Quantum Zeno effect in correlated qubits

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Near-term quantum hardware promises to achieve quantum supremacy. From a quantum dynamical point of view, however, it is not unambiguously clear whether fundamental peculiarities of quantum physics permit any arbitrary speedups in real time. We show that an only recently unveiled property of the quantum Fisher information has profound implications for the rate of possible quantum information processing. To this end, we analyze an exemplary and pedagogical example for a quantum computer consisting of a computational qubit and a quantum memory. We find that frequent interaction between memory and device exhibit the quantum Zeno effect. In a second part, we show that the Zeno effect can be prevented by carefully designing the correlations and interaction between single elements of the quantum memory.

We are on the verge of a technological revolution. Over the last few years the first computational devices have become available that promise to exploit so-called quantum supremacy [1]. While big corporations such as Google and IBM, and smaller start-ups such as Rigetti and DWave, announce new developments in frequent succession, many fundamental questions still remain to be answered.

Quantum supremacy means that quantum computers will be able to achieve specific tasks exponentially faster than any classical computer [2]. In this context, *faster* typically stands for *fewer single-qubit operations* that are necessary to obtain the computational result [3]. Therefore, the computational "speedup" in quantum physics is not necessarily related to physical time.

Ever since Heisenberg's original inception of an uncertainty relation between energy and time [4], it has been apparent that classical notions such as "speed" and "speedup" need to be carefully reconsidered. Following the seminal works of Mandelstam and Tamm [5] and Margolous and Levitin [6] it has been well-established that the rate of any quantum evolution is limited by the *quantum speed limit*. In the simplest case of Hamiltonian dynamics, the minimal time a quantum system needs to evolve between orthogonal states is determined by $\tau_{\mathrm{QSL}} = \max\{\pi\hbar/(2\Delta E), \pi\hbar/(2E)\}$, where ΔE is the variance of the energy of the initial state and E its mean energy above the energy of the ground state. In more general situations the energy is replaced by the geometric properties of the considered quantum dynamics. For details and a concise history of the quantum speed limit we refer to a recent review and references therein [7].

The natural question arises as to what extent the quantum speed limit constrains the performance of prospective quantum computers. Already before Feynman's inception of quantum computing [8], Bremermann [9] and Bekenstein [10,11]

realized that the quantum speed limit sets an upper bound on the rate with which information can be processed in and transmitted between quantum systems. However, it has also been realized that in purposefully designed quantum systems, correlations can be utilized to speed up quantum dynamics [12,13].

The purpose of the present work is to carefully analyze how correlations affect the rate of quantum computation. To this end, we study a fully solvable model which is inspired by recent work in thermodynamics of information [14] and quantum Maxwell demons [15]. Loosely speaking such systems consist of a device, i.e., the part of the system that *does* the computation, and a quantum hard disk consisting of a set of qubits. For such a simple and pedagogical model of a quantum computer, we compute the quantum speed limit from which we draw conclusions about the limitations on possible computations. As a first main result, we will see that in the absence of correlations in the quantum hard disk, the rate of computation is severely limited by fundamental properties of quantum dynamics. In particular, we will see that frequent interaction of the computation device and the hard disk leads to the *quantum Zeno effect*. This means that if the computational device and the quantum hard disk "talk to each other" too frequently, the quantum state of the computer freezes out and all computation is inhibited. As a second main result, we will then see that correlations between the qubits in the quantum hard disk assist in overcoming the Zeno effect, but also that there is a trade-off between the strength of the interaction and the maximal rate with which a computation can be successfully performed.

Preliminaries. We begin by outlining our model for a simple quantum computer, and by establishing notions and notations. The computational device is a single qubit $\mathcal{Q}$ that is initially prepared in the mixed state

$$\rho_0 = (1 - \nu)|0\rangle\langle 0| + \nu|1\rangle\langle 1|, \qquad (1)$$

where $0 \leqslant \nu \leqslant 1$, and ν is close to 1.

