}_\theta(k,u)|\mathcal{E}_\theta|\cos(kx-\alpha)\\ &\quad+\boldsymbol{f}_{\theta_\perp}(k,u)|\mathcal{E}_{\theta_\perp}|\sin(kx-\alpha)\\ &\equiv \boldsymbol{f}_\theta(k,u)E_\theta+\boldsymbol{f}_{\theta_\perp}(k,u)E_{\theta_\perp},\end{aligned} \tag{33}$$

$$\begin{aligned}\boldsymbol{B} &= \mathrm{Re}\mathfrak{B} = \boldsymbol{f}_\theta(k,u)|\mathcal{E}_{\theta_\perp}|\sin(kx-\alpha)\\ &\quad-\boldsymbol{f}_{\theta_\perp}(k,u)|\mathcal{E}_\theta|\cos(kx-\alpha)\\ &\equiv \boldsymbol{f}_\theta(k,u)B_\theta+\boldsymbol{f}_{\theta_\perp}(k,u)B_{\theta_\perp}.\end{aligned} \tag{34}$$

In consequence, the polarization equations are obtained in the form

$$\frac{E_\theta^2}{|\mathcal{E}_\theta|^2}+\frac{E_{\theta_\perp}^2}{|\mathcal{E}_{\theta_\perp}|^2}=1;\qquad \frac{B_\theta^2}{|\mathcal{E}_\theta|^2}+\frac{B_{\theta_\perp}^2}{|\mathcal{E}_{\theta_\perp}|^2}=1.$$

Hence, if $|\mathcal{E}_\theta|\neq|\mathcal{E}_{\theta_\perp}|$ the light is polarized elliptically, if $|\mathcal{E}_\theta|=|\mathcal{E}_{\theta_\perp}|$, circularly, and if $\mathcal{E}_{\theta_\perp}=0$, linearly. In the latter case, one has for the electromagnetic vectors

$$\boldsymbol{E}_\theta=\boldsymbol{E}(\theta;k,u,x)=\boldsymbol{f}_\theta E\cos(kx-\alpha);$$
$$\boldsymbol{B}_\theta=\boldsymbol{B}(\theta;k,u,x)=-\boldsymbol{f}_{\theta_\perp}E\cos(kx-\alpha), \tag{35}$$

where $E=|\mathcal{E}_\theta|$. Now, with the help of (18), we can derive the form of the linearly polarized electromagnetic field by means of (26) and (35),

$$\boldsymbol{E}_\theta=E\cos(kx-\alpha)(\boldsymbol{f}_0\cos\theta+\boldsymbol{f}_0\times\hat{\boldsymbol{k}}\sin\theta), \tag{36}$$

$$\boldsymbol{B}_\theta=E\cos(kx-\alpha)(\boldsymbol{f}_0\sin\theta-\boldsymbol{f}_0\times\hat{\boldsymbol{k}}\cos\theta). \tag{37}$$

It should be noted that the above results regarding the PF classical electrodynamics may be obtained in an alternative way which consists in adopting (12)–(14) as natural postulates for the classical four-potential, which transforms under the Lorentz group as a usual four-vector, supplemented with the classical variant of (15) and (16) which allow us to reconstruct the physical electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$, respectively, in the form given by (33) and (34). Let us also bring to attention the fact that the formalism presented above resolves into the standard quantum or classical electrodynamics in the limit $u\to(1;0,0,0)$ or equivalently $\boldsymbol{u}\to 0$, which besides corresponds to a transition to the standard relativity, valid in the PF.

Let us note the following transformation rules for vectors $\boldsymbol{f}_\theta$ and $\boldsymbol{g}_\theta$ which follow from (23) for $\Lambda=\Omega$, where Ω denotes a rotation,

$$\boldsymbol{f}_{\theta+\phi(\Omega,k,u)}(\Omega k,\Omega u)=\Omega\boldsymbol{f}_\theta(k,u);$$
$$\boldsymbol{g}_{\theta+\phi(\Omega,k,u)}(\Omega k,\Omega u)=\Omega\boldsymbol{g}_\theta(k,u).$$

