

Quantum teleportation in vacuum using only Unruh-DeWitt detectors

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We consider entanglement extraction into two two-level Unruh-DeWitt detectors from a vacuum of a neutral massless quantum scalar field in a four-dimensional space-time, where the general monopole coupling to the scalar field is assumed. Based on the reduced density matrix of the two detectors derived within the perturbation theory, we show that *the single copy* of the entangled pair of the detectors can be utilized in quantum teleportation, even when the detectors are separated acausally, while we observe no violation of the Bell–CHSH inequality. In the case of the Minkowski vacuum, in particular, we find that entanglement usable in quantum teleportation is extracted due to the special relativistic effect when the detectors are in a relative inertial motion, while it is not when they are comoving inertially and the switching of the detectors is executed adiabatically at infinite past and future.

DOI: [10.1103/PhysRevA.97.062338](https://doi.org/10.1103/PhysRevA.97.062338)**I. INTRODUCTION**

Quantum entanglement in the relativistic quantum field theory is a developing field of research. From the point of view of theoretical physics, in particular, the information loss paradox in black hole physics (see, e.g., Refs. [1,2]) and the entanglement entropy in the context of the correspondence between an anti-de Sitter space-time and a conformal field theory [3] have attracted much attention recently.

It has been shown by Summers and Werner [4–7] that, with *suitable* observables in local space-time regions, the Bell–Clauser-Holt-Shimony-Horne (CHSH) inequality is maximally violated in a vacuum of any quantum field theory, which has led to the observation that the vacuum intrinsically contains the “nonlocal” correlations, hence also entanglement, that cannot be explained from a local realistic view [8–10]. Then, much effort has been made to analyze entanglement extraction from a vacuum with detectors so that it can be useful for several quantum information processing methods. (For entanglement extraction from the Minkowski vacuum, see, e.g., [11–31].) In particular, following the pioneering paper by Reznik [16], two-level Unruh-DeWitt detectors [32–35] are frequently employed to analyze entanglement extracted from a vacuum, where we can apply the well-established results in quantum information theory for qubits. In such extraction of entanglement, which is called harvesting, Unruh-DeWitt detectors are assumed to be localized not only spatially but also temporally. In particular, Reznik [16] considered Unruh-DeWitt detectors that interact

with a quantum field for a strictly finite period so that the future light cones of the detectors do not intersect within the period of interaction. It thus has revealed that entanglement is generated between detectors even if they are separated acausally, i.e., located at causally disconnected regions, and has corroborated the result by Summers and Werner [4–7]. Since the maximally entangled state can be distilled from an ensemble of any two-qubit entangled states [36], one can in principle make use of the extracted entanglement for some information processes such as quantum teleportation. Notice, however, such distillation requires the preparation of infinitely many copies of vacua and detectors, which may be rather unrealistic. Therefore, it is still meaningful to ask whether the single copy of the entangled state has potential abilities, especially in the case where the detectors are separated acausally. The first purpose in this paper is then to understand in a general context the usability of the entanglement between the Unruh-DeWitt detectors coupled to a neutral massless quantum scalar field through a monopole coupling, i.e., without internal structures of a detector. In particular, we will show that although the entanglement does not violate the Bell-CHSH inequality, a quantum teleportation with the use of the single copy of the entangled Unruh-DeWitt detectors is still possible. To see this, we will assume neither the geometry of a space-time, the form of the monopole coupling, the world lines of the detectors, nor the switching functions of the detectors.

The notion of the Unruh-DeWitt detector has been introduced as a theoretical device that probes the nature of a quantum state. In the Minkowski vacuum, in particular, it detects no excitation when it is carried by an inertial observer, and its spectrum is thermal if it is carried by a uniformly accelerated observer. The latter particularly has corroborated the observation that the Minkowski vacuum looks

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to be a thermal bath for a uniformly accelerated observer, which is the so-called Unruh effect [32]. These results follow when one switches on and off the detector adiabatically at infinite past and future, which is implicitly assumed in the textbooks [33,34], and computes the excitation probability by considering practically detectors that interact with a quantum field for infinitely long time, and thus globally in time. As for entanglement, one might expect that a sufficiently long interaction time will naturally enable entanglement extraction. This is not necessarily the case, however. In the realistic model [19–22], for example, each of a pair of atoms with the electric dipole \mathbf{d} is coupled with the electric field \mathbf{D} , whose interaction Hamiltonian is given as $H_I = -(1/\epsilon_0)\mathbf{d} \cdot \mathbf{D}$, and the initial quantum state of the electromagnetic field is set to be the vacuum, i.e., without applying any external electric field. If the initial states of the atoms are prepared to be the ground state, it might be conceivable that the uncertainty relation in time and energy suppresses quantum fluctuation after sufficiently long time, the energy conservation being recovered, and the whole system returns back to the ground state. Then, the interaction might be expected as ineffective. In fact, as we will show in this paper, when two Unruh-DeWitt detectors are comoving inertially and interact with a quantum field for infinitely long time, entanglement is *not* generated between the Unruh-DeWitt detectors. Thus, the second purpose of this paper is to demonstrate it explicitly and further explore noncomoving inertial motions of Unruh-DeWitt detectors that interact with a quantum field for infinitely long time.

We will thus consider in this paper two two-level Unruh-DeWitt detectors coupled to a neutral massless quantum scalar field with a general monopole coupling, where the initial states of the detectors are prepared to be the ground state and the initial state of the scalar field is set to be a vacuum. The model is presented in Sec. II, where we will also derive the reduced density matrix of the two Unruh-DeWitt detectors in an arbitrary four-dimensional space-time in the perturbation theory, without assuming particular forms of the world lines of the detectors or the switching functions. Based on the reduced density matrix, in Sec. III, we will compute several entanglement measures, which include the bounds on the distillable entanglement, the entanglement cost, and the Bell-CHSH inequality. In particular, we will see the supremacy of the extracted entanglement in quantum teleportation with the single entangled pair, compared to the teleportation only via classical channels. In Sec. IV, we will turn to the issue of entanglement generated between inertial Unruh-DeWitt detectors with adiabatic switching. To perform explicit computation, we need to specify a space-time and a vacuum, and thus we will focus on the Minkowski vacuum. We will consider not only the case of infinitely long interaction with switching executed implicitly at infinite past and future, but also the case where the effect of switching is taken into account. We will conclude and discuss in Sec. V, which includes a preliminary result for the case where one of the detectors runs with uniform acceleration. Throughout this paper, we adopt natural units $c = \hbar = 1$.

II. REDUCED DENSITY MATRIX OF TWO DETECTORS

We consider two two-level Unruh-DeWitt detectors, A carried by Alice and B by Bob, with the discrete energy

eigenvalues $E_n^{(A)}$ and $E_n^{(B)}$, respectively, where $n = 0, 1$, which are thus considered as two qubits. The excitation energy $\Delta E^{(I)} \equiv E_1^{(I)} - E_0^{(I)}$ is assumed to be positive $\Delta E^{(I)} > 0$, where the index I stands for A or B , here and hereafter. We denote the coordinate variables of the world lines of the detectors with a bar as $\bar{x}_I^\mu(\tau_I)$, where τ_I is the proper time of the detector I . These detectors are assumed to be coupled with a neutral massless quantum scalar field $\phi(x)$, with the coupling being governed by the perturbation action

$$\begin{aligned} \mathcal{S}_{\text{int}} = & \int c \chi_A(\tau_A) m_A(\tau_A) \phi(\bar{x}_A) d\tau_A \\ & + \int c \chi_B(\tau_B) m_B(\tau_B) \phi(\bar{x}_B) d\tau_B, \end{aligned} \quad (1)$$

where c is the coupling constant, $m_I(\tau_I)$ is the monopole operator of the detector I , which commutes with that of the other detector and with the scalar field $\phi(\bar{x}_I(\tau_I))$. Note that we do not assume any particular form of the monopole coupling, guaranteeing the generality of the following discussion. The switching function $\chi_I(\tau_I)$ describes how the coupling between the detectors and the scalar field is implemented as a function of the proper time τ_I .

We choose the initial quantum state $|\text{in}\rangle$ of the whole system at infinite past as

$$|\text{in}\rangle = |0\rangle |E_0^{(A)}\rangle |E_0^{(B)}\rangle, \quad (2)$$

where $|0\rangle$ is the vacuum of the scalar field and $|E_n^{(I)}\rangle$ is the n th state of the detector I . Then, the quantum state in the asymptotic future is given by

$$|\text{out}\rangle = T e^{i\mathcal{S}_{\text{int}}} |\text{in}\rangle, \quad (3)$$

where T stands for time ordering. Since we are interested in the state of the two Unruh-DeWitt detectors, we trace out the degrees of freedom of the scalar field ϕ . As is shown in Appendix A, the reduced density matrix ρ_{AB} of the two detectors is then derived from the perturbation theory as

$$\begin{aligned} \rho_{AB} \equiv & \text{Tr}_\phi |\text{out}\rangle \langle \text{out}| \\ = & \begin{pmatrix} 0 & 0 & 0 & c^2 \mathcal{E} \\ 0 & c^2 \mathcal{P}_A & c^2 \mathcal{P}_{AB} & c^2 \mathcal{W}_A \\ 0 & c^2 \mathcal{P}_{AB}^* & c^2 \mathcal{P}_B & c^2 \mathcal{W}_B \\ c^2 \mathcal{E}^* & c^2 \mathcal{W}_A^* & c^2 \mathcal{W}_B^* & 1 - c^2(\mathcal{P}_A + \mathcal{P}_B) \end{pmatrix} \\ & + O(c^4), \end{aligned} \quad (4)$$

in the bases of $\{|E_1^{(A)}\rangle |E_1^{(B)}\rangle, |E_1^{(A)}\rangle |E_0^{(B)}\rangle, |E_0^{(A)}\rangle |E_1^{(B)}\rangle, |E_0^{(A)}\rangle |E_0^{(B)}\rangle\}$, where $*$ stands for the complex conjugate. Among the matrix elements in Eq. (4), \mathcal{P}_I and \mathcal{E} are given by

$$\begin{aligned} \mathcal{P}_I &= \langle E_1^{(I)} | m_I(0) | E_0^{(I)} \rangle^2 \mathcal{I}_I, \\ \mathcal{E} &= \langle E_1^{(B)} | m_B(0) | E_0^{(B)} \rangle \langle E_1^{(A)} | m_A(0) | E_0^{(A)} \rangle \mathcal{I}_E, \end{aligned} \quad (5)$$

where \mathcal{I}_I and \mathcal{I}_E are defined as

$$\mathcal{I}_I \equiv \int_{-\infty}^{\infty} d\tau'_I \int_{-\infty}^{\infty} d\tau_I \chi_I(\tau'_I) \chi_I(\tau_I) e^{i\Delta E^{(I)}(\tau_I - \tau'_I)} G_W(\bar{x}'_I, \bar{x}_I), \quad (6)$$

$$\begin{aligned} \mathcal{I}_E \equiv & -i \int_{-\infty}^{\infty} d\tau_B \int_{-\infty}^{\infty} d\tau'_A \chi_B(\tau_B) \chi_A(\tau'_A) e^{i\Delta E^{(B)}\tau_B} e^{i\Delta E^{(A)}\tau'_A} \\ & \times G_F(\bar{x}_B, \bar{x}'_A), \end{aligned} \quad (7)$$

and $G_W(x, x')$ and $G_F(x, x')$ are the Wightman function and the Feynman propagator, defined as

$$G_W(x, x') \equiv \langle 0 | \phi(x) \phi(x') | 0 \rangle, \quad (8)$$

$$G_F(x, x') \equiv -i \langle 0 | T \phi(x) \phi(x') | 0 \rangle. \quad (9)$$

We emphasize here that \mathcal{P}_I is nothing but the excitation probability of the detector I from the ground state to the excited state, and thus we have

$$\mathcal{P}_I \geq 0. \quad (10)$$

We note also that Eq. (7) is rewritten in terms of the Wightman function and the retarded Green function, as Eq. (A11) in Appendix A, by using the relation among the Green functions (A5). The forms of other elements in Eq. (4) are given in Appendix A. Note that the density matrix (4) generalizes those derived in [16,17,23,24,26,31] in the sense that \mathcal{W}_I appears in our case. Interestingly, we will see that \mathcal{W}_I does not play any role in this paper, similarly to the case where the initial state of the scalar field is set to the coherent state and the detectors have spacial extension but with a restricted form of the monopole coupling [29,30].

As we describe in Appendix A, the reduced density matrix (4) is derived by employing only the relations between the Green functions derived from Eqs. (8) and (9), along with the condition that the world lines of the detectors are causal, i.e., the time coordinate $t(\tau)$ of a single timelike world line is a monotonically increasing function of the proper time τ and hence

$$\begin{aligned} \Theta(\tau - \tau') \Theta(t(\tau) - t(\tau')) &= \Theta(\tau - \tau'), \\ \Theta(\tau - \tau') \Theta(t(\tau') - t(\tau)) &= 0, \end{aligned} \quad (11)$$

where τ and τ' are proper times along the single timelike world line, and thus we see that the reduced density matrix given by Eq. (4) is valid for arbitrary timelike world lines of the detectors in an arbitrary space-time, once a vacuum $|0\rangle$ is well defined, not necessarily uniquely.

