Experimental test of a stronger multiobservable uncertainty relation

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The Heisenberg-Robertson uncertainty relation is the hallmark of quantum physics and has been widely investigated. However, it does not capture the concept of incompatible observables because it can be trivial even for incompatible observables. Recently some stronger uncertainty relations relating the sums of variances were proposed. Here we experimentally demonstrate that these stronger multiobservable uncertainty relations are valid in a state-dependent manner and that the lower bound is guaranteed to be nontrivial for multiple observables that are incompatible on the state of the system being measured. We find that the behavior of multiple high-dimensional observables agrees with the predictions of quantum theory. Our experimental results not only foster insight into a fundamental limitation of measurements with multiple observables but also contribute to the study of the precision measurement technology in quantum information processing.

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I. INTRODUCTION

The Heisenberg-Robertson uncertainty relation [1–3] is the hallmarks of quantum physics and has been widely investigated since its original formulation, as it quantitatively expresses the impossibility of jointly sharp preparation of incompatible observables. The uncertainty relation plays an important role in quantum technologies including quantum cryptography [4,5], quantum entanglement [6–8], and general physics [9,10]. The uncertainty relation has been tested experimentally in many physics systems, such as neutronic [11–13] and photonic qubits [14–20].

If the measurement on a given particle is chosen from a set of two possible observables A and B, the resulting bound on the uncertainty can be expressed in terms of the commutator,

$$\Delta A^2 \Delta B^2 \geqslant \left|\tfrac{1}{2}\langle[A,B]\rangle\right|^2, \tag{1}$$

which is the so-called Heisenberg-Robertson uncertainty relation. However, it does not capture the concept of incompatible observables because it can be trivial even for incompatible observables. When either of two variances is zero the uncertainty relation becomes trivial even if the other variance is nonzero.

To overcome this limitation of the Heisenberg-Robertson uncertainty relation, Maccone and Pati proposed two uncertainty relations to overcome the flaw in the Heisenberg-Robertson relation [21], employing the sum of variances of measurements of general observables instead of product of them. The first inequality is

$$\Delta A^2 + \Delta B^2 \geqslant \pm i\langle[A,B]\rangle + |\langle\psi|A \pm iB|\psi^{\perp}\rangle|^2, \tag{2}$$

where the signs $\pm$ should be chosen so that $\pm i\langle[A,B]\rangle$ is positive, $|\psi\rangle$ is an arbitrary state on which A and B are incompatible, and $|\psi^{\perp}\rangle$ is orthogonal to $|\psi\rangle$. It becomes an equality by maximizing over $|\psi^{\perp}\rangle$. The second inequality is

$$\Delta A^2 + \Delta B^2 \geqslant \tfrac{1}{2}|_{A+B}\langle\psi^{\perp}|A+B|\psi\rangle|^2, \tag{3}$$

where $|\psi^{\perp}\rangle_{A+B} \propto (A+B-\langle A+B\rangle)|\psi\rangle$ is orthogonal to $|\psi\rangle$. It becomes an equality if the state $|\psi\rangle$ is an eigenstate of $A-B$. Both of these uncertainty relations, based on the sum of variances of two observables, have been tested experimentally [19].

The uncertainty relation based on the sum of variances of measurements of general observables can be extended to a multiobservable uncertainty relation [22]. For the case of N incompatible observables A_i $(i=1,\dots,N)$, there are two inequalities:

$$\sum_{i=1}^{N}(\Delta A_i)^2 \geqslant \frac{1}{2(N-1)}\sum_{1\leqslant i<j\leqslant N}[\Delta(A_i+A_j)]^2 \tag{4}$$

and

$$\sum_{i=1}^{N}(\Delta A_i)^2 \geqslant \frac{1}{2(N-1)}\sum_{1\leqslant i<j\leqslant N}[\Delta(A_i-A_j)]^2. \tag{5}$$

