

Exotic dynamical evolution in a secant-pulse-driven quantum system

Peng-Ju Zhao, Wei Li, Hong Cao, Shao-Wu Yao, and Li-Xiang Cen*

Center of Theoretical Physics, College of Physical Science and Technology, Sichuan University, Chengdu 610065, China



(Received 11 June 2018; published 28 August 2018)

We investigate an explicitly time-dependent quantum system driven by a secant-pulse external field. By solving the Schrödinger equation exactly, we elucidate exotic properties of the system with respect to its dynamical evolution: On the one hand, the system is shown to be essentially nonadiabatic, which prohibits an adiabatic approximation for its dynamics; on the other hand, the loop evolution of the model can induce a geometric phase which, analogous to the Berry phase of the cyclic adiabatic evolution, is in direct proportion to the solid angle subtended by the path of the state vector. Moreover, we extend the model and show that the described properties coincide in a special family of secant-pulse-driven models.

DOI: [10.1103/PhysRevA.98.022136](https://doi.org/10.1103/PhysRevA.98.022136)

I. INTRODUCTION

Exact solution to driven quantum systems with time-dependent external fields has long been a subject of particular interest in the field of quantum mechanics [1,2]. The motivation of the study comes not only from the fundamental interest about the solvability of the driven quantum system itself, but also from the nontrivial aspect of the dynamics that may be generated through an explicitly time-dependent Hamiltonian. For example, the cyclic adiabatic evolution of a time-dependent quantum system may induce a geometric phase, the so-called Berry phase [3,4], which indicates an intriguing connection between quantum physics and the gauge field theory. The Berry phase has been demonstrated to play important roles in various areas of physics, e.g., in exploring the property of electrons in crystals [5,6] and in designing fault-tolerant quantum operations for information processing [7–10].

In light of the application to quantum control, the driven systems of the following form have attracted considerable attention [11–19]:

$$H(t) = \Omega_x J_x + \Omega_z(t) J_z, \quad (1)$$

where $J_{x,z}$ denote the angular-momentum operators satisfying $[J_i, J_j] = i\epsilon_{ijk} J_k$, the field component $\Omega_z(t)$ takes a time-dependent form, and Ω_x is assumed to be a constant. Relevant study on this type of driving protocol can be retrospectively to the original Landau-Zener model [11,12]; the model and associated variants have been extensively investigated and applied to, e.g., the controllable quantum state transfer [13,15–19], Landau-Zener interferometry [20–23], and the charge transfer and chemical reactions [2,24–26]. To our best knowledge, the previous studies on this kind of driven system had not involved the geometric phase. In fact, a simple analysis on the adiabatic solution of the Hamiltonian (1) displays that no geometric phase could be generated in any such scanning protocols through the adiabatic process. Let $|m\rangle$ denote the eigenstate of J_z with magnetic quantum number m .

The instantaneous eigenstate of the Hamiltonian is given by $|\psi_m^{ad}(\theta_{ad})\rangle = e^{i\theta_{ad} J_y} |m\rangle$ with $\theta_{ad} = \arccos \frac{-\Omega_z}{\sqrt{\Omega_x^2 + \Omega_z^2}}$. For any cyclic evolution the parameter θ_{ad} should retrace itself and no Berry connection could be induced in the parametric space:

$$A_m \equiv i \langle \psi_m^{ad}(\theta_{ad}) | \partial_{\theta_{ad}} | \psi_m^{ad}(\theta_{ad}) \rangle = -\langle m | J_y | m \rangle = 0. \quad (2)$$

At this stage, we mention that this consequence resulted from the adiabatic evolution and it does not indicate the necessity of restriction to the nonadiabatic quantum process. So the question arises naturally: Could one find a system with the form of Eq. (1) that can generate nonadiabatic geometric phase during its evolution?