The eigenvalues of the reduced density matrix (4) are derived by using Eq. (B3) in Appendix B as

$$\begin{aligned} & 1 + O(c^2), \\ & \frac{c^2}{2} [\mathcal{P}_A + \mathcal{P}_B \pm \sqrt{(\mathcal{P}_A - \mathcal{P}_B)^2 + 4|\mathcal{P}_{AB}|^2}] + O(c^4), \\ & c^4(\mathcal{X} - |\mathcal{E}|^2) + O(c^6), \end{aligned} \quad (12)$$

where \mathcal{X} arises in the contribution of order of c^4 as

$$c^4 \mathcal{X} \equiv \langle E_1^{(A)} | \langle E_1^{(B)} | \rho_{AB} | E_1^{(A)} \rangle | E_1^{(B)} \rangle. \quad (13)$$

From the positivity of the density matrix ρ_{AB} and Eq. (10), we have

$$\mathcal{P}_A \mathcal{P}_B \geq |\mathcal{P}_{AB}|^2, \quad \text{and} \quad \mathcal{X} \geq |\mathcal{E}|^2. \quad (14)$$

The reduced density matrix ρ_B of the detector B is obtained by further tracing out over the states of the detector A , as

$$\rho_B \equiv \text{Tr}_A \rho_{AB} = \begin{pmatrix} c^2 \mathcal{P}_B & c^2 \mathcal{W}_B \\ c^2 \mathcal{W}_B^* & 1 - c^2 \mathcal{P}_B \end{pmatrix}, \quad (15)$$

whose eigenvalues are derived as

$$1 - c^2 \mathcal{P}_B + O(c^4), \quad c^2 \mathcal{P}_B + O(c^4). \quad (16)$$

III. PROPERTIES OF EXTRACTED ENTANGLEMENT

Based on the reduced density matrix (4) of the two Unruh-DeWitt detectors, we here consider the possibility of entanglement extraction and its general properties by computing entanglement measures.

The necessary and sufficient condition for a two-qubit system to be entangled, as two two-level Unruh-DeWitt detectors, is given by the famous positive partial transpose (PPT) criterion [37,38]: a two-qubit state ρ_{AB} is entangled if and only if its partial transpose has negative eigenvalues. The partial transpose $\rho_{AB}^{T_A}$ of ρ_{AB} with respect to the detector A is given as

$$\begin{aligned} \rho_{AB}^{T_A} = & \begin{pmatrix} 0 & 0 & 0 & c^2 \mathcal{P}_{AB}^* \\ 0 & c^2 \mathcal{P}_A & c^2 \mathcal{E}^* & c^2 \mathcal{W}_A^* \\ 0 & c^2 \mathcal{E} & c^2 \mathcal{P}_B & c^2 \mathcal{W}_B \\ c^2 \mathcal{P}_{AB} & c^2 \mathcal{W}_A & c^2 \mathcal{W}_B^* & 1 - c^2(\mathcal{P}_A + \mathcal{P}_B) \end{pmatrix} \\ & + O(c^4). \end{aligned} \quad (17)$$

The eigenvalues of $\rho_{AB}^{T_A}$ are derived by applying again Eq. (B3) in Appendix B as

$$\begin{aligned} & 1 + O(c^2), \\ & \frac{c^2}{2} [\mathcal{P}_A + \mathcal{P}_B \pm \sqrt{(\mathcal{P}_A - \mathcal{P}_B)^2 + 4|\mathcal{E}|^2}] + O(c^4), \\ & c^4(\mathcal{X} - |\mathcal{P}_{AB}|^2) + O(c^6). \end{aligned} \quad (18)$$

By noting Eq. (10) again, the condition for the two detectors to be entangled is then described as

$$\mathcal{P}_A \mathcal{P}_B < |\mathcal{E}|^2 \quad (19a)$$

or

$$\mathcal{X} < |\mathcal{P}_{AB}|^2, \quad (19b)$$

where \mathcal{X} is defined in Eq. (13). It is worth mentioning that these conditions coincide exactly with those derived by Reznik for the restricted form of monopole coupling [16], where \mathcal{W}_I is absent; hence those conditions are general enough for the extraction of entanglement from a vacuum as long as one considers a monopole coupling.

We notice that conditions (19a) and (19b) are not compatible with each other, and hence either of them, but not both, is the condition for the two detectors to be entangled. Indeed, by taking into account the positivity (14) of the reduced density matrix ρ_{AB} , we find that Eq. (19a) gives

$$\mathcal{X} \geq |\mathcal{E}|^2 > \mathcal{P}_A \mathcal{P}_B \geq |\mathcal{P}_{AB}|^2, \quad (20)$$

which contradicts the second condition (19b), while Eq. (19b) yields

$$\mathcal{P}_A \mathcal{P}_B \geq |\mathcal{P}_{AB}|^2 > \mathcal{X} \geq |\mathcal{E}|^2, \quad (21)$$

which is incompatible with the first condition (19a).

In order to discuss the nature of extracted entanglement, we shall compute below several entanglement measures for arbitrary switching functions and arbitrary world lines of the detectors in an arbitrary space-time. In the first two sections, we will consider the bounds on the distillable entanglement and the entanglement cost. In the third section, we will turn to the fundamental issue of the Bell-CHSH inequality and investigate whether Unruh-DeWitt detectors are suitable in the sense of Summers and Werner [4–7] in the general context. Note that the distillable entanglement and the entanglement cost assume many copies of entangled states, which would be rather unrealistic in our case since one needs to prepare many vacuum states and many pairs of Unruh-DeWitt detectors. Motivated by this, in the last section we will consider the quantum teleportation that is implemented with the single copy of entangled detectors and compute the teleportation fidelity.

A. Bounds on distillable entanglement—Negativity and coherent information

The distillability of singlet states (two-qubit maximally entangled states) is important for many applications, such as

$$\mathcal{N}(\rho_{AB}) = \begin{cases} -\frac{c^2}{2}[\mathcal{P}_A + \mathcal{P}_B - \sqrt{(\mathcal{P}_A - \mathcal{P}_B)^2 + 4|\mathcal{E}|^2}] + O(c^4) & \text{for condition (19a),} \\ c^4(|\mathcal{P}_{AB}|^2 - \mathcal{X}) + O(c^6) & \text{for condition (19b).} \end{cases} \quad (23)$$

Therefore, this gives an upper bound of possible distillation (22) of singlet states from our extracted entanglement.

Surely, a more interesting estimation is a lower bound of $E_D(\rho_{AB})$, as it will guarantee the amount of extraction of singlet states at lowest. It was shown [41] that the coherent information $I_c(A \rangle B)$ (from B to A) gives a lower bound of the one-way entanglement capacity $D_{\rightarrow}(\rho_{AB})$, i.e., entanglement distillation with one-way communication:

$$E_D(\rho_{AB}) \geq D_{\rightarrow}(\rho_{AB}) \geq I_c(A \rangle B). \quad (24)$$

The coherent information $I_c(A \rangle B)$ of a state ρ_{AB} is defined by

$$I_c(A \rangle B) \equiv H(\rho_B) - H(\rho_{AB}), \quad (25)$$

where $H(\rho) \equiv -\text{Tr} \rho \log_2 \rho$ is the von Neumann entropy, and thus $I_c(A \rangle B)$ coincides with the negative of the conditional entropy $I_c(A \rangle B) = -H(A|B)$. From Eqs. (12) and (16), we obtain, for the reduced density matrix (4),

$$I_c(A \rangle B) = 2 \mathcal{P}_A c^2 \log_2 |c| + O(c^2). \quad (26)$$

Since $|c| < 1$ within the perturbation theory and \mathcal{P}_A is positive (10), we find that $I_c(A \rangle B)$ is negative. Therefore, the fact that the excitation probability is non-negative unfortunately prohibits us from obtaining a meaningful lower bound for entanglement distillation.

quantum key distribution and quantum teleportation. While there generally exists an entangled state from which no singlet states can be extracted, i.e., a bound entangled state, it is always possible to distill singlet states from many copies of an arbitrary two-qubit entangled state [36]. Hence, in principle, one can distill singlet states from the extracted entangled states also in our case. However, beyond such a qualitative discussion, we still need to give a quantitative estimation for the distillability. The distillable entanglement $E_D(\rho_{AB})$, defined asymptotically ($n \rightarrow \infty$) as the optimal rate m/n to extract m copies of a singlet state from n copies of a states ρ_{AB} through the local operations and classical communication (LOCC), provides such an operationally motivated measure. Unfortunately, the measure is generally known to be difficult to compute even for two-qubit cases. So, here we shall focus on other computable measures that give an upper bound and a lower bound of the distillable entanglement.

The negativity $\mathcal{N}(\rho_{AB})$ of a density operator ρ_{AB} is defined as minus the sum of the negative eigenvalues of its partial transpose $\rho_{AB}^{T_A}$ [39]. Moreover, the logarithmic negativity $\log_2[2\mathcal{N}(\rho_{AB}) + 1]$ gives an upper bound of the distillable entanglement [40]:

$$E_D(\rho_{AB}) \leq \log_2(2\mathcal{N}(\rho_{AB}) + 1). \quad (22)$$

For the reduced density matrix derived in Eq. (4), we immediately obtain from Eqs. (18) and (19),

B. Entanglement cost—Entanglement of formation and concurrence

Another operationally motivated measure of entanglement is the entanglement of cost $E_C(\rho_{AB})$, which is defined as the optimal rate to cost copies of a singlet state in order to obtain the copies of a state ρ_{AB} . The related measure is the entanglement of formation $E_F(\rho_{AB})$ [42]:

$$E_F(\rho_{AB}) \equiv \inf \sum_j p_j E(\phi_j), \quad (27)$$

where \inf is taken over all possible decompositions into pure states $|\phi_j\rangle$ as $\rho_{AB} = \sum_j p_j |\phi_j\rangle\langle\phi_j|$, and $E(\phi_j)$ is the entropy of entanglement of a pure state $|\phi_j\rangle$. In particular, $E_C(\rho_{AB})$ and $E_F(\rho_{AB})$ are related as

$$E_C(\rho_{AB}) = \lim_{n \rightarrow \infty} \frac{E_F(\rho_{AB}^{\otimes n})}{n}. \quad (28)$$

Although both $E_C(\rho_{AB})$ and $E_F(\rho_{AB})$ are generally difficult to compute, it is known [43] in the case of a two-qubit system that $E_F(\rho_{AB})$ is given by a computable quantity, the concurrence $C(\rho_{AB})$, through the formula

$$E_F(\rho_{AB}) = h\left(\frac{1 + \sqrt{1 - C^2(\rho_{AB})}}{2}\right), \quad (29)$$

where $h(x) \equiv -x \log_2 x - (1-x) \log_2(1-x)$ is the binary entropy. The concurrence $C(\rho_{AB})$ for a two-qubit state ρ_{AB} is defined by

$$C(\rho_{AB}) \equiv \max[0, \tilde{\lambda}_1 - \tilde{\lambda}_2 - \tilde{\lambda}_3 - \tilde{\lambda}_4], \quad (30)$$

where $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3$, and $\tilde{\lambda}_4$ are the square roots of the eigenvalues of $\rho_{AB} \tilde{\rho}_{AB}$ in the descending order and $\tilde{\rho}_{AB}$ is defined as

$$\tilde{\rho}_{AB} \equiv \sigma_y \otimes \sigma_y \rho_{AB}^* \sigma_y \otimes \sigma_y. \quad (31)$$

The square roots of the eigenvalues of $\rho_{AB} \tilde{\rho}_{AB}$ for the reduced density matrix (4) are calculated from the eigenvalue

equation (B5) in Appendix B as

$$c^2(\sqrt{\mathcal{P}_A \mathcal{P}_B} \pm |\mathcal{P}_{AB}|) + O(c^4), \quad c^2(\sqrt{\mathcal{X}} \pm |\mathcal{E}|) + O(c^4). \quad (32)$$

As we noticed in Eq. (20), when $|\mathcal{E}| > \sqrt{\mathcal{P}_A \mathcal{P}_B}$, we have $\sqrt{\mathcal{X}} > |\mathcal{P}_{AB}|$, and hence $\sqrt{\mathcal{X}} + |\mathcal{E}| > \sqrt{\mathcal{P}_A \mathcal{P}_B} + |\mathcal{P}_{AB}|$. In this case, the maximal eigenvalue $\tilde{\lambda}_1$ is found to be $\tilde{\lambda}_1 = c^2(\sqrt{\mathcal{X}} + |\mathcal{E}|) + O(c^4)$. When $\sqrt{\mathcal{X}} < |\mathcal{P}_{AB}|$, on the other hand, we have $\sqrt{\mathcal{P}_A \mathcal{P}_B} > |\mathcal{E}|$ from Eq. (21), which implies $\sqrt{\mathcal{P}_A \mathcal{P}_B} + |\mathcal{P}_{AB}| > \sqrt{\mathcal{X}} + |\mathcal{E}|$, and thus $\tilde{\lambda}_1 = c^2(\sqrt{\mathcal{P}_A \mathcal{P}_B} + |\mathcal{P}_{AB}|) + O(c^4)$. Therefore, the concurrence $C(\rho_{AB})$ associated with the reduced density matrix (4) is computed as

$$C(\rho_{AB}) = \begin{cases} 2c^2(|\mathcal{E}| - \sqrt{\mathcal{P}_A \mathcal{P}_B}) + O(c^4) & \text{for condition (19a),} \\ 2c^2(|\mathcal{P}_{AB}| - \sqrt{\mathcal{X}}) + O(c^4) & \text{for condition (19b).} \end{cases} \quad (33)$$

It is interesting to notice that when the second condition (19b) for entanglement holds, the contribution to the concurrence is at order of c^2 while the contribution to the negativity is at order of c^4 [see Eq. (23)]. On the other hand, when the first condition (19a) is met, we see from Eqs. (23) and (33) that in the particular case of $\mathcal{P}_A = \mathcal{P}_B$, which we thus denote as \mathcal{P}_I , the concurrence and the negativity are related and given as

$$C(\rho_{AB}) = 2\mathcal{N}(\rho_{AB}) = 2c^2(|\mathcal{E}| - \mathcal{P}_I) + O(c^4). \quad (34)$$

The same relation has been derived in Ref. [26] for a restricted form of the monopole coupling. We will use Eq. (34) in the last section below.

