

Fast and robust control of two interacting spinsXiao-Tong Yu,¹ Qi Zhang,¹ Yue Ban,^{2,3} and Xi Chen^{1,*}¹*Department of Physics, Shanghai University, 200444 Shanghai, People's Republic of China*²*Department of Electronic Information Materials, Shanghai University, 200444 Shanghai, People's Republic of China*³*Instituto de Ciencia de Materiales de Madrid, CSIC, Sor Juana Inés de la Cruz 3, E-28049 Madrid, Spain*

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Rapid preparation, manipulation, and correction of spin states with high fidelity are requisite for quantum information processing and quantum computing. In this paper, we propose a fast and robust approach for controlling two spins with Heisenberg and Ising interactions. By using the concept of shortcuts to adiabaticity, we first inverse design the driving magnetic fields for achieving fast spin flip or generating the entangled Bell state, and further optimize them with respect to the error and fluctuation. In particular, the designed shortcut protocols can efficiently suppress the unwanted transition or control error induced by anisotropic antisymmetric Dzyaloshinskii-Moriya exchange. Several examples and comparisons are illustrated, showing the advantages of our methods. Finally, we emphasize that the results can be naturally extended to multiple interacting spins and other quantum systems in an analogous fashion.

DOI: [10.1103/PhysRevA.97.062317](https://doi.org/10.1103/PhysRevA.97.062317)**I. INTRODUCTION**

Efficient initialization and manipulation of quantum states have been long pursued in the fields of quantum optics, quantum control, and even quantum simulation, due to their significance for information storage, processing, and computing in various systems [1,2]. Along this research line, different approaches including π (or $\pi/2$) resonant pulse [3], adiabatic passages, and its variations [4,5] have been proposed to achieve such desirable goal. As compared to simple resonant pulses, adiabatic passages are usually robust against the systematic errors, but take a long time, due to adiabatic criteria. The shortcoming is that the long-time state evolution will be spoiled due to decoherence effects in a noisy environment. To remedy it, several complementary methods, i.e., composite pulses [6–8] and optimal control [9–11], have been proposed.

In recent years, an alternative concept of “shortcuts to adiabaticity” (STA) [12] has been put forward to speed up the slow adiabatic process without final excitation, with broad applications, ranging from atomic, molecular, and optical physics to solid-state physics. Among them, two specific protocols, inverse engineering [13] and counterdiabatic driving (or equivalently the quantum transitionless algorithm) [14–16], are popular for different motivations and proposals. In principle these two methods are mathematically equivalent, but the physical implementations are quite different [17]. For instance, ultrafast internal state manipulation [18–21] and ion transport in phase space [22] have been realized in state-of-the-art experimental implementations. The disadvantage, however, is that the counterdiabatic terms are sometimes infeasible or completely unphysical [23]. Also, the inverse engineering stems from the Lewis-Riesenfeld (LR) dynamics invariant [24], which was first proposed for fast frictionless

atom cooling in harmonic traps [13] and demonstrated experimentally as well [25,26]. More importantly, such shortcut provides more freedom for further optimization when the dynamics is designed only from the appropriate initial and final boundary conditions [27–29]. Actually, other methods can also reproduce similar results from the inverse engineering strategy [30–33].

Particularly, the applications of STA in spin-1/2 systems are always amazing, since the (optimal) control of its dynamics is demanding in nuclear magnetic resonance (NMR) [34–36], nitrogen-vacancy centers in diamond [19,21], and quantum dots [37–39], and such system also resembles ubiquitous two-level quantum systems, for instance, superconducting circuits [40], and optomechanical systems [41]. For single spin, the controllable magnetic fields are shaped, respectively, from STA methods of counterdiabatic driving [15], inverse engineering [30,42–44], and fast-forward scaling [45–47]. Also, the manipulation of two spins with isotropic and anisotropic exchange interactions becomes naturally attractive for entangled state generation and quantum annealing. The system of two interacting spins with four internal levels, satisfying SU(4) Lie algebra, allows one to design the STA by using the LR invariant [48]. Recently, the inverse engineering for four-level systems with specific coupling configurations was further developed by using four-dimensional double rotation [49]. But, in certain cases the four-level system can be simplified to three-level or two-level systems in terms of adiabatic elimination. As a consequence, the counterdiabatic terms [50,51] are easily calculated and implemented. Again, we shall emphasize that inverse engineering [43,52] really provides an efficient way for speeding up the conventional adiabatic passage [53], and producing the efficient quantum gates as well [44]. However, the optimization of STA with respect to the error and fluctuation, contributed from control field and perturbative interaction, has not been explored yet, which is an important issue in practice.