The Aharonov-Anandan phase [27] has often been used to describe the geometric phase for nonadiabatic processes. Differing from the Berry phase in the cyclic adiabatic evolution, the Aharonov-Anandan phase can hardly be interpreted as a geometric object because it usually depends on certain dynamical quantities, e.g., the rotating angular speed of the evolving state vector [28–30]. In this article we shall present a driven model with the form of Eq. (1) and demonstrate that its nonadiabatic evolution can generate the geometric phase. Astonishingly, we show that the nonadiabatic geometric phase induced here is in close analogy to the adiabatic Berry phase: A curvature vector could be identified for the loop evolution in the Bloch space (instead of the parametric space); the phase factor can then be understood as the geometric object of the solid angle subtended by the closed path of the state vector. On the other hand, our study reveals that the system is essentially nonadiabatic, which prohibits an adiabatic approximation for its evolution. So the existence of the geometric phase in the present model distinguishes itself from the conventional Berry phase as it is rooted in the nonadiabatic dynamics.

The rest of the article is organized as follows. In Sec. II we will introduce the secant-pulse-driven model and solve the Schrödinger equation analytically by invoking a gauge transformation approach. The explicit form of the dynamical invariant is achieved and the solution of the system is then characterized in virtue of the Lewis-Riesenfeld (L-R) theory

*lixiangcen@scu.edu.cn

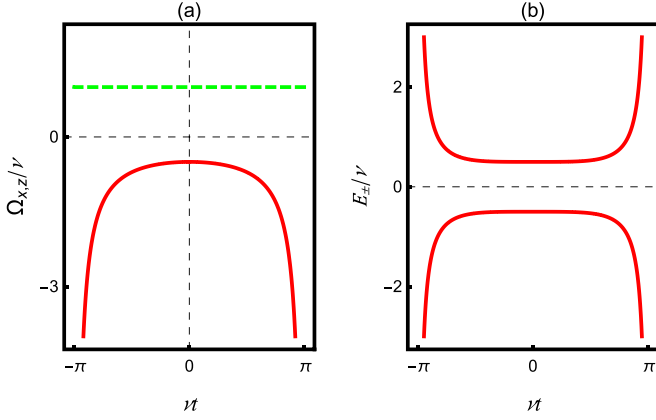


FIG. 1. The scanning process of the secant-pulse-driven model specified by Eq. (3). (a) The time dependency of the field component $\Omega_z(t)/v$ during the interval $t \in (-\frac{\pi}{v}, \frac{\pi}{v})$. (b) The diabatic energy levels $E_{\pm}(t)$ over v for the $j = \frac{1}{2}$ case. The energy difference at $t = 0$ is given by $E_+(0) - E_-(0) = -v$.

[31]. In Sec. III A we investigate the nonadiabatic geometric phase induced by the loop evolution and elucidate the linkage between it and the geometry of the evolving path in the Bloch space. In Sec. III B we describe the essential characteristic of the nonadiabaticity of the model. In Sec. IV we extend the system to a more general form and show that the previously described properties coincide in a family of secant-pulse-driven models. Finally, a summary of the manuscript is presented in Sec. V.

II. EXACT SOLUTION TO THE SECANT-PULSE-DRIVEN QUANTUM MODEL

We address the driven model described by

$$H(t) = \hbar v \left(J_x - \frac{1}{2} \sec \frac{vt}{2} J_z \right), \quad (3)$$

where the field component $\Omega_z(t)$ assumes a secant-shape pulse (see Fig. 1) and the x component Ω_x is fixed by $\Omega_x/\hbar = v$ (we set $\hbar = 1$ afterwards) with v the scanning frequency of $\Omega_z(t)$. Consider the time evolution of the system during the pulsing interval $t \in (t_0, t_f)$ with $|t_{0,f}| \leq \frac{\pi}{v}$. In view of the Lie algebraic structure of the Hamiltonian, a potentially effective way to solve the Schrödinger equation,

$$i \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle, \quad (4)$$

is to find out a specific gauge transformation [32], $|\psi^g(t)\rangle = G^\dagger(t) |\psi(t)\rangle$, via which the system is transformed into a new representation with a simpler Hamiltonian. Most of the successful cases in previous studies [14, 17–19] have exploited the gauge transformation $G(t)$ in the form of $e^{iz(t)J_z} e^{iy(t)J_y}$. Nevertheless, here we adopt a slightly different form (although equivalent mathematically), $G(t) = e^{i\alpha(t)J_x} e^{i\beta(t)J_y}$, and show that it is an efficient and convenient choice for this particular model. Under this transformation, one obtains a new Schrödinger equation, $i \partial_t |\psi^g(t)\rangle = H^g(t) |\psi^g(t)\rangle$, and the