These equations, together with (36) and (37), lead to the following form of passive transformations for linearly polarized electric and magnetic fields:

$$\begin{aligned}\boldsymbol{E}'_\theta &= E\cos(kx-\alpha)[\boldsymbol{f}_0\cos[\theta+\phi(\Omega,k,u)]\\ &\quad+\boldsymbol{f}_0\times\hat{\boldsymbol{k}}\sin[\theta+\phi(\Omega,k,u)]],\end{aligned} \tag{38}$$

$$\begin{aligned}\boldsymbol{B}'_\theta &= E\cos(kx-\alpha)[\boldsymbol{f}_0\sin[\theta+\phi(\Omega,k,u)]\\ &\quad-\boldsymbol{f}_0\times\hat{\boldsymbol{k}}\cos[\theta+\phi(\Omega,k,u)]].\end{aligned} \tag{39}$$

The phase $\phi(\Omega,k,u)$ can be calculated from the Wigner rotation (2) which takes the following simple form for $\Lambda=\Omega$:

$$W(\Omega,k,u)=R^{-1}_{\Omega\boldsymbol{n}(k,u)}\,\Omega\,R_{\boldsymbol{n}(k,u)}. \tag{40}$$

As mentioned in Sec. II, the u-dependent Wigner phase $\phi(\Omega,k,u)$, appearing in a theory with a PF differs from the standard one, $\phi_S(\Omega,k)$ [21]. Therefore, Eqs. (38) and (39) imply a u-dependent rotation of the polarization plane of linearly polarized light under a passive transformation, Ω. This is a purely geometrical consequence derived from the properties of the space of photonic states [19]. Contrary to common expectations regarding similar phenomena, this particular effect would be evidently independent of the distance between the source and the detector.

VI. TESTING THE PF SCENARIO FOR PHOTONS

It has been shown in our recent paper [19] that existence of a PF at the quantum level could lead to modification of the quantum Malus law [23,24]. Now, it follows from (38) and (39) that a similar phenomenon might be expected at the classical level. In order to demonstrate this, let us consider a case when the beam of linearly polarized light, with the polarization angle chosen as zero ($\theta=0$), passes through an analyzer perpendicular to the wave vector, as depicted in Fig. 1. When the slit of the analyzer is rotated by an angle δ about the beam direction (passive transformation) the phase $\phi(\delta,k,u)$ derived from (40) is given by

$$\phi(\delta,k,u)=2\arctan\frac{\sqrt{1-V^2}+[(1-\sqrt{1-V^2})\cos\chi-V]\cos\chi}{(1-V\cos\chi)\cot(\delta/2)+[(1-\sqrt{1-V^2})\cos\chi-V]\sin\chi}, \tag{41}$$

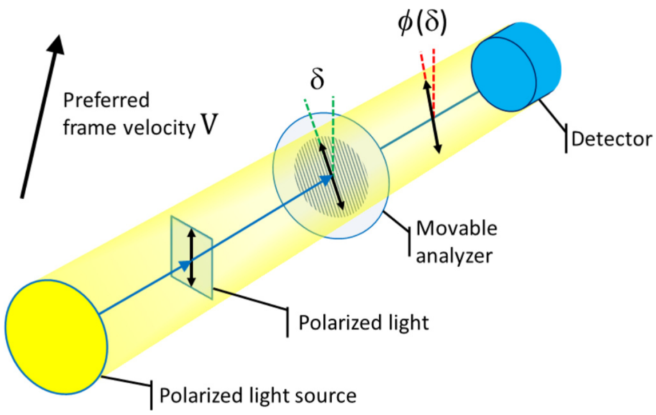

FIG. 1. Schematic presentation of a polarization experiment, involving the source and the detector at rest, to search for departures from the classical Malus law according to the PF electrodynamics, presented in this paper. Rotation of the analyzer slit by an angle δ (passive transformation) results in a change of intensity of light according to (42).

