Many-particle interferometry and entanglement by path identity

Mayukh Lahiri[*]

Faculty of Physics, University of Vienna, Boltzmanngasse 5, Vienna A-1090, Austria
and Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Boltzmanngasse 3, Vienna A-1090, Austria

We introduce a general scheme of many-particle interferometry in which two identical sources are used and "which-way information" is eliminated by making the paths of one or more particles identical (path identity). The scheme allows us to generate many-particle entangled states. We provide general forms of these states and show that they can be expressed as superpositions of various Dicke states. We illustrate cases in which the scheme produces maximally entangled two-qubit states (Bell states) and maximally three-tangled states (three-particle Greenberger-Horne-Zeilinger-class states). A striking feature of the scheme is that the entangled states can be manipulated without interacting with the entangled particles; for example, it is possible to switch between two distinct Bell states. Furthermore, each entangled state corresponds to a set of many-particle interference patterns. The visibility of these patterns and the amount of entanglement in a quantum state are connected to each other. The scheme also allows us to change the visibility and the amount of entanglement without interacting with the entangled particles and, therefore, has the potential to play an important role in quantum information science.

I. INTRODUCTION

In 1991, Zou, Wang, and Mandel reported observation of single-photon interference by using two identical two-photon sources [1,2]. A striking feature of their experiment, which was originally suggested by Ou, was to make the paths of the same photon generated by the two sources identical (Fig. 1). This path identity created coherence between the beams (b_1 and b_2) of the other photon and a single-photon pattern resulted. The interference pattern could be manipulated without interacting with the photon that was detected. In a recent series of work the concept of path identity has been applied to imaging [3,4], spectroscopy [5], generating a light beam in any state of polarization [6], fundamental test of quantum mechanics [7,8], measuring correlations between two photons [9,10], and generating multiphoton high-dimensional entangled states [11].

The aim of this paper is to introduce a general scheme of generating many-particle entangled states and many-particle interference patterns by applying the method of path identity. An important feature of this scheme is that the generated entangled states (and also the interference patterns) can be manipulated without interacting with the entangled particles.

The paper is organized as follows. In Sec. II, we introduce the scheme in its simplest form and show that it can be used to produce two-particle entangled sates and interference patterns. Then in Sec. III, we introduce the scheme in its most general form. In Sec. IV, we show that the scheme can produce three-particle GHZ-class states. In Sec. V, we illustrate how the scheme can be used to control the amount of entanglement in the generated quantum states. In Sec. VI,
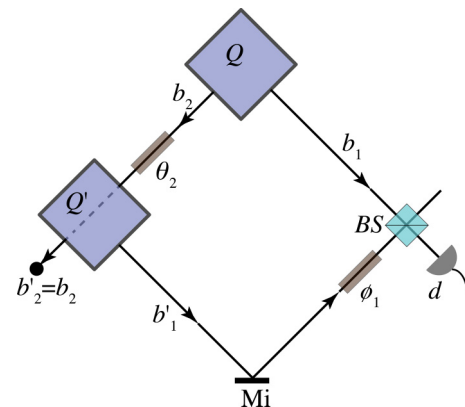
*mayukh.lahiri@univie.ac.at

FIG. 1. Zou-Wang-Mandel experiment. Q and Q' are two identical sources emitting two photons (1,2) into beams (b_1, b_2) and (b_1', b_2'). The paths of photon 2 are made identical by sending beam b_2 through Q' and aligning it with b_2'. Photon 2 is not detected. Single-photon interference is observed at detector d when b_1 and b_1' are superposed.

we briefly discuss the fidelity for the output states. Finally, we summarize the results in Sec. VII.

II. SIMPLEST CASE: TWO-PARTICLE INTERFERENCE AND ENTANGLEMENT

Case I (Fig. 2).—Suppose that a three-particle source Q emits particles 1, 2, and 3 into beams b_1, b_2, and b_3, respectively [Fig. 2(a)]. We now consider another identical source Q' whose emitted beams are denoted by b_1', b_2', and b_3'. If the two sources emit in quantum superposition [12], the three-particle state is given by