effective Hamiltonian $H^g(t)$ reads

$$H^g(t) = G^\dagger(t) H(t) G(t) - i G^\dagger(t) \partial_t G(t) = \vec{X}(t) \cdot \vec{J}, \quad (5)$$

in which $X_i(t)$'s are given by

$$\begin{aligned} X_1(t) &= \dot{\alpha} \cos \beta - \frac{v}{2} \sec \frac{vt}{2} \sin \beta \cos \alpha + v \cos \beta, \\ X_2(t) &= \dot{\beta} + \frac{v}{2} \sec \frac{vt}{2} \sin \alpha, \\ X_3(t) &= -\dot{\alpha} \sin \beta - \frac{v}{2} \sec \frac{vt}{2} \cos \beta \cos \alpha - v \sin \beta. \end{aligned} \quad (6)$$

From Eq. (6), one can verify that by setting

$$\alpha(t) = \beta(t) = \frac{1}{2}(\pi - vt), \quad (7)$$

there are $X_{1,2}(t) = 0$ and $X_3(t) = -\frac{v}{2} \sec(\frac{vt}{2})$, thus

$$H^g(t) = -\frac{v}{2} \sec \frac{vt}{2} J_z. \quad (8)$$

As a result, the dynamical basis in this transformed representation is obtained as $|\psi_m^g(t)\rangle = e^{-im \int_{t_0}^t X_3(\tau) d\tau} |m\rangle$ and the basic solution to the original Schrödinger equation (4) is then yielded via $|\psi_m(t)\rangle = G(t) |\psi_m^g(t)\rangle$. Not only that, the above gauge transformation approach also indicates that the system possesses a dynamical invariant, the so-called L-R invariant [31],

$$\begin{aligned} I(t) &= G(t) J_z G^\dagger(t) \\ &= -\sin \beta J_x + \cos \beta (\sin \alpha J_y + \cos \alpha J_z) \\ &= -\cos \frac{vt}{2} J_x + \sin \frac{vt}{2} \left(\cos \frac{vt}{2} J_y + \sin \frac{vt}{2} J_z \right), \end{aligned} \quad (9)$$

which satisfies $i \partial_t I(t) = [H(t), I(t)]$.

Let us express $I(t)$ as $I(t) \equiv \vec{R}(t) \cdot \vec{J}$ in which $\vec{R}(t) = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$ and the angles $\theta(t)$ and $\varphi(t)$ are given by

$$\theta(t) = \arccos \left(\sin^2 \frac{vt}{2} \right), \quad \varphi(t) = \pi - \arctan \left(\sin \frac{vt}{2} \right). \quad (10)$$

Note that these two equalities constitute the set of parametric equations for the evolving trajectory of $I(t)$. In the parametric space spanned by $\vec{R}(\theta, \varphi)$, $I(t)$ will evolve along a fixed path χ on the surface of the unit sphere (see Fig. 2). The orientation of $I(t)$ goes from $\theta = 0$ at $t_0 = -\frac{\pi}{v}$ to $(\theta, \varphi) = (\frac{\pi}{2}, \pi)$ at $t = 0$, and then returns to the initial orientation at $t_f = \frac{\pi}{v}$. According to the L-R theory, the eigenvector $|\phi_m(t)\rangle$ of $I(t)$, specified by $I(t) |\phi_m(t)\rangle = m |\phi_m(t)\rangle$, differs from the basic solution $|\psi_m(t)\rangle$ of the system only by a phase factor: $|\psi_m(t)\rangle = e^{i\Phi_m(t, t_0)} |\phi_m(t)\rangle$, where $\Phi_m(t, t_0)$ is given by

$$\Phi_m(t, t_0) = \int_{t_0}^t \langle \phi_m(\tau) | i \partial_\tau - H(\tau) | \phi_m(\tau) \rangle d\tau. \quad (11)$$

The two terms of the above integration represent the geometric phase and the dynamical phase, respectively. The kernel of the latter, accounting for the diabatic energy levels of the system,

