Charge-parity-violating effects in Casimir-Polder potentials

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We demonstrate under which conditions a violation of the charge-parity (*CP*) symmetry in molecules will manifest itself in the Casimir-Polder interaction of these with a magnetodielectric surface. Charge-parity violation induces a specific electric-magnetic cross polarizability in a molecule that is not chiral, but time-reversal (*T*) symmetry violating. As we show, a detection of such an effect via the Casimir-Polder potential requires a material medium that is also sensitive to time reversal, i.e., it must exhibit a nonreciprocal electromagnetic response. As simple examples of such media, we consider a perfectly reflecting nonreciprocal mirror that is a special case of a perfect electromagnetic conductor, as well as a Chern-Simons medium. In addition, we show that Chern-Simons and related media can induce unusual atom-surface interactions for anisotropic molecules with and without a chiral response.

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I. INTRODUCTION

The Casimir effect is an effective electromagnetic force between polarizable objects that is induced by the recently directly observed vacuum fluctuations of the quantum electromagnetic field [1]. Originally conceived by Casimir as an attractive force between two perfectly conducting mirrors [2], recent progress in material design and control has placed Casimir forces between materials exhibiting both more realistic and more complex electromagnetic responses in a focus of interest [3]. *Inter alia*, (para)magnetic media [4,5], chiral materials [6], topological insulators [7,8] and graphene [9,10] have been studied.

A major driving force behind such investigations continues to be the search for repulsive Casimir forces to overcome stiction in nanotechnology [11]. Inspired by Boyer's observation that the force between a perfectly conducting plate and an infinitely permeable one is repulsive [12], it was theoretically predicted that repulsion persists for combinations of purely electric and magnetic media with a more realistic response. Quite generally, repulsive Casimir forces arise whenever the nature of the electromagnetic response of the two interacting objects is diametrically opposite in a certain sense, for example, if the objects show electric versus magnetic responses, have opposite chirality, or represent topological insulators with different internal arrows of time.

Casimir-Polder forces between an atom or a molecule and a macroscopic body are a closely related type of dispersion force [13] and hence subject to the same phenomenology regarding their dependence on the electromagnetic nature of the interacting objects. However, they are (i) a local effect due to the small size of one of the two interacting objects, and (ii) they can be measured with far higher accuracy due to the superior techniques in manipulating and controlling single atoms or molecules [14]. This suggests that Casimir-Polder forces might (i) serve as a probe of the molecule's or the body's properties rather than a force that is to be manipulated and overcome, and that (ii) this probe could be applied to access very exotic properties of matter. In particular, we intend to apply this idea to the phenomenon of charge-parity violation in molecules, asking the hypothetical question: Would a potential charge-parity violation in a molecule become manifest in its Casimir-Polder interaction with a macroscopic body and, if so, under what conditions?

One of the fundamental symmetries of the standard model of particle physics is the invariance under combined charge (*C*), parity (*P*), and time (*T*) reversal. However, a physical system need not be invariant under each of these three symmetries individually. Examples of parity nonconservation include processes involving the weak force such as the β decay [15,16]. The weak interaction is also responsible for the broken parity invariance in rovibrational spectra of chiral molecules [17,18]. The antiferromagnet Cr_2O_3, on the other hand, is an example of a *P*-odd and *T*-odd system that exhibits a pseudoscalar response (see Ref. [19] and references therein) which provides a template for perfect electromagnetic conductors [20]. In the search for physics beyond the standard model, weak-scale

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supersymmetry would induce a *CP* violation that is reflected in an intrinsic electron dipole moment (EDM) of neutrons or electrons (see Ref. [21] for a review). Recent experiments place an upper limit of $|d_e| < 8.7 \times 10^{-29} e$ cm on the electron EDM [22].

Within a field-theoretic setting, it is known that a Chern-Simons interaction violates both *P* and *T* symmetries [23,24]. The Casimir interaction between two flat Chern-Simons layers was studied in Refs. [25–28], with its main feature being the prediction of both attractive and repulsive regimes of the Casimir force between Chern-Simons layers separated by a vacuum slit. A physical example showing an effective Chern-Simons interaction is a quantum Hall system consisting of a two-dimensional (2D) electron gas [29]. The coupling constant of the Chern-Simons action is different for each quantum Hall plateau, with its value being determined by the external magnetic field perpendicular to the quantum Hall layer. The implication is that the Casimir-Polder potential will be quantized at the quantum Hall plateaus [30].