$$|X_3\rangle = (|b_1\rangle_1|b_2\rangle_2|b_3\rangle_3 + e^{i\phi_0}|b_1'\rangle_1|b_2'\rangle_2|b_3'\rangle_3)/\sqrt{2}, \quad (1)$$
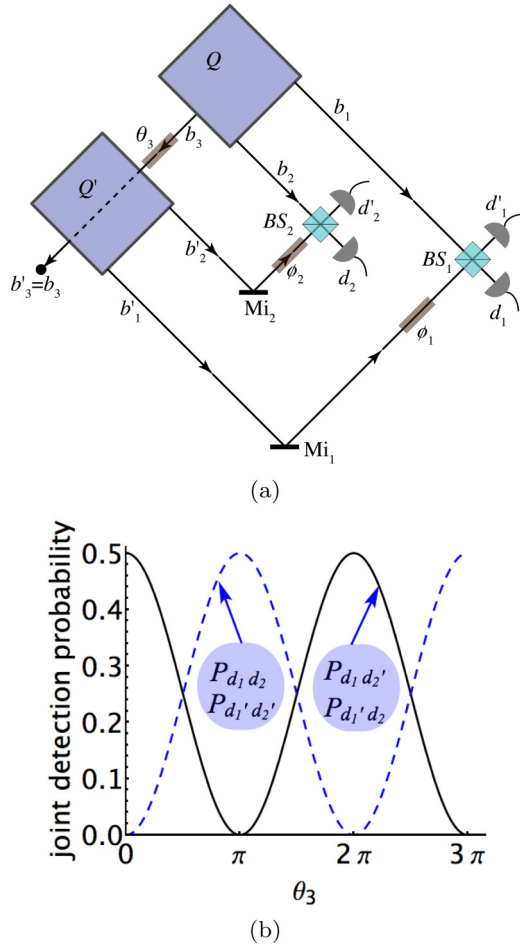
(a)

(b)

FIG. 2. (Case I) Two-particle interference and entanglement by one-particle path identity: (a) Schematic of the setup. Q and Q' are two identical three-particle sources emitting particles (1,2,3) into beams (b_1, b_2, b_3) and (b_1', b_2', b_3'). Beam b_3 is aligned with b_3' in such a way that it is not possible to determine the source of particle 3 if observed after Q'. The phase change along b_3 due to propagation from Q to Q' is θ_3. Beams b_1 and b_1' are superposed by BS1 (beam splitter or an equivalent device) with two outputs detected at d_1 and d_1'. The phase difference between b_1 and b_1' is ϕ_1. Likewise b_2 and b_2' are superposed by BS2 with two outputs at d_2 and d_2'; the corresponding phase difference is ϕ_2. Particle 3 is never detected. (b) Two-particle interference patterns. Probabilities ($P_{d_1 d_2}$, $P_{d_1 d_2'}$, $P_{d_1' d_2}$, $P_{d_1' d_2'}$) of joint detection at the pairs of detectors (d_1, d_2), (d_1, d_2'), (d_1', d_2), and (d_1', d_2') vary sinusoidally with phase θ_3 that can only be modulated using particle 3. Interference patterns $P_{d_1 d_2}$ and $P_{d_1' d_2'}$ are in phase (dashed line). These patterns are complementary to the patterns $P_{d_1 d_2'}$ and $P_{d_1' d_2}$ (solid line). We have set $\Phi^{(2)} = 2n\pi$, n being an integer. A maximum and a minimum of an interference pattern are attained for two distinct Bell states.

where $|b_1\rangle_1$ denotes particle 1 in beam b_1, etc., ϕ_0 is a phase factor, and we have assumed that emission probability at the two sources are equal. Note that $|X_3\rangle$ is a three-particle Greenberger-Horne-Zeilinger (GHZ) state [13,14].

Suppose now that the paths of particle 3 emitted by Q and Q' are made identical ($b_3 = b_3'$). This can be done by sending beam b_3 through Q' and aligning it with b_3' [Fig. 2(a)]. We therefore have $|b_3\rangle_3 \rightarrow \exp[i\theta_3]|b_3'\rangle_3$, where θ_3 can be

interpreted as the phase gained due to propagation from Q to Q'. Applying this transformation to Eq. (1), we find that $|X\rangle \rightarrow |\psi_0\rangle$, where [15]

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|b_1\rangle_1|b_2\rangle_2 + e^{i(\phi_0 - \theta_3)}|b_1'\rangle_1|b_2'\rangle_2)|b_3'\rangle_3. \quad (2)$$

This state is a tensor product of a "spin-free" two-particle entangled state [16] and a single third particle state.

Beams b_1 and b_1' are superposed by a 50-50 beam splitter (or an equivalent device), BS1, and the two outputs are received by detectors d_1 and d_1'. The phase difference between the beams b_1 and b_1' is given by ϕ_1. Likewise b_2 and b_2' are superposed (corresponding phase difference ϕ_2) by BS2 with outputs at d_2 and d_2'. The consequent transformations of the pairs of kets are therefore given by

$$|b_j\rangle_j \rightarrow (|d_j\rangle_j + i|d_j'\rangle_j)/\sqrt{2}, \quad (3a)$$

$$|b_j'\rangle_j \rightarrow e^{i\phi_j}(|d_j'\rangle_j + i|d_j\rangle_j)/\sqrt{2}, \quad (3b)$$