Recently, Song *et al.* [23] improved the multiobservable uncertainty relation and proposed a stronger one with a tighter lower bound:

$$\sum_{i=1}^{N}(\Delta A_i)^2 \geqslant \frac{1}{N}\left[\Delta\left(\sum_{i=1}^{N}A_i\right)\right]^2 + \frac{2}{N^2(N-1)}\left[\sum_{1\leqslant i<j\leqslant N}\Delta(A_i-A_j)\right]^2. \tag{6}$$

The uncertainty relations based on the sum of variances of three two-dimensional observables have been experimentally

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investigated [24,25]. The question has been raised of whether or not these uncertainty relations work for higher-dimensional observables.

Here, we experimentally demonstrate that the stronger uncertainty relations based on the sum of variances of multiple high-dimensional observables are valid in a state-dependent manner, and the lower bound is guaranteed to be nontrivial for multiple high-dimensional observables that are incompatible on the state of the system being measured. We use a qutrit as an example, realized by polarized photons in three different modes, and we demonstrate that these uncertainty relations are valid for states of a spin-1 particle. The behavior we find agrees with the predictions of quantum theory and obeys these uncertainty relations.

These uncertainty relations work well even for special states which trivialize the Heisenberg-Robertson relation. When either of the variances is zero, the Heisenberg-Robertson relation becomes an equality and is trivial even if the other variance is nonzero. This is the so-called complementarity—an extreme form of uncertainty. That is, one of properties of a system is perfectly known, and the others are completely uncertain. This is a situation which the Heisenberg-Robertson inequality fails to explain. However, the behavior of the system obeys the stronger uncertainty relations even in that situation.

Furthermore, in our experiment, every term can be obtained directly by the outcomes of the projective measurements applied on the state being measured. No quantum state tomography is required at all, which makes our experiment more easy to perform.

II. EXPERIMENTAL INVESTIGATIONS

We investigate the stronger uncertainty relations based on the sum of variants of three incompatible observables (6) by choosing three components of the angular momentum for spin-1 particle as three observables:

$$A = J_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad B = J_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & i & 0 \\ -i & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad C = J_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{7}$$

The inequality (6) can be written as

$$\Delta J_x^2 + \Delta J_y^2 + \Delta J_z^2 \geqslant \tfrac{1}{3}[\Delta(J_x + J_y + J_z)]^2 + \tfrac{1}{9}[\Delta(J_x - J_y) + \Delta(J_y - J_z) + \Delta(J_x - J_z)]^2. \tag{8}$$

It can be rewritten as

$$\begin{aligned}
&\Delta J_x^2 + \Delta J_y^2 + \Delta J_z^2 \\
&\quad \geqslant \frac{1}{3}\left[\langle J_x^2\rangle + \langle J_y^2\rangle + \langle J_z^2\rangle + \langle J'\rangle - (\langle J_x\rangle + \langle J_y\rangle + \langle J_z\rangle)^2\right] \\
&\qquad + \frac{1}{9}\left[\sqrt{\langle J_x^2\rangle + \langle J_y^2\rangle - (\langle J_x\rangle - \langle J_y\rangle)^2}\right. \\
&\qquad + \sqrt{\langle J_x^2\rangle + \langle J_z^2\rangle - \langle J'\rangle - (\langle J_x\rangle - \langle J_z\rangle)^2} \\
&\qquad \left. + \sqrt{\langle J_y^2\rangle + \langle J_z^2\rangle - (\langle J_z\rangle - \langle J_y\rangle)^2}\right]^2.
\end{aligned} \tag{9}$$

Here, we define an observable J' as

$$J' = J_x J_z + J_z J_x. \tag{10}$$

Thus to test the uncertainty relation (8), we need the expected values of observables J_x, J_y, J_z, and J'.

