

**Vacuum birefringence detection in all-optical scenarios**

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In this paper we propose an all-optical vacuum birefringence experiment and evaluate its feasibility for various scenarios. Many petawatt-class lasers became operational and many more are expected to enter operation in the near future, therefore unprecedented electromagnetic fields ( $E_L \sim 10^{14}$ – $10^{15}$  V/m and intensities  $I_L \sim 10^{21}$ – $10^{23}$  W/cm<sup>2</sup>) will become available for experiments. In our proposal a petawatt-class laser disturbs the quantum vacuum and creates a delay in a counterpropagating probe laser beam. Placing this delayed beam in one arm of a Mach-Zehnder interferometer (MZI), allows the measurement of the vacuum refraction coefficient via a phase shift. Coherent as well as squeezed light are both considered and the minimum phase sensitivity evaluated. We show that using existing technology with some moderately optimistic assumptions, at least part of the discussed scenarios are feasible for a vacuum birefringence detection experiment.

DOI: [10.1103/PhysRevA.97.063811](https://doi.org/10.1103/PhysRevA.97.063811)**I. INTRODUCTION**

It is a well known fact that Maxwell’s equations [1] are linear, therefore two or more electromagnetic fields propagating in vacuum—as far as classical electrodynamics is concerned—do not influence each other, no matter how intense they are. This state of facts would change just a few years after Dirac’s remarkable equation [2] and its prediction of antimatter.

Euler and Kockel [3] were the first to give a theoretical formulation of optical nonlinearities in the quantum vacuum at the lowest orders in the electromagnetic fields. Their work was extended in [4] where a complete theoretical study of what is now called the Heisenberg-Euler Lagrangian is given. Decades later, Schwinger [5], using the “proper-time method” from QED (quantum electrodynamics) reconfirmed the Heisenberg-Euler result.

One consequence of the Heisenberg-Euler Lagrangian [4] is vacuum birefringence [6,7]. Indeed, the quantum vacuum under the influence of external electric and/or magnetic fields behaves as if it were a birefringent (or anisotropic) material medium. Initially discussed for constant background fields only [5,6,8], vacuum birefringence is expected for an alternating background [9] or in an intense laser field [10–12], too.

On the experimental side, the PVLAS collaboration was started following a proposal from Iacopini and Zavattini [13] and with a realistic experimental method outlined in Ref. [14]. Their device (see, e.g., Fig. 2 in Ref. [15] or [16]) is based on a linearly polarized laser passing through a strong magnetic field. In order to enhance the effect, a Fabry-Perot cavity is introduced and the output laser signal is analyzed for small rotations of its initial polarization plane. Several upgrades of the experimental setup allowed the PVLAS team to report sensitivities three orders of magnitude above the QED limit [15] in 2012. The latest PVLAS results [16,17] place their experimental sensitivity at a factor of 50 above the QED limit,

with no vacuum birefringence signal detected. Other magnetic field based vacuum birefringence experiments include BMV (Birefringence Magnétique du Vide) [18] and Q&A [19]. None reported a vacuum birefringence signal.

The magnetoelectric birefringence in the quantum vacuum was discussed by Rikken and Rizzo [20,21]. In a configuration where the quantum vacuum is subjected to strong electric and magnetic fields (perpendicular to each other and perpendicular to the probe photon direction) the authors found besides the expected magnetic (“Cotton-Mouton”) and electric (“Kerr”) birefringences a magnetoelectric birefringence. The problem with the magneto-optical effect, as opposed to the purely magnetic case is that using a cavity actually cancels the effect. This is due to the fact that the magneto-optical term changes sign if the probe photon reverses its direction [20]. Therefore, experimental proposals focused on a hybrid, two cavity setup [22–24]. This design, however, does not seem to be adapted to very high intensity ( $\sim$ PW class) lasers, where all-optical elements are rated for a very limited number of laser shots.

The advent of petawatt-class laser facilities, for example Vulcan [25], Apollon [26], LFEX [27], ELI-NP [28,29] and ELI-BL [30] (the last two being part of the European “Extreme Light Infrastructure” project [31]), has boosted various proposals to probe quantum vacuum nonlinearities in high-intensity laser experiments [32]; vacuum birefringence is one of them.

Higher frequency probe beams (x rays, gamma rays) [33,34] have also been considered. At the HiBef consortium at DESY, the vacuum birefringence measurement was deemed a flagship experiment [35]. The proposal is based on the interaction of x rays with a high-power laser, already considered in [36]. At ELI-NP [37,38] a vacuum birefringence experiment based on the interaction of gamma rays with strong ( $\sim$ 1–10 PW) lasers was proposed [38,39] and is considered as a future experiment. It relies on the proposals of Dinu *et al.* [40,41].

To the best of our knowledge, no terrestrial vacuum birefringence experiment has reported a positive result. The only paper reporting a vacuum birefringence signal [42] is based

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on astrophysical observations of an isolated neutron star. However, it was met with criticism [43].

In this paper we discuss an interferometer-based and all-optical experimental setup able to measure a vacuum birefringence signal. The “pump” beam is a strong petawatt-class laser while the “probe” beam is a linearly polarized (cw or pulsed) optical laser beam. In some scenarios we also add squeezed vacuum into the second input port of the interferometer. The birefringence of the vacuum causes a small phase shift  $\Delta\varphi_{\text{QED}}$  in the Mach-Zehnder interferometer, an effect that should be measurable under certain circumstances.

The phase accuracy of a Mach-Zehnder interferometer is limited by the so-called shot noise or standard quantum limit [44–46] (i.e.,  $\Delta\varphi_{\text{SQL}} \sim 1/\sqrt{\langle N \rangle}$ , where  $\langle N \rangle$  is the average number of photons) if classical (coherent) light is used. This would make a vacuum birefringence experiment with femto- to picosecond scale pulses infeasible with realistic power estimates for the cw probe laser. A workaround could be found by employing pulsed probe beams reaching high peak powers [47]. In this scenario phase stability and synchronization with pump laser are issues to be solved.

Squeezed state of light (Yurke [48,49], see also [44]) can lower the phase sensitivity of an interferometer. This technique has been successfully implemented by the LIGO collaboration in order to detect gravitational waves [50,51]. Caves [52] was the first to show that introducing squeezing in the unused port of an interferometer can lower its phase sensitivity below the shot-noise limit. Experimental demonstration with a Mach-Zehnder interferometer [53] soon followed, proving that the concept is usable in practical measurements. Over the next decades, steady improvements in both theoretical and experimental aspects brought the sensitivity of a Mach-Zehnder interferometer to the so-called Heisenberg limit [46] (i.e.,  $\Delta\varphi_{\text{HL}} \sim 1/\langle N \rangle$ ).

Using the Cramér-Rao bound and Fisher information, Pezzé and Smerzi [54] showed that by injecting a coherent state in one port of an interferometer and squeezed vacuum in the other, the Heisenberg limit can be achieved if roughly half of the input power goes into squeezing. Lang and Caves [55,56] confirmed this result using Fisher information and showed it to be optimal for the class of coherent  $\otimes$  squeezed vacuum type of input states. Sparaciari, Olivares, and Paris [57,58] showed that the Heisenberg limit can be achieved in a Mach-Zehnder interferometer with squeezed coherent light in both inputs if the squeezing power is roughly 1/3 of the total power. NOON states [59,60] have been also shown to reach this limit, however, as pointed out in Ref. [61], a parity detection scheme is required.

