

Complete identification of nonclassicality of Gaussian states via intensity moments

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We present an experimental method for complete identification of the nonclassicality of Gaussian states in the whole phase space. Our method relies on nonclassicality witnesses written in terms of measured integrated intensity moments up to the third order, provided that appropriate local coherent displacements are applied to the state under consideration. The introduced approach, thus, only requires linear detectors for measuring intensities of optical fields, that is, very convenient and powerful from the experimental point of view. Additionally, we demonstrate that the proposed technique not only allows to completely identify the nonclassicality of the Gaussian states, but also to quantify it.

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Nonclassicality of light plays a crucial role in the field of quantum optics. The discovery of the nonclassical properties of light has led to the establishment of new branches of quantum physics, e.g., to quantum information theory [1]. One of the most known forms of the nonclassicality of light is the entanglement where different modes of quantum fields exhibit quantum correlations which have no analog in the classical optics [2–4]. The entangled states of light are now an indispensable source for quantum telecommunications and quantum computations [1,5].

The dimension of the Hilbert space of quantum systems can be either finite or infinite, as such, there are two most distinguished classes of the quantum states, namely, discrete variable and continuous variable (CV) states, respectively. A lot of quantum protocols are now based on CV systems [6–9]. Moreover, Gaussian states, which are the subclass of CV states, possess the qualities of mathematical and experimental handiness since the infinite Hilbert space for such states can be represented by a finite-dimensional covariance matrix (CM). Additionally, the Gaussian states are easy to generate in a laboratory, e.g., they are the common output in the quantum parametric nonlinear processes [10].

The global nonclassicality of the Gaussian states of light can be expressed as by the entanglement between optical modes and so by the local nonclassicalities in the form of field squeezing [11–16]. It is worth noting that for discrete variable quantum systems one can also quantify the global nonclassicality in terms of local coherence and entanglement [17,18].

An important question thus arises, namely, how can one fairly certify the nonclassicality of the Gaussian states in the experiment? One of the solutions, although experimentally complicated, can be a homodyne tomography [19,20], which enables one to reconstruct the state in the phase space [21,22] and, thus, to identify the state's nonclassicality by knowing the form of the reconstructed quasidistributing function. On the other hand, one would like to identify the nonclassicality

without source and time-demanding reconstruction techniques. Apparently, a direct measurement of the intensity moments of optical fields can be utilized using easily accessible quadratic detectors or intensified CCD cameras which allow for detecting nonclassicality of the weak and mesoscopic fields [23–26]. Recently, anomalous moments of the optical fields have been measured to identify the quantumness of light [27]. Also photon-number-resolving detectors yield a photocount histogram from which one can deduce the presence of the nonclassical correlations of the measured state with or without coherently displaced fields [28–40]. On the other hand by relying on the directly observed photocount statistics of the studied state one can lose the phase information of the system since it may turn out that the nonclassicality of the state resides only in the phase domain [41] or intensity measurements are simply unable to reveal the nonclassicality [42]. In parallel, by applying the dephased coherent displacement to the quantum field and using classical photodiodes one can retrieve intensity moments of higher orders from which the quantum properties of the given quantum state can be deduced, that is, the essence of an unbalanced homodyne technique [43].

In this Rapid Communication, we show that by applying only local displacements to the Gaussian states one can always identify the global nonclassicality of the state under consideration, i.e., its local squeezing and entanglement, by measuring only integrated intensity moments up to the second order. Moreover, for a single- or two-mode squeezed light, the proposed method allows not only to identify its nonclassicality, but also even to quantify it. Our approach requires only linear detectors since one has to measure coherently displaced fields, therefore one avoids the use of sophisticated detectors operating at the single-photon-level regime. Additionally, the presented method, as in the case of the unbalanced homodyne detection, is independent on quantum efficiencies of the detectors, compared to the balanced homodyne technique. In short, we present a convenient experimental tool in the extraction of the nonclassical correlations of, in general, the mixed Gaussian states in the whole phase space. Furthermore, by utilizing the fact that the nonclassicality of the multimode Gaussian states

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can be expressed by means of single-mode auto- and two-mode cross correlations and making use of the known multiport interferometric techniques [44] we show that our approach can be extended to the multimode case.