where $j = 1, 2$. Applying the evolution given by Eq. (3) to the state in Eq. (2), we find that

$$|\psi_0\rangle \rightarrow |\psi\rangle = \frac{1}{2}\left\{(1 - e^{i\zeta_1^{(3)}})\frac{1}{\sqrt{2}}(|d_1\rangle_1|d_2\rangle_2 - |d_1'\rangle_1|d_2'\rangle_2)\right.$$

$$\left. + i(1 + e^{i\zeta_1^{(3)}})\frac{1}{\sqrt{2}}(|d_1\rangle_1|d_2'\rangle_2 + |d_1'\rangle_1|d_2\rangle_2)\right\}|b_3'\rangle_3, \quad (4)$$

where $\zeta_1^{(3)} = \phi_0 + \phi_1 + \phi_2 - \theta_3$. The complex coefficients associated with $|d_1\rangle_1|d_2\rangle_2$, $|d_1\rangle_1|d_2'\rangle_2$, $|d_1'\rangle_1|d_2\rangle_2$, and $|d_1'\rangle_1|d_2'\rangle_2$ are the probability amplitudes of joint (coincidence) detection of particles 1 and 2 at the pairs of detectors (d_1, d_2), (d_1, d_2'), (d_1', d_2), and (d_1', d_2'), respectively. The coincidence detection rate at these pairs of detectors are given by the corresponding probabilities (square of the modulus of the probability amplitudes), i.e., by

$$P_{d_1 d_2} = P_{d_1' d_2'} = \frac{1}{4}[1 - \cos(\Phi^{(2)} - \theta_3)], \quad (5a)$$

$$P_{d_1 d_2'} = P_{d_1' d_2} = \frac{1}{4}[1 + \cos(\Phi^{(2)} - \theta_3)], \quad (5b)$$

where $\Phi^{(2)} = \phi_0 + \phi_1 + \phi_2$, i.e., $\zeta_1^{(3)} = \Phi^{(2)} - \theta_3$.

Clearly two-particle interference [17–20] involving 1 and 2 will occur. The fact that $|b_3'\rangle_3$ gets factored out in Eq. (18) implies that one does not need to detect particle 3 to observe the interference of 1 and 2. However, the two-particle interference patterns can be modulated by using this undetected particle [Fig. 2(b)], as is evident from the appearance of θ_3 in the joint-detection probabilities. Equation (5) shows that the two-particle interference patterns at the two pairs of detectors (d_1, d_2) and (d_1', d_2') are identical. Similarly, the patterns observed at (d_1', d_2) and (d_1, d_2') are also identical. The patterns observed in the former set of detector pairs are complementary to those observed in the latter set of detector pairs [Fig. 2(b)].

We now note that the pair of particles (1,2) will be the following two distinct Bell states for $\zeta_1^{(3)} = 2m\pi$ and $\zeta_1^{(3)} = (2m + 1)\pi$, respectively:

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|d_1\rangle_1|d_2'\rangle_2 + |d_1'\rangle_1|d_2\rangle_2), \quad (6a)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|d_1\rangle_1|d_2\rangle_2 - |d_1'\rangle_1|d_2'\rangle_2), \quad (6b)$$
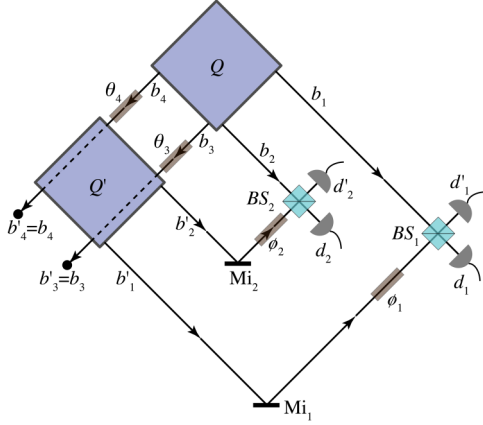
FIG. 3. (Case II) Two-particle interference and entanglement by two-particle path identity: Q and Q' are two identical four-particle sources emitting particles (1,2,3,4) into beams (b_1, b_2, b_3, b_4) and (b_1', b_2', b_3', b_4'), respectively. The beams b_3 and b_4 are aligned with b_3' and b_4', respectively; the corresponding phase changes are θ_3 and θ_4. Particles 3 and 4 are not detected. The rest of the notations are same as in Fig. 2(a). The two-particle interference patterns produced in this setup are identical to those shown in Fig. 2, except each pattern can now be modulated by both θ_3 and θ_4. The same Bell states are obtained under conditions strictly similar to case I.