In this article, we investigate the Casimir-Polder potential for an anisotropic molecule in the presence of a flat Chern-Simons layer. It is known that the *CP*-even part of the Casimir-Polder potential is quadratic in the coupling constant of the Chern-Simons layer [31]. On the other hand, the *CP*-odd part of the Casimir-Polder potential, which we derive here, is linear in the coupling constant. The sign of the Chern-Simons coupling can be altered by reversing the direction of an external magnetic field. As a result, the *CP*-odd part of the Casimir-Polder potential can be extracted from measurements at any plateau of a quantum Hall system performed at external magnetic fields with alternating spatial directions.

The article is organized as follows. We begin with a discussion of atom-field coupling and the resulting Casimir-Polder interaction potentials in the presence of nonreciprocal media. As a particular example, we consider a planar Chern-Simons layer that gives rise to nonreciprocal effects. In the following, we then construct the Casimir-Polder potentials for molecules with various anisotropic, asymmetric polarizabilities, including the particular case of *CP*-odd molecules. We close the article with some concluding remarks. Details regarding the Chern-Simons action and its influence on Maxwell's equations have been delegated to the Appendix.

II. ATOM-FIELD COUPLING

The Curie dissymmetry principle [32] suggests that *CP*-odd atomic properties can only couple to environments which are also *T* odd, i.e., nonreciprocal, so that the Green's tensor does not necessarily fulfill the Onsager relation $\mathbf{G}^{\mathsf{T}}(\boldsymbol{r}',\boldsymbol{r},\omega)=\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega)$ regarding the reversibility of optical paths [33]. This suggests a novel possibility for the detection of *CP*-odd atomic properties by studying the electromagnetic interaction of a *CP*-odd molecule with a macroscopic Chern-Simons layer by means of the Casimir-Polder potential.

The interaction of a molecule A with the electromagnetic field can be described by the multipolar Hamiltonian in long-wavelength approximation as [34]

$$\hat{H}_{\mathrm{AF}} = -\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{E}}(\boldsymbol{r}_A) - \hat{\boldsymbol{m}}\cdot\hat{\boldsymbol{B}}(\boldsymbol{r}_A) \tag{1}$$

($\hat{\boldsymbol{d}}$, $\hat{\boldsymbol{m}}$: molecular electric and magnetic dipole moments; $\boldsymbol{r}_A$: position) when neglecting the diamagnetic interaction. The position-dependent Casimir-Polder potential can be derived within second-order perturbation theory by replacing the full propagator of the electromagnetic field $\mathbf{G}$ with its scattering part $\mathbf{G}^{(1)}$ [34,35]. A general technique for calculating the Casimir-Polder potential in an arbitrary gauge of the vector potential was developed in Ref. [31], with example calculations in different gauges being provided in Ref. [35]. In the following, we will derive contributions to the Casimir-Polder potential from electric-electric and electric-magnetic terms in the presence of a planar Chern-Simons layer.

A. Casimir-Polder potential for a nonmagnetic molecule

The Casimir-Polder potential [13] for a nonmagnetic ground-state molecule arises from the second-order energy shift

$$U_{ee}(\boldsymbol{r}_A) = \frac{\hbar\mu_0}{2\pi}\int_0^\infty d\xi\,\xi^2\,\mathrm{tr}[\boldsymbol{\alpha}(i\xi)\cdot\mathbf{G}^{(1)}(\boldsymbol{r}_A,\boldsymbol{r}_A,i\xi)], \tag{2}$$

where

$$\boldsymbol{\alpha}(\omega) = \lim_{\epsilon\to 0+}\frac{1}{\hbar}\sum_k\left[\frac{\boldsymbol{d}_{k0}\boldsymbol{d}_{0k}}{\omega+\omega_k+i\epsilon} - \frac{\boldsymbol{d}_{0k}\boldsymbol{d}_{k0}}{\omega-\omega_k+i\epsilon}\right] \tag{3}$$

is the molecular polarizability, and $\mathbf{G}^{(1)}$ is the scattering part of the electromagnetic Green's tensor. In order to make the influence of nonreciprocal media more explicit, we decompose the polarizability and Green's tensors into their respective symmetric and antisymmetric parts,

$$U_{ee}(\boldsymbol{r}_A) = \frac{\hbar\mu_0}{2\pi}\int_0^\infty d\xi\,\xi^2\,\mathrm{tr}\big[\boldsymbol{\alpha}_S(i\xi)\cdot\mathbf{G}_S^{(1)}(\boldsymbol{r}_A,\boldsymbol{r}_A,i\xi) + \boldsymbol{\alpha}_A(i\xi)\cdot\mathbf{G}_A^{(1)}(\boldsymbol{r}_A,\boldsymbol{r}_A,i\xi)\big]. \tag{4}$$