The previously mentioned papers [54–60] showed mainly theoretical bounds in the phase sensitivity measurement. Practical detection schemes and their performance [62] as well as the effect of losses [62,63] have to be taken into account when a real-life experiment is described.

This is exactly the approach we shall take in this paper while discussing the MZI phase sensitivity with coherent as well as coherent and squeezed vacuum input light. We focus more on what is experimentally achievable insisting less on optimal but idealistic situations. The detection schemes are described and evaluated while the (high) power of the probe laser beam is not disregarded.

Magnetic field based vacuum birefringence experiments ( $B \sim 1\text{--}10\text{ T}$  [15–19]) scrutinize vacuum refraction coefficients on the order of  $\Delta n \sim 10^{-23}$ . This extremely small value is self-explanatory why decades of experimental effort failed to give a result. With a 1–10 PW laser we need to detect a refraction index  $\Delta n \sim 10^{-7}\text{--}10^{-9}$ . Assuming the pump laser to be focused on a few  $\mu\text{m}$  scale, the phase shift induced into a Mach-Zehnder interferometer is on the order of  $\Delta\varphi \sim 10^{-7}\text{--}10^{-9}$  rad. A phase shift measurement at this order of magnitude is challenging, especially if we consider that this phase shift lasts from tens of femtoseconds to tens of picoseconds (depending on the laser facility), however, it cannot be deemed as infeasible.

Contrary to a common practice in quantum field theories (where  $c = 1 = \hbar$ ), S.I. units will be employed throughout this paper. The reason for this choice is twofold. First, it avoids confusion among the many (S.I., Gaussian, Heaviside-Lorentz) systems of units found in various works. Second, it gives the final results and formulas to the experimentalists who are not particularly keen on keeping track of various factors that were absorbed in the redefinition of units.

This paper is structured as follows. In Sec. II we discuss the QED theoretical aspects of vacuum birefringence and the predictions for the vacuum refraction index ( $n_{\text{QED}}$ ). The proposed experimental setup is detailed in Sec. III. Maximum theoretical phase sensitivities for four petawatt-class laser facilities are computed and discussed in Sec. IV. A more realistic approach is taken in Sec. V, where emphasis is put on the achievable phase sensitivity given the experimental and technical constraints (detection scheme, maximum squeezing, available bandwidth, losses). Section VI concludes the paper.

## II. VACUUM BIREFRINGENCE IN A DISTURBED VACUUM

We consider a geometry where the intense linearly polarized petawatt beam [maximum electric (magnetic) field  $E_L$  ( $B_L = E_L/c$ ), both perpendicular on each other and perpendicular to the direction of propagation] collides almost head-on with the probe beam (see Fig. 1). We have the predicted values

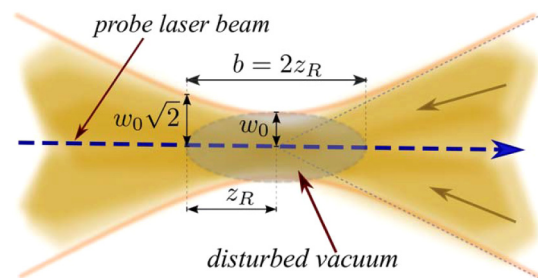


FIG. 1. The pump laser Gaussian beam (propagating from the right) is focused into a region with a waist  $w_0$ . We have the Rayleigh distance  $z_R = \pi w_0^2/\lambda_L$  and over the distance  $b = 2z_R$  we assume the pump laser’s electric field reaching its maximum value  $E_L$ . The probe beam (blue dashed line) is to a good approximation counterpropagating with respect to the pump beam.

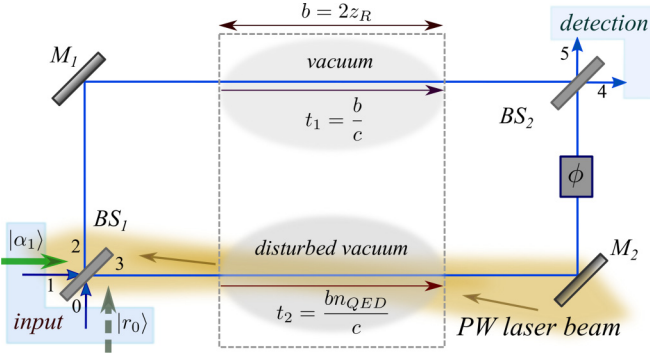


FIG. 2. Experimental setup of the proposed all-optical vacuum birefringence experiment. The petawatt-class (pump) laser is counterpropagating and intersecting the probe beam in the lower arm of the interferometer. The QED-induced phase shift  $\Delta\varphi_{\text{QED}}$  needs to be detected at the outputs 4–5 of the interferometer. The phase shift  $\phi$  is chosen by the experimenter in order to reach the MZI’s maximum sensitivity.

[20–24] (see Appendix A)

$$\left\{ \begin{array}{l} n_{\text{QED},\parallel} \\ n_{\text{QED},\perp} \end{array} \right\} = 1 + 2\xi \times \left\{ \begin{array}{l} 4 \\ 7 \end{array} \right\} \times E_L^2, \quad (1)$$

where  $n_{\parallel}$  ( $n_{\perp}$ ) is the vacuum refraction index when the probe beam’s polarization is parallel (perpendicular) to the pump beam’s polarization and we have the constant

$$\xi = \frac{\alpha}{45E_S^2} = \frac{\hbar e^4}{180\pi\epsilon_0 m_e^4 c^7} \approx 9.2039 \times 10^{-41} \frac{\text{m}^2}{\text{V}^2}, \quad (2)$$

where  $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137$  is the fine-structure constant and  $E_S = m_e^2 c^3 / e\hbar \approx 1.3 \times 10^{18}$  V/m is the Schwinger-Sauter critical electric field [5,8].  $\hbar$  denotes the reduced Planck constant,  $e$  ( $m_e$ ) is the charge (mass) of the electron,  $\epsilon_0$  is the vacuum electric permeability, and  $c$  is the speed of light in vacuum.