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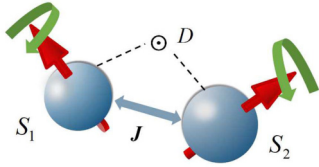


FIG. 1. Schematic diagram for the two-spin system with the Heisenberg or Ising interaction J and Dzyaloshinskii-Moriya interaction D , resulting from anisotropic antisymmetric exchange.

In this paper, we shall study systematically the optimal control of two interacting spins through time-dependent magnetic fields by using the technique of STA. For simplicity, we consider fast control of spin states in two coupled spins systems (see Fig. 1), in the presence of isotropic Heisenberg or anisotropic Ising interaction, and the controllable magnetic fields are designed inversely correspondingly. By combining with time-dependent perturbation theory, the spin dynamics is further optimized with respect to the errors and fluctuations. Moreover, we shall consider the influence of Dzyaloshinskii-Moriya (DM) interaction in two-spin systems, which is an anisotropic antisymmetric interaction, due to spin-orbit coupling. This non-negligible term contributes to control error in quantum information processing [54]. Our strategy is to treat it as perturbative error, rather than counterdiabatic term [51], and to improve the fidelity by suppressing the unwanted transition or canceling the control error. All these results are demonstrated by numerical examples, and compared with the resonant and composite pulses, showing the advantage of robustness.

The paper is organized as follows. In Sec. II, we consider the two Heisenberg-interacting spins to design the fast and robust spin flip with systematic errors and perturbative DM interactions. In Sec. III we design such shortcuts again for two Ising-interacting spins in the presence of systematic errors and DM interaction, to generate the entangled Bell state with a short time scale. We finally present the discussion and summary in Secs. IV and V.

II. TWO HEISENBERG-INTERACTING SPINS

First of all, we consider the two-spin system of spin quantum numbers \vec{S}_1 and \vec{S}_2 with isotropic exchange coupling in the presence of a time-dependent magnetic field, which is described by the Hamiltonian

$$H(t) = J\vec{S}_1 \cdot \vec{S}_2 + \vec{B}(t) \cdot (\vec{S}_1 + \vec{S}_2), \quad (1)$$

where $J > 0$ describes antiferromagnetic coupling ($J < 0$ ferromagnetic coupling), and $\vec{B}(t)$ is the rotating magnetic field with three components, B_i ($i = x, y, z$). (Noting that we consider the isotropic one for simplicity, and there also exist other anisotropic Heisenberg models, see the Appendix.) In the basis of $\{S_z, S^2\}$,

$$|\psi_{1,1}\rangle = |\uparrow\uparrow\rangle, \quad (2)$$

$$|\psi_{1,0}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \quad (3)$$

$$|\psi_{0,0}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \quad (4)$$

$$|\psi_{1,-1}\rangle = |\downarrow\downarrow\rangle, \quad (5)$$

the Hamiltonian has the following matrix form (by setting $\hbar \equiv 1$):

$$H(t) = \begin{pmatrix} \frac{J}{4} + B_z & \frac{B_x - iB_y}{\sqrt{2}} & 0 & 0 \\ \frac{B_x + iB_y}{\sqrt{2}} & \frac{J}{4} & 0 & \frac{B_x - iB_y}{\sqrt{2}} \\ 0 & 0 & -\frac{3J}{4} & 0 \\ 0 & \frac{B_x + iB_y}{\sqrt{2}} & 0 & \frac{J}{4} - B_z \end{pmatrix}, \quad (6)$$

where the time dependence is omitted for simplicity. By shifting the energy $J/4$, we can further simplify the Hamiltonian as follows:

$$H(t) = \begin{pmatrix} B_z & \frac{B_x - iB_y}{\sqrt{2}} & 0 & 0 \\ \frac{B_x + iB_y}{\sqrt{2}} & 0 & 0 & \frac{B_x - iB_y}{\sqrt{2}} \\ 0 & 0 & -J & 0 \\ 0 & \frac{B_x + iB_y}{\sqrt{2}} & 0 & -B_z \end{pmatrix}. \quad (7)$$