The first term is the ordinary Casimir-Polder potential in environments respecting the Onsager theorem [36,37]. The second term is due to the presence of nonreciprocal media; it only arises for molecules with an anisotropic, asymmetric polarizability. Examples for its relevance are the recently considered interaction of an atom with a plate exhibiting a Chern-Simons interaction [31] or with a topological-insulator plate [38].

It is instructive to study the effects of *P* and *CP* violation due to the presence of a flat Chern-Simons layer at $z=0$ described by the action

$$S = \frac{a}{2}\int dt\,dx\,dy\,\varepsilon^{z\nu\rho\sigma}A_\nu F_{\rho\sigma}, \tag{5}$$

with a dimensionless parameter a. The scattering part of a Green's function $\mathbf{G}^{(1)}$ above ($z_A>0$) a Chern-Simons plate in the gauge $A_0=0$ mixes s- and p-polarized waves,

$$\mathbf{G}^{(1)}(\boldsymbol{r},\boldsymbol{r}',i\xi) = \frac{1}{8\pi^2}\int\frac{d^2q}{\beta}\,e^{i\boldsymbol{q}\cdot(\boldsymbol{r}-\boldsymbol{r}')-\beta(z+z')}(\boldsymbol{e}_s^+\boldsymbol{e}_s^- r_s + \boldsymbol{e}_p^+\boldsymbol{e}_p^- r_p + \boldsymbol{e}_p^+\boldsymbol{e}_s^- r_{s\to p} + \boldsymbol{e}_s^+\boldsymbol{e}_p^- r_{p\to s}) \tag{6}$$

($\boldsymbol{q}\perp\boldsymbol{e}_z$, $\beta=\sqrt{\xi^2/c^2+q^2}$), where the polarization unit vectors for the s- and p-polarized waves read $\boldsymbol{e}_s^\pm = \boldsymbol{e}_q\times\boldsymbol{e}_z$,

$\boldsymbol{e}_p^{\pm} = -(c/\xi)(iq\boldsymbol{e}_z \pm \beta\boldsymbol{e}_q)]$ and the respective reflection coefficients of the half space are given in the Appendix.

The symmetric part of the polarizability leads in this case to a potential equal to the Casimir-Polder potential in front of a perfectly conducting plate multiplied by a factor $a^2/(1+a^2)$. The asymmetric part of the polarizability yields an additional interaction with the Chern-Simons plate [31], viz.,

$$U_{\mathrm{as}}(z_A) = \frac{\hbar}{32\pi^2\varepsilon_0 c}\frac{a}{1+a^2}\int_0^{+\infty} d\xi\,\epsilon_{jlz}\alpha_{jl}(i\xi)\xi \times \left(1+2\frac{\xi z_A}{c}\right)e^{-2\xi z_A/c}. \tag{7}$$

In the retarded limit, $\omega_k z_A/c \gg 1$, the potential (7) leads to an $1/z_A^5$ asymptote,

$$U_{\mathrm{as}}(z_A) = -\frac{c^2}{8\pi^2\varepsilon_0 z_A^5}\frac{a}{1+a^2}\sum_k \frac{\mathrm{Im}(d_{0k,x}d_{k0,y})}{\omega_k^2}, \tag{8}$$

whereas at short separations, $\omega_k z_A/c \ll 1$, it is well approximated by a $1/z_A^3$ potential,

$$U_{\mathrm{as}}(z_A) = -\frac{1}{16\pi^2\varepsilon_0 z_A^3}\frac{a}{1+a^2}\sum_k \mathrm{Im}(d_{0k,x}d_{k0,y}). \tag{9}$$

The limiting cases of a perfectly reflecting, nonreciprocal mirror ($r_{s\to p} = \pm 1, r_{p\to s} = \pm 1, r_s = 0, r_p = 0$) can be immediately obtained from Eqs. (7)–(9) by substituting $a/(1+a^2) \to \pm 1$. The latter is a specific example of a perfect electromagnetic conductor and emerges from a perfect electric conductor by means of a duality transformation [20].