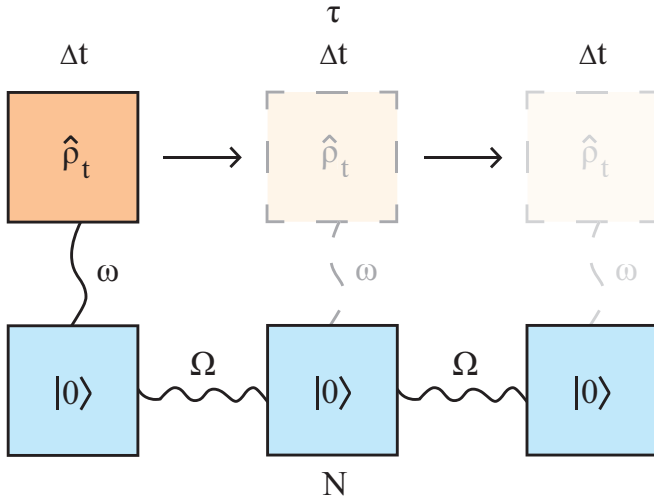
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FIG. 1. Sketch of simple quantum computer: A computational qubit $\mathcal{Q}$ interacts with the kth qubit of a quantum memory $\mathcal{M}$ consisting of N qubits for an interval of length Δt. The strength of interaction between $\mathcal{Q}$ and $\mathcal{M}$ is denoted by ω, and the strength of interaction between the N qubits in $\mathcal{M}$ is given by Ω.

This qubit $\mathcal{Q}$ interacts with a stream of N qubits, which in the limit of $N \to \infty$ constitutes a simple example of a quantum information reservoir [14,15]. For the present purposes and for finite N we call these qubits the "quantum hard disk" or "quantum memory" $\mathcal{M}$, which initially is assumed to be blank. More formally, any kth qubit of $\mathcal{M}$ is initially prepared in $\rho_k^{\mathcal{M}} = |0\rangle\langle 0|_k$.

For the sake of simplicity and in complete analogy to minimal models of quantum Maxwell demons, we further assume that in every instant $\mathcal{Q}$ interacts only with a single qubit of $\mathcal{M}$ described by the unitary evolution $U(t) = \exp(-i H_k^{\mathcal{Q}} t)$. After a time interval Δt the interaction between $\mathcal{Q}$ and the kth qubit of $\mathcal{M}$ is severed, and $\mathcal{Q}$ is put in interaction with the $(k + 1)$st qubit.

The interaction between $\mathcal{Q}$ and $\mathcal{M}$ is described by the Hamiltonian

$$H_k^{\mathcal{Q}} = i\omega(|0\rangle\langle 1|_{\mathcal{Q}} \otimes |1\rangle\langle 0|_k - |1\rangle\langle 0|_{\mathcal{Q}} \otimes |0\rangle\langle 1|_k), \quad (2)$$

which is a simple SWAP operation. A sketch of the system is depicted in Fig. 1.

Such an interaction (2) is only capable of the simplest computation, namely, partial qubit flip. After the first interval of length Δt, we have

$$\rho_t = \text{tr}_1\{U(\Delta t)\,\rho_0 \otimes \rho_1^{\mathcal{M}}\,U^\dagger(\Delta t)\}$$
$$= [1 - \nu(\cos \Delta t)^2]|0\rangle\langle 0| + \nu(\cos \Delta t)^2\,|1\rangle\langle 1|. \quad (3)$$

Thus, we have after N intervals, i.e., for times t such that $N\Delta t \leqslant t \leqslant (N+1)\Delta t$,

$$\rho_t = [1 - \nu_N[\cos(t - N\Delta t)]^2]\,|0\rangle\langle 0|$$
$$+ \nu_N[\cos(t - N\Delta t)]^2\,|1\rangle\langle 1|, \quad (4)$$

where we have introduced the *purity parameter* $\nu_N = \nu(\cos \Delta t)^{2N}$. From Eq. (4) it then becomes clear that the time to complete a full qubit flip is governed by the length of the interaction interval Δt.
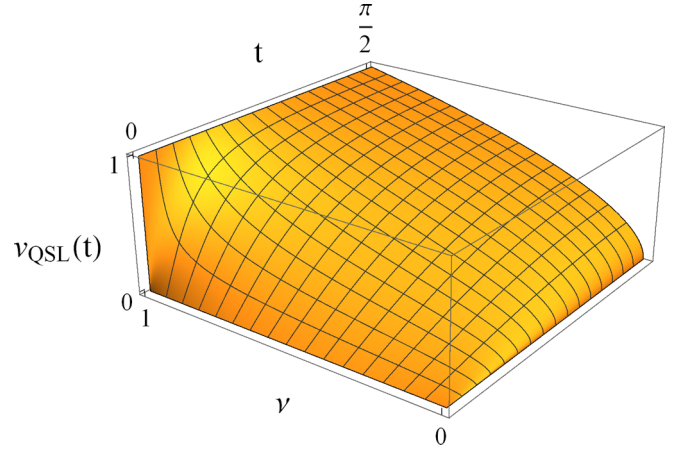

FIG. 2. Quantum speed limit [Eq. (6)] for the first interaction interval, that is, for $t \leqslant \Delta t$. Observe the discontinuity at $(t, \nu) = (0, 1)$.