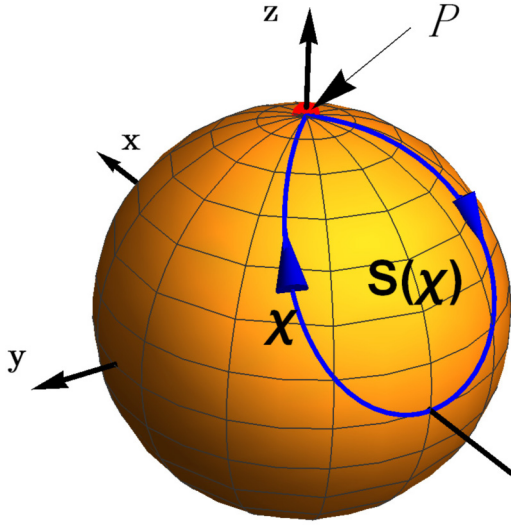


FIG. 2. Illustration of the loop evolution of the dynamical invariant $I(t)$ along the path χ in the parameter space spanned by $\vec{R}(\theta, \varphi)$. It also characterizes the trajectory of the state vector $|\phi_{\pm}(t)\rangle$ evolving on the surface of the Bloch sphere for the $j = \frac{1}{2}$ case. The enclosed surface of the loop evolution is denoted as $S(\chi)$.

is worked out to be

$$E_m(t) = \langle \phi_m(t) | H(t) | \phi_m(t) \rangle \\ = -\frac{mv}{2} \left(\cos \frac{vt}{2} + \sec \frac{vt}{2} \right), \quad (12)$$

of which the two-level case, i.e., with the azimuthal quantum number $j = \frac{1}{2}$, is depicted in Fig. 1(b).

III. EXOTIC DYNAMICAL PROPERTIES OF THE MODEL

A. Nonadiabatic Berry phase induced in the model

We now consider the geometric phase induced by the dynamical evolution of the model. To be specific, we focus on the case of $j = \frac{1}{2}$. The eigenstates of the dynamical invariant $I(t)$ in this case are expressed as

$$|\phi_{+}(t)\rangle = \cos \frac{\theta(t)}{2} |+\rangle + \sin \frac{\theta(t)}{2} e^{i\varphi(t)} |-\rangle, \\ |\phi_{-}(t)\rangle = \sin \frac{\theta(t)}{2} e^{-i\varphi(t)} |+\rangle - \cos \frac{\theta(t)}{2} |-\rangle, \quad (13)$$

in which we have used the notation “ \pm ” for $m = \pm \frac{1}{2}$, respectively. Up to a phase factor, the basis state $|\phi_{\pm}(t)\rangle$ undergoes a loop evolution from the spin-up (spin-down) state $|\pm\rangle$ at $t \rightarrow -\frac{\pi}{v}$ to a state $\frac{\sqrt{2}}{2}(|+\rangle \mp |-\rangle)$ at $t = 0$, and then returns to the initial spin-up (spin-down) state at the ending point $t \rightarrow \frac{\pi}{v}$. The total phase $\Phi_{\pm}(t, t_0)$ induced in the process can be written as $\Phi_{\pm}(t, t_0) \equiv \Phi_{\pm}^d(t, t_0) + \Phi_{\pm}^g(t, t_0)$, and the geometric phase $\Phi_{\pm}^g(t, t_0)$ is worked out to be

$$\Phi_{\pm}^g(t, t_0) = \int_{t_0}^t \langle \phi_{\pm}(\tau) | i \partial_{\tau} | \phi_{\pm}(\tau) \rangle d\tau \\ = \pm \int_{vt_0}^{vt} \frac{\cos^3 \frac{q}{2}}{4(1 + \sin^2 \frac{q}{2})} dq, \quad (14)$$

with $q \equiv v\tau$. For the overall evolution with $t_{0,f} = \mp \frac{\pi}{v}$, the above integration gives rise to $\Phi_{\pm}^g(t_f, t_0) = \pm \frac{1}{2}(\pi - 2)$.