For experimental demonstration, a qutrit is realized by single photons in three modes and the basis states $|0\rangle = (1,0,0)^{\mathrm{T}}$, $|1\rangle = (0,1,0)^{\mathrm{T}}$, and $|2\rangle = (0,0,1)^{\mathrm{T}}$ are encoded by the horizontally polarized photons in the upper mode, the vertically polarized photons in the upper mode, and the vertically polarized photons in the lower mode, respectively. Pairs of photons are generated via type-I spontaneous parametric down-conversion (SPDC). With the detection of a trigger photon, the other photon in one pair is heralded in the experimental setup shown in Fig. 1. The heralded single photons pass through a polarizing beam splitter (PBS) followed by a half-wave plate (HWP, H0) with the specific setting angle ϑ. Then a birefringent calcite beam displacer (BD1) splits them into two parallel spatial modes, i.e., upper and lower modes. The optical axis of the BD is cut so that vertically polarized photons are directly transmitted and horizontal photons undergo a lateral displacement into a neighboring mode. After passing through two HWPs (H1 at $\pi/8$ and H2 at $\pi/2$), the photons are prepared in the state

$$|\psi_\theta\rangle = \left(\frac{\sin\theta}{\sqrt{2}}, \frac{\sin\theta}{\sqrt{2}}, \cos\theta\right)^{\mathrm{T}}, \tag{11}$$

where $\theta = \pi/2 - 2\vartheta$. The matrix form of the operation of the HWP with the setting angle ϑ is

$$\begin{pmatrix} \cos 2\vartheta & \sin 2\vartheta \\ \sin 2\vartheta & -\cos 2\vartheta \end{pmatrix}.$$

The matrix form of the operation of the QWP is

$$\begin{pmatrix} \cos^2\vartheta + i\sin^2\vartheta & (1-i)\sin\vartheta\cos\vartheta \\ (1-i)\sin\vartheta\cos\vartheta & \sin^2\vartheta + i\cos^2\vartheta \end{pmatrix}.$$

We choose $\theta = j\pi/10$ $(j = 0,\ldots,10)$, a total of eleven states for testing the uncertainty relation (8).

To test the uncertainty relation (8), we need the expected values of observables J_x, J_y, J_z, and J'. An observable can be written as $M = \sum_i m_i |m_i\rangle\langle m_i|$, where $|m_i\rangle$ is the eigenstate of the observable M and m_i is the corresponding eigenvalue.