Since there exists one level decoupled to the other three, the Hamiltonian can be further reduced to

$$H(t) = \begin{pmatrix} \Delta & \frac{1}{\sqrt{2}}\Omega & 0 \\ \frac{1}{\sqrt{2}}\Omega & 0 & \frac{1}{\sqrt{2}}\Omega \\ 0 & \frac{1}{\sqrt{2}}\Omega & -\Delta \end{pmatrix}, \quad (8)$$

when we impose $\Omega = B_x$, $\Delta = B_z$, and $B_y = 0$. Here Ω and Δ refer to two components of magnetic fields, which resemble the Rabi frequency and detuning in quantum optics as well. With this interaction in such type of two coupled spins, one cannot reach the Bell state $|\psi_{1,0}\rangle$, from $|\psi_{1,1}\rangle$, since there is no energy gap between these two states. Instead, the time-dependent magnetic field can drive the states from $|\psi_{1,1}\rangle$ to $|\psi_{1,-1}\rangle$. But the adiabatic passage takes a long time to satisfy the widely used adiabatic condition, $T^{\text{ad}} \gg \Omega_0/a$ [55]. As an example, the Landau-Zener scheme, $\Omega = \Omega_0$ and $\Delta = a(t - T/2)$, requires $T^{\text{ad}} \simeq 20$, with constant Rabi frequency $\Omega_0 = 8$ and chirp $a = 4$. Our motivation is to speed up and optimize the spin-flip process by using inverse engineering.

This Hamiltonian (8) for the three-level model maps into a spin-1 system, and can thus be rewritten as $H = \Omega J_x + \Delta J_z$, where J_v ($v = x, y, z$) are spin-1 generator matrices, satisfying SU(2) algebra, $[J_\mu, J_\nu] = iJ_\gamma \varepsilon_{\mu\nu\gamma}$, with the structure constants $\varepsilon_{\mu\nu\gamma}$ (see [56]). Consequently, the dynamic invariant $I(t)$, satisfying $dI(t)/dt \equiv \partial I(t)/\partial t + (1/i\hbar)[H(t), I(t)] = 0$, can be constructed as [24]

$$I(t) = B_0 \begin{pmatrix} \cos \theta & \frac{1}{\sqrt{2}} \sin \theta e^{-i\beta} & 0 \\ \frac{1}{\sqrt{2}} \sin \theta e^{i\beta} & 0 & \frac{1}{\sqrt{2}} \sin \theta e^{-i\beta} \\ 0 & \frac{1}{\sqrt{2}} \sin \theta e^{i\beta} & -\cos \theta \end{pmatrix}, \quad (9)$$

where B_0 is a constant magnitude of magnetic field, guaranteeing the same dimension as $H(t)$. The eigenstates of the invariant $I(t)$, $I(t)|\phi_n(t)\rangle = \lambda_n|\phi_n(t)\rangle$ (we use the labels

$n = 0, 1, 2$), can be easily obtained as

$$|\phi_0(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sin \theta e^{-i\beta} \\ \sqrt{2} \cos \theta \\ \sin \theta e^{i\beta} \end{pmatrix}, \quad (10)$$

$$|\phi_1(t)\rangle = \begin{pmatrix} \cos^2 \frac{\theta}{2} e^{-i\beta} \\ \frac{1}{\sqrt{2}} \sin \theta \\ \sin^2 \frac{\theta}{2} e^{i\beta} \end{pmatrix}, \quad (11)$$

$$|\phi_2(t)\rangle = \begin{pmatrix} \sin^2 \frac{\theta}{2} e^{-i\beta} \\ -\frac{1}{\sqrt{2}} \sin \theta \\ \cos^2 \frac{\theta}{2} e^{i\beta} \end{pmatrix}, \quad (12)$$

with corresponding eigenvalues $\lambda_0 = 0$ and $\lambda_{1,2} = \pm B_0/2$. Based on LR theory, the dynamics of such three-level system, described by the time-dependent Schrödinger equation, $i\hbar \partial_t |\Psi(t)\rangle = H |\Psi(t)\rangle$, is in general governed by the superposition of orthogonal ‘‘dynamical mode’’ [24]

$$|\Psi(t)\rangle = \sum_n c_n e^{i\gamma_n} |\phi_n(t)\rangle, \quad (13)$$

where c_n is a time-independent constant and the LR phases γ_n are solved as $\gamma_0 = 0$:

$$\gamma_{1,2} = \pm \int_0^t dt' \left(\frac{\dot{\theta} \cot \beta}{\sin \theta} \right). \quad (14)$$