To manifest the geometric feature of the above phase factor, let us change the variable t into $\vec{R}(\theta, \varphi)$. In the Bloch space, the basis state $|\phi_{\pm}(t)\rangle$ will evolve along the same path χ as that of $I(t)$ depicted in Fig. 2. The definite integral in the first line of Eq. (14) can then be recast as the line integral along the path,

$$\Phi_{\pm}^g(t, t_0) = i \int_{\vec{R}_0}^{\vec{R}} \langle \phi_{\pm}(\vec{R}) | \vec{\nabla} | \phi_{\pm}(\vec{R}) \rangle \cdot d\vec{R}. \quad (15)$$

At this stage, it should be noted that the field component $\Omega_z(t)$ diverges as $t \rightarrow \pm \frac{\pi}{v}$ and the path of the state vector is not strictly closed owing to the singularity at the point $\theta = 0$ (the point P in Fig. 2). However, it is seen from Eqs. (11)–(15) that this divergency does not occur in the loop integral of the geometric phase $\Phi_{\pm}^g(\chi)$ but only affects that of the dynamical part of the total phase $\Phi_{\pm}(\chi)$. So we can regard the integral path of Eq. (15) a closed loop along which the geometric phase can be calculated by integrating the curvature over the enclosed surface,

$$\Phi_{\pm}^g(\chi) = i \oint_{\chi} \langle \phi_{\pm}(\vec{R}) | \vec{\nabla} | \phi_{\pm}(\vec{R}) \rangle \cdot d\vec{R} \\ = - \iint_{S(\chi)} \vec{\nabla} \times \vec{A}_{\pm}(\vec{R}) \cdot d\vec{S}, \quad (16)$$

where

$$\vec{A}_{\pm}(\vec{R}) \equiv i \langle \phi_{\pm}(\vec{R}) | \vec{\nabla} | \phi_{\pm}(\vec{R}) \rangle \quad (17)$$

denotes the nonadiabatic Berry connection. The occurring of the minus in the second line of Eq. (16) is due to the clockwise direction of the path χ . It is direct to obtain

$$\vec{A}_{\pm}(\vec{R}) = \mp \frac{\sin^2(\theta/2)}{R \sin \theta} \hat{\varphi}. \quad (18)$$

Thus the curvature $\vec{\nabla} \times \vec{A}_{\pm}(\vec{R}) = \mp \frac{\hat{R}}{2R^2}$ and the surface integral in Eq. (16) is obtained as

$$\Phi_{\pm}^g(\chi) = \pm \frac{1}{2} \iint_{S(\chi)} \sin \theta d\theta d\varphi = \pm \frac{1}{2} \Omega(\chi), \quad (19)$$

where $\Omega(\chi) = \pi - 2$ is just the solid angle swept by the loop evolution. It is remarkable that the nonadiabatic geometric phase induced here is independent of any dynamical quantity, e.g., the scanning frequency v or the angular speed of the evolving state vector.

In real physical systems the driving field cannot be infinite and the truncation of the secant pulse is inevitable. The expression of Eq. (14) then describes the nonadiabatic geometric phase for the noncyclic dynamical process. It is worthy to note that, as the integral kernel in Eq. (14) tends to zero as $\tau \rightarrow \pm \frac{\pi}{v}$, the truncation of the field pulse results in very small influence on the amount of the geometric phase. In Fig. 3 we depict the cutoff error to the geometric phase $\Phi_{\pm}^g(\chi)$ induced by the symmetric truncation of the filed pulse, i.e., with $vt_{0,f} = \mp |\pi - \delta|$. It is shown that the relative error is less than 10^{-3} even when there is dramatic truncation $\delta \sim \frac{\pi}{10}$.

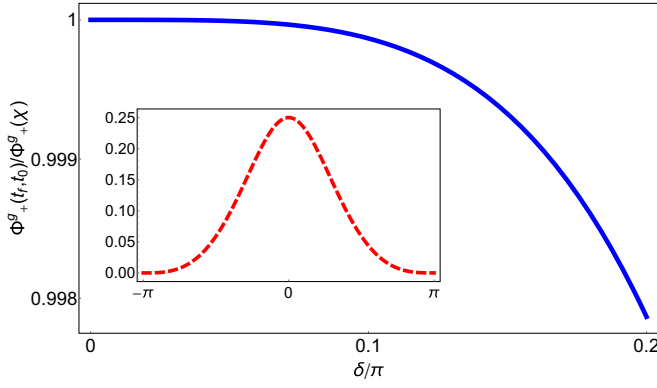


FIG. 3. Influence of the truncation of the field pulse on the geometric phase. The curve of the inset describes the integral kernel in Eq. (14) as a function of $q \equiv \nu\tau$. It is shown that the relative error [specified by $1 - \Phi_+^g(t_f, t_0)/\Phi_+^g(\chi)$] is almost negligible when the cutoff is within $\delta \lesssim 0.2\pi$.