B. Casimir-Polder potential for a molecule with CP-odd cross polarizabilities

For an electromagnetic molecule, both electric and magnetic dipole couplings contribute to the atom-field interaction (1). Beyond the purely electric interaction, we are now interested in the part of the Casimir-Polder potential due to the second-order energy shift arising from mixed electric-magnetic transitions,

$$\begin{aligned} U_{CP}(\boldsymbol{r}_A) &= U_{em}(\boldsymbol{r}_A) + U_{me}(\boldsymbol{r}_A) \\ &= -\frac{\hbar\mu_0}{2\pi}\int_0^{\infty} d\xi\,\xi\Big\{\mathrm{tr}\big[\boldsymbol{\chi}_{me}(i\xi)\cdot \mathbf{G}^{(1)}(\boldsymbol{r}_A,\boldsymbol{r}_A,i\xi)\overleftarrow{\nabla}'\big] \\ &\quad + \mathrm{tr}[\boldsymbol{\chi}_{em}(i\xi)\cdot\nabla\times\mathbf{G}^{(1)}(\boldsymbol{r}_A,\boldsymbol{r}_A,i\xi)]\Big\}, \end{aligned} \tag{10}$$

where we have introduced the cross polarizabilities

$$\boldsymbol{\chi}_{em}(\omega) = \lim_{\epsilon\to 0}\frac{1}{\hbar}\sum_k\left[\frac{\boldsymbol{d}_{k0}\boldsymbol{m}_{0k}}{\omega+\omega_k+i\epsilon} - \frac{\boldsymbol{d}_{0k}\boldsymbol{m}_{k0}}{\omega-\omega_k+i\epsilon}\right], \tag{11}$$

$$\boldsymbol{\chi}_{me}(\omega) = \lim_{\epsilon\to 0}\frac{1}{\hbar}\sum_k\left[\frac{\boldsymbol{m}_{k0}\boldsymbol{d}_{0k}}{\omega+\omega_k+i\epsilon} - \frac{\boldsymbol{m}_{0k}\boldsymbol{d}_{k0}}{\omega-\omega_k+i\epsilon}\right]. \tag{12}$$

It is customary to decompose the Hamiltonian $\hat{H}_A = \hat{H}_0 + \hat{V}^{CP}$ of the atomic subsystem into CP-even and -odd parts [39]. We can use perturbation theory to express the eigenstates $|n\rangle$ and energies E_n of $\hat{H}_A$ in terms of the eigenstates $|n^0\rangle$ and energies E_n^0 of $\hat{H}_0$. Assuming that $\langle n|\hat{V}^{CP}|n\rangle \equiv V_{nn}^{CP} = 0$ [39], we have $E_n = E_n^0$ in linear order in $\hat{V}^{CP}$, while the eigenstates acquire linear shifts due to the CP-odd interaction,

$$|n\rangle = |n^0\rangle + \sum_l |l^0\rangle\frac{\langle l^0|\hat{V}^{CP}|n^0\rangle}{E_n^0 - E_l^0}. \tag{13}$$

To linear order in $\hat{V}^{CP}$, we can hence expand

$$\begin{aligned} \boldsymbol{d}_{0k}\boldsymbol{m}_{k0} = &-\sum_l \frac{V_{0l}^{CP}\boldsymbol{d}_{kl}^0\boldsymbol{m}_{0k}^0}{\hbar\omega_l^0} - \sum_l \frac{\boldsymbol{d}_{0l}^0 V_{lk}^{CP}\boldsymbol{m}_{k0}^0}{\hbar(\omega_l^0-\omega_k^0)} \\ &- \sum_l \frac{\boldsymbol{d}_{0k}^0 V_{kl}^{CP}\boldsymbol{m}_{l0}^0}{\hbar(\omega_l^0-\omega_k^0)} - \sum_l \frac{\boldsymbol{d}_{0k}^0\boldsymbol{m}_{kl}^0 V_{l0}^{CP}}{\hbar\omega_l^0}, \end{aligned} \tag{14}$$

with the definitions $\omega_k^0 = (E_k^0 - E_0^0)/\hbar$, $\boldsymbol{d}_{nm}^0 = \langle n^0|\hat{\boldsymbol{d}}|m^0\rangle$, $\boldsymbol{m}_{nm}^0 = \langle n^0|\hat{\boldsymbol{m}}|m^0\rangle$, and $V_{nm}^{CP} = \langle n^0|\hat{V}^{CP}|m^0\rangle$.