Consider the probe beam (of wavelength  $\lambda_p$  and frequency  $\omega_p = 2\pi c/\lambda_p$ ) counterpropagating over a length  $b = 2z_R$  (see Fig. 1) with a pump laser that disturbed the vacuum. The parameter  $b$  is called depth of focus while  $z_R = \pi w_0^2/\lambda_L$  is called the Rayleigh distance. Here  $\lambda_L$  is the pump laser wavelength and  $w_0$  is the Gaussian beam’s waist. The phase shift of this beam with respect to the same beam propagated in an unperturbed vacuum (see Fig. 2) is  $\Delta\varphi_{\text{QED},\parallel/\perp} = \omega_p b/c(n_{\text{QED},\parallel/\perp} - 1)$ . Plugging in the values from Eq. (1) yields

$$\left\{ \begin{array}{l} \Delta\varphi_{\text{QED},\parallel} \\ \Delta\varphi_{\text{QED},\perp} \end{array} \right\} = \frac{8\pi^2 w_0^2 \xi}{\lambda_p \lambda_L} \times \left\{ \begin{array}{l} 4 \\ 7 \end{array} \right\} \times E_L^2. \quad (3)$$

The probe laser beam has a wavelength  $\lambda_p = 532$  nm in our numerical calculations. The pump laser beam has a wavelength  $\lambda_L = 820$  nm and a waist  $w_0 \approx 3\mu\text{m}$ . Throughout the depth of focus we assume a constant electrical field (equal to its maximum specified value  $E_L$ ). For ELI-NP [28] and ELI-BL [30] this amounts to  $E_L \sim 10^{15}$  V/m yielding the QED-predicted phase shifts

$$\left\{ \begin{array}{l} \Delta\varphi_{\text{QED},\parallel} \\ \Delta\varphi_{\text{QED},\perp} \end{array} \right\} \approx \left\{ \begin{array}{l} 6 \\ 10 \end{array} \right\} \times 10^{-7}. \quad (4)$$

For the Vulcan [25] and XFEL [27] facilities a factor of  $10^{-2}$  has to be multiplied to the values from Eq. (4). These predicted phase shifts ( $\Delta\varphi \sim 10^{-7}$  and, respectively,  $\Delta\varphi \sim 10^{-9}$ ) are indeed small, however they are not unrealistic to be measured, as we shall show in the following.

### III. EXPERIMENTAL SETUP

Our proposed experimental setup is based on a Mach-Zehnder interferometer [44,45] driven by a combination of coherent and/or squeezed light (see Fig. 2). The probe beam interferes in a Mach-Zehnder device formed by the beam splitters  $BS_1$  and  $BS_2$  (assumed balanced, i.e., transmission and reflection coefficients  $T = 1/\sqrt{2}/R = i/\sqrt{2}$ ) and the mirrors  $M_1$  and  $M_2$ . The experimentally adjustable phase shift  $\phi$  is used to bring the interferometer to its maximum sensitivity. The counterpropagating petawatt-class laser intersects the lower arm of the interferometer and generates a disturbed vacuum over a distance  $b = 2z_R$  (i.e., in its focus region). From the probe laser’s point of view, this is just a region with a birefringent medium that induces a phase shift. By rotating the polarization of the probe beam (parallel and, respectively, perpendicular to the pump beam) we can measure the QED induced phase shifts. We shall denote in the remainder of the paper the total phase shift by  $\varphi$ , where  $\varphi = \phi + \Delta\varphi_{\text{QED}}$  (where  $\Delta\varphi_{\text{QED}}$  stands for either  $\Delta\varphi_{\text{QED},\parallel}$  or  $\Delta\varphi_{\text{QED},\perp}$ , in function of the probe beam’s polarization).

Obviously the QED induced phase shift has to be larger than the phase sensitivity of our interferometer. It is well known that the phase sensitivity of a Mach-Zehnder is not constant [46] (unless special techniques are used, e.g., [64]). Depending on the input light and detection scenario, one usually has a phase shift  $\phi$  that minimizes the phase sensitivity. This topic shall be discussed at large in Secs. IV and V.

In order to remain realistic in our estimations, the petawatt-class (pump) laser is assumed to have two types of pulses:

(i) In the order of tens to hundreds of femtoseconds ( $\tau_L \sim 10^{-14}$  s for ELI-NP [28] and  $\tau_L \sim 10^{-13}$  s for ELI-BL [30]).

(ii) In the order of a fraction to tens of picoseconds ( $\tau_L \sim 10^{-12}$  s for Vulcan [25] and  $\tau_L \sim 10^{-12}$ – $10^{-11}$  s for LFX [27]).

Therefore, the output signal is expected to have a bandwidth  $B \sim 1/\tau_L$ .

### IV. THEORETICAL PHASE SENSITIVITY LIMITS

We consider in the following two cases:

- (1) Classical (coherent) input light.
- (2) Nonclassical (coherent  $\otimes$  squeezed vacuum) input light.

We focus in this section on theoretically achievable limits. Experimental issues like the actual detection schemes, signal bandwidth, and losses are relegated to Sec. V.

#### A. Theoretical phase sensitivity limits using classical light

##### 1. cw probe laser

We first consider an ideal single-mode cw laser applied at input 1 while input 0 is kept in the vacuum state. We have the

TABLE I. Comparison of QED-induced phase shifts  $\Delta\varphi_{\text{QED}}$  versus shot-noise (or SQL) limited phase sensitivities  $\Delta\varphi_{\text{SQL}}$  for four laser facilities: Extreme light infrastructure: nuclear physics (ELI-NP) 10 PW laser, extreme light infrastructure: beamlines (ELI-BL) 10 PW laser, Vulcan and LFEX (both 1 PW lasers). The probe beam wavelength was taken  $\lambda_p = 532$  nm. Bold numerical values denote measurement sensitivities within the QED-predicted vacuum birefringence effect.

	Pump laser parameters		QED predicted phase shift $\Delta\varphi_{\text{QED}}$ from Eq. (3)	Achievable phase sensitivity $\Delta\varphi_{\text{SQL}}$ given by Eq. (7) for a cw probe laser of		Achievable phase sensitivity $\Delta\varphi_{\text{SQL}}$ for a pulsed probe laser of	
	Maximum E field	Pulse duration		$P = 100$ W	$P = 500$ W	$P = 10^{10}$ W	$P = 10^{12}$ W
	$E_L$ (V/m)	$\tau_L$ (fs)					
ELI-NP	$10^{15}$	22	$\{6; 10\} \times 10^{-7}$	$4 \times 10^{-4}$	$1.8 \times 10^{-4}$	<b><math>4.1 \times 10^{-8}</math></b>	<b><math>4.1 \times 10^{-9}</math></b>
ELI-BL	$10^{15}$	150	$\{6; 10\} \times 10^{-7}$	$1.5 \times 10^{-4}$	$7 \times 10^{-5}$	<b><math>1.5 \times 10^{-8}</math></b>	<b><math>1.5 \times 10^{-9}</math></b>
Vulcan	$10^{14}$	500	$\{6; 10\} \times 10^{-9}$	$8.6 \times 10^{-5}$	$3.8 \times 10^{-5}$	$8.6 \times 10^{-9}$	<b><math>8.6 \times 10^{-10}</math></b>
LFEX	$10^{14}$	$10^4$	$\{6; 10\} \times 10^{-9}$	$1.9 \times 10^{-5}$	$8.6 \times 10^{-6}$	<b><math>1.9 \times 10^{-9}</math></b>	<b><math>1.9 \times 10^{-10}</math></b>

input state vector

$$|\psi_{\text{in}}\rangle = \hat{D}_1(\alpha)|0\rangle, \quad (5)$$

where the displacement operator [44,45] acting on input 1 is defined as  $\hat{D}_1(\alpha) = e^{\alpha\hat{a}_1^\dagger - \alpha^*\hat{a}_1}$ . The complex number  $\alpha = |\alpha|e^{i\theta_\alpha}$  (not to be confused with the fine structure constant from Sec. II and Appendix A) denotes the amplitude of the coherent state,  $|0\rangle$  represents the vacuum state, and  $\hat{a}_k$  ( $\hat{a}_k^\dagger$ ) denotes the annihilation (creation) operator in mode  $k$ . For the coherent state under consideration we have the average number of photons  $\langle N \rangle = \langle \psi_{\text{in}} | \hat{a}_1^\dagger \hat{a}_1 | \psi_{\text{in}} \rangle = |\alpha|^2$ .