Gaussian states and integrated intensity moments. Any n -mode Gaussian state $\hat{\rho}$ can completely be characterized by its first and second statistical moments, i.e., by the average values of the operator vector $\hat{\mathbf{X}} = (\hat{x}_1, \hat{p}_1, \dots, \hat{x}_n, \hat{p}_n)^T$ and by the covariance matrix $\hat{\sigma}$ with elements $\sigma_{jk} = \frac{1}{2}(\hat{X}_j \hat{X}_k + \hat{X}_k \hat{X}_j) - \langle \hat{X}_j \rangle \langle \hat{X}_k \rangle$, respectively, where the field quadratures \hat{x}_l and \hat{p}_l of the l th mode are related to the annihilation and creation operators \hat{a}_l and \hat{a}_l^\dagger as $\hat{x}_l = 1/\sqrt{2}(\hat{a}_l + \hat{a}_l^\dagger)$, $\hat{p}_l = -i/\sqrt{2}(\hat{a}_l - \hat{a}_l^\dagger)$. The characteristic function,

$$\chi_{\hat{\rho}}(\mathbf{\Lambda}) = \exp\left(-\frac{1}{2}\mathbf{\Lambda}^T \mathbf{\Omega} \sigma \mathbf{\Omega}^T \mathbf{\Lambda} - i\mathbf{\Lambda}^T \mathbf{\Omega} \langle \hat{\mathbf{X}} \rangle\right), \quad (1)$$

of state $\hat{\rho}$ in phase space is of Gaussian form where the vector $\mathbf{\Lambda} = (x_1, p_1, \dots, x_n, p_n)^T \in \mathbb{R}^{2n}$, $\mathbf{\Omega} = \bigotimes_{k=1}^n \omega_k$, and $\omega_k = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Moreover from the commutation relation $[\hat{x}_j, \hat{p}_k] = i\delta_{jk}$ it follows:

$$\sigma + \frac{i}{2}\mathbf{\Omega} \geq 0. \quad (2)$$

The inequality in Eq. (2) expresses the positivity of state $\hat{\rho}$.

Introducing a new complex vector $\boldsymbol{\beta} = \mathbf{\Theta} \mathbf{\Lambda}$, where $\mathbf{\Theta} = \bigotimes_{k=1}^n \theta_k$ and $\theta_k = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$, one can arrive at the normal characteristic function $C_{\mathcal{N}}(\boldsymbol{\beta}) = \chi_{\hat{\rho}}(\mathbf{\Theta}^{-1} \boldsymbol{\beta}) \exp(\frac{1}{2} \boldsymbol{\beta}^\dagger \boldsymbol{\beta})$, or explicitly,

$$C_{\mathcal{N}}(\boldsymbol{\beta}) = \exp\left(-\frac{1}{2} \boldsymbol{\beta}^\dagger \mathbf{\Omega} \mathbf{A}_{\mathcal{N}} \mathbf{\Omega}^T \boldsymbol{\beta} + \boldsymbol{\beta}^\dagger \mathbf{\Xi} \mathbf{\Xi}\right), \quad (3)$$

where $[\mathbf{A}_{\mathcal{N}}]_{jk} = \langle : \Delta \hat{A}_j^\dagger \Delta \hat{A}_k : \rangle = \langle : \hat{A}_j^\dagger \hat{A}_k : \rangle - \langle \hat{A}_j^\dagger \rangle \langle \hat{A}_k \rangle$ are the elements of the normal covariance matrix and $\hat{\mathbf{A}} = (\hat{a}_1^\dagger, \hat{a}_1, \dots, \hat{a}_n^\dagger, \hat{a}_n)^T$ is a vector of boson operators. The symbol $:$ accounts for normal ordering of operators, i.e., all creation operators \hat{a}^\dagger are put to the left with respect to annihilation operators \hat{a} . The complex vector $\mathbf{\Xi} = (\xi_1, \xi_1^*, \dots, \xi_n, \xi_n^*)^T \in \mathbb{C}^{2n}$ is in general a vector of displaced coherent fields.

The normal generating function for the n -mode Gaussian state is given as

$$G_{\mathcal{N}}(\boldsymbol{\lambda}) = \frac{1}{\pi^n} \int C_{\mathcal{N}}(\boldsymbol{\beta}) \prod_{j=1}^n (\lambda_j)^{-1} \exp\left(-\frac{|\beta_j|^2}{\lambda_j}\right) d^2 \beta_j, \quad (4)$$

where $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_n) \in \mathbb{R}^n$ is a real vector.