C. Bell-CHSH inequality

In this section, we shall ask whether the entanglement generated between the Unruh-DeWitt detectors can be explained by classical means, i.e., by the local realism. Indeed, Summers and Werner [4–7] showed the maximal violation of the Bell-CHSH inequality in a vacuum of any quantum field theory, which implies that a vacuum intrinsically includes the nonlocality that cannot be explained by any local realistic means. If one could observe the nonlocality in entanglement generated between detectors separated acausally, it would provide an indirect corroboration of their results. Notice, however, that the presence of entanglement does not necessary imply the nonlocality, since there is an entangled state the statistics of which can be still reproducible by some local hidden variable models [10,44]. Therefore, we still need to check the violation of a Bell inequality for the extracted entanglement. Here, we focus on the so-called Bell-CHSH inequality, which gives not only the necessary condition for the existence of the local hidden variable models [9], but also the sufficient condition [45] in the CHSH setting, i.e., the physical setting where there are two-valued dichotomic measurements in the bipartite system.

For a two-qubit state ρ_{AB} , letting the ‘‘optimal’’ CHSH quantity [46] be

$$\beta_{\text{CHSH}}(\rho_{AB}) \equiv \max_{a, a', b, b'} \text{Tr} \rho_{AB} [\sigma_a \otimes (\sigma_b + \sigma_{b'}) + \sigma_{a'} \otimes (\sigma_b - \sigma_{b'})], \quad (35)$$

where the maximization in Eq. (35) is taken over all spin observables $\sigma_a, \sigma_{a'}$ for one qubit and $\sigma_b, \sigma_{b'}$ for the other, the Bell-CHSH inequality is described as

$$\beta_{\text{CHSH}}(\rho_{AB}) \leq 2. \quad (36)$$

One can show [46] that

$$\beta_{\text{CHSH}}(\rho_{AB}) = 2\sqrt{M(\rho_{AB})}, \quad (37)$$

where $M(\rho_{AB})$ is the sum of the two greatest eigenvalues of

$$U_{\rho_{AB}} \equiv T_{\rho_{AB}}^T T_{\rho_{AB}}, \quad (38)$$

and $T_{\rho_{AB}}$ is the 3×3 matrix defined by

$$(T_{\rho_{AB}})_{ij} \equiv \text{Tr}(\rho_{AB} \sigma_i \otimes \sigma_j). \quad (39)$$

From the eigenvalue equation (B7) in Appendix B, the eigenvalues of the matrix $U_{\rho_{AB}}$ for the reduced density matrix (4) are derived as

$$1 - 4c^2(\mathcal{P}_A + \mathcal{P}_B) + O(c^4), \quad 4c^4(|\mathcal{E}| \pm |\mathcal{P}_{AB}|)^2 + O(c^6). \quad (40)$$

Consequently, we have

$$M(\rho_{AB}) = 1 - 4(\mathcal{P}_A + \mathcal{P}_B)c^2 + O(c^4), \quad (41)$$

and hence we obtain from Eq. (37),

$$\beta_{\text{CHSH}}(\rho_{AB}) = 2[1 - 2(\mathcal{P}_A + \mathcal{P}_B)c^2] + O(c^4) \leq 2. \quad (42)$$

Therefore, we see that the entanglement generated between the two Unruh-DeWitt detectors does not violate the Bell-CHSH inequality (36). The same conclusion was observed in [17,31]. We note again that our interaction is more general than those used there; hence the derived result here shows the fact of nonviolation of the Bell-CHSH inequality of the extracted entanglement in a more general context. Notice, however, that this result is still not sufficient to conclude that the extracted entanglement can be explainable by local realism. Indeed, the optimization in Eq. (35) is only through spin observables, while there are more general positive operator valued measure (POVM) measurements. Moreover, as is noticed above, the

Bell-CHSH inequality gives the sufficient condition for the local realism only in the restriction to the CHSH setting.¹ In this restricted sense, however, we see again that the non-negativity of the excitation probability (10) prevents the Bell-CHSH inequality from being violated, and hence Unruh-DeWitt detectors are found not to be suitable detectors in the sense of Summers and Werner [4–7], also for the general monopole coupling.

D. Optimal fidelity of teleportation

In this section, we consider the possibility of quantum teleportation only via the single copy of the entangled detectors. To do this, we compare the optimal teleportation fidelity with entanglement to that achievable only via classical channels. The (averaged) teleportation fidelity is given by

$$F = \int dM(\phi) \sum_k p_k \langle \phi | \rho_k | \phi \rangle, \quad (43)$$

where the integral is over the quantum state $|\phi\rangle$ to be teleported from the sender with the unitary invariant measure dM , and ρ_k is the state teleported to the receiver given the outcome k of the sender's measurement with the probability p_k . Let $F(\rho_{AB})$ denote the fidelity of teleportation using an entangled state

$$\text{Tr}\sqrt{U_{\rho_{AB}}} = \begin{cases} 1 + 4c^2 \left[|\mathcal{E}| - \frac{\mathcal{P}_A + \mathcal{P}_B}{2} \right] + O(c^4) & \text{for } |\mathcal{E}| > |\mathcal{P}_{AB}| \\ 1 + 4c^2 \left[|\mathcal{P}_{AB}| - \frac{\mathcal{P}_A + \mathcal{P}_B}{2} \right] + O(c^4) & \text{for } |\mathcal{E}| < |\mathcal{P}_{AB}|. \end{cases} \quad (46)$$

If the second condition (19b) for entanglement holds, and hence Eq. (21) is valid, we have $|\mathcal{E}| < |\mathcal{P}_{AB}|$. In this case, we have

$$\text{Tr}\sqrt{U_{\rho_{AB}}} = 1 + 4c^2 \left(|\mathcal{P}_{AB}| - \frac{\mathcal{P}_A + \mathcal{P}_B}{2} \right) + O(c^4). \quad (47)$$

However, Eq. (21) and $\sqrt{\mathcal{P}_A \mathcal{P}_B} \leq \frac{\mathcal{P}_A + \mathcal{P}_B}{2}$ show that condition (45) fails. Therefore, the entanglement extracted due to the second condition (19b) is not useful in quantum teleportation.

On the other hand, when the first condition (19a) is satisfied, and hence Eq. (20) is valid, we have $|\mathcal{E}| > |\mathcal{P}_{AB}|$. In this case, we have

$$\text{Tr}\sqrt{U_{\rho_{AB}}} = 1 + 4c^2 \left(|\mathcal{E}| - \frac{\mathcal{P}_A + \mathcal{P}_B}{2} \right) + O(c^4). \quad (48)$$

In particular, when the two detectors are symmetric in the sense $\mathcal{P}_A = \mathcal{P}_B$, which we write as \mathcal{P}_I , Eq. (20) shows $|\mathcal{E}| > \frac{\mathcal{P}_A + \mathcal{P}_B}{2} = \mathcal{P}_I$. Hence the condition (45) holds, implying that the extracted entangled state is indeed useful for the standard teleportation. Moreover, since the negativity $\mathcal{N}(\rho_{AB})$ and the

ρ_{AB} . The Horodecki family showed [49] the optimal fidelity for a two-qubit system is given as

$$F_{\max}(\rho_{AB}) = \frac{1}{2} \left(1 + \frac{1}{3} \text{Tr}\sqrt{U_{\rho_{AB}}} \right), \quad (44)$$

where $U_{\rho_{AB}}$ is given by Eq. (38). Here, we are concerned only with the standard teleportation, and then the optimization is over all local unitary operations on the receiver's side while the Bell measurement on the sender's side is fixed. (See [50] for general protocols based on LOCC.) On the other hand, one can show [50,51] that the classically achievable fidelity is $F_{cl} = 2/3$. Hence, the condition for the supremacy of teleportation using entanglement over classical channels is given by $F_{\max}(\rho_{AB}) > 2/3$, which turns out to be

$$\text{Tr}\sqrt{U_{\rho_{AB}}} > 1. \quad (45)$$

Note that while the violation of the Bell-CHSH inequality is sufficient to obtain the fidelity of the standard quantum teleportation larger than its classical value [49], the converse does not hold in general. Thus, in spite of the previous result in Sec. III C, it is still meaningful to investigate here whether the entanglement generated between the Unruh-DeWitt detectors can be utilized in quantum teleportation.

By summing the square roots of eigenvalues (40), we find, for the reduced density matrix (4),

concurrence $C(\rho_{AB})$ in this case are given by Eq. (34), we have

$$\begin{aligned} F_{\max}(\rho_{AB}) &= \frac{2}{3} + \frac{2}{3} \mathcal{N}(\rho_{AB}) + O(c^4) \\ &= \frac{2}{3} + \frac{1}{3} C(\rho_{AB}) + O(c^4). \end{aligned} \quad (49)$$

Therefore, we have shown that the extracted entanglement with the first condition (19a) for the symmetric case is useful actually for the standard teleportation only with the use of the single copy of the entangled Unruh-DeWitt detectors. Furthermore, we found the interesting characterization (49) of the optimal fidelity expressed in terms of the negativity and the concurrence.

IV. INERTIAL MOTIONS IN MINKOWSKI VACUUM

Now that we have seen that entanglement generated between Unruh-DeWitt detectors, if any, is useful in quantum teleportation, our interest is then whether and how entanglement is generated between the Unruh-DeWitt detectors. In this section, we focus on inertial motions of Unruh-DeWitt detectors in the Minkowski vacuum, where the Wightman function $G_W(x, x')$ of a neutral massless scalar field is given by

$$G_W(x, x') = \frac{-1}{(2\pi)^2} \frac{1}{(t - t' - i\epsilon)^2 - |\mathbf{x} - \mathbf{x}'|^2}, \quad (50)$$

where t and \mathbf{x} are Cartesian coordinates, and evaluate the entanglement generated between the Unruh-DeWitt detectors.

¹In [17,31], the authors also showed that there is a hidden nonlocality [47] by using a local filtering. However, we should be careful in concluding the nature of nonlocality. Similar to the argument of the detection loophole of Bell inequality, it is still not clear that the extracted entanglement cannot be explained by local realism [48].

We will consider in the first section the case where the switching functions $\chi_I(\tau_I)$ in Eqs. (6) and (7) are set as $\chi_I(\tau_I) = 1$ identically, while adiabatic switching on and off at infinite past and future is assumed implicitly, as in the textbooks on Unruh-DeWitt detectors [33,34], which we call the implicit adiabatic switching in what follows. In the second section, we will analyze the explicit effects of switching on and off of Unruh-DeWitt detectors, while being concerned only with the case of two comoving Unruh-DeWitt detectors.