B. Nonadiabaticity of the model

In spite of the geometric feature described above, the nonadiabatic Berry connection $\vec{A}_\pm(\vec{R})$ obtained in Eq. (17) could not recover the adiabatic one shown in Eq. (2). In particular, we show below that the dynamical evolution generated by the present model is essentially nonadiabatic and does not allow an adiabatic approximation. That is to say, the geometric phase induced in the model differs from the conventional Berry phase since it is rooted in the nonadiabatic dynamics but not the commonly known adiabatic process.

First, we mention that the basis state $|\phi_m(t)\rangle$ of $I(t)$ could never recover the adiabatic instantaneous eigenstate $|\psi_m^{ad}(t)\rangle$ of the Hamiltonian (3). This can be intuitively understood in view of the fact that in the parameter space $|\phi_m(t)\rangle$ always evolves along the fixed trajectory which is independent of the scanning frequency ν [cf. Eq. (10)]. More specifically, one can verify that the quantitative condition of the adiabatic approximation,

$$\left| \frac{\langle \psi_m^{ad}(t) | \dot{\psi}_n^{ad}(t) \rangle}{E_m^{ad}(t) - E_n^{ad}(t)} \right| = \left| \frac{\langle \psi_m^{ad}(t) | \dot{H}(t) | \psi_n^{ad}(t) \rangle}{[E_m^{ad}(t) - E_n^{ad}(t)]^2} \right| \ll 1, \quad (20)$$

cannot be fulfilled for the present model since both the numerator and denominator above are dominated by the same power of the parameter ν . For the $j = \frac{1}{2}$ case a straightforward calculation gives

$$\left| \frac{\langle \psi_+^{ad}(t) | \dot{\psi}_-^{ad}(t) \rangle}{E_+^{ad}(t) - E_-^{ad}(t)} \right| = \frac{|\sin \frac{\nu t}{2}|}{2} \sin \theta_{ad} \cos^2 \theta_{ad}, \quad (21)$$

with $\theta_{ad}(t) = \arccos[(1 + 4 \cos^2 \frac{\nu t}{2})^{-1/2}]$. So the adiabatic condition should be violated during the evolution, whatever how slow the scanning rate ν is.

Another perspective to exhibit the nonadiabaticity of the model is to compare the state evolution generated by $H(t)$ of Eq. (3) and that by $H'(t) \equiv -H(t)$. Although the two Hamiltonians $H(t)$ and $H'(t)$ possess completely identical instantaneous eigenvectors, the dynamical evolution generated by them is different. In view of the fact that $H'(t)$ relates to $H(t)$ by a transformation $\nu \rightarrow -\nu$, the dynamical invariant

$I'(t)$ (hence the basis state $|\phi'_m(t)\rangle$) of the model $H'(t)$ can be obtained from $I(t)$ of the original system $H(t)$ by changing $\varphi(t)$ into $\varphi'(t) = \pi + \arctan(\sin \frac{\nu t}{2})$ but with $\theta'(t) = \theta(t)$ [cf. Eq. (10)]. So the nonadiabatic effect can be displayed by the loss of fidelity between the basis sets of the two models: $\delta_m(t) \equiv 1 - |\langle \phi_m(t) | \phi'_m(t) \rangle|^2$. For the $j = \frac{1}{2}$ case, one obtains

$$\delta_\pm(t) = \sin^2 \theta(t) \sin^2 \varphi(t) = \sin^2 \frac{\nu t}{2} \cos^2 \frac{\nu t}{2}. \quad (22)$$

It is clear that the nonadiabaticity displayed above does not depend on the scanning frequency ν , which reconfirms that the model is essentially nonadiabatic.