where $\boldsymbol{V} = \boldsymbol{u}/u^0$ is the velocity vector of the preferred frame (in units of c) and χ is the angle between $\boldsymbol{k}$ and $\boldsymbol{V}$. Note that it also follows from (41) that in the limit $\boldsymbol{V} \to 0$ one has $\phi(\delta, k, u) \to \phi_S(\delta, k) = \delta$, i.e., the standard phase is equal just to the angle of rotation. Now, from the experimental point of view the consequence of (41) is as follows. If the slit of the analyzer is parallel to $\hat{\boldsymbol{f}}_0$ (i.e., $\delta = 0$) then the intensity of light, measured in the detector, reaches the maximum and equals I_0. However, if the slit of the analyzer is rotated by an angle δ, the intensity of light, according to the PF scenario, is a function of the phase $\phi(\delta, k, u)$,

$$I_{\mathrm{PF}}(\delta, k, u) = I_0 \cos^2 \phi(\delta, k, u), \tag{42}$$

rather than of δ alone as in the standard case, $I_S(\delta) = I_0 \cos^2 \delta$. Thus the Malus law takes a different form if existence of a PF is assumed. In particular, the condition for vanishing intensity of light, $\phi(\delta, k, u) = \pi/2$, would be satisfied by values of δ in general different than $\pi/2$. An illustration of the above phenomenon is shown in Fig. 2 where the relative difference $(I_{\mathrm{PF}} - I_S)/I_0$ is plotted vs angle δ for $\chi = 90^\circ$ (maximum effect) for two choices of PF velocities, including $|\boldsymbol{V}| = 0.001\,23$ (368 km/s) which corresponds to the velocity of Earth with respect to a frame in which the cosmic microwave background radiation (CMBR) is isotropic—an intriguing candidate for a PF. Thus it is tempting to test the preferred frame scenario for light by making use of an experimental implementation of (41) and (42).

VII. CONCLUSIONS

In our recent paper [19] we presented a quantum description of free photons, formulated on the grounds of the hypothesis that a preferred frame, affecting the photonic field, exists in nature. Now we have developed and laid down in detail a related approach regarding the quantum and classical electrodynamics. The main result of the present work is the following: when a classical, linearly polarized electromagnetic wave undergoes a passive Lorentz transformation, the phase of the polarization angle is predicted on these grounds to differ from that in the standard case. A departure from the Malus law is expected and a method of experimental verification suggested. The presented formalism can be treated as a unique test theory for the preferred frame hypothesis with regard to light.

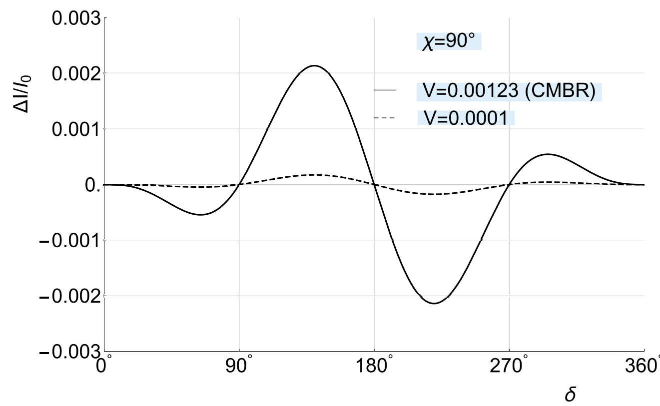

FIG. 2. Relative difference of intensities measured in the detector, $\Delta I/I_0 = \cos^2 \phi(\delta, k, u) - \cos^2 \delta$, predicted for the PF scenario, as a function of δ for $\chi = 90^\circ$ (maximum effect), assuming the CMBR frame for PF ($|V| = 0.001\,23$), solid line, and for comparison $|V| = 0.0001$, dashed line.

ACKNOWLEDGMENTS

The authors would like to thank P. Caban and K. A. Smoliński for discussions. This work has been supported by the Polish National Science Centre (PL) under Contract No. 2014/15/B/ST2/00117 and by the University of Lodz (PL).