A. Implicit adiabatic switching

In this section, we consider the implicit adiabatic switching $\chi_I(\tau_I) \equiv 1$, and thus the integrals in Eqs. (6) and (7) reduce to

$$\mathcal{I}_I = \int_{-\infty}^{\infty} d\tau'_I \int_{-\infty}^{\infty} d\tau_I e^{i\Delta E^{(I)}(\tau_I - \tau'_I)} G_W(\bar{x}'_I, \bar{x}_I), \quad (51)$$

$$\mathcal{I}_E = -i \int_{-\infty}^{\infty} d\tau_B \int_{-\infty}^{\infty} d\tau_A e^{i\Delta E^{(B)}\tau_B} e^{i\Delta E^{(A)}\tau_A} G_F(\bar{x}_B, \bar{x}_A). \quad (52)$$

For an inertial motion of the detector I , the coordinates of the world line are given by

$$\bar{t}_I(\tau_I) = \frac{1}{\sqrt{1 - v_{I0}^2}} \tau_I, \quad \bar{x}_I(\tau_I) = \mathbf{x}_{I0} + \frac{\mathbf{v}_{I0}}{\sqrt{1 - v_{I0}^2}} \tau_I, \quad (53)$$

where \mathbf{x}_{I0} and \mathbf{v}_{I0} are constants, and $v_{I0} \equiv |\mathbf{v}_{I0}|$. Then the Wightman function $G_W(\bar{x}'_I, \bar{x}_I)$ between the events \bar{x}'_I and \bar{x}_I along the world line of the detector I reduces to

$$G_W(\bar{x}'_I, \bar{x}_I) = \frac{-1}{(2\pi)^2} \frac{1}{(\tau'_I - \tau_I - i\varepsilon)^2}, \quad (54)$$

and thus \mathcal{I}_I is rewritten from Eq. (51) as

$$\mathcal{I}_I = \frac{-1}{(2\pi)^2} \int_{-\infty}^{\infty} d\tau'_I \int_{-\infty}^{\infty} d\Delta\tau e^{i\Delta E^{(I)}\Delta\tau} \frac{1}{(\Delta\tau + i\varepsilon)^2}, \quad (55)$$

where $\Delta\tau \equiv \tau_I - \tau'_I$. By analytically continuing onto the complex $\Delta\tau$ plane and taking as the integral path the infinite semicircle in the upper half of the complex $\Delta\tau$ plane, where the integrand in Eq. (55) is analytic, we obtain $\mathcal{I}_I = 0$. Thus, from Eq. (5) we have

$$\mathcal{P}_I = 0, \quad (56)$$

in this case. Actually, Eq. (56) is naturally required to hold for inertial detectors in the Minkowski vacuum if they properly probe the vacuum, because the Minkowski vacuum should be perceived by inertial observers as containing no excited particles, which means that the excitation probability vanishes.

We notice also that Eq. (56) implies that Eq. (21) is not satisfied, and hence that the second condition (19b) for entanglement does not hold. [This can be seen also from the facts that \mathcal{P}_{AB} vanishes due to Eqs. (61) and (A12) below, and that \mathcal{X} is non-negative, as we see from Eq. (14). For a restricted form of the monopole coupling and the gaussian switching function, the same result has been obtained when the two detectors are at rest [26].] Therefore, the condition for entanglement in this case reduces to the first condition (19a), which is rewritten as

$$\mathcal{E} = \langle E_1^{(B)} | m_B(0) | E_0^{(B)} \rangle \langle E_1^{(A)} | m_A(0) | E_0^{(A)} \rangle \mathcal{I}_E \neq 0. \quad (57)$$

When Eq. (57) holds, since $\mathcal{P}_A = \mathcal{P}_B = 0$, the concurrence and the negativity are related and given by Eq. (34), which is expressed as

$$\begin{aligned} C(\rho_{AB}) &= 2\mathcal{N}(\rho_{AB}) = 2c^2|\mathcal{E}| + O(c^4) \\ &= 2c^2 |\langle E_1^{(B)} | m_B(0) | E_0^{(B)} \rangle \langle E_1^{(A)} | m_A(0) | E_0^{(A)} \rangle| |\mathcal{I}_E| \\ &\quad + O(c^4), \end{aligned} \quad (58)$$

and then the optimal fidelity $F_{\max}(\rho_{AB})$ of quantum teleportation is computed by Eq. (49), with $C(\rho_{AB})$ and $\mathcal{N}(\rho_{AB})$ being proportional to $|\mathcal{I}_E|$ by Eq. (58). Therefore we see that whenever \mathcal{I}_E is nonvanishing, the entanglement is generated between the Unruh-DeWitt detectors and it is utilized in quantum teleportation.

In the remainder of this section, we choose the Cartesian coordinate system where Alice is at rest at the origin, and therefore

$$\bar{t}_A(\tau_A) = \tau_A, \quad \bar{x}_A(\tau_A) = 0. \quad (59)$$

In this case, the Wightman function $G_W(\bar{x}_B, \bar{x}_A)$ is written as

$$G_W(\bar{x}_B, \bar{x}_A) = \frac{-1}{(2\pi)^2} \frac{1}{(\tau_A - \bar{t}_B(\tau_B) + i\varepsilon)^2 - |\mathbf{x}_B(\tau_B)|^2}. \quad (60)$$

Since the poles are located in the lower half of the complex τ_A plane, by taking the infinite semicircle in the upper half as above, we obtain

$$\int_{-\infty}^{\infty} d\tau_A e^{i\Delta E^{(A)}\tau_A} G_W(\bar{x}_B, \bar{x}_A) = 0. \quad (61)$$

By noticing further that the Feynman propagator $G_F(x, x')$ is decomposed (see Appendix A), as

$$G_F(x, x') = -i G_W(x, x') + G_R(x', x), \quad (62)$$

we see that \mathcal{I}_E given by Eq. (52) is computed by

$$\mathcal{I}_E = -i \int_{-\infty}^{\infty} d\tau_B e^{i\Delta E^{(B)}\tau_B} \int_{-\infty}^{\infty} d\tau_A e^{i\Delta E^{(A)}\tau_A} G_R(\bar{x}_A, \bar{x}_B), \quad (63)$$

where the retarded Green function $G_R(x, x')$ of a neural massless scalar field in the Minkowski space-time is given as

$$\begin{aligned} G_R(x, x') &= -\frac{1}{2\pi} \Theta(t - t') \delta((t - t')^2 - |\mathbf{x} - \mathbf{x}'|^2) \\ &= -\frac{1}{4\pi} \frac{1}{|\mathbf{x} - \mathbf{x}'|} \delta(t - t' - |\mathbf{x} - \mathbf{x}'|). \end{aligned} \quad (64)$$

Therefore, contrary to the literature on entanglement harvesting, e.g., Ref. [16], where the contribution to entanglement results only from the Wightman function, entanglement extraction is possible only due to the causal propagation of the quantum field described by the retarded Green function in the case of the implicit adiabatic switching, while the quantum correlations due to vacuum fluctuation, i.e., virtual processes of the quantum field described by the Wightman function, do not contribute to entanglement extraction.

1. Comoving inertial motion

Here we consider the case where Alice and Bob are comoving. By choosing the coordinate system (59), Bob also

is at rest, and then the world line of Bob is described as

$$\bar{t}_B(\tau_B) = \tau_B, \quad \bar{\mathbf{x}}_B(\tau_B) = \mathbf{x}_0, \quad (65)$$

with $\mathbf{x}_0 \neq 0$ being constant, which we denote as $L = |\mathbf{x}_0|$. By substituting Eqs. (59) and (65) into Eqs. (63) and (64), we obtain

$$\mathcal{I}_E = -\frac{1}{2} \frac{e^{i \Delta E^{(A)} L}}{L} \delta(\Delta E^{(A)} + \Delta E^{(B)}) = 0, \quad (66)$$

where one notes $\Delta E^{(A)} > 0$. From Eq. (57), we thus find that entanglement is not extracted in this case. As the appearance of the δ function in Eq. (66) indicates, when considered as the distribution of the variable $\Delta E^{(A)} + \Delta E^{(B)}$, the reason why entanglement is not extracted in this case is understood from energy conservation as the consequence of an infinite amount of interaction time (transition time) in the uncertainty relation between time and energy. Transitions that violate energy conservation are not allowed after infinitely long interaction, and then the state of the two detectors is forced to return back to the ground state, which is not entangled.

2. Relative inertial motion

We next consider the case where Bob is in a relative inertial motion with respect to Alice, with a nonvanishing constant relative three-velocity \mathbf{v}_0 , which we assume not parallel or antiparallel to $\mathbf{x}_0 \neq 0$ so that the world lines of Alice and Bob do not intersect. In this case, the world line of Bob is given by

$$\bar{t}_B(\tau_B) = \frac{1}{\sqrt{1-v_0^2}} \tau_B, \quad \bar{\mathbf{x}}_B(\tau_B) = \mathbf{x}_0 + \frac{\mathbf{v}_0}{\sqrt{1-v_0^2}} \tau_B, \quad (67)$$

where $v_0 \equiv |\mathbf{v}_0|$. By substituting Eqs. (59) and (67) into Eqs. (63) and (64), we obtain

$$\mathcal{I}_E = \frac{i}{4\pi} \int_{-\infty}^{\infty} d\tau_B \frac{1}{|\bar{\mathbf{x}}_B(\tau_B)|} e^{i \epsilon \tau_B} e^{i \Delta E^{(A)} |\bar{\mathbf{x}}_B(\tau_B)|}, \quad (68)$$

where

$$\epsilon \equiv \Delta E^{(B)} + \frac{1}{\sqrt{1-v_0^2}} \Delta E^{(A)} \quad (69)$$

is the sum of the excitation energies of the two detectors measured in Bob's Lorentz frame. By changing the integration variable into ξ defined as

$$\xi \equiv \tau_B + \frac{\sqrt{1-v_0^2}}{v_0} |\mathbf{x}_0| \cos \theta, \quad (70)$$

Eq. (68) is written as

$$\begin{aligned} \mathcal{I}_E &= \frac{i}{4\pi} \frac{\sqrt{1-v_0^2}}{v_0} \exp \left[-i \frac{\sqrt{1-v_0^2}}{v_0} \epsilon |\mathbf{x}_0| \cos \theta \right] \\ &\times \int_{-\infty}^{\infty} d\xi \frac{1}{\sqrt{\xi^2 + \ell^2}} e^{i \epsilon \xi} e^{i p \sqrt{\xi^2 + \ell^2}}, \end{aligned} \quad (71)$$

where p and ℓ are defined by

$$p \equiv \frac{v_0}{\sqrt{1-v_0^2}} \Delta E^{(A)}, \quad \ell \equiv \frac{\sqrt{1-v_0^2}}{v_0} |\mathbf{x}_0| \sin \theta. \quad (72)$$

The integral in Eq. (71) is the zeroth modified Bessel function of the second kind $K_0(\eta)$ of the variable η defined as

$$\begin{aligned} \eta &\equiv \ell \sqrt{\epsilon^2 - p^2} = \frac{\sqrt{1-v_0^2}}{v_0} |\mathbf{x}_0| \sin \theta \\ &\times \sqrt{\Delta E^{(A)2} + \frac{2}{\sqrt{1-v_0^2}} \Delta E^{(A)} \Delta E^{(B)} + \Delta E^{(B)2}}, \end{aligned} \quad (73)$$

and $\theta (\neq 0, \pi)$ is the angle between \mathbf{x}_0 and \mathbf{v}_0 . From Eq. (67), we see that $|\mathbf{x}_0| \sin \theta$ is the closest approach between Alice and Bob. Therefore, η given by Eq. (73) is found to be invariantly defined under Lorentz transformations and affine reparametrizations. We thus obtain

$$|\mathcal{I}_E| = \frac{1}{2\pi} \frac{\sqrt{1-v_0^2}}{v_0} K_0(\eta). \quad (74)$$

We note that if $|\bar{\mathbf{x}}_B(\tau_B)|$ were constant in Eq. (68), \mathcal{I}_E would be proportional to the δ function of ϵ as in the comoving case. Thus the variable distance between Alice and Bob is responsible for the appearance of the modified Bessel function, instead of the δ function.

From the asymptotic behavior of $K_0(\eta)$ for $\eta \rightarrow \infty$, we immediately see that $|\mathcal{I}_E|$ vanishes and thus entanglement is not extracted in the comoving case $v_0 = 0$, as we have found above. On the other hand, since the leading behavior of $K_0(\eta)$ near $\eta = 0$ is given as $K_0(\eta) \approx -\ln \eta$, $|\mathcal{I}_E|$ drops to zero also in the limit of the speed of light $v_0 \rightarrow 1$. This will be naturally understood because the mutual causal contact between Alice and Bob diminishes in this limit. However, $|\mathcal{I}_E|$ does not vanish for $0 < v_0 < 1$. To demonstrate this simply, we consider the case of $\Delta E^{(A)} = \Delta E^{(B)}$, which we set as ΔE . Then, η is written as

$$\eta = \frac{a}{v_0} \sqrt{1-v_0^2 + \sqrt{1-v_0^2}}, \quad (75)$$

where $a \equiv \sqrt{2} |\mathbf{x}_0| \sin \theta \Delta E$ is the ratio of the closest approach to the de Broglie wavelength of a scalar particle with the resonance energy ΔE . The behavior of $|\mathcal{I}_E|$ in this case is shown in Fig. 1. As long as the relative velocity is nonrelativistic

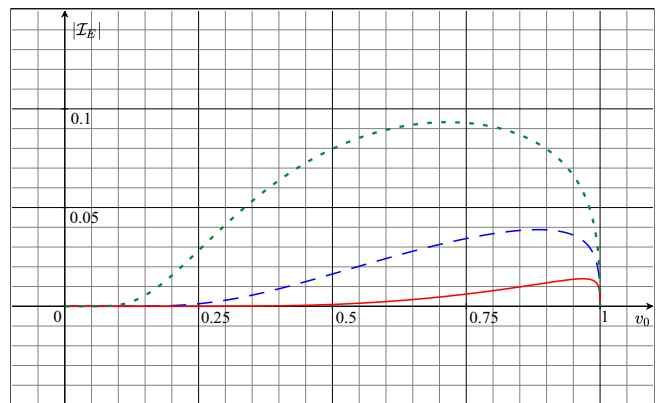


FIG. 1. The behavior of $|\mathcal{I}_E|$, where the horizontal axis is v_0 . The red solid line denotes $a = 2$, the blue dashed line $a = 1$, and the green dotted line $a = 0.5$.