IV. GENERALIZATION OF THE SECANT-PULSE-DRIVEN MODEL

We now extend the above proposed model to a more general form of which the field component Ω_x assumes an arbitrary time-dependent scanning pulse. Specifically, this family of driven models are shown as

$$\tilde{H}(t) = \Omega_x(t) \left\{ J_x - \frac{1}{2} J_z \sec \left[\frac{1}{2} \int \Omega_x(t) dt + \vartheta_0 \right] \right\}, \quad (23)$$

in which $\Omega_x(t)$ is a general function of t and the constant ϑ_0 in the integral $\vartheta(t) \equiv \frac{1}{2} \int \Omega_x(t) dt + \vartheta_0$ is set such that $|\vartheta(t)| < \frac{\pi}{2}$. By invoking a similar gauge transformation $\tilde{G}(t) = e^{i\tilde{\alpha}(t)J_x} e^{i\tilde{\beta}(t)J_y}$ with

$$\tilde{\alpha}(t) = \tilde{\beta}(t) = \frac{\pi}{2} - \vartheta(t), \quad (24)$$

one can obtain an effective Hamiltonian in the transformed representation:

$$\begin{aligned} \tilde{H}^g(t) &= \tilde{G}^\dagger(t) \tilde{H}(t) \tilde{G}(t) - i \tilde{G}^\dagger \partial_t \tilde{G}(t) \\ &= -\frac{1}{2} \Omega_x(t) \sec \vartheta(t) J_z. \end{aligned} \quad (25)$$

Subsequently the dynamical invariant of the system is achieved as

$$\tilde{I}(t) = -\cos \vartheta(t) J_x + \sin \vartheta(t) [\cos \vartheta(t) J_y + \sin \vartheta(t) J_z]. \quad (26)$$

For the time interval during which $\vartheta(t)$ goes from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, $\tilde{I}(t)$ will evolve along the same path χ as that of $I(t)$ shown in Fig. 2. So the geometric phase $\tilde{\Phi}_m(\chi)$ induced during the loop evolution is identical to $\Phi_m(\chi)$ [cf. Eq. (19)] obtained in the former model. That is to say, the sort of driven models possess the universal geometric property with respect to the dynamical evolution.

V. CONCLUSION

In summary, we have explored the dynamics generated by a secant-pulse-driven model. The Schrödinger equation of the system is solved exactly by virtue of the gauge transformation approach. The nonadiabatic Berry phase, or the so-called Aharonov-Anandan phase, induced by the loop evolution of the model is shown to possess quite exotic properties: It can be understood as the geometric object of the solid angle subtended by the evolution path and is independent of the evolving speed of the state vector in the Bloch space; on

the other hand, the geometric phase achieved in the present model distinguishes itself from the adiabatic Berry phase as the model does not allow the adiabatic assumption for the dynamical evolution. Furthermore, we have extended the system to a more general form and show that the described feature of the dynamics is universal in the specified family of secant-pulse-driven models.

For the potential application of the model, we note that the spin geometric phase driven by magnetic field textures has been exploited to manipulate electronic quantum states

in semiconducting nanostructures [33,34]. Very recently, the role of the nonadiabatic A-A phase of the spin carriers subject to in-plane magnetic textures has also been investigated in relation to the topological transition in electronic spin transport [35,36]. The model proposed in the present manuscript offers a renewed way to address the relevant issue. To this goal, a possible design of the described model in one-dimensional conducting rings, which takes into account the matching of the intrinsic Rashba field and the magnetic textures from an external source, should be a research topic in future study.