Substituting this result into Eq. (11), relabeling $l \leftrightarrow k$ in the third term, and using the identity

$$\begin{aligned} &\frac{1}{(\omega_l^0-\omega_k^0)(\omega_k^0\pm\omega+i\epsilon)} + \frac{1}{(\omega_l^0-\omega_k^0)(\omega_l^0\pm\omega+i\epsilon)} \\ &= \frac{1}{(\omega_k^0\pm\omega+i\epsilon)(\omega_l^0\pm\omega+i\epsilon)} \end{aligned} \tag{15}$$

results in the expression

$$\begin{aligned} \boldsymbol{\chi}_{em}(\omega) = &\lim_{\epsilon\to 0}\frac{1}{\hbar}\sum_{k,l}\left[\frac{V_{0l}^{CP}\boldsymbol{d}_{kl}^0\boldsymbol{m}_{0k}^0}{\omega_l^0(\omega+\omega_k^0+i\epsilon)} - \frac{V_{0l}^{CP}\boldsymbol{d}_{lk}^0\boldsymbol{m}_{k0}^0}{\omega_l^0(\omega-\omega_k^0+i\epsilon)}\right] \\ &+ \lim_{\epsilon\to 0}\frac{1}{\hbar}\sum_{k,l}\left[\frac{\boldsymbol{d}_{l0}^0 V_{lk}^{CP}\boldsymbol{m}_{0k}^0}{(\omega+\omega_l^0+i\epsilon)(\omega+\omega_k^0+i\epsilon)}\right. \\ &\left.- \frac{\boldsymbol{d}_{0l}^0 V_{lk}^{CP}\boldsymbol{m}_{k0}^0}{(\omega-\omega_l^0+i\epsilon)(\omega-\omega_k^0+i\epsilon)}\right] \\ &+ \lim_{\epsilon\to 0}\frac{1}{\hbar}\sum_{k,l}\left[\frac{\boldsymbol{d}_{k0}^0\boldsymbol{m}_{lk}^0 V_{l0}^{CP}}{\omega_l^0(\omega+\omega_k^0+i\epsilon)} - \frac{\boldsymbol{d}_{0k}^0\boldsymbol{m}_{kl}^0 V_{l0}^{CP}}{\omega_l^0(\omega-\omega_k^0+i\epsilon)}\right]. \end{aligned} \tag{16}$$

For a CP-odd system, the electric and magnetic transition dipole matrix elements have a vanishing relative phase, so that $\boldsymbol{\chi}_{me} = \boldsymbol{\chi}_{em}^{\mathsf{T}}$. Due to this symmetry, it is clear that the cross polarizabilities of a CP-odd system do not lead to an interaction with a perfectly conducting plate. However, they do provide an interaction with a Chern-Simons layer as well as with a perfectly reflecting nonreciprocal mirror. To evaluate this contribution of the cross polarizabilities of a CP-odd molecule to the Casimir-Polder interaction with a nonreciprocal medium, we use Eq. (10) with $\chi_{jl} \equiv \chi_{em,jl} = \chi_{me,lj}$ to find

$$\begin{aligned} U_{CP}(z_A) = &\frac{\hbar}{32\pi^2\varepsilon_0 c z_A^3}\frac{a}{1+a^2}\int_0^{\infty} d\xi\, e^{-2\xi z_A/c} \\ &\times\left\{[\chi_{xx}(i\xi)+\chi_{yy}(i\xi)]\left(1+2\frac{\xi z_A}{c}+4\frac{\xi^2 z_A^2}{c^2}\right)\right. \\ &\left.+2\chi_{zz}(i\xi)\left(1+2\frac{\xi z_A}{c}\right)\right\}. \end{aligned} \tag{17}$$