Quantum speed limit for qubit flips. Quantum dynamics can generally be characterized by the maximal rate of evolution, the quantum speed limit. This maximal rate is given by [7,16]

$$v_{\text{QSL}}(t) = \tfrac{1}{2}\sqrt{F_Q(t)}, \quad (5)$$

where $F_Q(t)$ is the quantum Fisher information.

Interestingly, $F_Q(t)$ is one of the best-studied quantities in quantum physics with a wide variety of applications. For instance, the quantum Fisher information sets bounds on the optimal estimation of parameters encoded in a quantum state [17–19], it helps to describe criticality and quantum phase transitions [20–26], it quantifies coherence and entanglement [27–29], it provides bounds on irreversibility in open quantum systems [30], and it also determines the best precision in thermometry [31].

Using well-known formulas from the literature [18], the quantum speed limit in Eq. (5) can be computed explicitly for the time-dependent state of $\mathcal{Q}$ [Eq. (4)]

$$v_{\text{QSL}}(t) = \frac{\sqrt{\nu}\,\left|(\cos \Delta t)^N \sin(t - N\Delta t)\right|}{\sqrt{1 - \nu(\cos \Delta t)^{2N}[\cos(t - N\Delta t)]^2}}. \quad (6)$$

The expression in Eq. (6) is plotted in Fig. 2 as a function of ν and t. Interestingly, $v_{\text{QSL}}(t)$ of Eq. (5) is discontinuous as a function of two variables at point $(t, \nu) = (0, 1)$. This discontinuity of the quantum Fisher information has been discussed in the literature [32]; in the present analysis we relate this property with the rate of information processing.

More formally, it can be shown that the quantum Fisher information is always discontinuous at points where the rank of the density matrix changes, i.e., when subject to a purity-changing channel dependent on the parameter t. Since for $(t, \nu) = (0, 1)$ the initial density matrix is pure, but for either $t > 0$ or $\nu < 1$ it is not, our situation is exactly the case where this property applies.

Quantum Zeno effect from uncorrelated memories. The mathematical observation that $v_{\text{QSL}}(t)$ [Eq. (5)] exhibits discontinuous behavior has profound, physical implications: for ν close to 1 and comparably small times t, the quantum speed limit $v_{\text{QSL}}(t)$ is very close to zero [33]. Therefore, frequent

switching between different qubits in $\mathcal{M}$ maintains a low rate of evolution of the reduced state of $\mathcal{Q}$. This switching corresponds to substituting the purity parameter ν_{k-1} for ν_k at each iteration, and as long as $t_k = t - k\Delta t$ is very small comparable to ν_k, $\nu_{\text{QSL}}(t)$ remains close to zero.

More conceptually, the discontinuous behavior of $\nu_{\text{QSL}}(t)$ [Eq. (5)] is a signature of the quantum Zeno effect [34]. In the quantum Zeno effect scenario, a state evolves with a finite (constant) speed when not measured, but stops evolving when frequently measured. The discontinuity of the quantum speed limit as shown in Fig. 2 exactly mimics this behavior. The interaction of $\mathcal{Q}$ with $\mathcal{M}$ can be understood as $\mathcal{M}$ taking N measurements on $\mathcal{Q}$. When $\mathcal{Q}$ is left alone, the quantum speed limit quickly rises to a finite and (approximately) constant value, but when $\mathcal{Q}$ is frequently measured by its interaction with $\mathcal{M}$, the quantum speed limit stays close to zero, indicating inhibition of evolution: $\mathcal{Q}$ "freezes out." This can be made even more obvious by introducing the *quantum speed limit time*, which is $\nu_{\text{QSL}}(t)$ averaged over an external time τ,

$$\tau_{\text{QSL}} = \frac{\tau}{\int_0^\tau dt \, \nu_{\text{QSL}}(t)}. \tag{7}$$

In general, τ_{QSL} depends on τ, which means that τ_{QSL} does not have a clear operational interpretation. Note, however, that for pure states evolving under time-independent Hamiltonians, τ_{QSL} becomes identical to the minimal time a quantum system needs to evolve between distinguishable states [7].