-
- [1] A. Bohm and M. Loewe, *Quantum Mechanics: Foundations and Applications*, 3rd ed. (Springer-Verlag, New York, 1993).
- [2] H. Nakamura, *Nonadiabatic Transitions: Concepts, Basic Theories and Applications* (World Scientific, Singapore, 2012).
- [3] M. V. Berry, *Proc. R. Soc. London A* **392**, 45 (1984).
- [4] B. Simon, *Phys. Rev. Lett.* **51**, 2167 (1983).
- [5] M.-C. Chang and Q. Niu, *J. Phys.: Condens. Matter* **20**, 193202 (2008).
- [6] D. Xiao, M.-C. Chang, and Q. Niu, *Rev. Mod. Phys.* **82**, 1959 (2010).
- [7] P. Zanardi and M. Rasetti, *Phys. Lett. A* **264**, 94 (1999).
- [8] J. A. Jones, V. Vedral, A. Ekert, and G. Castagnoli, *Nature (London)* **403**, 869 (2000).
- [9] L. M. Duan, J. I. Cirac, and P. Zoller, *Science* **292**, 1695 (2001).
- [10] L.-X. Cen, X. Q. Li, Y. J. Yan, H. Z. Zheng, and S. J. Wang, *Phys. Rev. Lett.* **90**, 147902 (2003).
- [11] L. D. Landau, *Phys. Z. Sowjetunion* **2**, 46 (1932).
- [12] C. Zener, *Proc. R. Soc. A* **137**, 696 (1932).
- [13] F. T. Hioe, *Phys. Rev. A* **30**, 2100 (1984).
- [14] S. J. Wang and L.-X. Cen, *Phys. Rev. A* **58**, 3328 (1998).
- [15] L. F. Wei, J. R. Johansson, L. X. Cen, S. Ashhab, and F. Nori, *Phys. Rev. Lett.* **100**, 113601 (2008).
- [16] E. Barnes and S. Das Sarma, *Phys. Rev. Lett.* **109**, 060401 (2012).
- [17] G. Yang, W. Li, and L.-X. Cen, *Chin. Phys. Lett.* **35**, 013201 (2018).
- [18] W. Li and L.-X. Cen, *Ann. Phys.* **389**, 1 (2018).
- [19] W. Li and L.-X. Cen, *Quantum Inf. Process.* **17**, 97 (2018).
- [20] W. D. Oliver, Y. Yu, J. C. Lee, K. K. Berggren, L. S. Levitov, and T. P. Orlando, *Science* **310**, 1653 (2005).
- [21] S. N. Shevchenko, S. Ashhab, and F. Nori, *Phys. Rep.* **492**, 1 (2010).
- [22] S. Gasparinetti, P. Solinas, and J. P. Pekola, *Phys. Rev. Lett.* **107**, 207002 (2011).
- [23] F. Forster, G. Petersen, S. Manus, P. Hänggi, D. Schuh, W. Wegscheider, S. Kohler, and S. Ludwig, *Phys. Rev. Lett.* **112**, 116803 (2014).
- [24] A. M. Kuznetsov, *Charge Transfer in Physics, Chemistry, and Biology* (Gordon and Breach, Reading, 1995).
- [25] C. Zhu and S. H. Lin, *J. Chem. Phys.* **107**, 2859 (1997).
- [26] A. Nitzan, *Chemical Dynamics in Condensed Phases* (Oxford University Press, Oxford, 2006).
- [27] Y. Aharonov and J. Anandan, *Phys. Rev. Lett.* **58**, 1593 (1987).
- [28] A. Bulgac, *Phys. Rev. A* **37**, 4084 (1988).
- [29] G. J. Ni, S. Q. Chen, and Y. L. Shen, *Phys. Lett. A* **197**, 100 (1995).
- [30] X.-Q. Li, L.-X. Cen, G. Huang, L. Ma, and Y. J. Yan, *Phys. Rev. A* **66**, 042320 (2002).
- [31] H. R. Lewis, Jr., *Phys. Rev. Lett.* **18**, 510 (1967).
- [32] S. J. Wang, F. L. Li, and A. Weiguny, *Phys. Lett. A* **180**, 189 (1993).
- [33] D. Loss, P. Goldbart, and A. V. Balatsky, *Phys. Rev. Lett.* **65**, 1655 (1990).
- [34] F. Nagasawa, D. Frustaglia, H. Saarikoski, K. Richter, and J. Nitta, *Nat. Commun.* **4**, 2526 (2013).
- [35] H. Saarikoski, J. E. Vázquez-Lozano, J. P. Baltanás, F. Nagasawa, J. Nitta, and D. Frustaglia, *Phys. Rev. B* **91**, 241406(R) (2015).
- [36] J. P. Baltanás, H. Saarikoski, A. A. Reynoso, and D. Frustaglia, *Phys. Rev. B* **96**, 035312 (2017).