$v_0 \ll 1$, we see from Fig. 1 that $|\mathcal{I}_E|$ remains quite small. As the relative velocity becomes relativistic, $|\mathcal{I}_E|$ starts to grow in an accelerated manner, which is thus certainly ascribed to the special relativistic effect. Although $|\mathcal{I}_E|$ decreases quickly as one further increases v_0 to the limit of $v_0 \rightarrow 1$, we see that the entanglement is extracted and enhanced by the special relativistic effect, unless the relative velocity is ultrarelativistic. From Eqs. (49) and (58), we thus find that the special relativistic effect enables quantum teleportation.

B. Switching effects

Now we analyze the effects on entanglement caused by switching on and off of Unruh-DeWitt detectors, by focusing on the case where Alice and Bob are comoving. For simplicity, we assume that the two Unruh-DeWitt detectors have identical structure and hence set $E_n^{(A)} = E_n^{(B)} \equiv E_n$, $\Delta E \equiv E_1 - E_0$, $|E_n\rangle \equiv |E_n^{(I)}\rangle$, and $m_A(\tau) = m_B(\tau) \equiv m(\tau)$. Furthermore, in order to take into account the effects of switching, we also prescribe the form of the switching function $\chi_I(\tau_I)$ as

$$\chi_I(\tau_I) = \chi(\tau_I) \equiv \tanh[\sigma(\tau_I + T)] - \tanh[\sigma(\tau_I - T)], \tag{76}$$

where the positive constants $1/\sigma$ and $2T$ denote the timescales of the switching on and off, and the duration that $\chi(\tau_I)$ stays above a half of its maximal value, which we call the effective interaction time, respectively. Thus, the timescale of the total interaction time is given by $2(T + 1/\sigma) = (2/\sigma)(\sigma T + 1)$. In Fig. 2, we show two typical cases of the behavior of $\chi(\tau_I)$.

In this case, we have $\mathcal{P}_A = \mathcal{P}_B$, and hence the concurrence $C(\rho_{AB})$ and the negativity $\mathcal{N}(\rho_{AB})$ when the first condition

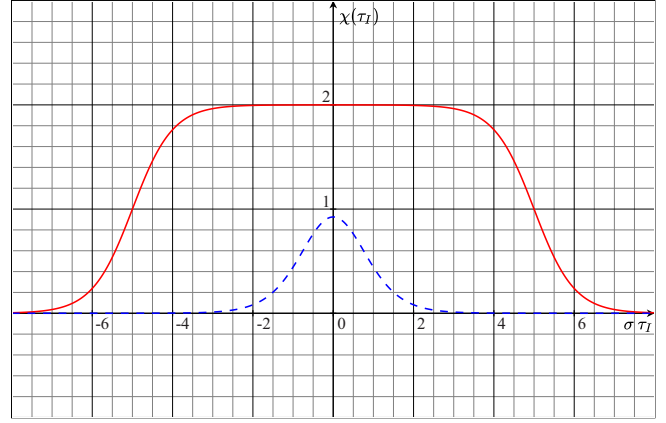


FIG. 2. The behavior of the switching function $\chi(\tau_I)$. The horizontal axis denotes τ_I normalized by σ and the vertical axis shows $\chi(\tau_I)$. The blue dashed line shows the behavior of $\chi(\tau_I)$ for $\sigma T = 0.5$, and the red solid line for $\sigma T = 5$.

(19a) for entanglement is valid are now given from Eq. (34) as

$$C(\rho_{AB}) = 2\mathcal{N}(\rho_{AB}) = 2c^2(|\mathcal{E}| - \mathcal{P}_I) + O(c^4) = 2c^2|E_1| m(0) |E_0|^2 (|\mathcal{I}_E| - \mathcal{I}_I) + O(c^4), \tag{77}$$

and the fidelity of quantum teleportation is given by Eq. (49), with $C(\rho_{AB})$ and $\mathcal{N}(\rho_{AB})$ taking the form of Eq. (77). Since quantum teleportation is not possible in the case of the second condition (19b) for entanglement, we focus in this section on the first condition (19a), which is rewritten in the present case as $|\mathcal{I}_E| - \mathcal{I}_I > 0$, and thus compute and analyze the behavior of $|\mathcal{I}_E| - \mathcal{I}_I$.

1. Computation

We first evaluate \mathcal{I}_I by employing the expansion of $\tanh z$ as

$$\tanh z = \sum_{k=1}^{\infty} \left[\frac{1}{z + i(k - \frac{1}{2})\pi} + \frac{1}{z - i(k - \frac{1}{2})\pi} \right]. \tag{78}$$

By substituting Eqs. (50), (59), (65), (76), and (78) into Eq. (6), we perform the first integration by considering the infinite semicircle in the upper half of the complex τ_I plane, within which only the poles of the switching function $\chi(\tau_I)$ located at

$$\tau_I = \mp T + i \left(k - \frac{1}{2} \right) \frac{\pi}{\sigma} \tag{79}$$

contribute to the integral. The second integration in Eq. (6) is performed by taking as the integration path the infinite semicircle in the lower half of the complex τ'_I plane. Again, only the poles of the switching function $\chi(\tau'_I)$ contribute, which are located at

$$\tau'_I = \mp T - i \left(k' - \frac{1}{2} \right) \frac{\pi}{\sigma}, \tag{80}$$

where k' is the summation index that appears in the expansion of $\chi(\tau'_I)$ as Eq. (78). Relabeling the summation indices as $\ell = k + k' - 1$ and $m = k - k'$, and performing the summation over m , we obtain

$$\begin{aligned} \mathcal{I}_I = & \frac{1}{\pi^2} \left[-2 \ln \left(1 - e^{-\frac{\Delta E}{\sigma} \pi} \right) - e^{2iT \Delta E} \Phi \left(e^{-\frac{\Delta E}{\sigma} \pi}, -2iT \frac{\sigma}{\pi}, 1 \right) - e^{-2iT \Delta E} \Phi \left(e^{-\frac{\Delta E}{\sigma} \pi}, 2iT \frac{\sigma}{\pi}, 1 \right) \right. \\ & \left. + 2iT \frac{\sigma}{\pi} \left\{ e^{-2iT \Delta E} \Phi \left(e^{-\frac{\Delta E}{\sigma} \pi}, 2iT \frac{\sigma}{\pi}, 2 \right) - e^{2iT \Delta E} \Phi \left(e^{-\frac{\Delta E}{\sigma} \pi}, -2iT \frac{\sigma}{\pi}, 2 \right) \right\} \right], \tag{81} \end{aligned}$$

where $\Phi(z, a, s)$ is the Hurwitz-Lerch ζ function defined as

$$\Phi(z, a, s) \equiv \sum_{n=0}^{\infty} \frac{z^n}{(a+n)^s}. \tag{82}$$

By decomposing the Feynman propagator $G_F(x, x')$ in Eq. (7) into the Wightman function $G_W(x, x')$ and the retarded Green function $G_R(x', x)$ as Eq. (62), we then compute \mathcal{I}_E . The contribution from the Wightman function $G_W(x, x')$ to \mathcal{I}_E is calculated similarly to the case of \mathcal{I}_I above, except that the integral path in the second integration is chosen to be the infinite semicircle in the upper half of the complex τ_B plane, which encircles the five series of poles located at

$$\begin{aligned} \tau_B &= \mp T + i \left(k' - \frac{1}{2} \right) \frac{\pi}{\sigma}, \quad \tau_B = -L - T + i \left\{ \left(k - \frac{1}{2} \right) \frac{\pi}{\sigma} + \varepsilon \right\}, \quad \tau_B = L - T + i \left\{ \left(k - \frac{1}{2} \right) \frac{\pi}{\sigma} + \varepsilon \right\}, \\ \tau_B &= -L + T + i \left\{ \left(k - \frac{1}{2} \right) \frac{\pi}{\sigma} + \varepsilon \right\}, \quad \tau_B = L + T + i \left\{ \left(k - \frac{1}{2} \right) \frac{\pi}{\sigma} + \varepsilon \right\}, \end{aligned} \tag{83}$$

where $L = |x_0|$ as above, and k and k' are the summation indices in the expansion of the switching function $\chi(\tau_A)$ and $\chi(\tau_B)$, respectively. We implement this integration by assuming that these poles do not coincide with each other. This requires $T \neq L$ and $2T \neq L$, but \mathcal{I}_E for these cases is determined by continuity. We note that the expansion

$$\coth z = \frac{1}{z} + \sum_{\ell=1}^{\infty} \left[\frac{1}{z + i \ell \pi} + \frac{1}{z - i \ell \pi} \right] = \sum_{\ell=-\infty}^{\infty} \frac{z}{z^2 + \ell^2 \pi^2}, \tag{84}$$

is helpful in order to simplify the expression.

On the other hand, the contribution from the retarded Green function $G_R(x', x)$ to \mathcal{I}_E is evaluated by substituting Eqs. (59), (64), (65), (76), and (78) into Eq. (7), and by using

$$\int_{-\infty}^{\infty} d\tau e^{i E \tau} \tanh[\sigma(\tau + T_1)] \tanh[\sigma(\tau + T_2)] = -\frac{2\pi}{\sigma} \frac{1}{\sinh\left[\frac{\pi E}{2\sigma}\right]} \coth[\sigma(T_2 - T_1)] \sin\left[E \frac{T_2 - T_1}{2}\right] e^{-i E \frac{T_1 + T_2}{2}} - 2\pi i \delta(E), \tag{85}$$

which is derived by considering the rectangle with the infinite width (along the real axis) and the height π/σ (along the imaginary axis) in the upper half of the complex τ plane. By adding these two parts, we finally obtain

$$\begin{aligned} \mathcal{I}_E &= \frac{1}{2 \sinh\left(\frac{\pi}{\sigma} \Delta E\right)} \frac{1}{\sigma L} \left(-\frac{2L}{\sigma(4T^2 - L^2)} - 2 \cos(2 \Delta E T) \left[\frac{1}{\sigma L} + \frac{i}{\pi} \left\{ \Phi\left(e^{-\frac{\pi}{\sigma} \Delta E}, -i \frac{\sigma}{\pi} L, 1\right) - \text{c.c.} \right\} \right] \right) \\ &\quad + e^{i \Delta E L} [\coth[\sigma(2T + L)] e^{2i \Delta E T} - \coth[\sigma(2T - L)] e^{-2i \Delta E T} - 2 \cos(2 \Delta E T) \coth(\sigma L)] \\ &\quad - \frac{i}{\pi} \left[\left\{ \Phi\left(e^{-\frac{\pi}{\sigma} \Delta E}, i \frac{\sigma}{\pi} (2T + L), 1\right) - \Phi\left(e^{-\frac{\pi}{\sigma} \Delta E}, i \frac{\sigma}{\pi} (2T - L), 1\right) \right\} - \text{c.c.} \right]. \end{aligned} \tag{86}$$

2. Behavior of entanglement

When the switching of the detectors is executed quickly enough, it will disturb the quantum state of the scalar field and excite the detectors. Since the timescale of the switching is given by $1/\sigma$, we may apply in this case the approximation $\Delta E/\sigma \ll 1$. When we keep $\Delta E T$ fixed, the Hurwitz-Lerch ζ functions are shown to be bounded, and hence Eq. (81) is approximated as

$$\mathcal{I}_I \simeq -\frac{2}{\pi^2} \ln \left(\frac{\pi}{\sigma} \Delta E \right). \tag{87}$$

Thus, \mathcal{I}_I logarithmically diverges in the limit of $\Delta E/\sigma \rightarrow 0$, as in Ref. [35]. The two factors in front of the outermost round bracket in Eq. (86) are approximated as $1/(2 \Delta E L)$, and $\sigma T = \Delta E T/(\Delta E/\sigma) \rightarrow \infty$ in this limit. Therefore, unless we set the distance between the two detectors to be vanishingly small, \mathcal{I}_E remains finite in this limit. Although entanglement will be naturally generated between detectors put so close, we see that the first condition (19a) for entanglement extraction

is not satisfied under physically plausible circumstances of the finite distance for the sudden switching limit $\Delta E/\sigma \rightarrow 0$. This is consistent with Ref. [25], where entanglement is shown not to be extracted in the sudden switching limit, while for a different form of the switching function.