The cw probe laser beam has an energy per light quantum  $\epsilon_p = \hbar\omega_p = hc/\lambda_p \approx 3.7 \times 10^{-19}$  J. The average number of photons in the interferometer over the relevant time scale (i.e., the pump laser pulse duration  $\tau_L$ ) is given by

$$\langle N \rangle = \frac{P\tau_L}{\hbar\omega_p} = \frac{P\tau_L}{\epsilon_p}. \quad (6)$$

It is common knowledge that the minimum phase measurement error of a standard MZI using coherent light is lower bounded by the standard quantum limit [46]

$$\Delta\varphi_{\text{SQL}} \geq \frac{1}{\sqrt{\langle N \rangle}} = \frac{1}{|\alpha|}. \quad (7)$$

From Table I it is clear that the maximum realistic powers ( $P \sim 100$ – $500$  W) for the cw probe beam are insufficient to detect the QED predicted phase shift  $\Delta\varphi_{\text{QED}}$ . Equation (7) also allows one to compute the minimum power for the probe laser in order to satisfy the QED-predicted phase shift ( $\Delta\varphi_{\text{QED}}$ ), namely

$$P \approx \frac{\epsilon_l}{\tau_L(\Delta\varphi_{\text{QED}})^2}, \quad (8)$$

and plugging in the values from Eq. (4) implies immediately  $P \sim 10^{10}$  W, a totally unrealistic power for a cw laser.

## 2. Pulsed probe laser

However, such (peak) powers are commonplace in table-top pulsed lasers. Powers of the order of  $P \sim 10^{12}$  W are readily available [47,65], pushing the theoretical phase sensitivity orders of magnitude below the required level. A few terawatt femto- to picosecond class laser (i.e., probe pulse duration  $\tau_p \sim 10^{-14}$ – $10^{-10}$  s) implies an average number of photons

over the relevant pump laser time scale (from  $\tau_L \sim 10^{-14}$  to  $\tau_L \sim 10^{-12}$  s) of  $\langle N \rangle \sim 10^{16}$ – $10^{18}$  photons implying a sensitivity  $\Delta\varphi \sim 10^{-8}$ – $10^{-9}$ , enough to detect the QED signal. These results are summarized in the last two columns of Table I.

The main challenge in this scenario is the synchronization with the pump laser and the phase stability of the (pulsed) probe laser.

## B. Theoretical phase sensitivity limits using nonclassical light

One can add squeezed vacuum into the unused port 0, therefore the input state vector can be written as

$$|\psi_{\text{in}}\rangle = |r_0\alpha_1\rangle = \hat{S}_0(r)\hat{D}_1(\alpha)|0\rangle, \quad (9)$$

with the squeeze operator [44]  $\hat{S}_0(r) = e^{r/2[\hat{a}_0^2 - (\hat{a}_0^\dagger)^2]}$ , where  $r \in \mathbb{R}^+$  is the squeezing factor. The best achievable phase sensitivity measurement with this type of input has been shown to be [54–56]

$$\Delta\varphi_{\text{CSV}} \geq \frac{1}{\sqrt{|\alpha|^2 e^{2r} + \sinh^2 r}}. \quad (10)$$

The phase sensitivity  $\Delta\varphi_{\text{CSV}}$  from Eq. (10) reaches the Heisenberg limit [46,54–60]

$$\Delta\varphi_{\text{HL}} \approx \frac{1}{\langle N \rangle} \quad (11)$$

only if we impose [54]

$$|\alpha|^2 \approx \sinh^2 r \approx \frac{\langle N \rangle}{2}. \quad (12)$$

In Table II the QED predicted phase shift and the Heisenberg-limited sensitivity for our MZI are given for four petawatt-class lasers (ELI-NP [37], ELI-BL [30], Vulcan [25], and LFEX [27]).

Using a cw coherent probe beam in port 1 with only  $P = 10$  W plus squeezing in port 0 (see Fig. 1) so that the phase sensitivity becomes Heisenberg limited (11), two facilities (ELI-BL and LFEX) are theoretically feasible for this experiment. If we push the power of our probe laser to  $P \sim 100$  W (similar to the one used by LIGO) and somehow manage to maintain the squeezing factor constraint (12), a simple calculation shows that all considered facilities are theoretically within the sensitivity range to detect the QED

TABLE II. Comparison of QED-induced phase shifts  $\Delta\varphi_{\text{QED}}$  versus theoretical best achievable sensitivities using nonclassical light  $\Delta\varphi_{\text{HL}}$ , as well as suboptimal but more realistic sensitivities for the coherent-squeezed vacuum scenario  $\Delta\varphi_{\text{CSV}}$ . Bold numerical values denote measurement sensitivities within the QED-predicted vacuum birefringence effect.

	Pump laser parameters		QED predicted phase shift $\Delta\varphi_{\text{QED}}$ from Eq. (3)	Theoretical phase sensitivity $\Delta\varphi_{\text{HL}}$ given by Eq. (11) for a cw probe laser power of		Phase sensitivity $\Delta\varphi_{\text{CSV}}$ given by Eq. (13) for a cw probe with $P = 200$ W and squeezing factor	
	Maximum E field $E_L$ (V/m)	Pulse duration $\tau_L$ (fs)		$P = 10$ W	$P = 100$ W	$r = 3.5$	$r = 6$
	ELI-NP	$10^{15}$	22	$\{6; 10\} \times 10^{-7}$	$1.6 \times 10^{-6}$	<b><math>1.6 \times 10^{-7}</math></b>	$8.7 \times 10^{-6}$
ELI-BL	$10^{15}$	150	$\{6; 10\} \times 10^{-7}$	<b><math>2.5 \times 10^{-7}</math></b>	<b><math>2.5 \times 10^{-8}</math></b>	$3.3 \times 10^{-6}$	<b><math>2.7 \times 10^{-7}</math></b>
Vulcan	$10^{14}$	500	$\{6; 10\} \times 10^{-9}$	$7 \times 10^{-8}$	<b><math>7 \times 10^{-9}</math></b>	$1.8 \times 10^{-6}$	$1.5 \times 10^{-7}$
LFEX	$10^{14}$	$10^4$	$\{6; 10\} \times 10^{-9}$	<b><math>3.7 \times 10^{-9}</math></b>	<b><math>3.7 \times 10^{-10}</math></b>	$4.7 \times 10^{-7}$	$3 \times 10^{-8}$

predicted effect. We shall critically discuss this scenario in Sec. VB.

If one cannot satisfy the Heisenberg scaling condition (12) and we assume  $|\alpha|^2 \gg \sinh^2 r$  (i.e., the available laser power is much higher than the squeezing), then we can approximate the minimum phase uncertainty with

$$\Delta\varphi_{\text{CSV}} \approx \frac{e^{-r}}{|\alpha|} = \frac{e^{-r}}{\sqrt{\langle N \rangle}}. \tag{13}$$

The squeezing still brings an  $e^{-r}$  gain in the phase sensitivity, i.e.,  $\Delta\varphi_{\text{CSV}} \approx e^{-r} \Delta\varphi_{\text{SQL}}$ . In today's technology this would mean an order of magnitude [66]. We shall discuss this case at large in Sec. VB, too.