Note that the symmetry of the cross polarizabilities implies that $\chi_{jl}(i\xi) = \frac{1}{\hbar}\sum_k \frac{2\omega_k}{\omega_k^2+\xi^2} d_{k0,j}m_{0k,l}$, where $d_{k0,j}m_{0k,l}$ is a real number. In the retarded limit, $\omega_k z_A/c \gg 1$, the approximation $\chi_{jl}(i\xi) \simeq \chi_{jl}(0)$ leads to the asymptote

$$U_{CP}(z_A) = \frac{\hbar}{16\pi^2\varepsilon_0 z_A^4}\frac{a}{1+a^2} \times \left[\chi_{xx}(0) + \chi_{yy}(0) + \chi_{zz}(0)\right]. \quad (18)$$

In the opposite nonretarded limit, we may approximate

$$U_{CP}(z_A) = \frac{\hbar}{32\pi^2\varepsilon_0 c z_A^3}\frac{a}{1+a^2}\int_0^\infty d\xi \times \left[\chi_{xx}(i\xi) + \chi_{yy}(i\xi) + 2\chi_{zz}(i\xi)\right]. \quad (19)$$

The limiting case of a perfectly reflecting, nonreciprocal mirror ($r_{s\to p} = \pm 1, r_{p\to s} = \pm 1, r_s = 0, r_p = 0$) can be immediately obtained from Eqs. (17)–(19) by replacing $a/(1+a^2) \to \pm 1$.

Upon substitution, $\boldsymbol{\chi} \to \boldsymbol{\alpha}$, the potential (17) coincides with the well-known Casimir-Polder potential [13] of a purely electric atom in front of a perfectly conducting plate, apart from a factor of two. This correspondence can be easily understood from the duality of electric and magnetic fields [33]. Under a duality transformation by an angle $\theta/4$, a perfectly conducting plate transforms into a perfect nonreciprocal reflector [20], while a purely dielectric atom transforms into one with cross polarizabilities. The factor of two stems from the fact that two cross polarizabilities contribute to U_{CP} as opposed to the single electric polarizability contributing to the ordinary Casimir-Polder potential.

Let us compare the magnitude of the *CP*-odd ground-state potential with that of the Casimir-Polder potential of an ordinary, purely electric atom. We note that when going from the ordinary to the *CP*-odd case, one electric dipole moment d is replaced by m/c, while the other electric dipole moment of the order of $d \sim ea_0$ (e: electron charge; a_0: Bohr radius) is replaced by the *CP*-odd electron EDM d_{CP}. The first replacement leads to a reduction by a factor of roughly the fine-structure constant $m/(cd) \sim \alpha$, while the second yields a factor $d_{CP}/(ea_0) \approx 1.6 \times 10^{-20}$ according to the most recent upper limit [22]. The *CP*-odd ground-state potential is thus smaller than the ordinary Casimir-Polder potential by a factor of roughly 10^{-22}.

In order to potentially use the Casimir-Polder potential as a probe for *CP*-odd effects, one needs to enhance the effect by means of cavity QED. To this end, consider an excited atom in resonance with a cavity. Starting from the known resonant Casimir-Polder frequency shift of an excited atom in state n [40],

$$\Delta f = -\frac{\mu_0}{h}\sum_{k<n}\omega_{nk}^2 \boldsymbol{d}_{nk}\,\mathrm{Re}\,\mathbf{G}^{(1)}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_{nk})\cdot\boldsymbol{d}_{kn}, \quad (20)$$

and applying the above replacements $d \mapsto m/c, d \mapsto d_{CP}$, we estimate the *CP*-odd resonant frequency shift to be of the order of

$$\Delta f_{CP} \sim \frac{\mu_0}{h}\,\omega^2\,\frac{m}{c}\,\mathrm{Re}\,G d_{CP}. \quad (21)$$

The Green's tensor scales as $\mathrm{Re}\,G \sim Q\omega/c$, where Q is the quality factor of the cavity [41]. In addition, we take into account the EDM enhancement which arises in heavy paramagnetic atoms so that $d_{CP} \sim K d_e$, where K is the atomic enhancement factor and $d_e \approx 1.6 \times 10^{-20} ea_0$ is the electron EDM, as above. In addition, we estimate $\omega \sim E_{\mathrm{R}}/\hbar$, where E_{R} is the Rydberg energy and, once more, $m/c \sim ea_0\alpha$. Combining these estimates and noting that the EDM enhancement factor can reach values of up to 10^3 [24] and that Q factors of up to 10^{11} have been reported, a frequency shift of the order of 50 Hz becomes conceivable, which is within reach of current experimental resolution [18]. Note that such a cavity-QED setup based on *CP*-odd Casimir-Polder shifts would easily outperform a number of early experiments that have placed upper limits on the electron EDM using paramagnetic atoms in a conventional setup. Here, the reported constraints range from $d_e \approx 4 \times 10^{-5} ea_0$ in the 1950s [42] to $d_e \approx 8 \times 10^{-19} ea_0$ in the early 1990s [43] (see Ref. [24] for an overview).