where $m = 0, \pm 1, \pm 2, \ldots$. A comparison between Eqs. (5) and (6) shows that when the state $|\Psi^+\rangle$ is obtained, the coincidence counts at (d_1', d_2) and (d_1, d_2') maximize and the coincidence counts at (d_1, d_2) and (d_1', d_2') minimize [see Fig. 2(b)]. Likewise, the state $|\Phi^-\rangle$ is obtained when coincidence counts are maximum at (d_1, d_2) and (d_1', d_2'), and minimum at (d_1', d_2) and (d_1, d_2'). *The system therefore allows one to switch between the two Bell states without any interaction with the pair of particles.*

Case II (Fig. 3).—We now consider two four-particle sources, Q and Q', emitting in quantum superposition. Q and Q' emit particles 1, 2, 3, and 4 into the beams (b_1, b_2, b_3, b_4) and (b_1', b_2', b_3', b_4'), respectively (Fig. 3). The resulting quantum state is given by

$$|X_4\rangle = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{4} |b_j\rangle_j + e^{i\phi_0} \prod_{j=1}^{4} |b_j'\rangle_j\right). \qquad (7)$$

Beams b_3 and b_4 are sent through Q' and are perfectly aligned with beams b_3' and b_4' (path identity). The corresponding transformations of kets are given by $|b_3\rangle_3 \rightarrow \exp[i\theta_3]|b_3'\rangle_3$ and $|b_4\rangle_4 \rightarrow \exp[i\theta_4]|b_4'\rangle_4$. The beams of particles 1 and 2 are superposed in the same way as in case I. Following theoretical steps which are strictly similar to case I, we find that the two-particle interference patterns are given by

$$P_{d_1 d_2} = P_{d_1' d_2'} = \tfrac{1}{4}\left[1 - \cos\left(\Phi^{(2)} - \theta_3 - \theta_4\right)\right], \qquad (8a)$$

$$P_{d_1 d_2'} = P_{d_1' d_2} = \tfrac{1}{4}\left[1 + \cos\left(\Phi^{(2)} - \theta_3 - \theta_4\right)\right], \qquad (8b)$$

where $\Phi^{(2)}$ is defined below Eq. (5).

Let us define $\zeta_2^{(4)} \equiv \Phi^{(2)} - \theta_3 - \theta_4$. It again follows that the pair of particles (1,2) will be in the Bell states given by

Eqs. (6a) and (6b) for $\zeta_2^{(4)} = 2m\pi$ and $\zeta_2^{(4)} = (2m + 1)\pi$, respectively.

Before introducing the general scheme, we compare cases I and II and note the following: (1) the difference between the number of particles produced by a source and the number of particles used for path identity is the same; (2) both setups produce the same entangled states; (3) an entangled state is obtained only when a maximum occurs in a set of interference patterns; and (4) the entangled states and the interference patterns can be modified without interacting with the associated particles.

III. GENERAL SCHEME

Let us consider two identical sources, Q and Q', each of which can emit N particles $(1, 2 \ldots, N)$, into beams $(b_1, b_2, \ldots, b_N)$ and $(b_1', b_2', \ldots, b_N')$, respectively. The sources emit in quantum superposition and thus produce the state

$$|X_N\rangle = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{N} |b_j\rangle_j + e^{i\phi_0} \prod_{j=1}^{N} |b_j'\rangle_j\right). \qquad (9)$$

Paths of the particles $N - M + 1, \ldots, N$ are made identical by sending the beams $b_{N-M+1}, \ldots, b_N$ through Q' and perfectly aligning them with $b_{N-M+1}', \ldots, b_N'$. These alignments lead to the set of transformations

$$|b_l\rangle_l \rightarrow \exp[i\theta_l]|b_l'\rangle_l, \quad l = N - M + 1, \ldots, N, \qquad (10)$$

where θ_l is the phase gained due to propagation from Q to Q' along b_l. The pairs of beams (b_1, b_1'), (b_2, b_2'), …, (b_{N-M}, b_{N-M}') are superposed by $N - M$ beam splitters, $BS_1, BS_2, \ldots, BS_{N-M}$. The outputs of the beam splitters are detected at the pairs of detectors (d_1, d_1'), (d_2, d_2'), …, (d_{N-M}, d_{N-M}'). The corresponding transformations of kets are given by Eq. (3) with $j = 1, 2, \ldots, N - M$. We measure $(N - M)$-fold coincidences at a set of $N - M$ detectors, each placed at an output of a distinct beam splitter; an example of a set of detectors is $(d_1, d_2, \ldots, d_{N-M})$.