Combining now Eqs. (3) and (4) we acquire

$$G_{\mathcal{N}}(\boldsymbol{\lambda}) = \frac{1}{\sqrt{\det \mathbf{A}'_{\mathcal{N}} \prod_{j=1}^n \lambda_j}} \exp\left(-\frac{1}{2} \boldsymbol{\Xi}^\dagger \mathbf{A}'_{\mathcal{N}}^{-1} \boldsymbol{\Xi}\right), \quad (5)$$

with $\mathbf{A}'_{\mathcal{N}} = \mathbf{A}_{\mathcal{N}} + \boldsymbol{\lambda}^{-1} \mathbb{I}_{2n}$, where \mathbb{I}_{2n} is an identity matrix of dimension $2n$ and we denote the matrix $\boldsymbol{\lambda}^{-1} = \text{diag}(1/\lambda_1, 1/\lambda_1, \dots, 1/\lambda_n, 1/\lambda_n)$.

The integrated intensity moments $\langle W_1^{k_1} \dots W_n^{k_n} \rangle$ are obtained along the formula,

$$\langle W_1^{k_1} \dots W_n^{k_n} \rangle = (-1)^{k_1 + \dots + k_n} \frac{\partial^{k_1 + \dots + k_n} G_{\mathcal{N}}(\boldsymbol{\lambda})}{\partial \lambda_1^{k_1} \dots \partial \lambda_n^{k_n}} \Big|_{\lambda_1 = \dots = \lambda_n = 0}. \quad (6)$$

Nonclassicality criteria based on integrated intensity moments. The very form of the CM of the n -mode Gaussian state

suggests that all the nonclassicality properties are encoded into single-mode squeezing of each mode and entanglement between two arbitrary modes of the state. Indeed, looking at the normal CM,

$$\mathbf{A}_{\mathcal{N}} = \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_{12} & \dots & \mathbf{A}_{1n} \\ \mathbf{A}_{12}^\dagger & \mathbf{A}_2 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{1n}^\dagger & \dots & \dots & \mathbf{A}_n \end{pmatrix}, \quad (7)$$

where \mathbf{A}_k and \mathbf{A}_{jl} are block 2×2 matrices,

$$\mathbf{A}_k = \begin{pmatrix} B_k & C_k \\ C_k^* & B_k \end{pmatrix}, \quad B_k = \langle : \Delta \hat{a}_k^\dagger \Delta \hat{a}_k : \rangle, \quad (8)$$

$$\mathbf{A}_{jl} = \begin{pmatrix} \bar{D}_{jl}^* & D_{jl} \\ D_{jl}^* & \bar{D}_{jl} \end{pmatrix}, \quad \begin{aligned} C_k &= \langle : \Delta \hat{a}_k^2 : \rangle, \\ D_{jl} &= \langle : \Delta \hat{a}_j \Delta \hat{a}_l : \rangle, \\ \bar{D}_{jl} &= \langle : \Delta \hat{a}_j^\dagger \Delta \hat{a}_l^\dagger : \rangle, \end{aligned} \quad (9)$$

describing the quantum autocorrelation of mode k and cross correlations between modes j and l , respectively, one can see that the single- and two-mode correlations explicitly determine the n -mode Gaussian state. Indeed, by means of appropriate unitary operations one can always reduce the n -mode nonclassical Gaussian state to the separable n -mode locally squeezed states or, as in the case of the pure Gaussian states, to the tensor product of $\frac{n}{2}$ ($n = 2k, k \in \mathbb{Z}$) two-mode squeezed states [45,46]. Below we show that such single-mode and two-mode nonclassicality correlations of multimode Gaussian states can be retrieved by integrated intensity moments up to the second order.

The single-mode nonclassicality witness (NW) expressed in terms of integrated intensity moments can be written as the following [47]:

$$R_k = \langle W_k \rangle \langle W_k^3 \rangle - \langle W_k^2 \rangle^2 < 0, \quad k = 1, \dots, n. \quad (10)$$

Whenever $R_k < 0$ a mode k exhibits nonclassicality in the form of squeezing.

To quantify the nonclassicality of two modes the following NW can be used [38]:

$$M_{jl} = \langle W_j^2 \rangle \langle W_l^2 \rangle - \langle W_j W_l \rangle^2 < 0. \quad (11)$$

Whenever $M_{jl} < 0$ the two-mode Gaussian state cannot be both locally classical and separable.

It is important to note that the NWs in Eqs. (10) and (11) can be used for the detection of nonclassicality for any kind of state of light, i.e., even for non-Gaussian states since the negativity of these NWs refers to nonclassical properties of the quasidistribution Glauber-Sudarshan P function [4,21,38]. But they become optimal for complete nonclassicality detection only for Gaussian states.