In the present case, where $\tau = N\Delta t$, we have explicitly

$$\tau_{\text{QSL}} = \frac{\tau}{\arcsin(\sqrt{\nu}) - \arcsin\{\sqrt{\nu}\,[\cos{(\tau/N)}]^N\}}. \tag{8}$$

We observe that no matter how we choose the total time of interaction τ, as long as it is fixed, τ_{QSL} diverges as N goes to infinity. This means that the characteristic time it takes for the initial quantum state to evolve to an orthogonal state goes to infinity. In other words, frequent interaction stops the density matrix from evolving. We could have immediately reached this conclusion from the reduced density matrix (3). The advantage of Eq. (8) is, however, that it gives a precise quantification of this effect.

From a practical point of view this means that in the limit of frequent interaction between $\mathcal{Q}$ and the quantum hard disk $\mathcal{M}$ successful computation is infeasible. In the remainder of this analysis we are going to show that if the qubits in $\mathcal{M}$ are allowed to interact and build correlations this failure of computation can be prevented.

Correlated quantum hard disks. It has been shown that a positive feedback loop caused by interaction with environments can lead to the so-called anti-Zeno effect [35–38], which means that quantum evolution can be sped up. Similarly, it has been experimentally demonstrated that quantum dynamics can experience an environment-assisted speedup in the presence of correlations [13]. Therefore, we will now show that a similar effect can be achieved for our present model by a judicious design of $\mathcal{M}$.

The idea behind a possible speedup of computation is that the leakage of the excited state from $\mathcal{Q}$ to $\mathcal{M}$ is faster when $\mathcal{Q}$ interacts with an excited state instead of a ground state. In other words, if we would allow for the excited state to be exchanged between different qubits of $\mathcal{M}$, then when $\mathcal{Q}$ starts interacting
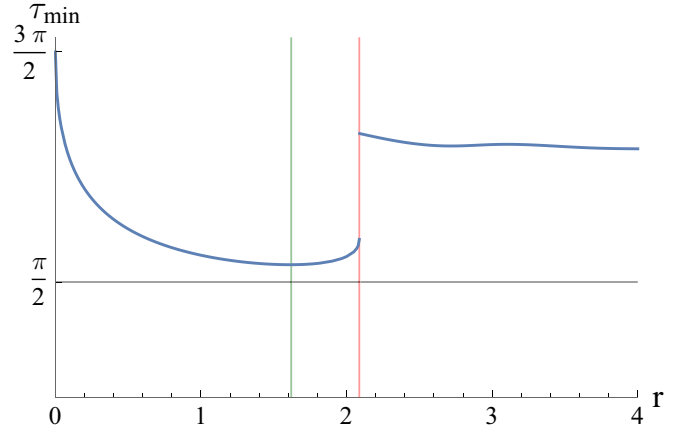

FIG. 3. Minimal time τ_{min} to complete a qubit flip in $\mathcal{Q}$ as a function of $r = \Omega/\omega$. The first vertical line (green) marks the optimal ratio $r_{\text{opt}} \approx 1.620000$ that leads to the fastest computation $\tau_{\text{min}}^{(\text{opt})} \approx 1.807022$. The second vertical line (red) denotes a critical point $r_{\text{crit}} \approx 2.087532$, where τ_{min} discontinuously increases. Further, for $r \to \infty$ we see that τ_{min} asymptotically approaches $\tau_{\text{min}}^{(r \to \infty)} \approx 3.332162$. The horizontal line is a reference line that represents the ideal case of a minimal τ_{min} for $N = 1$.

with the next qubit in $\mathcal{M}$, the $(k + 1)$st qubit in $\mathcal{M}$ has already a head start.