Applying transformations (3) and (17) to Eq. (9), we find that the quantum states becomes

$$|\psi_N\rangle = \left(\frac{1}{\sqrt{2}}\right)^{N-M+1}\left[\sum_{r=0}^{N-M} (i^r + i^{N-M-r} e^{i\xi_M^{(N)}})|D_r\rangle^{N-M}\right]$$

$$\otimes \prod_{j=1}^{M} |b_{N-M+j}'\rangle_{N-M+j}, \qquad (11)$$

where $\xi_M^{(N)} = \phi_0 + \sum_{k=1}^{N-M} \phi_k - \sum_{j=1}^{M} \theta_{N-M+j}$ and the $(N - M)$-particle state $|D_r\rangle^{N-M}$ is a Dicke state [21,22], i.e., a sum of $\binom{N-M}{r}$ terms (states), each being a product of r primed states $(|d_k'\rangle_k)$ and $N - M - r$ unprimed states $(|d_k\rangle_k)$; in our notation, $|D_0\rangle^{N-M} = \prod_{k=1}^{N-M} |d_k\rangle_k$ and $|D_{N-M}\rangle^{N-M} = \prod_{k=1}^{N-M} |d_k'\rangle_k$ have one term each.

It follows from Eq. (11) that when $N - M \geqslant 1$, the system produced $(N - M)$-particle interference patterns. The fact that the states $|b_{N-M+j}'\rangle_{N-M+j}$ factor out implies that in order to observe these patterns one does not need to detect the M particles used for path identity.

The $N - M$ particles emerging from the outputs of the beam splitters will be in different entangled states depending on the value of $\xi_M^{(N)}$. One can express these states in simplified forms by considering the cases $N - M = 4n, 4n + 1, 4n + 2, 4n + 3$, where $n = 0, 1, 2, \ldots$. It follows from Eq. (11) that the forms are [dropping the normalization constant $(1/\sqrt{2})^{N-M-1}$]:

(i) for $N - M = 4n > 0$, $\xi_M^{(N)} = 2m\pi$; and $N - M = 4n + 2$, $\xi_M^{(N)} = (2m + 1)\pi$:

$$|F_1\rangle = \sum_{r'=0}^{(N-M)/2} (-1)^{r'} |D_{2r'}\rangle^{N-M}, \qquad (12)$$

(ii) for $N - M = 4n > 0$, $\xi_M^{(N)} = (2m + 1)\pi$; and $N - M = 4n + 2$, $\xi_M^{(N)} = 2m\pi$:

$$|F_2\rangle = \sum_{r'=0}^{(N-M-2)/2} (-1)^{r'} |D_{2r'+1}\rangle^{N-M}, \qquad (13)$$

(iii) for $N - M = 4n + 1$, $\xi_M^{(N)} = (2m - 1/2)\pi$; and $N - M = 4n + 3$, $\xi_M^{(N)} = (2m + 1/2)\pi$:

$$|F_3\rangle = \sum_{r'=0}^{(N-M-1)/2} (-1)^{r'} |D_{2r'}\rangle^{N-M}, \qquad (14)$$

and

(iv) for $N - M = 4n + 1$, $\xi_M^{(N)} = (2m + 1/2)\pi$; and $N - M = 4n + 3$, $\xi_M^{(N)} = (2m - 1/2)\pi$:

$$|F_4\rangle = \sum_{r'=0}^{(N-M-1)/2} (-1)^{r'} |D_{2r'+1}\rangle^{N-M}, \qquad (15)$$

where $m = 0, \pm 1, \pm 2, \ldots$. These entangled states [Eqs. (12)–(15)] depend on the difference $N - M$, not on individual values of N and M.

It is important to note that the particles emerging from the beam splitters can be transformed from one entangled state to other by changing the phase $\xi_M^{(N)}$. Since $\xi_M^{(N)}$ contains the phases θ_{N-M+j}, it can be varied without interacting with the entangled particles. Therefore, the scheme allows us to modify a many-particle entangled state in an interaction-free way. Furthermore, each of these states is generated when a maximum occurs in a corresponding set of many-particle interference patterns. We made these observations in the special cases I and II discussed above.

IV. GHZ-CLASS STATE

As another example let us consider the case in which $N - M = 3$. It follows from Eq. (14) that the system produces the states of the form (replacing the unprimed states by 0 and primed states by 1)

$$\frac{1}{2}(|0\rangle_1 |0\rangle_2 |0\rangle_3 - |1\rangle_1 |1\rangle_2 |0\rangle_3 - |1\rangle_1 |0\rangle_2 |1\rangle_3 - |0\rangle_1 |1\rangle_2 |1\rangle_3). \qquad (16)$$

