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Deterministic quantum network for distributed entanglement and quantum computation

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We propose a simple interaction protocol to be implemented on a scalable quantum network, in which the quantum nodes consist of qubit systems confined in cavities. The nodes are deterministically coupled by transmission and reflection of a single photon, which is disentangled from the qubits at the end of the coupling operation. This single photon can generate an entangling controlled-PHASE gate between any selected number of qubits in the network. Our multiqubit gate reaches a much higher fidelity compared to schemes concatenating one-qubit and two-qubit gates; thus it forms an efficient basis for universal quantum computing distributed over multiple processor units. In our analysis we consider atomic qubits coupled to optical photons, while the scheme can be readily generalized to other architectures, such as superconducting qubit nodes coupled by microwave photons.

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Introduction. A quantum network consists of quantum nodes that locally process and store quantum information. The quantum information is then shared between the nodes by linking them via quantum channels [1] which mediate the interaction between the nodes [2,3]. The division of tasks between stationary and flying qubits provides a route to extend quantum computers to operation on large numbers of qubits [4], it is at the basis of quantum repeater networks for long-distance and multiuser quantum communication [5], and it has applications in metrology [6]. Several theoretical proposals use single photons to couple separated nodes, either in a deterministic [7] or a probabilistic (heralded) fashion [8,9]. Experimental realizations range from atoms [10–13], trapped ions [4,14–16], nitrogen-vacancy centers in diamond [17], to superconducting qubits [18–20]. So far, experimental studies were restricted to the coupling of pairs of nodes. While pairwise entanglement and gate schemes are theoretically sufficient to perform general operations on a larger network, this requires the concatenation of operations and drastically limits the experimental feasibility. Here, we propose a quantum network scheme, in which a single photon is sufficient to mediate a multinode interaction. Instead of applying a sequence of pairwise operations on quantum nodes consisting of atoms inside optical cavities, our scheme treats the entire network as an interferometer. Subject to the phase shifts incurred under reflection and transmission of a single photon, our scheme generates a controlled-PHASE (CPHASE) gate on all qubits involved. The scheme readily lends itself to application on nodes with several atomic qubits, and it should be equally well suited to superconducting circuit architectures with microwave excitation frequencies.

Basic idea. In our system we consider a single-photon wave packet that enters and leaves a quantum network through a single input-output channel (Fig. 1), such that the photon pulse duration ΔT is much longer than the temporal delay τ_i of the ith optical path inside the quantum network. In the N cavity configuration, the quantum network consists of a single one-sided and $N-1$ two-sided cavities containing atomic qubits [Fig. 2(a)]. We employ interactions, where the cavity mode couples the ground qubit state $|1\rangle$ and excited state $|e\rangle$, and does not couple to the qubit state $|0\rangle$, as shown in Fig. 2(b). Hence, the photon is reflected or transmitted from the cavities with phases that depend on the qubit states. In the ideal case, a qubit occupying a $|1\rangle$ state causes the reflection of the photon with a 0 phase, whereas when a $|0\rangle$ state is occupied, the photon is reflected with a π phase from the one-sided cavity, as demonstrated experimentally in Ref. [21], or transmitted with a π phase from the two-sided one. This permits the analysis of the entire network by concatenating the different components in analogy with the analysis of linear classical interferometers, treating each component of the state of the qubits in the product basis $|q_1, q_2, \ldots, q_N\rangle$, $q_i = 0$ or 1. The phase factor of this concatenation is a property belonging to the joint quantum state of the photon and the qubits. However, if the photon wave packets depending on the different qubit states sufficiently overlap after the photon leaves the system (see Fig. 1), the state factorizes and the qubits are disentangled from the photon at the end of the operation. Therefore, we can associate the qubit-dependent phase factors with a multiqubit operation.

Ideally, in the N cavity configuration [Fig. 2(a)], when all the N qubits occupy the state $|1\rangle$, the photon reflects from all cavities with no phase shift, and the output single-photon wave packet is in phase with the input. Otherwise, the photon is reflected until it encounters the first cavity with a qubit in a $|0\rangle$ state. Here, it is transmitted (or reflected if the first $|0\rangle$ qubit state is encountered in the one-sided cavity) with a π phase shift. The photon is reflected by its subsequent, second encounter with all the previous cavities since their qubits are still in the $|1\rangle$ states. We thus obtain a change of sign of the one-photon wave packet, and the photon mediates the multiqubit CPHASE gate between the qubits, where the state $|1\rangle^{\otimes N}$ acquires a π phase relative to all other qubit product states.