From the condition for the dynamical invariant, we have the following auxiliary differential equations:

$$\dot{\theta} = -\Omega \sin \beta, \quad (15)$$

$$\dot{\beta} = \Delta - \Omega \cot \theta \cos \beta, \quad (16)$$

by which the magnetic-field components, Δ and Ω , are connected with two parameters θ and β . Now it is ready to apply the inverse engineering for achieving the fast spin flip from $|\psi_{1,1}\rangle$ to $|\psi_{1,-1}\rangle$ within a short time. The unperturbative solution is along the single dynamical mode $|\psi_1(t)\rangle$, and we impose the boundary conditions

$$\theta(0) = 0, \theta(T) = \pi, \quad (17)$$

$$\dot{\theta}(0) = 0, \dot{\theta}(T) = 0. \quad (18)$$

Note that the boundary conditions (17) are necessary for spin flip, while the others (18) make the fields smooth at the edges. Once θ and β are interpolated by the polynomial ansatz [52] with the appropriate boundary conditions (17) and (18), the magnetic fields can be inversely designed from Eqs. (15) and (16). In principle, inverse engineering has more flexibilities than counterdiabatic driving, since there are thousands of possible paths to connect the boundary conditions. One has to optimize the shortcut by combining with the time-dependent perturbation theory [27], or numerical recipe [57]. In what follows, we shall optimize the magnetic fields with respect to the systematic error, and also suppress the unwanted transition, induced by anisotropic antisymmetric DM interaction.

A. Systematic error

The existence of noises, errors, and fluctuations is unavoidable during the state control for most practical quantum

systems. The optimization of STA with respect to systematic errors is helpful, since the rotating magnetic field might have imperfection, resulting in the shift δ in the amplitude of \vec{B} . Here we describe such sole systematic error by the perturbative Hamiltonian, that is,

$$H'(t) = \begin{pmatrix} \delta\Delta & \frac{1}{\sqrt{2}}\delta\Omega & 0 \\ \frac{1}{\sqrt{2}}\delta\Omega & 0 & \frac{1}{\sqrt{2}}\delta\Omega \\ 0 & \frac{1}{\sqrt{2}}\delta\Omega & -\delta\Delta \end{pmatrix}, \quad (19)$$

which means the two components of the magnetic field are simultaneously shifted as $\Delta \rightarrow \Delta(1 + \delta)$ and $\Omega \rightarrow \Omega(1 + \delta)$.

By using time-dependent perturbation theory, we have

$$\begin{aligned} |\Psi(T)\rangle &= |\Psi_1(T)\rangle - i \int_0^T dt U_0(T,t) H' |\Psi_1(t)\rangle - \int_0^T dt \\ &\times \int_0^t dt' U_0(T,t) H' U_0(t,t') H' |\Psi_1(t')\rangle + \dots, \end{aligned} \quad (20)$$

where the unperturbed time evolution operator $U_0(T,t) = \sum_n |\Psi_n(T)\rangle \langle \Psi_n(t)|$ with $|\Psi_n(t)\rangle = e^{i\gamma_n(t)} |\phi_n(t)\rangle$ where $n = 0, 1, 2$.

The fidelity to find the final state $|\psi_{1,-1}\rangle$ from initial state $|\psi_{1,1}\rangle$ along one of the dynamical modes, $|\Psi_1(t)\rangle$, is defined as $F = |\langle \Psi_1(T) | \Psi(T) \rangle|^2$, and can be further estimated as, by keeping the second order,

$$F \simeq 1 - \sum_{n \neq 1} \left| \int_0^T dt \langle \Psi_1(t) | H' | \Psi_n(t) \rangle \right|^2. \quad (21)$$

By defining the systematic error sensitivity as [27]

$$q_S = -\frac{1}{2} \frac{\partial^2 F}{\partial \delta^2} \Big|_{\delta=0}, \quad (22)$$

we have

$$q_S = \frac{1}{2} \left| \int_0^T dt (-\dot{\beta} \sin \theta - i\dot{\theta}) e^{im(t)} \right|^2, \quad (23)$$

with $m(t) = -\gamma_1$. This quantity resembles the fidelity susceptibility [58], which is the second-order derivative of the fidelity with respect to δ , and describes the response of the fidelity to a small error. So, to minimize or nullify the systematic error sensitivity can somehow improve the fidelity.

In the simplest case of a flat π pulse, we have $\theta = \pi t/T$ and $\beta = -\pi/2$, therefore the error sensitivity gives $q_S = \pi^2/2$, independent of T . It is consistent with other results that the error sensitivity is only relevant to boundary conditions but irrelevant to the duration time T [27,29,44].