APPENDIX A: DERIVATION OF THE RELATION $(\boldsymbol{k}, \boldsymbol{u}) = ((L_u R_n)q, (L_u R_n)u_{\mathrm{PF}})$

The Lorentz boost L_u is defined by the relation $L_u u_{\mathrm{PF}} = u$, where

$$u_{\mathrm{PF}} = \begin{pmatrix} 1\\0\\0\\0 \end{pmatrix} \equiv \begin{pmatrix} 1\\ \boldsymbol{0} \end{pmatrix}, \quad u = \begin{pmatrix} u^0\\u^1\\u^2\\u^3 \end{pmatrix} \equiv \begin{pmatrix} u^0\\ \boldsymbol{u} \end{pmatrix},$$

$$u^{02} - \boldsymbol{u}^2 = 1, \tag{A1}$$

and has the form

$$L_u = \begin{pmatrix} u^0 & \boldsymbol{u}^T\\ \boldsymbol{u} & I + \frac{\boldsymbol{u}\boldsymbol{u}^T}{1+u^0} \end{pmatrix}. \tag{A2}$$

The rotation R_n, defined by the relation

$$R_n q = \kappa \begin{pmatrix} 1\\ \boldsymbol{n} \end{pmatrix},$$

where

$$q = \kappa \begin{pmatrix} 1\\0\\0\\1 \end{pmatrix}$$

and $\boldsymbol{n}$ is a unit vector, takes the form

$$R_{\boldsymbol{n}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-\frac{(n^1)^2}{1+n^3} & \frac{-n^1n^2}{1+n^3} & n^1 \\ 0 & \frac{-n^1n^2}{1+n^3} & 1-\frac{(n^2)^2}{1+n^3} & n^2 \\ 0 & -n^1 & -n^2 & n^3 \end{pmatrix}. \tag{A3}$$

Notice that $R_{\boldsymbol{n}} u_{\rm PF} = u_{\rm PF}$. We see that $(L_u R_{\boldsymbol{n}})u_{\rm PF} = L_u u_{\rm PF} = u$ as well as

$$(L_u R_{\boldsymbol{n}})q = \kappa L_u \begin{pmatrix} 1 \\ \boldsymbol{n} \end{pmatrix} = \kappa \begin{pmatrix} u^0 + \boldsymbol{un} \\ \boldsymbol{u}\left(\frac{1+u^0+\boldsymbol{un}}{1+u^0}\right) + \boldsymbol{n} \end{pmatrix}. \tag{A4}$$

To fulfill the relation $(k,u) = ((L_u R_{\boldsymbol{n}})q, (L_u R_{\boldsymbol{n}})u_{\rm PF})$ we demand $(L_u R_{\boldsymbol{n}})q = k$. Consequently,

$$(u^0 + \boldsymbol{un})\kappa = k^0 = |\boldsymbol{k}| \quad \text{and} \quad \left[\boldsymbol{u}\left(\frac{1+u^0+\boldsymbol{un}}{1+u^0}\right) + \boldsymbol{n}\right]\kappa = \boldsymbol{k}$$

so

$$\kappa = u^0k^0 - \boldsymbol{uk} = uk \quad \text{and} \quad \boldsymbol{n} = \frac{1}{uk}\left[\boldsymbol{k} - \left(\frac{uk+k^0}{1+u^0}\right)\boldsymbol{u}\right].$$

Concluding, $(k,u) = ((L_u R_{\boldsymbol{n}})q, (L_u R_{\boldsymbol{n}})u_{\rm PF})$ provided

$$\boldsymbol{n} = \boldsymbol{n}(k,u) = \frac{1}{uk}\left[\boldsymbol{k} - \left(\frac{uk+|\boldsymbol{k}|}{1+u^0}\right)\boldsymbol{u}\right].$$

APPENDIX B: WIGNER-MACKEY DERIVATION OF THE UNITARY ACTION OF THE LORENTZ GROUP

Using the above result, we can define the four-momentum eigenvectors as follows:

$$|k,u,\lambda\rangle = |(L_u R_{\boldsymbol{n}})q, (L_u R_{\boldsymbol{n}})u_{\rm PF}, \lambda\rangle = U(L_u R_{\boldsymbol{n}})|q, u_{\rm PF}, \lambda\rangle.$$