Another proposal employs squeezing in both ports. Instead of using the coherent  $\otimes$  squeezed input state configuration, Sparaciari, Olivares, and Paris considered squeezed coherent states in both inputs [57,58]. Therefore, we consider the input state

$$|\psi_{\text{in}}\rangle = |\gamma, \zeta\rangle_0 \otimes |\alpha, \xi\rangle_1, \tag{14}$$

where  $|a, b\rangle_j = \hat{D}_j(a)\hat{S}_j(b)|0\rangle$ ,  $a \in \{\alpha, \gamma\}$ ,  $b \in \{\zeta, \xi\}$ , and  $j = 0, 1$ . In [57] it is shown that the maximum Fisher information (and thus the best phase sensitivity) is achieved when  $\alpha = \gamma$  and  $\xi = \zeta = r$  (all taken real in their paper). In other words, the optimum case is when both input lasers have the same power and both squeezings are identical. If we denote the total squeezing factor  $\beta_{\text{tot}} = 2 \sinh^2 r / N_{\text{tot}}$ , where  $N_{\text{tot}} = 2(|\alpha|^2 + \sinh^2 r)$  is the total number of photons inside the interferometer, we have the phase sensitivity given by [57]

$$\Delta\varphi_{\text{SQC}} \geq \frac{1}{\sqrt{\frac{8N_{\text{tot}}^2(2 + \sqrt{1 + 3N_{\text{tot}}^{-1}})}{9} + 4N_{\text{tot}}}} \approx \frac{1}{\frac{4}{3}N_{\text{tot}}}, \tag{15}$$

where in this case  $\beta_{\text{tot}} = 2/3$  and we assumed a large energy regime ( $N_{\text{tot}} \gg 1$ ). This result leads indeed to a Heisenberg scaling with an improved proportionality constant (compared to the coherent  $\otimes$  squeezed vacuum case).

V. REALISTIC PHASE SENSITIVITY ESTIMATION

It is self-understood that the interferometer can be operated in vacuum only. Due to the smallness of the expected phase shift, seismic isolation and environmental vibration damping have to be considered, too. The mirrors and beam splitters from

Fig. 2 should be suspended by double pendulums [67] or using stabilized platforms with actuators [68].

In order not to average out the QED-induced signal, the photodetector(s) should be triggered (with a suitable delay) after the moment of interaction of the probe beam with the pump laser pulse. The integration time of the photodetector should also be matched with the pump pulse duration. This is not a problem for the picosecond pump pulses, however it is rather problematic for the femtosecond one. Depending on the experimental scenario, more complex detection schemes might be considered [69].

A. Classical light input

1. cw probe laser

The sensitivity of a Mach-Zehnder given by Eq. (7) is a lower bound. If we use as an observable the difference in photocounts  $\hat{N}_d = \hat{a}_4^\dagger \hat{a}_4 - \hat{a}_5^\dagger \hat{a}_5$  at detectors  $D_4$  and  $D_5$  (see Fig. 3), one actually has (see Appendix B) the phase sensitivity

$$\Delta\varphi = \frac{1}{|\alpha| |\sin \varphi|} = \frac{1}{\sqrt{\langle N \rangle} |\sin \varphi|}, \tag{16}$$

where we remind that  $\varphi = \Delta\varphi_{\text{QED}} + \phi$ . Since  $\Delta\varphi_{\text{QED}} \ll 1$ , the optimum point is when the experimenter imposes  $\phi = \pi/2$ . This scenario implies large signals on both detectors

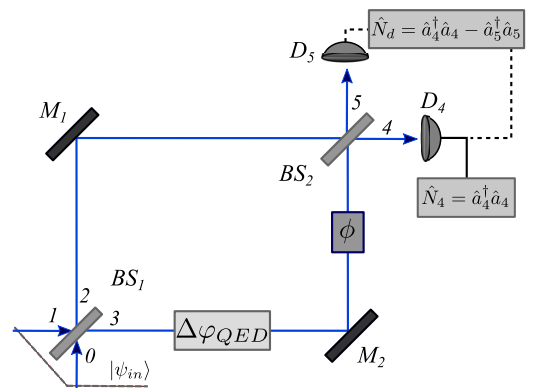


FIG. 3. The actual detection schemes used for our setup. The “differential detection” subtracts the photocurrents from the detectors  $D_4$  and  $D_5$ , therefore the corresponding operator is  $\hat{N}_d$ . The second option is to use a single detector and we chose  $D_4$ , therefore the relevant operator is  $\hat{N}_4$ . The experimenter-controlled phase shift  $\phi$  brings the Mach-Zehnder to its optimum phase sensitivity so that the very small QED-induced phase shift  $\Delta\varphi_{\text{QED}}$  can be detected.

being impractical for sensitive photodetector PIN diodes. (The highest efficiency for PIN diodes is in the  $\mu\text{W}$  range [70].)

A single detector setup employing the observable  $\hat{N}_4 = \hat{a}_4^\dagger \hat{a}_4$  (see Fig. 3) yields a phase sensitivity (details are given in Appendix B)

$$\Delta\varphi = \frac{1}{\sqrt{\langle N \rangle} \left| \sin\left(\frac{\varphi}{2}\right) \right|}. \quad (17)$$

This time the optimum is when  $\varphi = \pi$  and this implies a very small signal at the detector  $D_4$  (“dark port”). This is a desirable scenario since a high-efficiency PIN diode can be used as a detector [70].

## 2. Pulsed probe laser

As discussed in Sec. IV A, a pulsed laser has orders of magnitude higher peak powers compared to its cw counterpart, therefore a noticeable improvement in the phase sensitivity can be expected.

Indeed, quantum-limited measurements with a femtosecond pulsed laser have been reported [71,72] and their scaling yields

$$\Delta\varphi \sim \frac{1}{2\sqrt{N} \sqrt{\frac{\Delta\omega^2}{\omega_l^2} + 1}}, \quad (18)$$

where  $\Delta\omega$  is the spectral width of the pulsed probe laser. Hence, the SQL scaling should be practically achievable for the pulsed probe laser, too.

To the synchronization between the probe laser and the detector trigger discussed in the beginning of this section, one has to add now the synchronization in order to obtain a “head-on” collision of the probe and the pump pulses at the focus point of the pump laser. This synchronization requirement is on the order of  $\tau_L$ . For the femtosecond laser facilities [28,30] this is a serious issue. It is less of a problem for the picosecond pump pulses [25,27].

## B. Nonclassical light input

In Sec. IV B we concluded that if we arrive at a Heisenberg scaling even a cw laser with  $P = 10\text{ W}$  could provide enough sensitivity to detect the vacuum birefringence signal. While theoretically possible, a closer look satisfying Eq. (12) implies a squeezing factor beyond today’s technological possibilities.

The current record for the squeezing factor (15.3 dB or  $r = 1.7$ ) is reported in [66]. For example, a probe laser power  $P = 10\text{ W}$  would imply  $\langle N \rangle \approx 4 \times 10^6$  photons during the relevant time scale for ELI-BL, therefore according to Eq. (12) a value of  $r \approx 8$  would be needed to attain the Heisenberg limit. This is an unrealistic value from an experimental point of view.