Inspired by [28,29], we assume that

$$m(t) = 2\theta + 2\alpha \sin 2\theta, \quad (24)$$

to nullify the error sensitivity (23), by choosing an appropriate α . After substituting Eq. (14) into the equation $\gamma_1 = -2(\theta + \alpha \sin 2\theta)$, we can solve for

$$\beta = -\text{arccot} [2(1 + 2\alpha \cos 2\theta) \sin \theta]. \quad (25)$$

To achieve the goal of spin flip, the form of θ is assumed to be

$$\theta = 3\pi \left(\frac{t}{T} \right)^2 - 2\pi \left(\frac{t}{T} \right)^3, \quad (26)$$

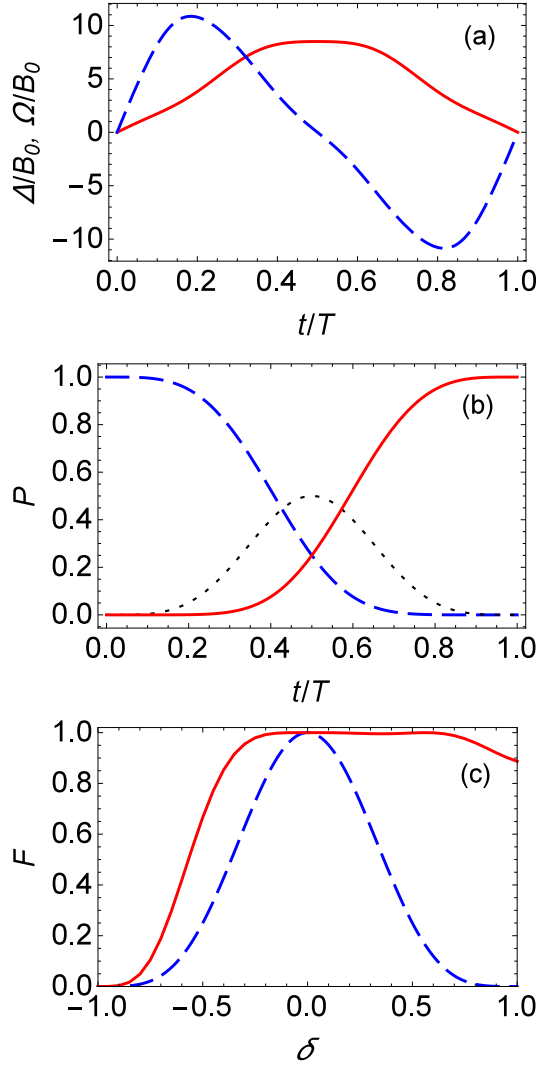


FIG. 2. Optimal shortcut for spin flip in two Heisenberg-interacting spins with rotating magnetic fields, by taking into account the systematic error. (a) Optimal protocol of two components of magnetic field, Ω (red, solid line) and Δ (blue, dashed line), in the unit of B_0 . (b) Dynamics of spin, described by the probability of state $|\psi_{1,1}\rangle$ (blue, dashed line), $|\psi_{1,0}\rangle$ (black, dotted line), and $|\psi_{1,-1}\rangle$ (red, solid line). (c) Fidelity vs the systematic error δ in the amplitude of rotating magnetic field, where the optimal shortcut (red, solid line) and flat π pulse (blue, dashed line) are compared. Parameters: $T = 1$ and $\alpha = 0.125$.

from which we obtain $\alpha = 0.125$ in Eq. (25) to make $q_S = 0$. By using Eqs. (15) and (16), the two components of the magnetic field, Ω and Δ , can be further inferred for optimal shortcut with respect to the systematic error.

Figure 2 presents the optimal protocol of spin flip in such two-coupled spins. The two components, Ω and Δ , of the rotating magnetic field [see Fig. 2(a)] are smooth enough to implement easily in practice. By using the designed magnetic field, the spin dynamics is also illustrated in Fig. 2(b), where the spin flip from $|\psi_{1,1}\rangle$ to $|\psi_{1,-1}\rangle$ is perfectly achieved. Moreover, Fig. 2(c) illustrates that the fidelity is better than the flat π pulse, where the fidelity is calculated by solving the time-dependent Schrödinger equation with Hamiltonian (8), by using the designed magnetic fields.

B. Dzyaloshinskii-Moriya interaction

Next, we shall consider the optimal shortcuts, by taking account of the DM term, since the non-negligible magnitude results in the control errors in quantum information processing [54]. This DM interaction, originally introduced by Dzyaloshinskii [59] and Moriya [60], is an anisotropic antisymmetric exchange interaction arising from the spin-orbit coupling, and has the following form:

$$H_{\text{DM}}(t) = \vec{D} \cdot (\vec{S}_1 \times \vec{S}_2), \quad (27)$$

where \vec{D} can be simply chosen as the constant DM vector D along the z axis. In the basis of $\{S_z, S^2\}$, the Hamiltonian is written as

$$H_{\text{DM}}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{i}{2}D & 0 \\ 0 & \frac{i}{2}D & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (28)$$

which leads to the total Hamiltonian $\tilde{H}(t) = H(t) + H_{\text{DM}}(t)$, by combining with Eq. (8):

$$\tilde{H}(t) = \begin{pmatrix} \Delta & \frac{1}{\sqrt{2}}\Omega & 0 & 0 \\ \frac{1}{\sqrt{2}}\Omega & 0 & -\frac{i}{2}D & \frac{1}{\sqrt{2}}\Omega \\ 0 & \frac{i}{2}D & -J & 0 \\ 0 & \frac{1}{\sqrt{2}}\Omega & 0 & -\Delta \end{pmatrix}. \quad (29)$$