On the other hand, when the switching is performed adiabatically compared to the excitation energy ΔE , we employ the approximation $\Delta E/\sigma \gg 1$. In this case, Eq. (81) is approximated as

$$\begin{aligned} \mathcal{I}_I &\simeq e^{-\frac{\Delta E}{\sigma} \pi} \left[\frac{2}{\pi^2} - 2 \frac{\pi^2 - 4 \sigma^2 T^2}{(\pi^2 + 4 \sigma^2 T^2)^2} \cos(2 \Delta E T) \right. \\ &\quad \left. + \frac{8 \pi \sigma T}{(\pi^2 + 4 \sigma^2 T^2)^2} \sin(2 \Delta E T) \right], \end{aligned} \tag{88}$$

while the approximate form of Eq. (86) is given by

$$\begin{aligned} \mathcal{I}_E &\simeq e^{-\frac{\Delta E}{\sigma} \pi} \frac{1}{\sigma L} \left(\frac{2L}{\sigma(4T^2 - L^2)} + \frac{2}{\sigma L} \cos(2 \Delta E T) \right) \\ &\quad + e^{i \Delta E L} \{ \coth[\sigma(2T + L)] e^{2i \Delta E T} \} \end{aligned}$$

$$\begin{aligned}
 & - \coth[\sigma(2T-L)] e^{-2i\Delta ET} \\
 & - 2 \cos(2\Delta ET) \coth(\sigma L) \Big). \quad (89)
 \end{aligned}$$

In particular, in the limit of $\sigma T \rightarrow 0$, by taking the limit $\Delta E/\sigma \rightarrow \infty$ while keeping ΔET small, Eqs. (88) and (89) yield

$$|\mathcal{I}_E| - \mathcal{I}_I = \frac{4}{\pi^2} e^{-\frac{\Delta E}{\sigma} \pi} \left[\left(\frac{\pi}{\sigma L} \right)^2 - 1 \right] \sin^2(\Delta ET). \quad (90)$$

We thus see that the entanglement generated between the detectors falls off as L^{-2} , which is understood from the behavior of the Wightman function $G_W(x, x')$ in Eq. (50) for the massless scalar field, whose Compton wavelength is infinite. Since the timescale of the total interaction time $(2/\sigma)(\sigma T + 1)$ (see above) in this case is approximated as $2/\sigma$, and $|\mathcal{I}_E| - \mathcal{I}_I$ is positive if $L < \pi/\sigma = (2/\sigma)(\pi/2)$, we also confirm that the entanglement is generated between the detectors even if they are separated acausally $L > 2/\sigma$, i.e., even when one of them is put outside the other's light cone within the total interaction time. Although the switching function $\chi(\tau_I)$ in

this paper has an exponential tail, our analytical treatment complements the numerical investigation by Reznik [16], where the switching function is nonvanishing only for a strictly finite period and hence the detectors are separated acausally in the rigorous sense. We note from Eqs. (49) and (77) that the entanglement extracted in both of these cases is useful in quantum teleportation.

However, in the case of adiabatic switching $\Delta E/\sigma \gg 1$, the maximal extraction of entanglement occurs around $L = 2T$, which we expediently call the light cone within the effective interaction time (L.E.). (As we described above, $2T$ is the effective interaction time.) To see this, we depict in Fig. 3 the behavior of $|\mathcal{I}_E| - \mathcal{I}_I$, multiplied by $e^{\pi \Delta E/\sigma}$ so that the magnitude is not too small, on the $\sigma L - \Delta ET$ plane. Although we employ Eqs. (81) and (86) in the case where $\Delta E/\sigma$ is small, we need to resort to the approximate forms (88) and (89) when $\Delta E/\sigma$ is large, due to the apparent numerical divergences in each of the Hurwitz-Lerch ζ functions that are analytically found to cancel among them. In Fig. 3, along with the line of L.E. described by $\Delta ET = (\Delta E/2\sigma)\sigma L$, we plot what we call the light cone within the total interaction time $2(T + 1/\sigma)$ (L.T.), which is defined by the line

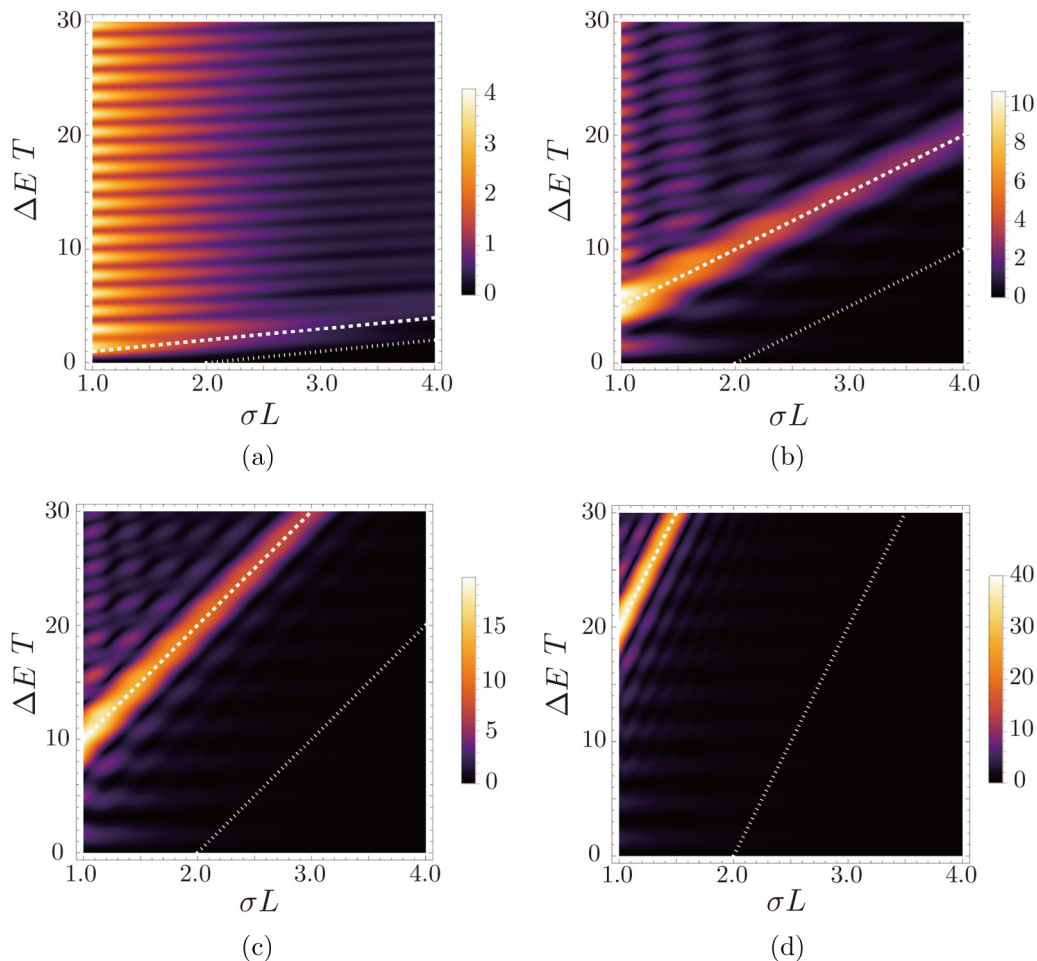


FIG. 3. The behavior of $|\mathcal{I}_E| - \mathcal{I}_I$ multiplied by $e^{\pi \Delta E/\sigma}$, (a) for $\Delta E/\sigma = 2$, (b) for $\Delta E/\sigma = 10$, (c) for $\Delta E/\sigma = 20$, and (d) for $\Delta E/\sigma = 40$. The horizontal axis is the distance L between the detectors normalized by σ , and the vertical axis is half the effective interaction time T normalized by ΔE . The region of $\sigma L < 1$ is not shown because too large a magnitude near $\sigma L = 0$ obscures the detailed structure elsewhere. L.E. and L.T. (see the main text) are denoted by a white dashed line and a white dotted line, respectively.

$\Delta E T = (\Delta E/2\sigma)(\sigma L - 2)$. The entanglement in the region between L.E. and L.T. is considered as arising from the disturbance due to switching on and off of the Unruh-DeWitt detectors.

We see from Fig. 3 that there exist two periods generally, one along the $\Delta E T$ axis [as we see also from Eq. (90)], and the other along the normal direction to L.E., which is normal to L.T. also. As we decrease $\Delta E/\sigma$, L.E. and L.T. get tilted horizontally, and then the two periods become almost degenerate. We see also that the magnitude of $|\mathcal{I}_E| - \mathcal{I}_I$ gets flatter as $\Delta E/\sigma$ becomes small, which is in accord with the above argument on the sudden switching limit.

On the other hand, for large values of $\Delta E/\sigma$, the periodic behavior in the two different directions manifests itself. In particular, the maximum of the amount of the extracted entanglement is found to occur on L.E., and the oscillation in the normal direction to L.E. is drastically damped away from L.E.. Although the region between L.E. and L.T. gets wider and hence one can extract entanglement due to switching effects, it is quite small compared with the entanglement extracted due to causal propagation of the quantum field during the effective interaction time, i.e., on and inside L.E. The entanglement extraction outside L.T. is possible, as we have seen from Eq. (90), but even smaller. This implies that entanglement is extracted essentially due to causal propagation of the quantum field when we implement the switching of the Unruh-DeWitt detectors adiabatically, and it is consistent with the analysis in Sec. IV A, where we have seen that the only contribution comes from the retarded Green function in the case of the implicit adiabatic switching, while it actually vanishes due to the energy conservation for the comoving case. Indeed, as we see from Eq. (89), \mathcal{I}_E vanishes in the limit of $T \rightarrow \infty$, because the first term falls off as T^{-2} , and the rest terms oscillate infinitely rapidly in the same manner as the δ function arises in Eq. (66), i.e., when those are considered as the distribution of the variable ΔE . Since \mathcal{I}_I in Eq. (88) contains the term that does not vanish in the limit $T \rightarrow \infty$, we see that by explicitly considering the adiabatic switching effect and taking the limit of infinitely long exposure of the Unruh-DeWitt detectors to the interaction with the quantum field, entanglement is not generated between the comoving detectors, as the analysis in Sec. IV A for the implicit adiabatic switching.

V. CONCLUSION AND DISCUSSION

We considered in this paper two two-level Unruh-DeWitt detectors, as a pair of qubits, with an arbitrary monopole coupling to a neutral massless quantum scalar field in an arbitrary four-dimensional space-time, and analyzed entanglement generated between the Unruh-DeWitt detectors from a vacuum. We first derived the general form of the reduced density matrix of the two Unruh-DeWitt detectors in the perturbation theory, for arbitrary world lines of the detectors and arbitrary switching functions.

We then considered the entanglement measures. Although we have not obtained evidences for usability of the entanglement from the analyses of the bounds on the distillable entanglement or the Bell-CHSH inequality, we did find that the single copy of the entangled pair of the Unruh-DeWitt detectors alone serves as the resource for quantum teleportation. More

precisely, the optimal fidelity of the standard teleportation exceeds the classical value whenever a nonvanishing value of the concurrence (and hence the negativity) of the entanglement between symmetric ($\mathcal{P}_A = \mathcal{P}_B$) Unruh-DeWitt detectors arises from the first condition (19a) for entanglement, which is found within the second-order perturbation theory. It is worthwhile to emphasize that this result is valid for an arbitrary monopole coupling with arbitrary switching functions and for arbitrary world lines of the detectors in an arbitrary four-dimensional space-time, while its extension into higher dimensionality will be straightforward.

In order to find whether and how entanglement is actually generated between the Unruh-DeWitt detectors, we then focused on inertial motions of the Unruh-DeWitt detectors in the Minkowski vacuum. When we assume the implicit adiabatic switching, we found that no entanglement is extracted when Alice and Bob are comoving inertially. This is interpreted as resulting from the energy conservation due to an infinite amount of interaction time (transition time) and the uncertainty relation between time and energy. On the other hand, if Alice and Bob are in a relative inertial motion, entanglement was found to be extracted and enhanced by the special relativistic effect, obeying the first condition (19a) for entanglement, unless the relative velocity is ultrarelativistic. Therefore, we found that one can perform quantum teleportation by using the entanglement extracted in this manner without invoking many copies of the entangled pair or preparing an entangled state initially. Bob's desperate run in a relativistic speed in the vacuum suffices!

By assuming the form (76) of the switching function $\chi_I(\tau_I)$, we considered explicitly the switching effects also for the case where Alice and Bob are comoving inertially. In the case of adiabatic switching $\Delta E/\sigma \gg 1$, in particular, we found that entanglement arises primarily from causal propagation of the quantum field, which validated the analyses in the case of the implicit adiabatic switching. However, we noted that entanglement generation between the Unruh-DeWitt detectors separated acausally is possible also. Although our form of the switching function has the exponential tail and hence the terminology "acausal" does not have a rigorous sense, the analysis on the fidelity of quantum teleportation in this paper applies also to the case where the detectors are located in causally disconnected regions, as in Ref. [16]. Therefore, we see that quantum teleportation is possible even if Alice and Bob are separated acausally in the strict sense.