This state is a three-particle Greenberger-Horne-Zeilinger-class state (see, for example, [23]). It has highest (unit) "three-tangle" or "residual entanglement" (proposed by Coffman, Kundu, and Wooters [24]): the concurrence [25,26] of each
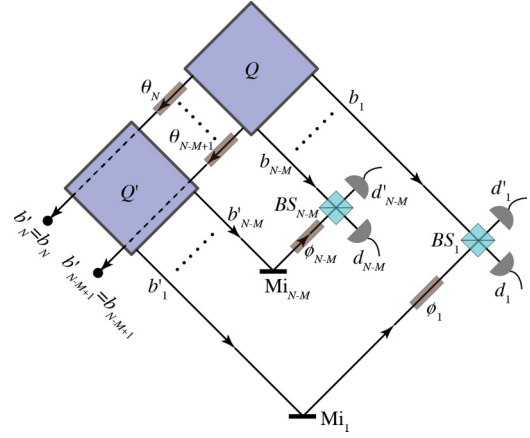

FIG. 4. The general scheme (notations are analogous to Figs. 2 and 3). Two identical N-particle sources emit particles $(1, 2, \ldots, N)$ into beams $(b_1, b_2, \ldots, b_N)$ and $(b_1', b_2', \ldots, b_N')$, respectively. Paths of M particles $(N - M + 1, \ldots, N)$ are made identical by aligning the corresponding beams and these particles are not detected. Rest of the particles $(1, 2, \ldots, N - M)$ produce $N - M$-particle interference patterns and entangled states when the corresponding beams are superposed.

qubit with the rest of the system is 1, and all the pairwise concurrences are 0. A three-particle GHZ-class state is also obtained from Eq. (15).

V. CONTROLLING THE AMOUNT OF ENTANGLEMENT

In an actual experiment, the path identity can be partially (or fully) lost. Importantly, the loss of path identity can be controlled by inserting an attenuator (neutral density filter for photons) in the path of aligned particles between the two sources. We now analyze such a situation and show that it is possible to control the amount of entanglement without interacting with the entangled particles.

We consider the general scheme (Fig. 4) and in addition we assume that attenuators are placed between Q and Q' in each of the beams b_l, where $l = N - M + 1, \ldots, N$. The quantum state generated by the two N-particle sources is again given by Eq. (9). However, the transformation of the states due to alignment of particle paths is now given by [27]

$$|b_l\rangle_l \rightarrow \exp[i\theta_l] \left(T_l |b_l'\rangle_l + \sqrt{1 - T_l^2} |v\rangle_l \right), \qquad (17)$$

where $0 \leqslant T_l \leqslant 1$ is the amplitude transmission coefficient of an attenuator ($1 - T_l^2$ is the probability of particle l getting lost before arriving at Q'), $|v\rangle_l$ represents the state of a lost particle, and $l = N - M + 1, \ldots, N$. Clearly $T_l = 1$ implies no loss of path identity (for particle l) and $T_l = 0$ implies complete loss of path identity.

The transformations of the states due to beam splitters are given by Eq. (3), where $j = 1, 2, \ldots, N - M$. The many-particle interference patterns and the many-particle entangled states are obtained by applying Eqs. (3), (9), and (17). It is to be noted that the particles emerging from the beam splitters are in a mixed state when $T_l \neq 1$ for any l. The density operator representing this state is obtained by taking partial

trace over the undetected modes and the loss modes. Below we illustrate the method by an example.

Let us consider the situation illustrated by Fig. 2(a) with the additional assumption that an attenuator is placed in beam b_3 between Q and Q'. In this case, $N = 3$ and $M = 1$. Applying Eqs. (3), (9), and (17), we find that

$$|\psi_0\rangle \rightarrow |\psi\rangle = \frac{1}{2}\left[\left(T_3 - e^{i\zeta_1^{(3)}}\right)|b_3'\rangle_3 + \sqrt{1 - T_3^2}|v\rangle_3\right]|\Phi^-\rangle$$
$$+ \frac{i}{2}\left[\left(T_3 + e^{i\zeta_1^{(3)}}\right)|b_3'\rangle_3 + \sqrt{1 - T_3^2}|v\rangle_3\right]|\Psi^+\rangle,$$
(18)

where $\zeta_1^{(3)} = \phi_0 + \phi_1 + \phi_2 - \theta_3$; and $|\Psi^+\rangle$ and $|\Phi^-\rangle$ are given by Eq. (6). The density operator, $\widehat{\rho}$, representing the quantum state of the particles emerging from the beam splitters is obtained by taking the partial trace of $|\psi\rangle\langle\psi|$ over $|b_3'\rangle_3$ and $|v\rangle_3$. We thus have

$$\widehat{\rho} = \mathrm{tr}\{|\psi\rangle\langle\psi|\}_{b_3',v} = \frac{1}{2}\left(1 - T_3\cos\zeta_1^{(3)}\right)|\Phi^-\rangle\langle\Phi^-|$$
$$+ \frac{1}{2}\left(1 + T_3\cos\zeta_1^{(3)}\right)|\Psi^+\rangle\langle\Psi^+|.$$
(19)