Obviously, the DM interaction has the imaginary couple, which plays the same role as counterdiabatic driving, for example, when flipping the spin from $|\uparrow\downarrow\rangle$ to $|\downarrow\uparrow\rangle$ [51]. But in this case, we would like to design the shortcut to adiabatic state evolution along the dynamical mode $|\Psi_1(t)\rangle$ for achieving the spin flip from $|\psi_{1,1}\rangle$ to $|\psi_{1,-1}\rangle$. If one looks at the dynamical mode in Eq. (11), the state $|\psi_{1,0}\rangle$ is involved and populated during the shortcut path. Therefore, such additional coupling between $|\psi_{1,0}\rangle$ and $|\psi_{0,0}\rangle$ definitely induces an unwanted transition, and finally lowers the fidelity as a consequence.

Now, we shall apply the STA, combining with optimization, by reducing the effect of the DM interaction. According to LR dynamical invariant theory, the solutions of the time-dependent Schrödinger equation with $D = 0$ are the set of orthogonal solutions:

$$|\Psi_0(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sin\theta e^{-i\beta} \\ \sqrt{2}\cos\theta \\ \sin\theta e^{i\beta} \\ 0 \end{pmatrix} e^{i\gamma_0}, \quad (30)$$

$$|\Psi_1(t)\rangle = \begin{pmatrix} \cos^2\frac{\theta}{2} e^{-i\beta} \\ \frac{1}{\sqrt{2}}\sin\theta \\ \sin^2\frac{\theta}{2} e^{i\beta} \\ 0 \end{pmatrix} e^{i\gamma_1}, \quad (31)$$

$$|\Psi_2(t)\rangle = \begin{pmatrix} \sin^2\frac{\theta}{2} e^{-i\beta} \\ -\frac{1}{\sqrt{2}}\sin\theta \\ \cos^2\frac{\theta}{2} e^{i\beta} \\ 0 \end{pmatrix} e^{i\gamma_2}, \quad (32)$$

$$|\Psi_3(t)\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ e^{iJt} \end{pmatrix}, \quad (33)$$

where the ancillary functions of θ , β , and γ fulfill Eqs. (14)–(16) with the boundary conditions (17) and (18).

We treat the DM interaction as perturbation, and apply time-dependent perturbation theory to obtain estimated fidelity to find the final state $|\psi_{1,-1}\rangle$ from initial state $|\psi_{1,1}\rangle$ along the state evolution $|\Psi_1(t)\rangle$:

$$F \simeq 1 - \sum_{n \neq 1} \left| \int_0^T dt \langle \Psi_1(t) | H_{DM}(t) | \Psi_n(t) \rangle \right|^2, \quad (34)$$

from which the definition of transition sensitivity [61],

$$q_D = - \frac{1}{2} \frac{\partial^2 F}{\partial D^2} \Big|_{D=0}, \quad (35)$$

is calculated as

$$q_D = \frac{1}{8} \left| \int_0^T dt \sin \theta(t) e^{iJt} e^{im(t)} \right|^2, \quad (36)$$

with $m(t) = -\gamma_1(t)$ as before. This quantifies how sensitive a given protocol is concerning the unwanted transition to the state $|\psi_{0,0}\rangle$. Similar to the previous part, we can have the same functions of θ and β as Eqs. (25) and (26), by imposing $m(t)$ [see Eq. (24)]. The numerical calculation gives $\alpha = 0.059$ to nullify the transition sensitivity (36), i.e., $q_D = 0$.

Figures 3(a) and 3(b) demonstrate the shortcut to adiabatic spin flip driven by the optimal magnetic fields, with two components Ω and Δ , with respect to the influence of the DM term. By using the designed protocol, we solve the time-dependent Schrödinger equation with Hamiltonian (29). As a result, the unwanted transition to the state $|\psi_{0,0}\rangle$ can be efficiently suppressed, and the fidelity for spin flip can be above > 0.9999 , when $D/B_0 \leq 2$. In addition, Fig. 3(c) shows that the robustness of our designed protocol is better than the flat π pulse, in which q_D is constant, $q_D = 1/2\pi^2$, irrelevant to final time T . As a matter of fact, the optimal shortcut is quite insensitive to DM interaction, since the fidelity is reasonable when DM interaction is less than the magnitude of magnetic fields, $\Omega/B_0 \simeq 10$.