C. Chern-Simons interaction with chiral molecules

For chiral, time-reversal invariant molecules, Lloyd's theorem states that electric and magnetic transitions carry a relative phase factor, $i = e^{i\pi/2}$ [44], so that the relation $\boldsymbol{\chi}_{me} = -\boldsymbol{\chi}_{em}^{\mathsf{T}}$ holds between the cross polarizabilities. The case of an isotropic chiral polarizability and the respective chiral Casimir-Polder potential was studied in Ref. [45]. Here we demonstrate that the interaction of a molecule with an anisotropic, asymmetric chiral polarizability and nonchiral media leads to an additional component of the Casimir-Polder potential. As an example, we evaluate this component of the Casimir-Polder potential for a molecule with anisotropic, asymmetric chiral polarizability in front of a Chern-Simons layer.

Using Eq. (10), one obtains the Chern-Simons interaction with *P*-odd chiral molecules now satisfying $\chi_{jl} \equiv \chi_{em,jl} = -\chi_{me,lj}$,

$$U_P(z_A) = -\frac{\hbar}{64\pi^2\varepsilon_0 z_A^4}\frac{a^2}{1+a^2}\int_0^{+\infty} d\xi\,\epsilon_{jlz}\chi_{jl}(i\xi) \times \frac{e^{-2\xi z_A/c}}{\xi}\left(3 + 6\frac{\xi z_A}{c} + 8\frac{\xi^2 z_A^2}{c} + 8\frac{\xi^3 z_A^3}{c}\right), \quad (22)$$

where a summation over j,l is implied. Note that due to the symmetry $\chi_{em,jl} = -\chi_{me,lj}$, one can write $\chi_{jl}(i\xi) = \frac{1}{\hbar}\sum_k \frac{2\xi}{\omega_k^2+\xi^2}\mathrm{Im}(d_{k0,j}m_{0k,l})$, where $d_{k0,j}m_{0k,l}$ is purely imaginary. As a result, in the retarded limit, $\omega_k z_A/c \gg 1$, we obtain

$$U_P(z_A) = -\frac{c}{4\pi^2\varepsilon_0 z_A^5}\frac{a^2}{1+a^2}\sum_k \frac{\epsilon_{jlz}\mathrm{Im}(d_{k0,j}m_{0k,l})}{\omega_k^2}. \quad (23)$$

In the opposite nonretarded limit, we may approximate

$$U_P(z_A) = -\frac{3\hbar}{64\pi^2\varepsilon_0 z_A^4}\frac{a^2}{1+a^2}\int_0^{+\infty} d\xi\,\frac{\epsilon_{jlz}\chi_{jl}(i\xi)}{\xi} = -\frac{3}{64\pi\varepsilon_0 z_A^4}\frac{a^2}{1+a^2}\sum_k \frac{\epsilon_{jlz}\mathrm{Im}(d_{k0,j}m_{0k,l})}{\omega_k}. \quad (24)$$

In the limit $a \to \pm\infty$, we obtain the potential of a chiral molecule in front of a perfectly conducting plate. Note, however, that the quantity $\chi_{xy}(i\xi) - \chi_{yx}(i\xi)$ should be different from zero to obtain a nonvanishing potential.

III. CONCLUSIONS

With regard to answering our central question, we have shown that charge-parity-violating effects in molecules can indeed be manifest in their Casimir-Polder interaction with a surface. As anticipated from the Curie dissymmetry principle, this requires the surface to also possess *CP*-odd or *T*-odd properties. We have shown this explicitly for a nonreciprocal perfect reflector as an example of a perfect electromagnetic conductor medium, as well as for a Chern-Simons medium. The result in the former case is strikingly similar in form to the well-known formula by Casimir and Polder, which can readily be understood from the duality invariance of QED in the absence of free charges or currents.