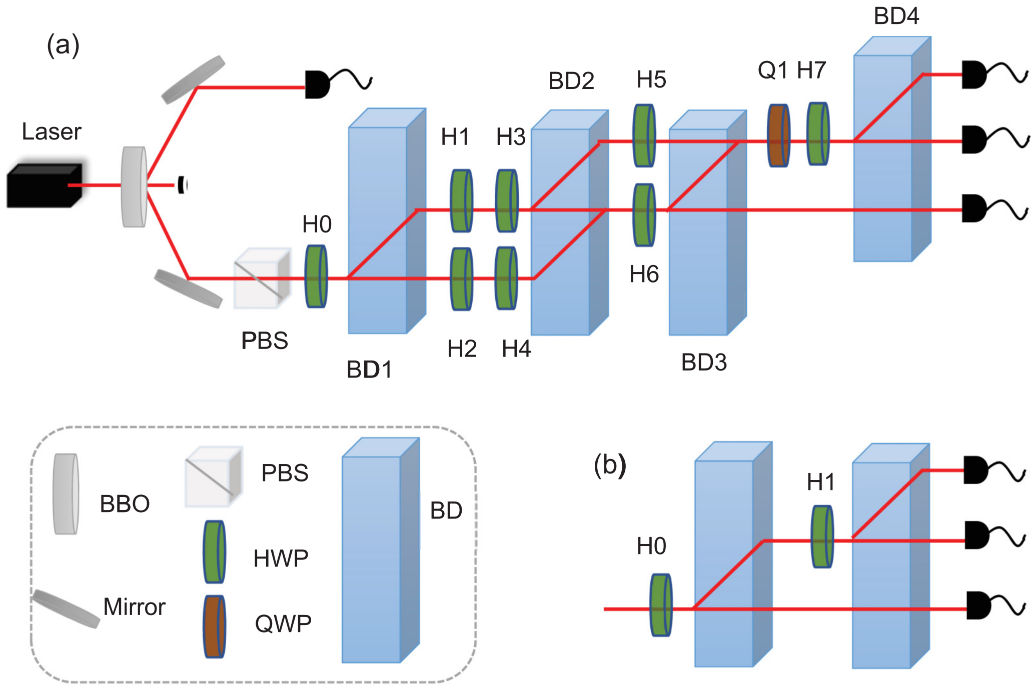

FIG. 1. Experimental setup for testing the uncertainty relations based on the sum of variants of three incompatible observables J_x, J_y, and J_z. Photon pairs are generated via type-I SPDC using a 0.5-mm-thick β-barium-borate (BBO) crystal, pumped by a continuous wave diode laser with 80 mW of power. The pump is filtered out by an interference filter which restricts the photon bandwidth to 3 nm. With the detection of the trigger, the heralded single photon is injected into the optical network. The polarizing beam splitter (PBS), half-wave plates (HWPs; H0, H1, and H2) and beam displacer (BD1) are used to generate the qutrit state $|\psi_\theta\rangle$ being measured. The rest HWPs, quarter-wave plates (QWPs) and BDs are used to realize the projective measurements of the observables J_x, J_y, and J' in (a) and J_z in (b), which are applied on the state $|\psi_\theta\rangle$.

The expected value of the observable M is

$$\langle M\rangle = \langle\psi_\theta|M|\psi_\theta\rangle = \sum_i m_i\langle\psi_\theta|m_i\rangle\langle m_i|\psi_\theta\rangle = \sum_i m_i|\langle\psi_\theta|m_i\rangle|^2. \quad (12)$$

Then we can define a unitary operator $U=\sum_i |i\rangle\langle m_i|$. We apply the unitary operator U on the initial state $|\psi_\theta\rangle$ and then project the final state into the basis states $|i\rangle$ $(i=0,1,2)$. The value $|\langle\psi_\theta|m_i\rangle|^2$ equals the probability of the photons being measured in the state $|i\rangle$,

$$|\langle\psi_\theta|m_i\rangle|^2 = \mathrm{Tr}(|\psi_\theta\rangle\langle\psi_\theta|U^\dagger|i\rangle\langle i|U). \quad (13)$$

Similarly, one can calculate the variance $\Delta M = \langle M^2\rangle - \langle M\rangle^2$ of the observable M with the outcome of the projective measurement on the state $|\psi_\theta\rangle$. Thus to obtain the expected values of the observables J_x, J_y, J_z, and J', we only need to realize four unitary operators applied on the state $|\psi_\theta\rangle$ being measured and apply the projective measurement $|i\rangle\langle i|$ $(i=0,1,2)$ on the final state $U|\psi_\theta\rangle$.

The unitary operator belong to $SU(3)$ can be decomposed into three unitary operators, each of which applies a rotation on just two of the basis states, leaving the other unchanged. Each of them can be realized by wave plates and beam displacers. One of the HWPs is used to apply a rotation on two modes of the qutrit state and the others are use to compensate for the optical delay. The BDs are used to split the photons with different polarizations into different spatial modes and then combine the photons with two specific polarization modes in the same spatial mode. Then two-mode transformations can be implemented via wave plates acting on the two polarization modes propagating in the same spatial mode.