III. TWO ISING-INTERACTING SPINS

In this section, we shall consider the two coupled spins, described by a simple transverse Ising model, a minimum model for quantum annealing [45,47]. The anisotropic interaction allows us to generate the entangled Bell state, i.e., $|\psi_{1,0}\rangle$, which is crucial in quantum information processing with atoms or spins [52,53]. The Hamiltonian has the simple form

$$H(t) = JS_1^z \cdot S_2^z + \vec{B}(t) \cdot (\vec{S}_1 + \vec{S}_2), \quad (37)$$

where J is the exchange interaction, \vec{S}_1 and \vec{S}_2 are the two respective spin operators, and $\vec{B}(t)$ is the time-dependent rotating magnetic field. Similar to Sec. II, in the basis of $\{S_z, \tilde{S}^2\}$, the Hamiltonian is rewritten as

$$H(t) = \begin{pmatrix} At & \frac{\Omega}{\sqrt{2}} e^{-i\omega t} & 0 & 0 \\ \frac{\Omega}{\sqrt{2}} e^{i\omega t} & -J/2 & 0 & \frac{\Omega}{\sqrt{2}} e^{-i\omega t} \\ 0 & 0 & -J/2 & 0 \\ 0 & \frac{\Omega}{\sqrt{2}} e^{i\omega t} & 0 & -At \end{pmatrix}, \quad (38)$$

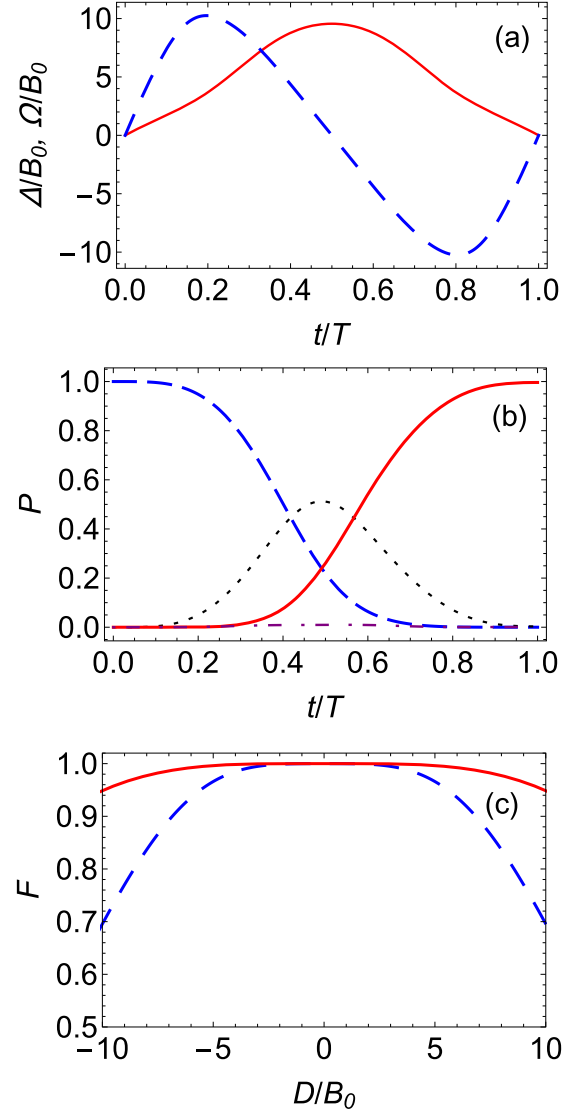


FIG. 3. Optimal shortcut for spin flip in two Heisenberg-interacting spins with rotating magnetic fields, by taking into account the perturbative DM interaction. (a) Optimal protocol of two components of magnetic field, Ω (red, solid line) and Δ (blue, dashed line), in the unit of B_0 . (b) Dynamics of spin, described by the probability of state $|\psi_{1,1}\rangle$ (blue, dashed line), $|\psi_{1,0}\rangle$ (black, dotted line), $|\psi_{0,0}\rangle$ (purple, dot-dashed line), and $|\psi_{1,-1}\rangle$ (red, solid line). (c) Fidelity vs constant vector D , representing the magnitude of such anisotropic antisymmetric exchange, where the optimal shortcut (red, solid line) and flat π pulse (blue, dashed line) are compared. Parameters: $J/B_0 = 10$, $D/B_0 = 1$, $T = 1$, and $\alpha = 0.059$.