Qubit-light interfaces. To study the physical case, where the reflection and transmission processes incorporate delays,

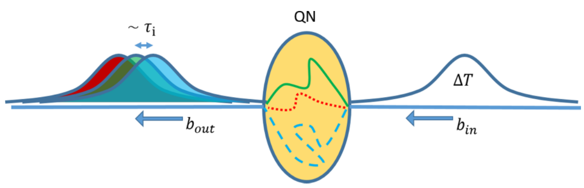

FIG. 1. A single photon with a long-wave packet ΔT interacts with a quantum network (QN) containing qubits. For different qubit states the photon propagates through different paths and accumulates different phases, giving rise to a multiqubit phase gate. Depending on the qubit states, the photon wave packet is distorted and delayed (by τ_i), and close to unitary (deterministic) operation on the qubit register requires a high overlap of the different transmitted wave packets.

decays, and losses, we shall employ the input-output formalism [22,23]. The one-sided cavity [Fig. 2(c)] with a single qubit with states $|q\rangle$ is illuminated by a single photon, occupying a wave-packet mode function, specified by the time-dependent amplitude $b_{\text{in}}(t)$ arriving at the cavity input mirror. Using the input-output formalism, detailed in Ref. [24], we obtain the relation between the input and output wave packets in frequency domain $b_{\text{out}}(\omega) = R_{1q}(\omega) b_{\text{in}}(\omega)$, with the reflection coefficient

$$R_{1,q}(\omega) = \left[1 - \frac{2\kappa(\gamma - i\omega)}{2\left({g_q}^2 - \omega^2\right) + \gamma(\kappa + \kappa' - 2i\omega) - i(\kappa + \kappa')\omega}\right]. \quad (1)$$

Here, ω denotes the detuning from the cavity resonance, g_q is the coupling ($g_0 = 0$, $g_1 = g$) between the qubit and the cavity mode, $\kappa(\kappa')$ is the cavity damping rate by transmission (absorption) losses, and γ is the decay rate of the excited atomic state [25]. Following the convention in Refs. [21,26],

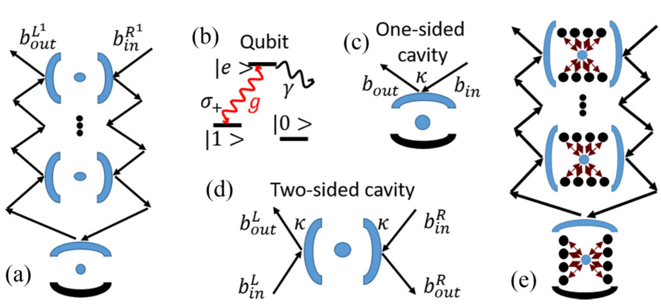

FIG. 2. (a) A network configuration with N qubits (blue circles) located in separated cavities. A CPHASE gate between the qubits is mediated by a photon entering from the upper right in the figure. (b) Qubit lambda system: The excited state $|e\rangle$ with decay rate γ is coupled with strength g to the qubit state $|1\rangle$ by resonant exchange of a photon with the cavity mode, whereas the qubit state $|0\rangle$ is uncoupled. (c) Input-output fields of a one-sided cavity with damping rate κ, containing a single qubit that is coupled to the cavity mode. (d) Two input and two output fields of a symmetric two-sided cavity $\kappa_L = \kappa_R = \kappa$, containing a single qubit. (e) Scaling to more qubits with registers consisting of K qubits inside each cavity, where a single qubit (blue circle) interacts with the cavity mode and is controlled by the other qubits (black circles).

when $|1\rangle$ is populated, the cavity resonance frequency is split by the strong coupling $g_1 = g \gg \gamma, \kappa, \omega$, and the incident photon is reflected at the input mirror with a reflection coefficient $R_{1,1}(\omega) \sim +1$ close to resonance. When the qubit populates the noncoupled state $|0\rangle$ with $g_0 = 0$, the photon enters the resonant cavity and is reflected with $R_{1,0}(\omega) \sim -1$ close to resonance.