It follows from Eqs. (6) and (19) that the rate of coincidence detection rate of particles 1 and 2 at the pairs of detectors (d_1, d_2), (d_1, d_2'), (d_1', d_2), and (d_1', d_2') are given by

$$P_{d_1 d_2} = P_{d_1' d_2'} = \frac{1}{4}[1 - T_3\cos\zeta_1^{(3)}],$$
(20a)
$$P_{d_1 d_2'} = P_{d_1' d_2} = \frac{1}{4}[1 + T_3\cos\zeta_1^{(3)}].$$
(20b)

These two-particle interference patterns are similar to the ones given by Eq. (5), except they no longer have unit visibility. The visibility is now given by

$$\mathcal{V} = T_3.$$
(21)

If we choose $\zeta_1^{(3)} = 2m\pi$, Eq. (19) reduces to

$$\widehat{\rho}_{\mathrm{even}} = \frac{1}{2}(1 - T_3)|\Phi^-\rangle\langle\Phi^-| + \frac{1}{2}(1 + T_3)|\Psi^+\rangle\langle\Psi^+|,$$
(22)

and for $\zeta_1^{(3)} = (2m + 1)\pi$, we get

$$\widehat{\rho}_{\mathrm{odd}} = \frac{1}{2}(1 + T_3)|\Phi^-\rangle\langle\Phi^-| + \frac{1}{2}(1 - T_3)|\Psi^+\rangle\langle\Psi^+|.$$
(23)

Clearly when the coincidence detection rates at (d_1, d_2') and (d_1', d_2) maximize, the state given by Eq. (22) is obtained. Similarly, when the coincidence detection rates at (d_1, d_2) and (d_1', d_2') maximize, the state given by Eq. (23) is obtained.

We now investigate the amount of entanglement in these mixed states. For simplicity of notation we represent the unprimed state by 0 and primed states by 1. In this notation we have $|d_1\rangle_1|d_2\rangle_2 \equiv |0,0\rangle$, $|d_1\rangle_1|d_2'\rangle_2 \equiv |0,1\rangle$, $|d_1'\rangle_1|d_2\rangle_2 \equiv |1,0\rangle$, and $|d_1'\rangle_1|d_2'\rangle_2 \equiv |1,1\rangle$. In this basis, the mixed states given by Eqs. (22) and (23) take the following matrix forms:

$$[\widehat{\rho}_{\mathrm{even}}] = \begin{pmatrix} \frac{1-T_3}{4} & 0 & 0 & -\frac{1-T_3}{4} \\ 0 & \frac{1+T_3}{4} & \frac{1+T_3}{4} & 0 \\ 0 & \frac{1+T_3}{4} & \frac{1+T_3}{4} & 0 \\ -\frac{1-T_3}{4} & 0 & 0 & \frac{1-T_3}{4} \end{pmatrix},$$
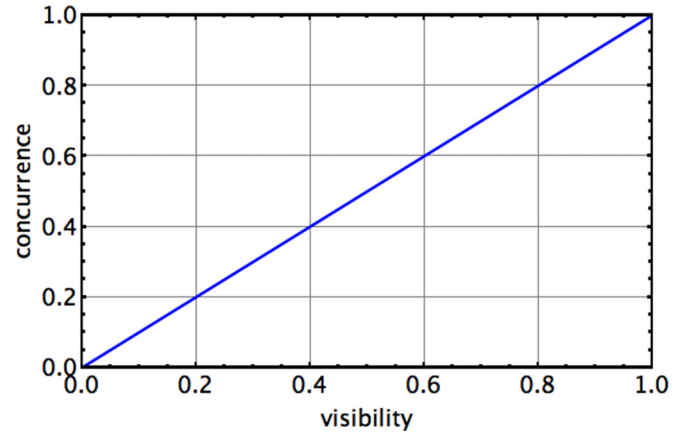(24a)

FIG. 5. Controlling the amount of entanglement. Two-particle entangled states are produced using the setup illustrated by Fig. 2(a). The concurrence is equal to the visibility of the two-particle interference pattern. Both concurrence and visibility are equal to the amplitude transmission coefficient of the attenuator (when there is no experimental loss).

$$[\widehat{\rho}_{\mathrm{odd}}] = \begin{pmatrix} \frac{1+T_3}{4} & 0 & 0 & -\frac{1+T_3}{4} \\ 0 & \frac{1-T_3}{4} & \frac{1-T_3}{4} & 0 \\ 0 & \frac{1-T_3}{4} & \frac{1-T_3}{4} & 0 \\ -\frac{1+T_3}{4} & 0 & 0 & \frac{1+T_3}{4} \end{pmatrix}.$$
(24b)

We determine the concurrence using the standard procedure [26] and find that both states have the same concurrence

$$\mathcal{C}(\widehat{\rho}) = T_3.$$
(25)

Comparing Eqs. (21) and (25), it becomes clear that

$$\mathcal{C}(\widehat{\rho}) = \mathcal{V},$$
(26)

i.e., in this case the concurrence is equal to the visibility of the two-particle interference pattern (Fig. 5). We note that one can change both the concurrence and the visibility by varying T_3. Since the attenuator never interacts with the entangled particles, *the scheme allows us to control the amount of entanglement in an interaction-free way*.