Motivated by this observation and for the sake of simplicity we thus *design* the interaction between the N qubits in $\mathcal{M}$ by pairwise SWAP operations,

$$H_{k,k+1}^{\mathcal{M}} = i\Omega(|0\rangle\langle 1|_k \otimes |1\rangle\langle 0|_{k+1} - |1\rangle\langle 0|_k \otimes |0\rangle\langle 1|_{k+1}), \tag{9}$$

where the strength of interaction is denoted by Ω. Consequently, the total Hamiltonian of the "universe" at time t, $k\Delta t \leqslant t \leqslant (k + 1)\Delta t$ consisting of $\mathcal{Q}$ and N qubits in the quantum memory $\mathcal{M}$, can be written as

$$H^{\mathcal{Q} \otimes \mathcal{M}} = H_{k+1}^{\mathcal{Q}} + \sum_{l=1}^{N-1} H_{l,l+1}^{\mathcal{M}}, \tag{10}$$

which is illustrated in Fig. 1. Although still conceptually simple, the Hamiltonian $H^{\mathcal{Q} \otimes \mathcal{M}}$ is complicated enough that its dynamics cannot be solved analytically for general N in a closed form. Therefore, we continue with a discussion of numerical findings for $N = 3$.

The first interesting question to ask is whether the minimal time, $\tau_{\text{min}} = 3\Delta t$, necessary to perform a qubit flip can be minimized as a function of the relative interaction strength $r = \Omega/\omega$. The result of this optimization problem is summarized in Fig. 3. In this and all the following examples, we took $\nu = 1$.

We observe that τ_{min} and thus the optimal lengths of interaction Δt strongly depend on $r = \Omega/\omega$. Remarkably, there is a unique minimum that marks the optimal ratio r_{opt} at which the qubit flip is performed the fastest. Quite remarkably, the minimal value τ_{min} can even become close to the ideal noninteracting case. In this limit, $\mathcal{Q}$ interacts with only a single qubit for the entire time of interacting, and hence the time for a qubit flip is determined by the "eigentime," i.e., the time for the unperturbed system to complete one full oscillation
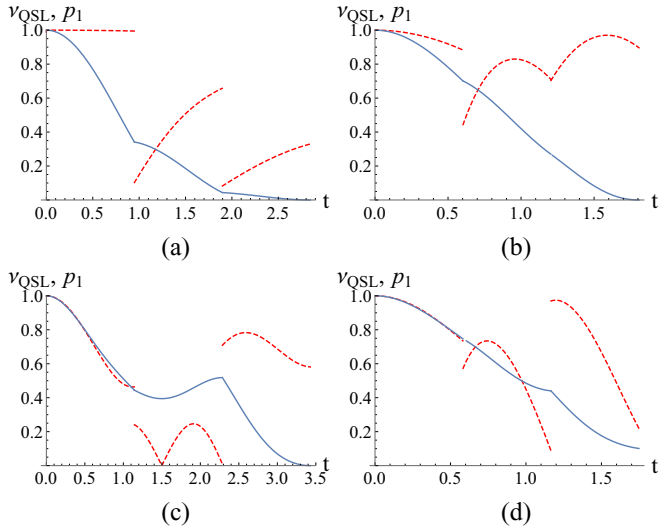
FIG. 4. Quantum speed limit v_{QSL} of $\mathcal{Q}$ [Eq. (6)] (red, dashed line), and fidelity $p_1(r, t) = \langle 1|\rho_t|1\rangle$ [Eq. (4)] (blue, solid line) for various ratios of interaction strengths $r = \Omega/\omega$ and total times of interaction τ. (a) Small interaction ratio ($r = 0.2$, $\tau = \tau_{\min}$). (b) Optimal interaction ratio ($r = r_{opt} \approx 1.62$, $\tau = \tau_{\min}$). (c) Supercritical interaction ratio ($r = 2.7 > r_{crit}$, $\tau = \tau_{\min}$). (d) Supercritical interaction ratio $r = 2.7$, τ given by the first local minimum of $p_1(\tau)$. In (a)–(c), the system qubit is successfully erased at the end of the cycle. However, since the ratio r in (c) is above the critical value, the time it takes to erase is much longer than in (a) or (b). In (d), the fidelity achieves a local minimum at the end of the cycle, but the system qubit fails to be fully erased.

FIG. 5. Fidelity $p_1(r, \tau) = \langle 1|\rho_\tau|1\rangle$ as a function of τ and r. The constant function $f = 0.001$ (blue) illustrates the minimums of the fidelity. The first (green) and the second (red) vertical line denote the optimal ratio r_{opt} and the critical ratio r_{crit} respectively. The horizontal line is a reference line that represents the ideal case of a minimal $\tau_{\min}$ for $N = 1$.

in Hilbert space. Thus, for r_{opt} the quantum Zeno effect is effectively prevented, even though $\mathcal{Q}$ is frequently interrogated by $\mathcal{M}$. A second observation is that at a critical value r_{crit} the minimal time $\tau_{\min}$ jumps to a value which is only slightly better than the worst value given by the case when no interaction between the qubits in $\mathcal{M}$ is present. In this limit, the rate of interaction between the N qubits in $\mathcal{M}$ is faster than the exchange of information of the memory with $\mathcal{Q}$. Thus, $\mathcal{Q}$ effectively interacts with the whole memory $\mathcal{M}$ and no longer qubit by qubit.