The results in this paper may shed light on the physical process behind the entanglement generation between Unruh-DeWitt detectors. By regarding the quantum field as the continuum limit of discretized particles connected with springs, one may consider the entanglement extraction from a vacuum as resulting from the entanglement between these particles. Although a vacuum in the second quantization is the continuum limit of the product state of the ground state of each normal mode, it may be regarded as an entangled state by considering the Hilbert space of each *particle*. However, the transformation between the position operators of the particles and the normal modes is time independent, and then this picture of entanglement does not seem to be compatible with our result that entanglement generated between the comoving Unruh-DeWitt detectors depends on the interaction time T , especially in the

limit of $T \rightarrow \infty$. As a more plausible picture of entanglement extraction, it may be possible to consider that entanglement is transferred from vacuum fluctuation [52], whose correlation length extends acausally as described by the explicit form of the Wightman function Eq. (50). Vacuum fluctuation is nothing but virtual processes of creation and annihilation of quanta, and its effect is suppressed when interaction lasts for infinite time, due to the uncertainty relation between time and energy, which also is understood from Eq. (50). This will be the reason why the Wightman function does not contribute to entanglement extraction in the case of the implicit adiabatic switching, and the entanglement extracted outside L.E. in the case of explicit adiabatic switching is small. From the same reason, the contribution from the retarded Green function vanishes in the comoving case, because infinitely long interaction suppresses the effect of the virtual processes and then leads to the energy conservation, as indicated by the appearance of the δ function in Eq. (66). However, this mechanism of suppression does not work completely when the detectors are in a relative motion because of the variable distance between the detectors, as in the Doppler effect, which gives the modified Bessel function instead of the δ function. This will explain why entanglement is extracted and enhanced by the special relativistic effect when the detectors are in a relative inertial motion.

The analyses in this paper will provide applications and extensions. It may be interesting to investigate the relation of the entanglement between the detectors in a relative motion to the mechanism that gives rise to the revival of entanglement after entanglement sudden death. It also seems valuable to extend the analyses to the case of accelerated observers. In particular, when Alice is at rest and Bob is uniformly

accelerated with the magnitude of the acceleration κ , our preliminary calculation in the case of the Minkowski vacuum gives

$$|\mathcal{I}_E| = \left| \frac{e^{\pi \frac{\Delta E^{(B)}}{\kappa}}}{e^{2\pi \frac{\Delta E^{(B)}}{\kappa}} - 1} \sinh \left(\frac{\pi}{2} \frac{\Delta E^{(B)}}{\kappa} - \frac{\Delta E^{(A)}}{\kappa} \right) \right|, \quad (91)$$

for the implicit adiabatic switching $\chi_I(\tau_I) \equiv 1$. Since $\mathcal{P}_A = 0$ in this case (arbitrarily small even in the case of the explicit adiabatic switching), we see from Eq. (19a) that entanglement is extracted if Eq. (91) is nonvanishing. Although we have not arrived at complete understanding of this result, it is interesting to note that even when Bob's world line is very close to the Rindler horizon, where $\kappa \rightarrow \infty$, entanglement is generated between the Unruh-DeWitt detectors. (In this case, massless quanta emitted from Bob reach Alice, even after Alice passes across Bob's event horizon, in contrast to the case of the $v_0 \rightarrow 1$ limit of relative inertial motion.) In a recent paper [53], the authors considered the case where both of the detectors are accelerated in the same direction in the B.T.Z. black hole space-time. Comparison of Ref. [53] and our preliminary result above may provide a clue to the information loss paradox.

Note added in proof. Recently, a paper by Ng *et al.* [54] appeared, which also derived the expressions equivalent to Eqs. (A10) and (A11) for spatially extended detectors.

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APPENDIX A: EXPLICIT FORM OF REDUCED DENSITY MATRIX

In this Appendix, we describe briefly the computation of the elements $\langle E_{n_A}^{(A)} | \langle E_{n_B}^{(B)} | \rho_{AB} | E_{\hat{n}_A}^{(A)} \rangle | E_{\hat{n}_B}^{(B)} \rangle$ of the reduced density matrix (4) and present their explicit forms.

Expanding $e^{iS_{\text{int}}}$ in Eq. (3) to second order in c , the matrix elements of the reduced density matrix $\rho_{AB} \equiv \text{Tr}_\phi | \text{out} \rangle \langle \text{out} |$ are found to be written as

$$\langle E_{n_A}^{(A)} | \langle E_{n_B}^{(B)} | \rho_{AB} | E_{\hat{n}_A}^{(A)} \rangle | E_{\hat{n}_B}^{(B)} \rangle = [\delta_0^{n_A} \delta_0^{n_B} \delta_0^{\hat{n}_A} \delta_0^{\hat{n}_B} + c^2 \{ R_{n_A, n_B, \hat{n}_A, \hat{n}_B}^{(1)} + R_{n_A, n_B, \hat{n}_A, \hat{n}_B}^{(2)} + R_{\hat{n}_A, \hat{n}_B, n_A, n_B}^{(2)*} \}] + O(c^4), \quad (A1)$$

where

$$\begin{aligned} R_{n_A, n_B, \hat{n}_A, \hat{n}_B}^{(1)} &= \delta_0^{n_B} \delta_0^{\hat{n}_B} \langle E_{n_A}^{(A)} | m_A(0) | E_0^{(A)} \rangle \langle E_{\hat{n}_A}^{(A)} | m_A(0) | E_0^{(A)} \rangle^\dagger \\ &\quad \times \int_{-\infty}^{\infty} d\tau'_A \int_{-\infty}^{\infty} d\tau_A \chi_A(\tau'_A) \chi_A(\tau_A) e^{i(E_{n_A}^{(A)} - E_0^{(A)})\tau_A} e^{-i(E_{\hat{n}_A}^{(A)} - E_0^{(A)})\tau'_A} \langle 0 | \phi(\bar{x}'_A) \phi(\bar{x}_A) | 0 \rangle \\ &\quad + \delta_0^{n_A} \delta_0^{\hat{n}_B} \langle E_{n_B}^{(B)} | m_B(0) | E_0^{(B)} \rangle \langle E_{\hat{n}_A}^{(A)} | m_A(0) | E_0^{(A)} \rangle^\dagger \\ &\quad \times \int_{-\infty}^{\infty} d\tau'_A \int_{-\infty}^{\infty} d\tau_B \chi_A(\tau'_A) \chi_B(\tau_B) e^{i(E_{n_B}^{(B)} - E_0^{(B)})\tau_B} e^{-i(E_{\hat{n}_A}^{(A)} - E_0^{(A)})\tau'_A} \langle 0 | \phi(\bar{x}'_A) \phi(\bar{x}_B) | 0 \rangle \\ &\quad + \delta_0^{n_B} \delta_0^{\hat{n}_A} \langle E_{n_A}^{(A)} | m_A(0) | E_0^{(A)} \rangle \langle E_{\hat{n}_B}^{(B)} | m_B(0) | E_0^{(B)} \rangle^\dagger \\ &\quad \times \int_{-\infty}^{\infty} d\tau'_B \int_{-\infty}^{\infty} d\tau_A \chi_B(\tau'_B) \chi_A(\tau_A) e^{i(E_{n_A}^{(A)} - E_0^{(A)})\tau_A} e^{-i(E_{\hat{n}_B}^{(B)} - E_0^{(B)})\tau'_B} \langle 0 | \phi(\bar{x}'_B) \phi(\bar{x}_A) | 0 \rangle \\ &\quad + \delta_0^{n_A} \delta_0^{\hat{n}_A} \langle E_{n_B}^{(B)} | m_B(0) | E_0^{(B)} \rangle \langle E_{\hat{n}_B}^{(B)} | m_B(0) | E_0^{(B)} \rangle^\dagger \\ &\quad \times \int_{-\infty}^{\infty} d\tau'_B \int_{-\infty}^{\infty} d\tau_B \chi_B(\tau'_B) \chi_B(\tau_B) e^{i(E_{n_B}^{(B)} - E_0^{(B)})\tau_B} e^{-i(E_{\hat{n}_B}^{(B)} - E_0^{(B)})\tau'_B} \langle 0 | \phi(\bar{x}'_B) \phi(\bar{x}_B) | 0 \rangle \end{aligned} \quad (A2)$$

and

$$\begin{aligned}
 R_{n_A, n_B, \hat{n}_A, \hat{n}_B}^{(2)} &= -\delta_0^{n_B} \delta_0^{\hat{n}_A} \delta_0^{\hat{n}_B} \sum_k \langle E_{n_A}^{(A)} | m_A(0) | E_k^{(A)} \rangle \langle E_k^{(A)} | m_A(0) | E_0^{(A)} \rangle \\
 &\times \int_{-\infty}^{\infty} d\tau_A \int_{-\infty}^{\infty} d\tau'_A \chi_A(\tau_A) \chi_A(\tau'_A) \Theta(\tau_A - \tau'_A) e^{i(E_{n_A}^{(A)} - E_k^{(A)})\tau_A} e^{i(E_k^{(A)} - E_0^{(A)})\tau'_A} \langle 0 | T \phi(\bar{x}_A) \phi(\bar{x}'_A) | 0 \rangle \\
 &- \delta_0^{\hat{n}_A} \delta_0^{\hat{n}_B} \langle E_{n_B}^{(B)} | m_B(0) | E_0^{(B)} \rangle \langle E_{n_A}^{(A)} | m_A(0) | E_0^{(A)} \rangle \\
 &\times \int_{-\infty}^{\infty} d\tau_B \int_{-\infty}^{\infty} d\tau'_A \chi_B(\tau_B) \chi_A(\tau'_A) e^{i(E_{n_B}^{(B)} - E_0^{(B)})\tau_B} e^{i(E_{n_A}^{(A)} - E_0^{(A)})\tau'_A} \langle 0 | T \phi(\bar{x}_B) \phi(\bar{x}'_A) | 0 \rangle \\
 &- \delta_0^{n_A} \delta_0^{\hat{n}_A} \delta_0^{\hat{n}_B} \sum_k \langle E_{n_B}^{(B)} | m_B(0) | E_k^{(B)} \rangle \langle E_k^{(B)} | m_B(0) | E_0^{(B)} \rangle \\
 &\times \int_{-\infty}^{\infty} d\tau_B \int_{-\infty}^{\infty} d\tau'_B \chi_B(\tau_B) \chi_B(\tau'_B) \Theta(\tau_B - \tau'_B) e^{i(E_{n_B}^{(B)} - E_k^{(B)})\tau_B} e^{i(E_k^{(B)} - E_0^{(B)})\tau'_B} \langle 0 | T \phi(\bar{x}_B) \phi(\bar{x}'_B) | 0 \rangle. \quad (\text{A3})
 \end{aligned}$$

Here we employ the causality relations Eq. (11) and the properties of the Feynman propagator $G_F(x, x')$ and the Wightman function $G_W(x, x')$,

$$G_F(x, x') = -i [\Theta(t - t') G_W(x, x') + \Theta(t' - t) G_W(x', x)], \quad (\text{A4})$$

$$G_W(x, x') - i G_F(x, x') = -i G_R(x', x), \quad (\text{A5})$$

$$G_F(x, x') = G_F(x', x), \quad G_R^*(x, x') = G_R(x, x'), \quad G_W^*(x, x') = G_W(x', x), \quad (\text{A6})$$

which are derived from their definitions Eqs. (8) and (9), along with

$$G_R(x, x') \equiv -i \Theta(t - t') \langle 0 | [\phi(x), \phi(x')] | 0 \rangle = i \Theta(t - t') [G_W(x', x) - G_W(x, x')]. \quad (\text{A7})$$