We use J' as an example. The corresponding unitary operator is

$$U = \begin{pmatrix} -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (14)$$

which can be decomposed as

$$U = U_3U_2U_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (15)$$

A similar unitary transformation was realized in our previous experiment in [19], and our setup can actually realize an arbitrary unitary transformation on a qutrit. In this experiment, a HWP (H3) is used to realize a two-mode transformation on the basis states $|0\rangle$ and $|1\rangle$, and then a BD (BD2) is used to split the photons with different polarizations into different spatial modes and then combine the photons in states $|1\rangle$ and $|2\rangle$ in the same spatial mode. We use a HWP (H6) to realize another two-mode transformation on the basis states $|1\rangle$ and $|2\rangle$ and the following BD3 splits and recombines the photons. After a HWP (H7) is applied on the basis states $|0\rangle$

TABLE I. The setting angles of wave plates for the projective measurements of the observables J_x, J_y, J_z, and J'. Here "-" denotes that the corresponding wave plate is not used for the certain measurement and has been removed from the optical circuit.

Observable	H3	H4	H5	H6	H7	Q1
J_x	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$-\frac{\pi}{4}$	$\frac{\pi}{8}$	$\frac{\pi}{8}$	-
J_y	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{8}$	$\frac{\pi}{8}$	0
J'	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$-\frac{\pi}{4}$	$\frac{\pi}{8}$	$\frac{\pi}{8}$	-
J_z	-	-	-	-	-	-

and $|1\rangle$, the unitary operation has been accomplished. The last BD4 is used to project the photons into the basis states $|i\rangle$ ($i = 0, 1, 2$). The probability of the photons being measured in $|i\rangle$ is obtained by normalizing photon counts in the specific spatial mode to total photon counts. Angles of the wave plates are shown in Table I.

In Fig. 2, we show the experimental demonstration of a three-observable uncertainty relation (8). Our experimental results agree with the theoretical predictions well. For comparison, we also show the experimental results of the uncertainty relations (4) and (5). We find that the three-observable uncertainty relation (8) is much stronger compared to the other uncertainty relations (4) and (5). The right-hand side of the inequality (8) is much closer to the left-hand side of the inequalities $\Delta J_x^2 + \Delta J_y^2 + \Delta J_z^2$.

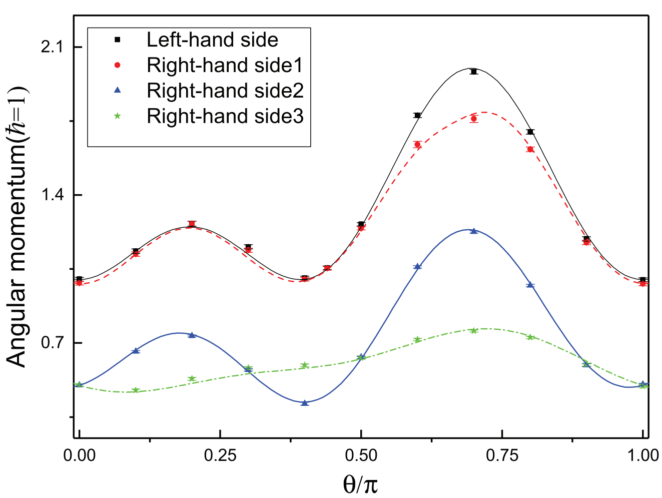

FIG. 2. Experimental results of the uncertainty relations based on the sum of variants of three incompatible observables J_x, J_y, and J_z. The solid black curve represents theoretical predictions of the left-hand side of the inequality (8), i.e., $\Delta J_x^2 + \Delta J_y^2 + \Delta J_z^2$ with eleven states $|\psi_\theta\rangle$. Black squares represent the experimental results of the left-hand side of the inequality (8), i.e., the sum of the measured uncertainties ΔJ_x^2, ΔJ_y^2, and ΔJ_z^2. The red dashed curve represents theoretical predictions of the right-hand side of the inequality (8). Red dots indicate the experimental results of the right-hand side of the inequality (8) with eleven states $|\psi_\theta\rangle$. Blue triangles and green stars indicate the experimental result of the right-hand side of the inequalities (4) and (5), respectively. The corresponding curves represent the theoretical predictions. Error bars indicate the statical uncertainty which is obtained based on assuming Poissonian statistics.