Finally, we also computed the quantum speed limit $v_{QSL}(t)$ and the fidelity between instantaneous state ρ_t and target state $|1\rangle$, $p_1(t) = \langle 1|\rho_t|1\rangle$. Results are collected in Fig. 4 for four different values of r. The first graph shows the case of small ratio of interaction $r \ll r_{opt}$, the second the optimal interaction ratio $r = r_{opt}$. There, the total time of interaction was chosen as the minimal time of erasure $\tau_{\min}$. Thus, the $\mathcal{Q}$ is flipped when the interaction stops, $\rho_{\tau_{\min}} = |0\rangle\langle 0|$. The third example shows the case when the ratio is larger than the critical ratio, $r > r_{crit}$. As a result, $\tau = \tau_{\min}$ is much longer. In the fourth example, we picked the same supercritical ratio, but we chose τ such that $p_1(\tau)$ achieves its first local minimum at τ ($\tau_{\min}$ corresponds to the second local minimum). In this graph, $\mathcal{Q}$ gets quite close to $|0\rangle$, but fails to be fully erased at the end of the cycle.

To gain further insight into the dynamics of $\mathcal{Q}$ at $t = \tau$, we also plotted the dependence of the fidelity $p_1(\tau) = \langle 1|\rho_\tau|1\rangle$ as a function of total time of interaction τ and ratio of interaction
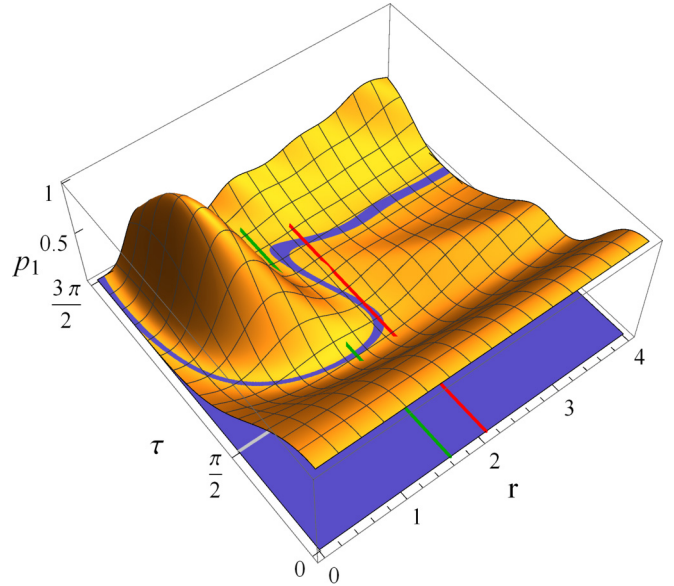
strengths r in Fig. 5. This fairly complicated function shows that only carefully chosen combinations of τ and r lead to a successful prevention of the quantum Zeno effect, which means here successfully performing a qubit flip.

Concluding remarks. The present analysis revealed that frequent interactions between a computational device $\mathcal{Q}$ and a quantum hard disk $\mathcal{M}$ can lead to a failure of computation. In particular, we saw that this "failure" is a consequence of the quantum Zeno effect, which is governed by a discontinuity of the quantum speed limit, or more generally by a discontinuity of the quantum Fisher information. Since it was shown that there is a wide class of generic systems for which the Fisher information exhibits discontinuous behavior, our findings are exemplary for what one would expect in more general settings.

In a second part of the analysis we showed that correlations and interaction between the elements of $\mathcal{M}$ can prevent the failure of computation, but also that there is a trade-off between the interaction strength between the qubits in $\mathcal{M}$ and the rate with which a computation can be successfully performed. Therefore, we would expect our findings to have profound implication in the design of quantum computers: accessing quantum information frequently by reading into a quantum memory can effectively stall the computation and lead to errors due to time mismatch. However, carefully designed side interaction could help to sustain the computation.

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