The remark previously stated holds for the squeezed coherent scenario [57,58], too: the high squeezing factors (i.e., the  $\beta_{\text{tot}}$  factor in this case) needed to follow a Heisenberg scaling for powers in the Watt range (and above) are technically difficult today.

We note, nonetheless, that there is *no theoretical limitation* on the value of  $r$  (or  $\beta_{\text{tot}}$ ), the experimental limitations arising mainly from the losses in the generation and detection of this state [73]. The rapid progress of squeezing in the last decade (7 dB ( $r = 0.8$ ) in 2007, 10 dB ( $r = 1.15$ ) in 2010 [74], 15 dB in 2016 [66]) suggests further technological improvements

[73]. We can speculate that a 20 dB ( $r = 2.3$ ) squeezing is within reach in the near future and a more optimistic 25–30 dB ( $r = 2.8$ –3.5) squeezing is not excluded in the next decade.

In the last two columns of Table II we compute the achievable phase sensitivities for a coherent  $\otimes$  squeezed vacuum input light. We take our probe cw laser with  $P = 200\text{ W}$  and consider two squeezing factors:  $r = 3.5$  (optimistic, however foreseeable in the future) and  $r = 6$  (rather idealistic). One notes that roughly one order of magnitude is missing in order to meet the required QED phase sensitivity for  $r = 3.5$  and for the rather too optimistic squeezing factor  $r = 6$  despite not being Heisenberg limited, the sensitivity is reached for the ELI lasers.

In the following we consider the realistic detection schemes depicted in Fig. 3 and evaluate their performance. For a difference detection scheme, the phase sensitivity of a coherent  $\otimes$  squeezed vacuum input is given by [46,62] (see also Appendix B)

$$\Delta\varphi = \frac{\sqrt{\frac{|\alpha|^2 + \frac{\sinh^2 2r}{2}}{\tan^2 \varphi} + \sinh^2 r + |\alpha|^2 e^{-2r}}}{||\alpha|^2 - \sinh^2 r|}, \quad (19)$$

and for  $\varphi = \pi/2$  we have the optimum phase sensitivity

$$\Delta\varphi = \frac{\sqrt{\sinh^2 r + |\alpha|^2 e^{-2r}}}{||\alpha|^2 - \sinh^2 r|}. \quad (20)$$

In our setup we have  $|\alpha|^2 \gg \sinh^2 r$  and imposing this condition to Eq. (20) leads to the result from Eq. (13). Thus, we are not Heisenberg limited but we gain an  $e^{-r}$  factor compared to the SQL.

However, a total phase shift  $\varphi = \pi/2$  inside the MZI implies equal output powers at the detectors  $D_4$  and  $D_5$  causing additional problems for a high-efficiency detection in the case of a high-power probe beam. We thus focus on the single-detection case. If we consider detection events solely at  $D_4$ , we arrive at a phase sensitivity (see calculations in Appendix B)

$$\Delta\varphi = \frac{\sqrt{\frac{\sinh^2 2r \tan^2(\frac{\varphi}{2})}{2} + \sinh^2 r + \frac{|\alpha|^2}{\tan^2(\frac{\varphi}{2})} + \frac{|\alpha|^2}{e^{2r}}}}{||\alpha|^2 - \sinh^2 r|}. \quad (21)$$

An optimum point can be found for the total phase shift inside the MZI:

$$\varphi_{\text{opt}} = \pm 2 \arctan \left( \sqrt{\frac{2|\alpha|}{\sinh 2r}} \right) + 2k\pi, \quad (22)$$

with  $k \in \mathbb{Z}$  and introducing this value into Eq. (21) gives the best achievable sensitivity in the case of a single detector scheme namely

$$\Delta\varphi = \frac{\sqrt{\sinh^2 r + \sqrt{2}|\alpha| \sinh 2r + |\alpha|^2 e^{-2r}}}{||\alpha|^2 - \sinh^2 r|} \quad (23)$$

result also found in Ref. [62]. For a strong coherent source and a realistic squeezing factor we have  $|\alpha|^2 \gg |\alpha|$  and  $|\alpha| \gg \sinh r$ , therefore the phase sensitivity from Eq. (23), similar to the difference detection scheme, migrates to Eq. (13). It is noteworthy to point out that we have again a dark port scenario ( $D_4$  gets

almost no light) and this is the optimum situation for sensitive photodetectors. Thus, for high input powers, the single detector scheme is preferable to the difference detection setup.

### C. Detection bandwidth

The fact that the useful signal at the outputs 4 and 5 lasts only for a short time (depending on the laser facility, from dozens of femtoseconds to tens of picoseconds) implies a detection scheme having the necessary bandwidth to cope with this signals.

For coherent light, the generation [47] and detection [69] of femtosecond pulses do not pose a problem. This assertion is no longer true for squeezed states. For example, adding in the detection scheme beam splitters with one input port “empty” will introduce extra vacuum fluctuations, thus reducing or even canceling the advantage brought by squeezed light.

The typical scheme used (see, e.g., Fig. 20 in Ref. [73]) to generate squeezed vacuum is based on a SHG (second harmonic generation) followed by a subthreshold pumped OPA (optical parametric amplifier) [50,66,74,75]. Among the reasons for placing the nonlinear crystal in an OPA cavity are an enhanced down-conversion efficiency and confining the transversal mode (usually TEM<sub>00</sub>). The output spectrum of the squeezed ( $\Delta^2 \hat{X}_+$ ) and antisqueezed ( $\Delta^2 \hat{X}_-$ ) quadrature variances are given by

$$\Delta^2 \hat{X}_{\pm}(\Omega) = 1 \pm \eta_d \frac{4\sqrt{p}}{(1 \mp \sqrt{p})^2 + (\Omega/\Gamma)^2}, \quad (24)$$

where  $p = P/P_{\text{th}}$ ,  $P$  is the second harmonic pump power,  $P_{\text{th}}$  is the OPA threshold power,  $\Omega$  is the sideband frequency, and  $\eta_d$  denotes all-optical losses. The cavity linewidth (also called decay rate) is  $\Gamma = c(T + L)/l$  where  $l$  is the cavity length,  $T$  denotes the cavity mirror’s power transmission coefficient and  $L$  is the round-trip loss.

For the detection of the vacuum birefringence signal created by a pump laser pulse of duration  $\tau_L$  we need a bandwidth on the order of  $B \sim 1/\tau_L$ . This would imply for example for the LFEX laser a minimum of  $B \sim 50$  GHz and for the ELI-BL laser  $B \sim 6.7$  THz.

The “natural bandwidth” of the OPA generated squeezed states is in the tens to hundreds of megahertz [66,74,75]. It is here that the record in squeezing has been achieved. However, ultrabroadband squeezed light ( $B \sim 13.4$  THz in the telecommunication band  $\lambda = 1535$ ) has been reported [76]. Therefore, bandwidth does not seem to pose a particular problem to squeezed states, the technological problem to answer is the amount of squeezing achievable in the ultrabroadband case.

### D. The impact of losses on the phase sensitivity

All results discussed up to this point assume no losses. In some scenarios this might not be realistic.