Now, we would like to show that NWs R_k and M_{jl} can be used as genuine nonclassicality identifiers for single- and two-mode Gaussian states, provided that appropriate local unitary operations are applied to the corresponding modes. Moreover, we will demonstrate that the NW M_{jl} is useful even for verification of the nonclassicality of the single-mode states.

Theorem 1. A local nonclassicality of the single-mode Gaussian state ρ_k can be revealed and quantified by the

nonclassicality witness R_k by means of the appropriate coherent displacement operator of the given mode.

Proof. By applying a coherent displacement operator $\hat{D}(\xi'_k) = e^{\hat{a}_k^\dagger \xi'_k - \hat{a}_k \xi_k^*}$ to the single-mode Gaussian state $\hat{\rho}_k$, i.e., $\hat{\rho}_k \rightarrow \hat{\rho}'_k = \hat{D}(\xi'_k) \hat{\rho}_k \hat{D}(\xi'_k)^\dagger$ such that the coherent shifting vector Ξ in the normal characteristic function $C_{\mathcal{N}}[\hat{\rho}'_k]$ takes the form $\Xi \rightarrow \Xi' = (|\xi|e^{i\alpha_k}, |\xi|e^{-i\alpha_k})$, one then can rewrite the NW R_k for state $\hat{\rho}'_k$ in the polynomial form

$$R_k = ax^3 + bx^2 + cx + d, \quad (12)$$

where $x = |\xi|^2$ and a , b , c , and d are functions of B_k , C_k , which are given in Eq. (8) and of phase α_k . The condition at which R_k may acquire negative values can automatically be satisfied whenever $a = 2(B_k + \text{Re}[C_k e^{-2i\alpha_k}]) < 0$ since in that case one can always find such $x \in [0, \infty)$ for which $R_k < 0$. The complex parameter C_k can be presented as $C_k = |C_k|e^{i\phi_k}$, and by setting $\alpha_k = 1/2(\phi_k - \pi)$ one arrives at

$$a \equiv B_k - |C_k|. \quad (13)$$

The expression in Eq. (13) is nothing else but the condition of the Gaussian single-mode nonclassicality when negative [16,48]. Moreover, the negative values of a in Eq. (13) are a monotone of Lee's nonclassicality depth τ , which is a good nonclassicality monotone for the Gaussian states [16,49]. Therefore, if state $\hat{\rho}_k$ is nonclassical, one can always make NW R_k not only detect its nonclassicality, but also quantify it. This completes the proof of the theorem.

When the free coefficient is

$$d = 2B_k^4 + 5|C_k|^2 B_k^2 - |C_k|^4 < 0,$$

in Eq. (12), then R_k becomes negative even with $\Xi' = 0$. In that case the coherent displacement can be used for the enhancement of the nonclassicality detection since $\lim_{|\xi| \rightarrow \infty} R_k = -\infty$. If initially $d > 0$ then by choosing such $|\xi| > |\xi|_{\text{cr}}$, one eventually can access the negative values of R_k . The critical values $|\xi|_{\text{cr}}$ are found as one of the real positive roots of the NW R_k ,

$$|\xi|_{\text{cr}} = [-1/3a(b + \eta^h F + \Delta_0/\eta^h F)]^{1/2}, \quad h = 0-2, \quad (14)$$

where

$$\eta = \frac{-1 + i\sqrt{3}}{2}, \quad F = \sqrt[3]{\frac{\Delta_1 \pm \sqrt{\Delta_1^2 - 4\Delta_0^2}}{2}},$$

and $\Delta_0 = b^2 - 3ac$, $\Delta_1 = 2b^3 - 9abc + 27a^2d$.

Lemma 1. The NW M_{jl} in Eq. (11) is invariant with respect to the local phase-shifting operations $\hat{S} = \hat{S}_j(\phi_j) \otimes \hat{S}_l(\phi_l)$ applied to modes j and l of the two-mode Gaussian state $\hat{\rho}_{jl}$.