For some states, the inequality (8) becomes an equality, which means the uncertainty inequality (8) is tight. However, as the uncertainty relation (8) is state-dependent, with the choice of the state $|\psi_\theta\rangle$ being measured in our experiment, for the angle parameter of the state $\theta = 0.44\pi$, the left-hand and right-hand sides of (8) are 1.0494 and 1.0465, respectively. Thus the inequality is almost saturated with this specific type of states $|\psi_\theta\rangle$. In Fig. 2, we show the experimental results of the left-hand and right-hand sides of (8) with the angle parameter of the state $\theta = 0.44\pi$; they are 1.0557 ± 0.0084 and 1.0534 ± 0.0083, respectively. Error bars indicate the statical uncertainty, which is obtained based on assuming Poissonian statistics. Total coincidence counts are about 10 000 over a collection time of 5 s.

III. CONCLUSION

A correct understanding and experimental confirmation of a fundamental limitation of measurements will not only foster insight into foundational problems but also advance precision measurement technology in quantum information processing. In this work, we experimentally demonstrate that the stronger multiple high-dimensional observable uncertainty relations are valid in a state-dependent manner and the lower bound is guaranteed to be nontrivial for multiple high-dimensional observables that are incompatible on the state of the system being measured. With the measurement of three observables as an example, we find that the behavior of multiple high-dimensional observables agrees with the predictions of quantum theory. All experimental results agree well with theoretical predictions. Our experimental results provide a correct understanding and confirmation of a fundamental limitation of measurements with multiple high-dimensional observables.

Furthermore, Our achievement relies on a stable interferometric network with simple linear optical elements. In our setup, a high-dimensional system can always be realized by polarized single photons in different spatial modes. With wave plates, a qutrit or qudit can be prepared in an arbitrary state, which is important for testing state-dependent uncertainty relations. To realize an arbitrary unitary transformation on a qutrit or qudit, we can decompose it into several unitary operators, each of which applied a rotation on just two modes, leaving the other modes unchanged. Each of them can be realized by wave plates and BDs. Thus we can use simple optical elements to realize arbitrary state preparation and unitary transformation and then demonstrate the interesting phenomena. Our demonstration is simple and programmable, and serves as an ideal platform for demonstrating uncertainty relations and the other protocols of quantum information science.

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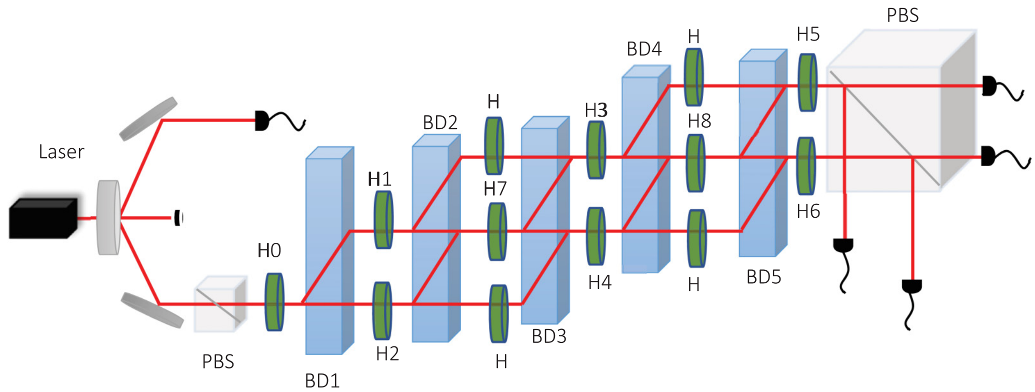

FIG. 3. Experimental setup for testing the uncertainty relations based on the sum of variants of multiple four-dimensional observables. The polarizing beam splitter (PBS), half-wave plates (HWP, H0), and beam displacer (BD1) are used to generate a family of qudit states $|\psi_\theta^d\rangle$ being measured. The rest of the HWPs, BDs, and PBS are used to realize the projective measurements of the four-dimensional observable J_x, as an example, which is applied on the qudit state.