One finds that the reduced density matrix ρ_{AB} takes the form of Eq. (4) with the nonvanishing elements given by

$$\mathcal{P}_A \equiv |\langle E_1^{(A)} | m_A(0) | E_0^{(A)} \rangle|^2 \int_{-\infty}^{\infty} d\tau'_A \int_{-\infty}^{\infty} d\tau_A \chi_A(\tau'_A) \chi_A(\tau_A) e^{i \Delta E^{(A)}(\tau_A - \tau'_A)} G_W(\bar{x}'_A, \bar{x}_A), \quad (\text{A8})$$

$$\mathcal{P}_B \equiv |\langle E_1^{(B)} | m_B(0) | E_0^{(B)} \rangle|^2 \int_{-\infty}^{\infty} d\tau'_B \int_{-\infty}^{\infty} d\tau_B \chi_B(\tau'_B) \chi_B(\tau_B) e^{i \Delta E^{(B)}(\tau_B - \tau'_B)} G_W(\bar{x}'_B, \bar{x}_B), \quad (\text{A9})$$

$$\mathcal{E} \equiv -\langle E_1^{(B)} | m_B(0) | E_0^{(B)} \rangle \langle E_1^{(A)} | m_A(0) | E_0^{(A)} \rangle \int_{-\infty}^{\infty} d\tau_B \int_{-\infty}^{\infty} d\tau'_A \chi_B(\tau_B) \chi_A(\tau'_A) e^{i \Delta E^{(B)}\tau_B} e^{i \Delta E^{(A)}\tau'_A} i G_F(\bar{x}_B, \bar{x}'_A) \quad (\text{A10})$$

$$= -\langle E_1^{(B)} | m_B(0) | E_0^{(B)} \rangle \langle E_1^{(A)} | m_A(0) | E_0^{(A)} \rangle \int_{-\infty}^{\infty} d\tau_B \int_{-\infty}^{\infty} d\tau'_A \chi_B(\tau_B) \chi_A(\tau'_A) e^{i \Delta E^{(B)}\tau_B} e^{i \Delta E^{(A)}\tau'_A} [G_W(\bar{x}_B, \bar{x}'_A) + i G_R(\bar{x}'_A, \bar{x}_B)], \quad (\text{A11})$$

$$\mathcal{P}_{AB} \equiv \langle E_1^{(A)} | m_A(0) | E_0^{(A)} \rangle \langle E_1^{(B)} | m_B(0) | E_0^{(B)} \rangle^\dagger \int_{-\infty}^{\infty} d\tau'_B \int_{-\infty}^{\infty} d\tau_A \chi_B(\tau'_B) \chi_A(\tau_A) e^{i \Delta E^{(A)}\tau_A - i \Delta E^{(B)}\tau'_B} G_W(\bar{x}'_B, \bar{x}_A), \quad (\text{A12})$$

$$\begin{aligned}
 \mathcal{W}_A &\equiv -\langle E_1^{(A)} | m_A(0) | E_0^{(A)} \rangle [\langle E_1^{(A)} | m_A(0) | E_1^{(A)} \rangle - \langle E_0^{(A)} | m_A(0) | E_0^{(A)} \rangle] \\
 &\times \int_{-\infty}^{\infty} d\tau_A \int_{-\infty}^{\infty} d\tau'_A \chi_A(\tau_A) \chi_A(\tau'_A) \Theta(\tau'_A - \tau_A) e^{i \Delta E^{(A)}\tau_A} G_W(\bar{x}'_A, \bar{x}_A) \\
 &- i \langle E_1^{(A)} | m_A(0) | E_0^{(A)} \rangle \langle E_0^{(B)} | m_B(0) | E_0^{(B)} \rangle \int_{-\infty}^{\infty} d\tau'_B \int_{-\infty}^{\infty} d\tau_A \chi_B(\tau'_B) \chi_A(\tau_A) e^{i \Delta E^{(A)}\tau_A} G_R(\bar{x}_A, \bar{x}'_B) \\
 &- i \langle E_1^{(A)} | m_A(0) | E_0^{(A)} \rangle \langle E_0^{(A)} | m_A(0) | E_0^{(A)} \rangle \int_{-\infty}^{\infty} d\tau'_A \int_{-\infty}^{\infty} d\tau_A \chi_A(\tau'_A) \chi_A(\tau_A) e^{i \Delta E^{(A)}\tau_A} G_R(\bar{x}_A, \bar{x}'_A), \quad (\text{A13})
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{W}_B &\equiv -\langle E_1^{(B)} | m_B(0) | E_0^{(B)} \rangle [\langle E_1^{(B)} | m_B(0) | E_1^{(B)} \rangle - \langle E_0^{(B)} | m_B(0) | E_0^{(B)} \rangle] \\
 &\times \int_{-\infty}^{\infty} d\tau_B \int_{-\infty}^{\infty} d\tau'_B \chi_B(\tau_B) \chi_B(\tau'_B) \Theta(\tau'_B - \tau_B) e^{i \Delta E^{(B)}\tau_B} G_W(\bar{x}'_B, \bar{x}_B)
 \end{aligned}$$

$$\begin{aligned}
 & -i \langle E_1^{(B)} | m_B(0) | E_0^{(B)} \rangle \langle E_0^{(A)} | m_A(0) | E_0^{(A)} \rangle \int_{-\infty}^{\infty} d\tau'_A \int_{-\infty}^{\infty} d\tau_B \chi_A(\tau'_A) \chi_B(\tau_B) e^{i \Delta E^{(B)} \tau_B} G_R(\bar{x}_B, \bar{x}'_A) \\
 & -i \langle E_1^{(B)} | m_B(0) | E_0^{(B)} \rangle \langle E_0^{(B)} | m_B(0) | E_0^{(B)} \rangle \int_{-\infty}^{\infty} d\tau'_B \int_{-\infty}^{\infty} d\tau_B \chi_B(\tau'_B) \chi_B(\tau_B) e^{i \Delta E^{(B)} \tau_B} G_R(\bar{x}_B, \bar{x}'_B). \quad (\text{A14})
 \end{aligned}$$

APPENDIX B: EIGENVALUES OF MATRICES

In this Appendix, we outline the derivations of the eigenvalues of the matrices in the main text. In particular, we shall see that \mathcal{W}_I does not appear in the leading contributions to the eigenvalues.

The density matrix ρ_{AB} given in Eq. (4) and its partial transpose $\rho_{AB}^{T_A}$ in Eq. (17) take the same form,

$$P = \begin{pmatrix} 0 & 0 & 0 & c^2 \alpha \\ 0 & c^2 A & c^2 \beta & c^2 \sigma \\ 0 & c^2 \beta^* & c^2 B & c^2 \kappa \\ c^2 \alpha^* & c^2 \sigma^* & c^2 \kappa^* & 1 - c^2(A + B) \end{pmatrix} + c^4 \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} \\ Q_{21} & Q_{22} & Q_{23} & Q_{24} \\ Q_{31} & Q_{32} & Q_{33} & Q_{34} \\ Q_{41} & Q_{42} & Q_{43} & Q_{44} \end{pmatrix}, \quad (\text{B1})$$

where A and B are real, α , β , σ , and κ are complex constants, and $Q_{ij} = Q_{ji}^*$ is conditioned to satisfy $Q_{11} + Q_{22} + Q_{33} + Q_{44} = 0$. Thus, their eigenvalues are derived in a single stroke. A straightforward calculation shows that the eigenvalue equation is given by

$$0 = \det(P - \lambda \mathbb{I}) = \lambda^4 - \lambda^3 + [c^2(A + B) + O(c^4)]\lambda^2 - [c^4(A B - |\beta|^2) + O(c^6)]\lambda + c^8(A B - |\beta|^2)(Q_{11} - |\alpha|^2) + O(c^{10}). \quad (\text{B2})$$

Since σ or κ do not contribute to leading order in Eq. (B2), we expect that they will not appear in leading order of the eigenvalues, either. Indeed, Eq. (B2) is found to factorize as

$$0 = \det(P - \lambda \mathbb{I})$$

$$= \{\lambda^2 - [c^2(A + B) + O(c^4)]\lambda + c^4(AB - |\beta|^2) + O(c^6)\} \{\lambda^2 - [1 - c^2(A + B) + O(c^4)]\lambda + c^4(Q_{11} - |\alpha|^2) + O(c^6)\},$$

and thus the leading terms of the eigenvalues of P are derived as

$$1 + O(c^2), \quad \frac{c^2}{2} [A + B \pm \sqrt{(A - B)^2 + 4|\beta|^2}] + O(c^4), \quad c^4(Q_{11} - |\alpha|^2) + O(c^6). \quad (\text{B3})$$

We now set as $A = \mathcal{P}_A$, $B = \mathcal{P}_B$, and $Q_{11} = \mathcal{X}$. When we further set as $\alpha = \mathcal{E}$, $\beta = \mathcal{P}_{AB}$, $\sigma = \mathcal{W}_A$, and $\kappa = \mathcal{W}_B$, we obtain Eq. (12), while $\alpha = \mathcal{P}_{AB}^*$, $\beta = \mathcal{E}^*$, $\sigma = \mathcal{W}_A^*$, and $\kappa = \mathcal{W}_B$ give Eq. (18).

Similarly, the eigenvalue equation of $\rho_{AB} \tilde{\rho}_{AB}$ computed from Eqs. (4) and (31) is found to be given as

$$\begin{aligned}
 0 = \det(\rho_{AB} \tilde{\rho}_{AB} - \lambda \mathbb{I}) &= \lambda^4 - [2c^4(\mathcal{X} + |\mathcal{E}|^2 + \mathcal{P}_A \mathcal{P}_B + |\mathcal{P}_{AB}|^2) + O(c^6)]\lambda^3 \\
 &+ [c^8\{(\mathcal{X} - |\mathcal{E}|^2)^2 + (\mathcal{P}_A \mathcal{P}_B - |\mathcal{P}_{AB}|^2)^2 + 4(\mathcal{X} + |\mathcal{E}|^2)(\mathcal{P}_A \mathcal{P}_B + |\mathcal{P}_{AB}|^2)\} + O(c^{10})]\lambda^2 \\
 &- [2c^{12}\{(\mathcal{X} + |\mathcal{E}|^2)(\mathcal{P}_A \mathcal{P}_B - |\mathcal{P}_{AB}|^2)^2 + (\mathcal{X} - |\mathcal{E}|^2)^2(\mathcal{P}_A \mathcal{P}_B + |\mathcal{P}_{AB}|^2)\} + O(c^{14})]\lambda \\
 &+ c^{16}(\mathcal{X} - |\mathcal{E}|^2)^2(\mathcal{P}_A \mathcal{P}_B - |\mathcal{P}_{AB}|^2)^2 + O(c^{18}), \quad (\text{B4})
 \end{aligned}$$

which is found to be factorized as

$$\det(\rho_{AB} \tilde{\rho}_{AB} - \lambda \mathbb{I}) = [\lambda^2 - \{2c^4(\mathcal{X} + |\mathcal{E}|^2) + O(c^6)\}]\lambda + c^8(\mathcal{X} - |\mathcal{E}|^2)^2 + O(c^{10}) \quad [\lambda^2 - \{2c^4(\mathcal{P}_A \mathcal{P}_B + |\mathcal{P}_{AB}|^2) + O(c^6)\}]\lambda + c^8(\mathcal{P}_A \mathcal{P}_B - |\mathcal{P}_{AB}|^2)^2 + O(c^{10})]. \quad (\text{B5})$$

Then, we see that the square roots of the eigenvalues of $\rho_{AB} \tilde{\rho}_{AB}$ are given as Eq. (32).

The matrix $T_{\rho_{AB}}$ defined by Eq. (39) is computed from Eq. (4) as

$$T_{\rho_{AB}} = \begin{pmatrix} c^2(\mathcal{E} + \mathcal{E}^* + \mathcal{P}_{AB} + \mathcal{P}_{AB}^*) & i c^2(\mathcal{E} - \mathcal{E}^* - \mathcal{P}_{AB} + \mathcal{P}_{AB}^*) & -c^2(\mathcal{W}_A + \mathcal{W}_A^*) \\ i c^2(\mathcal{E} - \mathcal{E}^* + \mathcal{P}_{AB} - \mathcal{P}_{AB}^*) & c^2(-\mathcal{E} - \mathcal{E}^* + \mathcal{P}_{AB} + \mathcal{P}_{AB}^*) & i c^2(-\mathcal{W}_A + \mathcal{W}_A^*) \\ -c^2(\mathcal{W}_B + \mathcal{W}_B^*) & i c^2(-\mathcal{W}_B + \mathcal{W}_B^*) & 1 - 2c^2(\mathcal{P}_A + \mathcal{P}_B) \end{pmatrix} + O(c^4), \quad (\text{B6})$$

and then the eigenvalue equation of $U_{\rho_{AB}}$ is derived and factorized as

$$\begin{aligned}
 0 = \det(U_{\rho_{AB}} - \lambda \mathbb{I}) &= -\lambda^3 + [1 - 4c^2(\mathcal{P}_A + \mathcal{P}_B) + O(c^4)]\lambda^2 - [8c^4(|\mathcal{E}|^2 + |\mathcal{P}_{AB}|^2) + O(c^6)]\lambda + 16c^8(|\mathcal{E}|^2 - |\mathcal{P}_{AB}|^2)^2 + O(c^{10}) \\
 &= -\{\lambda - [1 - 4c^2(\mathcal{P}_A + \mathcal{P}_B) + O(c^4)]\}\{\lambda^2 - [8c^4(|\mathcal{E}|^2 + |\mathcal{P}_{AB}|^2) + O(c^6)]\lambda + 16c^8(|\mathcal{E}|^2 - |\mathcal{P}_{AB}|^2)^2 + O(c^{10})\}, \quad (\text{B7})
 \end{aligned}$$

where $U_{\rho_{AB}} = T_{\rho_{AB}}^T T_{\rho_{AB}}$ as defined in Eq. (38). Then one finds that the eigenvalues of $U_{\rho_{AB}}$ are given by Eq. (40).

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