Photons far from resonance are reflected with $R_{1,q}(\omega) \sim +1$ irrespective of the qubit state, emphasizing that our scheme will not work with large frequency bandwidth photon pulses. We note that by writing the solution in the frequency domain we do not assume steady-state driving of the cavity at detuning ω. A Fourier transform back to the time domain yields the buildup and decay dynamics of the cavity excitation amplitude [24]. In addition to the desired phase shift, the reflection coefficient $R_{1,0}(\omega)$ causes a delay by $\sim 1/\kappa$ of the wave packet, and reduces its amplitude by $\sim\kappa'/\kappa$ due to cavity absorption loss, while $R_{1,1}(\omega)$ reduces the wave packet's amplitude by $\sim\kappa\gamma/g^2$ due to atomic decay, but causes little or no delay. These effects reduce the fidelity of the qubit gate operations and use of our proposal is restricted to long incident photon wave packets and qubit and cavity systems that operate in the high cooperativity ($C \equiv g^2/\kappa\gamma \gg 1$) and overcoupled ($\kappa \gg \kappa'$) regime, which is indeed the regime explored in experiments [21,26].

The two-sided cavity [Fig. 2(d)] has input and output ports on both sides of the cavity (index L, R). Using the input-output formalism as detailed for the one-photon wave packets in Ref. [24], we obtain the effective beam-splitter relation, which for identical transmission and absorption rates $\kappa = \kappa_L = \kappa_R$, $\kappa' = \kappa'_L = \kappa'_R$ of the left and right mirror reads

$$\begin{pmatrix} b^R_{\text{out}}(\omega) \\ b^L_{\text{out}}(\omega) \end{pmatrix} = \begin{pmatrix} R_{2,q} & T_{2,q} \\ T_{2,q} & R_{2,q} \end{pmatrix} \begin{pmatrix} b^R_{\text{in}}(\omega) \\ b^L_{\text{in}}(\omega) \end{pmatrix}, \quad (2)$$

where the transmission coefficient reads

$$T_{2,q}(\omega) = -\frac{\kappa(\gamma - i\omega)}{\left({g_q}^2 - \omega^2\right) + \gamma(\kappa + \kappa' - i\omega) - i(\kappa + \kappa')\omega} \quad (3)$$

and the reflection coefficient is $R_{2,q}(\omega) = 1 + T_{2,q}(\omega)$. Ideally, the qubit state $|1\rangle$ splits the cavity resonance and prevents entrance of the resonant photon into the cavity, giving rise to $R_{2,1} = 1$, and $T_{2,1} = 0$, while the inert qubit state $|0\rangle$ corresponds to an empty cavity which transmits a resonant classical field (and hence the wave-packet mode function) with a negative amplitude, $T_{2,0} = -1$ and $R_{2,0} = 0$ [27]. Similar as for the one-sided cavity, delay of the reflected or transmitted wave packet by $\sim 1/\kappa$ should be kept much shorter than the pulse duration and damping should be minimized.

Interferometer analysis. The complex frequency-dependent transmission and reflection coefficients permit calculation of the output field of the setup in Fig. 2(a), that takes into account the distortion and damping of the photon wave packet. This is done by either summing the amplitude contributions from the multiple photon paths through the system, or by solving consistently for the field amplitudes by matching the incident and outgoing wave-packet amplitudes to the reflection and transmission coefficients at all optical components. This yields a complex transmission coefficient between the input wave packet at the first input port R^1 and the output wave packet at the last output port L^1, $b_{\rm out}^{L^1}(\omega) = T(\omega, \{g_{q_i}\}_1^N) b_{\rm in}^{R^1}(\omega)$, depending on the N qubit states. For the two-cavity configuration, the transmission coefficient reads