Proof. The local phase-shifting operations $\hat{S} = \hat{S}_j(\phi_j) \otimes \hat{S}_l(\phi_l)$ applied to the two-mode Gaussian state $\hat{\rho}_{jl}$ transform the corresponding boson operators of the modes as $\hat{a}_k \rightarrow \hat{a}_k e^{i\phi_k}$, where $k = j, l$. By putting the latter into Eq. (11) and utilizing the following expression for integrated intensity moments $\langle W_j^m W_l^n \rangle = \langle \hat{a}_j^{\dagger m} \hat{a}_l^{\dagger n} \hat{a}_j^m \hat{a}_l^n \rangle$, one makes sure that NW M_{jl} remains unchanged under such transformations.

Theorem 2. The nonclassicality of the two-mode Gaussian state $\hat{\rho}_{jl}$ expressed solely in the form of the entanglement can be completely detected by the nonclassicality witness M_{jl}

provided that appropriate local coherent displacements are applied to the state.

Proof. With the appropriate combination of the local phase-shifting $\hat{S} = \hat{S}_j \otimes \hat{S}_l$ and coherent displacements $\hat{D} = \hat{D}_j(\xi'_j) \hat{D}_l(\xi'_l)$, where subscripts j (l) denote an operator acting on the j th (l)th mode of the entangled two-mode Gaussian state $\hat{\rho}_{jl}$, one can transform state $\hat{\rho}_{jl} \rightarrow \hat{\rho}'_{jl} \equiv \hat{D} \hat{S} \hat{\rho}_{jl} \hat{S}^\dagger \hat{D}^\dagger$ such that the normal covariance matrix $A'_{\mathcal{N}} = \hat{S}_{jl} A_{\mathcal{N}} \hat{S}_{jl}^\dagger$ attains real nonzero elements B_j , B_l , \bar{D}_{jl} , and D_{jl} with $C_j = C_l = 0$ by default (for nonzero C_j and C_l the proof is straightforward), and the coherent shifting vector $\Xi \rightarrow \Xi'$ in the normal characteristic function $C_{\mathcal{N}}[\hat{\rho}'_{jl}]$ becomes of the form $\Xi' = (|\xi|e^{i\alpha_j}, |\xi|e^{-i\alpha_j}, |\xi|e^{i\alpha_l}, |\xi|e^{-i\alpha_l})$, i.e., with equal amplitudes but different phases. The form of the matrix $A'_{\mathcal{N}}$ is such that it corresponds to the standard form of the symmetrical covariance matrix σ_{st} ,

$$\sigma_{\text{st}} = \begin{pmatrix} q_j & 0 & q_{jl} & 0 \\ 0 & q_j & 0 & q'_{jl} \\ q_{jl} & 0 & q_l & 0 \\ 0 & q'_{jl} & 0 & q_l \end{pmatrix}, \quad (15)$$

i.e., the applied phase-shifting operation \hat{S} does not affect the global nonclassicality of the state [50]. The relations between the elements of σ_{st} and $A'_{\mathcal{N}}$ are given as the following:

$$B_k = q_k - \frac{1}{2}, \quad k = j, l, \\ D_{jl} = \frac{q_{jl} - q'_{jl}}{2}, \quad \bar{D}_{jl} = \frac{q_{jl} + q'_{jl}}{2}. \quad (16)$$

The NW M_{jl} in Eq. (11) for state $\hat{\rho}'_{jl}$ with the normal characteristic function $C_{\mathcal{N}}[\hat{\rho}'_{jl}]$ then can be presented as a polynomial $M_{jl} = M_{jl}(x)$ with $x = |\xi|^2$, which has the same form as in Eq. (12), where a , b , c , and d , now are real functions of α_j , α_l and B_j , B_l , \bar{D}_{jl} , and D_{jl} . A sufficient condition for which $M_{jl}(x)$ can acquire negative values for nonclassical state $\hat{\rho}_{jl}$ is when the coefficient a is negative, i.e.,

$$a \equiv B_j + B_l - 2D_{jl} \cos(\alpha_j + \alpha_l) + 2\bar{D}_{jl} \cos(\alpha_j - \alpha_l) < 0. \quad (17)$$