Following [63], we can define  $\sigma$  the loss ratio, i.e., the probability that any photon is lost between generation and detection. Then, for a coherent state we have to make the replacement  $\alpha' \rightarrow \sqrt{1 - \sigma}\alpha$  resulting in a bound

$$\Delta\varphi_{\text{SQL}}^l = \frac{1}{\sqrt{1 - \sigma}|\alpha|} = \frac{\Delta\varphi_{\text{SQL}}}{\sqrt{1 - \sigma}}. \quad (25)$$

Especially for low losses (i.e.,  $\sigma \ll 1$ ) we have a small effect for the phase sensitivity. Therefore, in an experimental setup employing only classical light, losses are not of paramount importance.

For the coherent  $\otimes$  squeezed vacuum case we can approximate [63] the phase sensitivity

$$\Delta\varphi_{\text{CSV}}^l \approx \frac{\sqrt{\sigma + (1 - \sigma)e^{-2r}}}{\sqrt{(1 - \sigma)|\alpha|^2 + \sigma(1 - \sigma)\sinh^2 r}}. \quad (26)$$

The effect of losses is more vicious for high squeezing factors. Indeed, for  $\sigma \gg e^{-2r}$ , we arrive at the SQL scaling given by Eq. (25) for  $|\alpha|^2 \gg \sinh^2 r$  and the whole interest of squeezing is lost.

For the squeezed coherent case [57], too, it has been shown in [58] that including losses degrades the performance from a Heisenberg scaling to a shot-noise scaling. A lengthier discussion about the effect of losses can be found the literature [62,63,77].

### E. Repeated measurements

All results discussed up to this point assumed a single measurement. For  $N_{\text{expt}}$  independent measurements having the same probability distribution of the measured observable, the Fisher information is additive [46], resulting in  $F(\varphi) \rightarrow N_{\text{expt}}F(\varphi)$ . The Cramér-Rao lower bound is therefore given by

$$\Delta\varphi_{\text{multi}} = \frac{1}{\sqrt{N_{\text{expt}}}} \Delta\varphi_{\text{single}}. \quad (27)$$

The scaling with  $1/\sqrt{N_{\text{expt}}}$  has been shown both theoretically and experimentally for coherent light [64] and for coherent and squeezed vacuum [54].

While theoretically the limit  $N_{\text{expt}} \rightarrow \infty$  can be taken, in realistic experiments many constraints limit the acquisition time. In a high-power laser facility, one of the most stringent constraints is the number of PW laser shots available for a given experiment. Realistically we can estimate this number between dozens and thousands, depending on the laser facility.

With an order of magnitude sensitivity gain from repeated measurements and data processing, the figures from Tables I and II become more optimistic with feasible scenarios including, e.g., the cw  $P = 200$  W probe beam and a squeezing factor of  $r = 3.5$ .

### F. Boosting the phase sensitivity with SU(1,1) interferometers

On a more speculative note, one can also consider the active interferometers setup [57,58] also called SU(1,1) interferometer where the beam splitters ( $BS_1$  and/or  $BS_2$ ) are replaced by nonlinear crystals acting as OPAs. These types of interferometers have a long history, being first discussed in the 1980s (Wodkiewicz and Eberly [78], Yurke *et al.* [79]). Theoretical phase sensitivity studies [80] as well as experimental demonstrations [81,82] confirmed the better performance of this setup versus the passive Mach-Zehnder, especially when losses are involved. The main drawback of the active SU(1,1) interferometer is its very low operational power, therefore for our purpose it seems not feasible for the moment. Further improvements in its operational power could make it an interesting candidate in the future.

## VI. CONCLUSIONS

In this paper we introduced and discussed an all-optical experimental setup for the detection of the QED-predicted vacuum birefringence. The proposal is based on a Mach-Zehnder interferometer operated between the standard quantum limit and the Heisenberg limit through the use of classical and nonclassical states of light. The vacuum birefringence signal is created by a petawatt-class laser and its signature is a delay (phase shift) detectable at the output of the interferometer.

Multipetawatt-class lasers can induce a significant delay on a counterpropagating probe laser beam, therefore its detection becomes—despite the technological difficulties—foreseeable. Various scenarios have been shown to be feasible, including terawatt pulsed lasers for the probe beam and the use of squeezed states of light.

We conclude by remarking that this experiment does not “consume” at all power from the pump laser. It can be placed upstream, immediately after the compressor of the high power laser system. The petawatt laser pulse, after disturbing the quantum vacuum inside the Mach-Zehnder interferometer, can be used for other downstream experiments.

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## APPENDIX A: DERIVATION OF THE VACUUM REFRACTION INDEX

The effective QED Lagrangian including nonlinearities at all orders (i.e., the Heisenberg-Euler Lagrangian, see, e.g., Eq. (14) in [83]) can be expanded in its lowest order nonlinear contributions yielding the so-called Euler-Kockel Lagrangian [3]

$$\mathcal{L}_{\text{EK}} = \frac{\mathcal{F}}{2} + \kappa(\mathcal{F}^2 + 7\mathcal{G}^2), \quad (\text{A1})$$

where we introduced the relativistic invariants  $\mathcal{F} = \varepsilon_0(E^2 - c^2B^2)$ ,  $\mathcal{G} = \sqrt{\varepsilon_0/\mu_0}(\mathbf{E} \cdot \mathbf{B})$ , and  $X^2 = \mathbf{X} \cdot \mathbf{X}$ , where  $\mathbf{X} \in \{\mathbf{E}, \mathbf{B}\}$ . Also, for compactness we made the notation  $\kappa = 2\alpha^2\hbar^3/45m_e^4c^5$ . One can now write the constitutive relations from the above Lagrangian, i.e.,  $\mathbf{D}_{\text{EK}} = \partial\mathcal{L}_{\text{EK}}/\partial\mathbf{E}$  and  $\mathbf{H}_{\text{EK}} = -\partial\mathcal{L}_{\text{EK}}/\partial\mathbf{B}$  yielding

$$\begin{aligned} \mathbf{D}_{\text{EK}} &= \varepsilon_0(1 + 4\kappa\mathcal{F})\mathbf{E} + 14\varepsilon_0\kappa\mathcal{G}\mathbf{B}, \\ \mathbf{H}_{\text{EK}} &= \frac{1}{\mu_0}(1 + 4\kappa\mathcal{F})\mathbf{B} - 14\varepsilon_0\kappa\mathcal{G}\mathbf{E}. \end{aligned} \quad (\text{A2})$$

The total electric field is the (vectorial) sum of the probe ( $\mathbf{E}_p$ ) and pump electric fields  $\mathbf{E} = \mathbf{E}_p + \mathbf{E}_L$  and the same goes for the magnetic field  $\mathbf{B} = \mathbf{B}_p + \mathbf{B}_L$ . We use the relation  $E_p^2 - c^2B_p^2 = 0$  and retain from Eq. (A2) only the components at the probe frequency (i.e., linear in  $\mathbf{E}_p$  and  $\mathbf{B}_p$ ). Next, following Ref. [20], we use Maxwell’s equations in the classical limit and assume that the monochromatic probe field “sees” a refraction index  $n_{\text{QED}}$  (to be determined). Combining

the previous findings allows one to write an equation whose eigenvalues yield the refraction indices [20]. In the case of two, counterpropagating light beams, one finds the parallel and perpendicular indices of refraction [20,24] seen by the probe beam

$$\begin{aligned} n_{\text{QED},\parallel} &= 1 + 2\xi(E_L^2 + 2cE_LB_L + c^2B_L^2), \\ n_{\text{QED},\perp} &= 1 + \frac{7}{2}\xi(E_L^2 + 2cE_LB_L + c^2B_L^2), \end{aligned} \quad (\text{A3})$$

and since we have for the pump laser beam  $E_L = cB_L$  we arrive at the expression from Eq. (1).