$$T(\omega, g_{q_1}, g_{q_2}) = \frac{2g_{q_1}^2\big[g_{q_2}^2 - (\gamma - i\omega)(2\kappa + i\omega)\big] - (\gamma - i\omega)\big[2g_{q_2}^2(\kappa + i2\omega) + \omega(\gamma - i\omega)(2\omega - i5\kappa)\big]}{2g_{q_1}^2\big[g_{q_2}^2 + (\gamma - i\omega)(2\kappa - i\omega)\big] + (\gamma - i\omega)\big[2g_{q_2}^2(\kappa - i\omega) - \omega(\gamma - i\omega)(2\omega + i5\kappa)\big]}, \tag{4}$$

where we have assumed the same damping parameters κ and γ of both cavities and qubit atoms, and (for simplicity of the expression) $\kappa' = 0$. One can see that if the qubits populate the qubit product state $|11\rangle$, with couplings $g_{q_1} = g_{q_2} = g \gg \gamma, \kappa, \omega$, the global state acquires no phase shift whereas in the other three qubit cases, $|10\rangle$, $|01\rangle$, or $|00\rangle$, either g_{q_1}, g_{q_2} or both vanish, and the global states acquire a π phase change.

In addition to the delay and loss by cavities, the photon wave packets suffer temporal delay and loss associated with the propagation between the cavities. These are incorporated by introducing a frequency-dependent phase factor $e^{i\omega\tau}$ and an amplitude factor $e^{-\eta}$ for each optical path traversed by the photon [24]. The photon follows different paths through the network and for the outgoing pulses to be disentangled from the qubits, the propagation delays should all be kept shorter than the duration of the pulse (cf. Fig. 1). Further below, we shall separately discuss qubit gate errors caused by optical phase fluctuations, e.g., due to mirror positioning errors in the different arms of our interferometric setup.

Fidelity. The fidelity of the gate is calculated by evaluating the overlap between the various output wave packets $T(\omega, \{g_{q_i}\}_1^N)\Phi^{\rm in}(\omega)$, and a single, normalized, reference function $\Phi^{\rm ref}(\omega)$. Note that it is convenient to evaluate these overlaps in frequency domain, and while the input mode function $\Phi^{\rm in}(\omega)$ is normalized to unity, the output field may have reduced norm, due to loss of the photon. We conservatively associate photon loss with a complete gate error, and we hence obtain a lower bound for the multiqubit CPHASE gate fidelity, averaged over all $d = 2^N$ register qubit basis states $|q_1, q_2, \ldots, q_N\rangle$,

$$F_N = \left| \frac{1}{2^N} \sum_{\{q_i\}_1^N} \int d\omega\, e^{i\omega\xi} |\Phi^{\rm in}(\omega)|^2 T\big(\omega, \{g_{q_i}\}_1^N\big) \mathrm{CP}\big(\{q_i\}_1^N\big) \right|^2, \tag{5}$$

where, for simplicity, we assume the desired reference output photon mode function to be on the form $\Phi^{\rm ref}(\omega) = \Phi^{\rm in}(\omega) e^{-i\omega\xi}$, and we optimize the expression with respect to the real variable $\xi \in \Re$. The ideal multiqubit CPHASE unitary operator is described by its action on the qubit product states, $\mathrm{CP}(\{1\}_1^N) = -1$ (all qubits in state $|1\rangle$) or $\mathrm{CP}(\{q_i\}_1^N) = 1$ (otherwise).

We have calculated the fidelity for the two-, three-, and four-cavity cases, as a function of the following physical parameters: the cooperativity parameter $C = g^2/\kappa\gamma$, a Gaussian incident wave packet $|\Phi(\omega)|^2 = \exp(-\omega^2/2\Delta\Omega^2)/\Delta\Omega\sqrt{2\pi}$ with a bandwidth $\Delta\Omega = 2\pi/\Delta T$, identical propagation delays τ between neighboring cavities, identical cavity transmission and absorption loss rates $\kappa_i = \kappa$, $\kappa_i' = \kappa'$ for all $i \in \{1, 2, \ldots, N\}$, and identical photon losses between the cavities η. We provide here the first-order expansion of the fidelity in the small parameters $1/C$, $\Delta\Omega/\kappa$, $\tau\Delta\Omega$, κ'/κ, $\eta \ll 1$,