In addition, we have shown that Chern-Simons media lead to a different Casimir-Polder potential for anisotropic chiral molecules. In this case, the respective power laws of the potential in the short- and long-distance limits differ from those previously predicted for isotropic molecules. Our findings can be generalized to topological insulator media, where we expect similar new potential components for *CP*-odd or anisotropic molecules.

Although the *CP*-odd polarizability that induces the respective ground-state potential is very small [46], the resonant Casimir-Polder potential for an excited atom may be considerably larger. We have estimated that using an optimal combination of a high-Q cavity and a strong EDM enhancement factor as found in heavy paramagnetic atoms, the *CP*-odd resonant Casimir-Polder frequency shift is within reach of current experimental resolution, even when taking into account the most recent upper limits on the electron EDM.

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APPENDIX: DIFFRACTION FROM A PLANAR CHERN-SIMONS LAYER

The action for a planar Chern-Simons layer at $z=0$ has the form [see Eq. (5)]

$$S=\frac{a}{2}\int dt\,dx\,dy\;\varepsilon^{z\nu\rho\sigma}A_\nu F_{\rho\sigma}, \tag{A1}$$

where the current due to the Chern-Simons interaction is $J^\nu=a\varepsilon^{z\nu\rho\sigma}F_{\rho\sigma}$. Maxwell's equations for the electromagnetic field in the presence of the action (A1) are subsequently modified to

$$\partial_\mu F^{\mu\nu}+a\,\varepsilon^{z\nu\rho\sigma}F_{\rho\sigma}\delta(z)=0. \tag{A2}$$

From Eq. (A2), the continuity conditions follow as (see, e.g., Ref. [47])

$$E_z|_{z=0^+}-E_z|_{z=0^-}=-2aH_z|_{z=0}, \tag{A3}$$

$$H_x|_{z=0^+}-H_x|_{z=0^-}=2aE_x|_{z=0}, \tag{A4}$$

$$H_y|_{z=0^+}-H_y|_{z=0^-}=2aE_y|_{z=0}. \tag{A5}$$

Consider an s-polarized plane electromagnetic wave impinging onto a planar Chern-Simons layer at $z=0$,

$$E_x=\exp(-ik_zz)+r_s\exp(ik_zz),\ z>0, \tag{A6}$$

$$E_x=t_s\exp(-ik_zz),\ z<0, \tag{A7}$$

$$H_x=r_{s\to p}\exp(ik_zz),\ z>0, \tag{A8}$$

$$H_x=t_{s\to p}\exp(-ik_zz),\ z<0, \tag{A9}$$

where the factor $\exp[i(\omega/c)t+ik_yy]$ is omitted for simplicity. From the continuity condition (A4), it follows that

$$r_{s\to p}-t_{s\to p}=2a\,t_s. \tag{A10}$$

From the continuity condition $E_x|_{z=0^+}=E_x|_{z=0^-}$, one obtains

$$1+r_s=t_s. \tag{A11}$$

The continuity condition $E_y|_{z=0^+}=E_y|_{z=0^-}$ and the Maxwell equation $E_y=-\frac{1}{i\omega}\partial_zH_x$ yield

$$r_{s\to p}=-t_{s\to p}. \tag{A12}$$

From the continuity condition (A5) and the Maxwell equation $H_y=\frac{1}{i\omega}\partial_zE_x$, one gets

$$1-r_s-t_s=-2a\,t_{s\to p}. \tag{A13}$$

By solving (A10)–(A13), one obtains the reflection and transmission coefficients of an s-polarized wave as

$$r_s=-\frac{a^2}{1+a^2},\quad t_s=\frac{1}{1+a^2},$$
$$r_{s\to p}=\frac{a}{1+a^2},\quad t_{s\to p}=-\frac{a}{1+a^2}. \tag{A14}$$

By a duality transformation, i.e., by exchanging the fields E, H as well as the indices s, p in Eqs. (A6)–(A9), we obtain the reflection and transmission coefficients for the diffraction of a p-polarized wave as

$$r_p=\frac{a^2}{1+a^2},\quad t_p=\frac{1}{1+a^2},$$
$$r_{p\to s}=\frac{a}{1+a^2},\quad t_{p\to s}=\frac{a}{1+a^2}. \tag{A15}$$

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