We mention that other, more elaborate, methods to compute the vacuum refraction indices exist, mainly based on the polarization tensor [11,84].

## APPENDIX B: SHORT THEORETICAL DISCUSSION OF THE MZI PHASE SENSITIVITY

Let a Hermitian operator  $\hat{O}$  depend on a parameter we wish to measure, say  $\varphi$ . We can measure the average of this operator as  $\langle\hat{O}(\varphi)\rangle = \langle\psi|\hat{O}(\varphi)|\psi\rangle$ . A small variation  $\delta\varphi$  of the parameter  $\varphi$  causes a change

$$\langle\hat{O}(\varphi + \delta\varphi)\rangle \approx \langle\hat{O}(\varphi)\rangle + \frac{\partial\langle\hat{O}(\varphi)\rangle}{\partial\varphi}\delta\varphi, \quad (\text{B1})$$

thus we have the approximative result  $\langle\hat{O}(\varphi + \delta\varphi)\rangle - \langle\hat{O}(\varphi)\rangle \approx \frac{\partial\langle\hat{O}(\varphi)\rangle}{\partial\varphi}\delta\varphi$ . In order to detect this difference of averages, we have to require that

$$|\langle\hat{O}(\varphi + \delta\varphi)\rangle - \langle\hat{O}(\varphi)\rangle| \geq \Delta\hat{O}(\varphi), \quad (\text{B2})$$

where the standard deviation  $\Delta\hat{O}$  of the operator  $\hat{O}$  is defined as the square root of its variance,  $\Delta^2\hat{O} = \langle\hat{O}^2\rangle - \langle\hat{O}\rangle^2$ . The value of  $\delta\varphi$  that saturates inequality (B2) is called sensitivity (denoted  $\Delta\varphi$  throughout this paper) and is given by

$$\Delta\varphi = \frac{\Delta\hat{O}}{\left|\frac{\partial}{\partial\varphi}\langle\hat{O}\rangle\right|}. \quad (\text{B3})$$

In the difference detection scheme (see Fig. 3), we have the observable  $\hat{N}_d = \hat{a}_4^\dagger\hat{a}_4 - \hat{a}_5^\dagger\hat{a}_5$ . Using standard field operator transformations in a balanced MZI [44]:

$$\begin{aligned} \hat{a}_4^\dagger &= -\sin\left(\frac{\varphi}{2}\right)\hat{a}_0^\dagger + \cos\left(\frac{\varphi}{2}\right)\hat{a}_1^\dagger, \\ \hat{a}_5^\dagger &= \cos\left(\frac{\varphi}{2}\right)\hat{a}_0^\dagger + \sin\left(\frac{\varphi}{2}\right)\hat{a}_1^\dagger \end{aligned} \quad (\text{B4})$$

(some global phase factors have been ignored), we get

$$\hat{N}_d = \cos\varphi(\hat{a}_1^\dagger\hat{a}_1 - \hat{a}_0^\dagger\hat{a}_0) - \sin\varphi(\hat{a}_1^\dagger\hat{a}_0 + \hat{a}_1\hat{a}_0^\dagger). \quad (\text{B5})$$

In the case of the input state vector from Eq. (5) we immediately have  $|\partial\langle\hat{N}_d\rangle/\partial\varphi| = |\sin\varphi||\alpha|^2$  and the variance yields  $\Delta^2\hat{N}_d = |\alpha|^2$ . Applying these results to Eq. (B3) gives  $\Delta\varphi$  from Eq. (16). With a single detector scheme we have the observable  $\hat{N}_4 = \hat{a}_4^\dagger\hat{a}_4$  which, after performing the MZI field operator transformations given by Eq. (B4) yields

$$\hat{N}_4 = \sin^2\left(\frac{\varphi}{2}\right)\hat{a}_0^\dagger\hat{a}_0 + \cos^2\left(\frac{\varphi}{2}\right)\hat{a}_1^\dagger\hat{a}_1 - \frac{\sin\varphi}{2}(\hat{a}_1^\dagger\hat{a}_0 + \hat{a}_1\hat{a}_0^\dagger), \quad (\text{B6})$$



thus  $|\partial\langle\hat{N}_4\rangle/\partial\varphi| = |\sin\varphi||\alpha|^2/2$  and the variance is  $\Delta^2\hat{N}_4 = \cos^2\varphi|\alpha|^2$ . Plugging these results into Eq. (B3) yields  $\Delta\varphi$  from Eq. (17).

In the case of a coherent  $\otimes$  squeezed vacuum input state given by Eq. (9), for a differential detection scheme we have

$$\left|\frac{\partial\langle\hat{N}_d\rangle}{\partial\varphi}\right| = |\sin\varphi||\alpha|^2 - \sinh^2 r| \quad (\text{B7})$$

and a variance

$$\begin{aligned} \Delta^2\hat{N}_d &= \cos^2\varphi(2\sinh^2 r \cosh^2 r + |\alpha|^2) \\ &+ \sin^2\varphi(\sinh^2 r + |\alpha|^2 e^{-r}) \\ &+ \sin^2\varphi|\alpha|^2 \sinh 2r[1 - \cos(2\theta_\alpha)], \end{aligned} \quad (\text{B8})$$

where  $\theta_\alpha$  is the phase of the coherent light. Squeezed states are typically characterized by  $\xi = r e^{i\theta}$  [44], however, for simplicity, we considered  $\xi = r \in \mathbb{R}^+$ , therefore the phase of the squeezed vacuum was taken to be  $\theta = 0$ . The minimum

variance is obtained if  $\theta_\alpha = 0$  and using again Eq. (B3) we arrive at the phase sensitivity given by Eq. (19). For a single detection scheme we have

$$\left|\frac{\partial\langle\hat{N}_4\rangle}{\partial\varphi}\right| = \frac{|\sin\varphi|}{2}||\alpha|^2 - \sinh^2 r| \quad (\text{B9})$$

and a variance

$$\begin{aligned} \Delta^2\hat{N}_4 &= \sin^4\left(\frac{\varphi}{2}\right)\frac{\sinh^2 2r}{2} + \cos^4\left(\frac{\varphi}{2}\right)|\alpha|^2 \\ &+ \frac{\sin^2\varphi}{4}\left(\sinh^2 r + \frac{|\alpha|^2}{e^{2r}}\right) \\ &+ \frac{\sin^2\varphi}{4}\sinh 2r|\alpha|^2(1 - \cos 2\theta_\alpha). \end{aligned} \quad (\text{B10})$$

Setting again  $\theta_\alpha = 0$  and employing the sensitivity definition from Eq. (B3) takes us to the result (21).

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