If one puts $\alpha_q = (2k_q + 1)\pi/4$, $k_q \in \mathbb{Z}$, $q = j, l$ into Eq. (17) the coefficient a becomes equivalent to the condition for inseparability of the state. Indeed, in that case Eq. (17) along with Eq. (16) can be rewritten as

$$\langle (\Delta \hat{u})^2 \rangle + \langle (\Delta \hat{v})^2 \rangle < 1, \quad (18)$$

where $\hat{u} = (|h\rangle \hat{x}_j + \hat{x}_l/h)/\sqrt{2}$, $\hat{v} = (|h\rangle \hat{p}_j - \hat{p}_l/h)/\sqrt{2}$, and $h = \pm 1$. Equation (18) is the inseparability condition written for fields' quadratures and was first derived in Ref. [50]. Now, by applying the inverse phase-shifting operations \hat{S}^{-1} to the modified state $\hat{\rho}'_{jl}$ in order to come back to the initial form of the covariance matrix $A_{\mathcal{N}}$ and by making use of *Lemma 1*, one concludes that for any entangled two-mode Gaussian state $\hat{\rho}_{jl}$ there are always such local coherent displacements for which the NW M_{jl} is able to detect the entanglement of the state. This completes the proof of the theorem.

If some coherent displacement operations are applied to the state, i.e., $\hat{\rho}'_{jl} = \hat{D} \hat{\rho}_{jl} \hat{D}^\dagger$, meaning that only the vector $\Xi \rightarrow \Xi'$ is modified in the normal characteristic function $C_{\mathcal{N}}[\hat{\rho}'_{jl}]$

and as before $\Xi' = (|\xi\rangle e^{i\alpha_j}, |\xi\rangle e^{-i\alpha_j}, |\xi\rangle e^{i\alpha_l}, |\xi\rangle e^{-i\alpha_l})$, then the coefficient a in $M_{jl}(x = |\xi|^2)$ takes the following form:

$$a \equiv B_j + B_l + \text{Re}[C_j e^{-2i\alpha_j}] + \text{Re}[C_l e^{-2i\alpha_l}] - 2 \text{Re}[\bar{D}_{jl} e^{i(\alpha_j - \alpha_l)}] - 2 \text{Re}[D_{jl} e^{-i(\alpha_j + \alpha_l)}]. \quad (19)$$

One can immediately see that in that case it is possible to retrieve the nonclassicality of the state by means of M_{jl} , which is expressed by the entanglement if choosing the right phases of the displaced coherent fields.

For instance, for two-mode squeezed vacuum states or so-called twin beam states which exhibit entanglement, the Eq. (19) can be reduced to

$$a \equiv B_j + B_l - 2|D_{jl}|, \quad (20)$$

since for such states $C_j = C_l = \bar{D}_{jl} = 0$ and where we define $\arg(D_{jl}) = \alpha_j + \alpha_l$, \arg stands for the argument of a complex number. Equation (20) represents the entanglement condition for twin beam states when negative [51]. Additionally, the negative values of the coefficient a in Eq. (20) are a monotone of the entanglement negativity, which is, by itself, a good entanglement monotone for two-mode Gaussian states [52]. Thus, for twin beams the NW M_{jl} can also be used as an entanglement quantifier.

Most importantly, the NW M_{jl} can detect, apart from the entanglement, also the local squeezing. For example, for the two-mode state $\hat{\rho}_{jl}$ which is both locally nonclassical and entangled then by suitably chosen α_j and α_l one can obtain

the condition of the nonclassicality of such a state. Namely, for $\alpha_j = \phi_j - \pi(n_2 - n_1 + 1/2)$ and $\alpha_l = \phi_l + \pi(n_1 + n_2 + 1/2)$, where ϕ_j (ϕ_l) is the phase of the boson operator \hat{a}_j (\hat{a}_l) of the mode j (l) and $n_1, n_2 \in \mathbb{Z}$, Eq. (19) attains the form

$$a \equiv B_j - |C_j| + B_l - |C_l| - 2(|D_{jl}| - |\bar{D}_{jl}|). \quad (21)$$

Equation (21) is especially easy to read for pure Gaussian states since the first four terms in Eq. (21) are responsible as before for the local nonclassicality, the differences in the last two terms in the parentheses are responsible for the entanglement [16,48], and thus $a < 0$ for such states.

Moreover, the form of the coefficient a in Eq. (19) suggests that the NW M_{jl} allows for detecting the nonclassicality of the single-mode state. For example, to detect the nonclassicality of the single-mode j one needs to replace mode l just with the coherent field $\xi_l = |\xi\rangle e^{i\alpha_l}$, where α_l can be arbitrary since in that case Eq. (19) reduces to Eq. (13).

We would also like to note that the coherent displacements of the Gaussian states generated in the spontaneous parametric processes could be encompassed by means of the stimulated emission of the corresponding parametric process. As such, by means of an appropriate choice of the initial phase and intensity of the stimulating coherent fields one can completely reveal the nonclassicality of the given Gaussian state [53].

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