The method also applies when the number of entangled particles is more than two. This is because for any number of particles, the placement of the attenuators results in the conversion of a pure output state to a mixed one.

VI. FIDELITY

Finally, we briefly discuss the fidelity for the output states. Determining fidelity is relevant when the loss of path identity is unintended and due to experimental imperfections. Equation (17) again applies in this case but T_l now signifies the quality of alignment or other experimental losses.

Let us once again consider the situation illustrated by Fig. 2(a). Without any experimental imperfections, one would
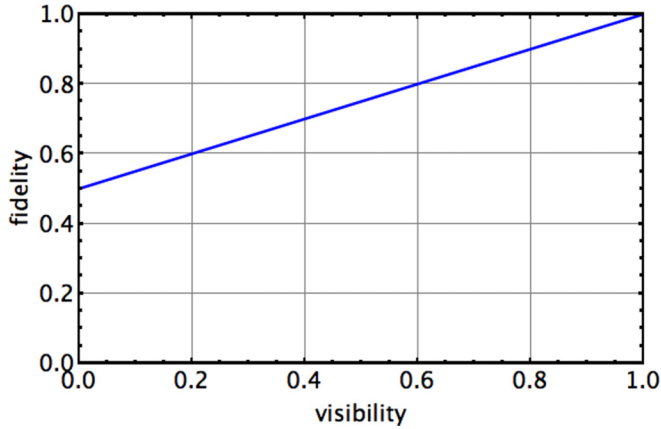
FIG. 6. Fidelity and loss of path identity. We consider the case illustrated by Fig. 2(a). The fidelity varies linearly with the visibility of the two-particle interference pattern. No path identity results in zero visibility and perfect path identity results in highest possible visibility.

expect the output state to be $|\Psi^+\rangle$ for $\zeta_1^{(3)} = 2m\pi$. However, when there is a loss of path identity, the output state is represented by $\widehat{\rho}_{\text{even}}$ [Eq. (22)]. The fidelity is, therefore, given by [28]

$$\mathcal{F} \equiv \langle \Psi^+ | \widehat{\rho}_{\text{even}} | \Psi^+ \rangle = \tfrac{1}{2}(1 + \mathcal{V}), \qquad (27)$$

where we have used Eq. (21). Similarly, for $\zeta_1^{(3)} = (2m + 1)\pi$, the fidelity is again equal to $(1 + \mathcal{V})/2$. Clearly the fidelity is directly related to the visibility of the two-particle interference pattern (Fig. 6).

The loss of path identity results in the conversion of a pure output state into a mixed one for any number of particles. We therefore expect that a relationship between fidelity and visibility also exists when the number of particle increases.

VII. CONCLUSIONS

We have introduced a novel scheme of many-particle interferometry that can be used for producing many-particle entangled states. In contrast to a series of notable studies (see, for example, [13,14,20,29–32]) that have already emphasized the connection between entanglement and interference, our work uses the concept of path identity.

In our scheme, path identity is a result of the fact that both sources can emit a certain number (M) of particles into the same modes of the associated quantum field. Therefore, the scheme is applicable to any quantum system (e.g., atoms, fundamental particles) that can be treated in the framework of quantum field theory [33].

Our scheme produces many-particle entangled states that are superpositions of different Dicke states. We have also shown that using this scheme, maximally entangled two-qubit states (Bell states) and maximally three-tangled quantum states (Greenberger-Horne-Zeilinger-class states) can be produced. We expect that further investigations regarding the states produced by our scheme will lead to promising results.

An important feature of our scheme is that the generated entangled states can be manipulated without interacting with the entangled particles. Furthermore, the scheme also allows us to control the amount of entanglement in a quantum state. We hope that this type of quantum state control and engineering will have a significant impact in quantum information science.

Finally, our scheme can be further generalized by including other degrees of freedom, for example, polarization, orbital angular momentum, etc., for the photonic cases. Another generalization will be the use of multiport beam splitters [34] instead of the standard two-port beam splitters. It will also be interesting to investigate whether our scheme can be represented and further analyzed by the graph theoretical technique that has recently been introduced by Krenn, Gu, and Zeilinger [35].

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