APPENDIX: PROPOSAL OF TESTING MULTIPLE FOUR-DIMENSIONAL OBSERVABLES UNCERTAINTY RELATION

We show our setup can encode information of qudits in different freedoms of single photons including polarizations and spatial modes and realize measurements of four-dimensional observables.

The basis states of qudits $|0\rangle$, $|1\rangle$, $|2\rangle$ and $|3\rangle$ are encoded by the horizontally polarized photons in the upper mode, the vertically polarized photons in the upper mode, the horizontally polarized photons in the lower mode and the vertically polarized photons in the lower mode, respectively. We can prepare an arbitrary qudit state with a PBS, several wave plates with certain setting angles and a BD. For example, a family of qudit states $|\psi_\theta^d\rangle = (\cos\theta, 0, 0, \sin\theta)^{\mathrm{T}}$ can be prepared with a PBS, HWP (H0) with certain setting angle and BD1 shown in Fig. 3.

Then we choose the components of the angular momentum for spin-3/2 particle as observables. Here, we use

$$J_x = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

as an example to show how to realize the measurement of J_x via single photons and linear optics. First we calculate the eigenstates $|m_j\rangle$ of J_x ($j = 0, 1, 2, 3$) and 4×4 unitary transformation is

$$U_4 = \sum_j |j\rangle\langle m_j| = \frac{\sqrt{2}}{4}\begin{pmatrix} -1 & \sqrt{3} & -\sqrt{3} & 1 \\ 1 & \sqrt{3} & \sqrt{3} & 1 \\ \sqrt{3} & -1 & -1 & \sqrt{3} \\ -\sqrt{3} & -1 & 1 & \sqrt{3} \end{pmatrix}$$

which can be decomposed as

$$U_4 = \begin{pmatrix} H_5 & 0 \\ 0 & H_6 \end{pmatrix} P \begin{pmatrix} H_3 & 0 \\ 0 & H_4 \end{pmatrix} Q \begin{pmatrix} H_1 & 0 \\ 0 & H_2 \end{pmatrix},$$

where

$$Q(P) = \begin{pmatrix} 0 & \sin 2\theta_{7(8)} & \cos 2\theta_{7(8)} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\cos 2\theta_{7(8)} & \sin 2\theta_{7(8)} & 0 \end{pmatrix}.$$

As

$$H_i = \begin{pmatrix} \cos 2\theta_i & \sin 2\theta_i \\ \sin 2\theta_i & -\cos 2\theta_i \end{pmatrix} \quad (i = 1, \ldots, 6)$$

is the transformation on two dimensions, i.e., the degrees of freedom of polarizations of the photons in either the upper mode or the lower mode, we can realize them via a half-wave plate with the setting angle θ_i. Q (P) can be realized by two beam displacers BD2 and BD3 (BD4 and BD5) and three half-wave plates [two of them with the setting angle 45° and the other with the setting angle θ_7 (θ_8)]. The setting angles of wave plates for the projective measurement of a four-dimensional observable J_x are shown in Table II.

Similar to the realization of measurements of three-dimensional observables, after applying the unitary transformation U_4, we project the qudit state into the four basis states $|0\rangle$, $|1\rangle$, $|2\rangle$, and $|3\rangle$, and the probabilities of projective measurements can be used to calculate the expected values and variances of four-dimensional observables. This method is general and can be used to realize the measurement of arbitrary four-dimensional observables. Thus our setup can be used to test uncertainty relations of multiple higher-dimensional observables.

TABLE II. The setting angles of wave plates for the projective measurement of a four-dimensional observable J_x.

Observable	H1	H2	H3	H4	H5	H6	H7	H8	H
J_x	27.311°	−6.011°	0.781°	−18.133°	122.622°	2.945°	16.896°	24.862°	45°

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