$$\begin{aligned} 1 - F_2 &\approx 2.5/C + (2.3/\kappa^2 + 1.1\tau/\kappa + 0.95\tau^2)\Delta\Omega^2 \\ &\quad + 1.8\kappa'/\kappa + 3.7\eta, \\ 1 - F_3 &\approx 2.25/C + (1.3/\kappa^2 + 1.5\tau/\kappa + 2.1\tau^2)\Delta\Omega^2 \\ &\quad + 1.6\kappa'/\kappa + 5\eta, \\ 1 - F_4 &\approx 1.6/C + (0.76/\kappa^2 + 1.4\tau/\kappa + 2.7\tau^2)\Delta\Omega^2 \\ &\quad + 1.3\kappa'/\kappa + 6.3\eta. \end{aligned} \tag{6}$$

As expected from our analysis, the optimal value of the adjustable phase shift variable ξ of the reference mode function leading to these expressions represents a suitable median delay with $\sim 1/\kappa$ and $\sim\tau$ contributions [24]. We recall that with photon pulses of duration longer than μs, their spatial extent of several hundred meters readily exceeds realistic distances between cavities in laboratories, and the main time delays are caused by the reflection and transmission processes. In Ref. [24] we show by comparison with a numerical evaluation of Eq. (5) that the analytical expressions provide correct lowest-order approximations of the gate fidelities.

There is no principal lower limit to the different terms in Eq. (6). We have arranged the terms in descending order according to typical current experiments, e.g., in Refs. [21,26], $\{g, \gamma, \kappa, \kappa'\} = \{7.9, 3, 2.3, 0.2\} \times 2\pi$ MHz and $\{\Delta T, \tau\} = \{5\ \mu\text{s}, 10\ \text{ns}\}$, leading to a two-qubit CPHASE gate fidelity of $F_2 = 0.65$.

As the cooperativity parameter C clearly constitutes a main current limitation to the fidelity of our gate and other matter-light interface protocols, it is important to note that this parameter may have more favorable values, e.g., in circuit QED implementations with superconducting qubits and microwave photons. We may also use other mechanisms to control the cavity reflection and transmission of optical photons by atomic qubits, such as recent theoretical proposals, increasing C by applying the collectively enhanced cavity coupling to an ensemble of atoms, which is, in turn, controlled by the Rydberg excitation of a single atom qubit [28–31]. It is thus possible to obtain higher fidelities than the ones pertaining to our single atom example. For example, for $N_{Re} = 2500$ Rydberg atoms, the coupling strength is increased by a factor of $\sqrt{N_{Re}} = 50$; thus, increasing κ by the same factor would result in $F_2 > 0.99$, perfectly sufficient for the distribution of entanglement between network nodes [32], and even above the threshold for direct implementation of the surface code [33].

It may appear surprising that the fidelity of the multiqubit CPHASE gate is higher for an increasing number of qubits N. The multiqubit CPHASE gate fidelity indeed increases with N

as $F_N \approx 1 - (4N-3)/2^{N-1}C$ in the $\Delta\Omega \to 0$ limit. Such favorable scaling is due, in parts, to the fact that only one out of 2^N states has a different output than the others. A more crucial observation for applications is the favorable scaling of our multiqubit gate in comparison with the growing loss of fidelity by sequential application of $O(N)$ two-qubit gates [34].

Dynamical decoupling against phase fluctuations. If the optical paths of the network are stabilized, e.g., with a classical continuous-wave beam [35], small phase fluctuations $\delta \ll 1$ will reduce the fidelity $F \approx 1 - O(\delta^2)$ [24]. Otherwise, we can use the fact that such fluctuations have a finite bandwidth and can hence be compensated by a dynamical decoupling approach in a manner inspired by Refs. [17,36,37]. This can be done due to the ability to rotate qubits, and to exempt selected cavities from the multiqubit CPHASE operation [38], in combination with the transmission of two or more single-photon wave packets. Let us explain the protocol for the case of two cavities. Assuming ϕ_1 and ϕ_2 random phases of the two optical paths between the cavities (see the figure in Ref. [24]), our CPHASE operation yields the unitary operator $U_{\mathrm{CP}_2} = \exp\left[i\pi|1\rangle_1\langle 1| \otimes |1\rangle_2\langle 1| + i(\phi_1+\phi_2)|1\rangle_1\langle 1|\right]$. The following sequence yields the CPHASE gate and refocuses the random phases,