Therefore

$$\begin{aligned} U(\Lambda)|k,u,\lambda\rangle &= U(L_{\Lambda u} R_{\boldsymbol{n}(\Lambda k,\Lambda u)})U^\dagger(L_{\Lambda u} R_{\boldsymbol{n}(\Lambda k,\Lambda u)}) \\ &\quad \times U(\Lambda)U(L_u R_{\boldsymbol{n}})|q, u_{\rm PF}, \lambda\rangle \\ &= U(L_{\Lambda u} R_{\boldsymbol{n}(\Lambda k,\Lambda u)})U[(L_{\Lambda u} R_{\boldsymbol{n}(\Lambda k,\Lambda u)})^{-1} \\ &\quad \times \Lambda(L_u R_{\boldsymbol{n}})]|q, u_{\rm PF}, \lambda\rangle \\ &= e^{i\lambda\phi(\Lambda,k,u)}U(L_{\Lambda u} R_{\boldsymbol{n}(\Lambda k,\Lambda u)})|q, u_{\rm PF}, \lambda\rangle \\ &= e^{i\lambda\phi(\Lambda,k,u)}|\Lambda k, \Lambda u, \lambda\rangle, \end{aligned}$$

where $e^{i\lambda\phi(\Lambda,k,u)}$ is the phase factor related to the subgroup SO(2) of the little group O(2), corresponding to the Wigner rotation $W(\Lambda,k,u) = (L_{\Lambda u} R_{\boldsymbol{n}(\Lambda k,\Lambda u)})^{-1}\Lambda L_u R_{\boldsymbol{n}(k,u)}$.

[1] S. Liberati, Class. Quantum Grav. **30**, 133001 (2013).
[2] S. Liberati and L. Maccione, Annu. Rev. Nucl. Part. Sci. **59**, 245 (2009).
[3] D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997).
[4] D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).
[5] S. R. Coleman and S. L. Glashow, Phys. Rev. D **59**, 116008 (1999).
[6] A. G. Cohen and S. L. Glashow, Phys. Rev. Lett. **97**, 021601 (2006).
[7] T. Jacobson, Center for Fundamental Physics, University of Maryland, Report No. QG-PH:020, 2007.
[8] P. Hořava, Phys. Rev. D **79**, 084008 (2009).
[9] J. Rembieliński, Phys. Lett. B **730**, 67 (2014).
[10] G. Amelino-Camelia, Living Rev. Relativ. **16**, 5 (2013).
[11] V. A. Kostelecky and M. Mewes, Astrophys. J. **689**, L1 (2008).
[12] P. A. M. Dirac, Nature (London) **168**, 906 (1951).
[13] P. A. M. Dirac, Proc. R. Soc. London, Ser. A **209**, 291 (1951).
[14] *The Ghost in the Atom*, edited by P. C. W. Davies and J. R. Brown (Cambridge University Press, Cambridge, UK, 1986).
[15] D. Bohm, Phys. Rev. **85**, 166 (1952).
[16] J. S. Bell, in *Quantum Gravity*, edited by C. J. Isham, R. Penrose and D. W. Sciama (Clarendon, Oxford, 1981), Vol. 2, pp. 611–637.
[17] N. Gisin, in *Quantum Theory: A Two-Time Success Story*, edited by D. C. Struppa and J. M. Tollaksen (Springer, Berlin, 2014), pp. 185–204.
[18] P. Caban and J. Rembieliński, Phys. Rev. A **59**, 4187 (1999).
[19] J. Rembieliński and J. Ciborowski, Phys. Rev. A **97**, 062106 (2018).
[20] G. Mackey, *Induced Representations of Groups and Quantum Mechanics* (W. A. Benjamin, Turin: Editore Boringhieri, 1968); A. O. Barut and R. Raczka, *Theory of Group Representations and Applications* (PWN, Warsaw, 1977).
[21] P. Caban and J. Rembieliński, Phys. Rev. A **68**, 042107 (2003).
[22] S. Weinberg, *Lectures on Particles and Field Theory*, Lectures Delivered at Brandeis Summer Institute in Theoretical Physics, edited by S. Deser and K. W. Ford (Prentice-Hall, Englewood Cliffs, NJ, 1964), Vol. II, p. 405.
[23] K. Wódkiewicz, Phys. Rev. A **51**, 2785 (1995).
[24] M. O. Scully and K. Wódkiewicz, in *Coherence and Quantum Optics VI*, edited by J. H. Eberly, L. Mandel, and E. Wolf (Plenum, New York, 1990), p. 1047.