$$\Pi_1 \cdot U_{B_2} \cdot \Pi_1 \cdot \Pi_2 \cdot U_{\mathrm{CP}_2} \cdot \Pi_2 = -e^{i(\phi_1+\phi_2)} e^{i\pi|1\rangle_1\langle 1|\otimes|1\rangle_2\langle 1|}, \tag{7}$$

where Π_i denotes π pulse rotations of the ith qubit, and $U_{B_2} = \exp\left[i|1\rangle_1\langle 1|(\pi+\phi_1+\phi_2)\right]$ results from the transmission of a second photon while detuning the second cavity [38]. In Ref. [24] we show numerical and analytical results for the accomplishments of the phase cancellation in the N cavity case by refocusing sequences, involving the transmission of N photons, while detuning specific cavities.

Universality. Single-qubit continuous rotations can be realized using separate driving fields on the qubits, and together with our proposed multiqubit CPHASE gates they constitute a universal set of gates for quantum computation [39], e.g., a multi-controlled-NOT (CNOT) gate can be performed by operating with Hadamard (H) gates on the target qubit, before and after a multi-CPHASE operation on the target and the selected set of control qubits [38]. The fidelity of a two-node entangled Bell state prepared this way is exactly F_2 calculated for the two-node CPHASE operation [Eq. (6)], while neglecting errors of initialization and single-qubit H operations [24]. Note that the fact that the photon wave packet is much longer than the physical setup implies that we cannot separate the interaction of the photon with the different qubits in time and, e.g., perform gates on individual qubits between their interactions with the same photon.

Increasing the Hilbert space. To expand the number of qubits, we can incorporate local K-qubit quantum registers [Fig. 2(e)] on which we can perform mutual quantum gates, e.g., by short-range Rydberg blockade or state-selective contact atomic interactions, motional gates in ion traps, or on-chip gates between superconducting qubits. A single qubit may then be assigned the role of communicating with the other registers via its selective interaction with the cavity field, and our single-photon protocol coupling any number out of N such cavities. We note that the local K-qubit registers may also be employed to correct errors and distill the entanglement provided by the photon scattering [32]. Finding the optimum trade-off between the use of fast local gates and relatively slow, but simultaneous, multiqubit entangling gates presents an interesting challenge for distributed quantum registers [4].

Let us conclude the analysis of our proposal by recalling how our multiqubit CPHASE on all qubits is, indeed, the only entangling gate needed for an efficient implementation of the Grover search algorithm [40]. This is because the Grover algorithm assumes a π phase shift on the targeted element $|x^0\rangle \equiv |q_1^0 q_2^0 \cdots q_N^0\rangle$ relative to all other states, which we obtain by first applying the rotation that takes all qubit $|q_i^0\rangle$ states into $|1_i\rangle$, then applying the CPHASE gate, and finally rotating the qubits back. The crucial inversion about the mean in the Grover algorithm is obtained in a similar manner, by application of Hadamard gates to all qubits before and after the CPHASE gate [41]. See Ref. [42] for the related implementation and application of collective C_K-NOT (Toffoli) gates, which, together with intercavity C_N-NOT gates, can form the basis of error correction protocols.

Summary. We have proposed a network architecture where a single photon generates a deterministic multiqubit CPHASE gate between qubits embedded in different cavities. We stress that this architecture only relies on the well-established technology of atom-photon interfaces [21,26]. Since a single photon suffices to mediate the multiqubit CPHASE gate, the fidelity of our scheme is higher than other schemes involving one- and two-node gates alone. Therefore, our proposed architecture is a promising candidate for distributed quantum computing applications such as implementation of the Grover search algorithm. Our scheme needs single photons as a quantum resource as a weak coherent state contains both odd and even photon number components, the latter acquiring identical phase factors by the reflection and transmission processes. Fourier-limited single photons are becoming readily available, also in architectures that lend themselves to integrate cavities and transmission lines [43], and we recall that although we considered an optical setup with single atoms coupled to optical cavities in our quantitative analysis, our derivation is readily generalized to other quantum platforms that interact with traveling quanta of excitation, e.g., including microwave photons, spin waves, and phonons.

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