in which the magnetic-field components we choose are $B_x = \Omega \cos \omega t$, $B_y = \Omega \sin \omega t$, and $B_z = At$. Here the time dependence is omitted for simplicity. Obviously, one of the states is decoupled from the other three, thus the Hamiltonian, after the phase transformation, can be reduced to [53]

$$H(t) = \begin{pmatrix} -\omega + At & \frac{1}{\sqrt{2}}\Omega & 0 \\ \frac{1}{\sqrt{2}}\Omega & -\frac{J}{2} & \frac{1}{\sqrt{2}}\Omega \\ 0 & \frac{1}{\sqrt{2}}\Omega & \omega - At \end{pmatrix}, \quad (39)$$

from which we learn that level crossing happens at three different times, $t_{12} = (\omega - \frac{J}{2})/A$, $t_{13} = \omega/A$, and $t_{23} = (\omega + \frac{J}{2})/A$. Actually, the energies of the adiabatic states have avoided crossings at certain times, which helps us treat them as an effective two-level problem (see the details in [52,53]).

To prepare the entangled Bell state, we choose $|\psi_{1,1}\rangle$ and $|\psi_{1,0}\rangle$ as initial and final state, respectively. So, in order to avoid other crossings except for these two states, we can choose a Ω centered at t_{12} . And the Hamiltonian, involving only these two states, can be eventually simplified as

$$H(t) = \frac{1}{2} \begin{pmatrix} \Delta & \sqrt{2}\Omega \\ \sqrt{2}\Omega & -\Delta \end{pmatrix}, \quad (40)$$

where $\Delta = At - \omega + J/2$. Here Ω and Δ are slightly different from the previous section, and refer to the amplitude and frequency of the magnetic field. Typically, the adiabatic passage for transferring the state from $|\psi_{1,1}\rangle$ to $|\psi_{1,0}\rangle$ requires a time scale of $30/J$ [43,52], which gives $T^{\text{ad}} \approx 3$ when $J = 10$. Later, we shall apply STA to shorten the time, and choose $T = 1$.

For the effective two-level system, there exist dynamical invariants as follows [27,29]:

$$I(t) = \frac{B_0}{2} \begin{pmatrix} \cos \theta & \sin \theta e^{-i\beta} \\ \sin \theta e^{i\beta} & -\cos \theta \end{pmatrix}, \quad (41)$$

where θ and β are the polar and azimuthal angles in the Bloch sphere. The eigenstates are

$$|\phi_+(t)\rangle = \begin{pmatrix} \cos \frac{\theta}{2} e^{-i\beta} \\ \sin \frac{\theta}{2} \end{pmatrix}, \quad |\phi_-(t)\rangle = \begin{pmatrix} \sin \frac{\theta}{2} \\ -\cos \frac{\theta}{2} e^{i\beta} \end{pmatrix}, \quad (42)$$

corresponding to the eigenvalues, $\lambda_{\pm} = \pm B_0/2$. Similar to Sec. II, the solution of the time-dependent Schrödinger equation, $i\hbar\partial_t|\Psi(t)\rangle = H|\Psi(t)\rangle$, can be written as

$$|\Psi(t)\rangle = \sum_n c_n |\phi_n(t)\rangle e^{i\gamma_n}, \quad (43)$$

where c_n is a time-independent constant and the LR phases γ_n are calculated as

$$\gamma_{\pm} = \pm \frac{1}{2} \int_0^t dt' \left(\dot{\beta} + \frac{\dot{\theta} \cot \beta}{\sin \theta} \right). \quad (44)$$

To satisfy the condition for the dynamical invariant, the time-dependent control parameters θ and β are connected to Ω and Δ by the following equations:

$$\dot{\theta} = -\sqrt{2}\Omega \sin \beta, \quad (45)$$

$$\dot{\beta} = \Delta - \sqrt{2}\Omega \cot \theta \cos \beta, \quad (46)$$

from which we can design state evolution inferred for the driving fields, fulfilling the appropriate boundary conditions. This invariant-based inverse engineering provides an efficient way to design a shortcut for generating the entangled Bell state. To do this, we assume the state evolves along the dynamical mode $|\phi_+(t)\rangle$, and the same boundary conditions, Eqs. (17) and (18), are also imposed. Once the functions of θ and β satisfy the boundary conditions, the driving magnetic field can be designed to prepare the entangled Bell state from initial state $|\psi_{1,1}\rangle$ to final state $|\psi_{1,0}\rangle$ within a short time scale [43,52]. However, the freedom to choose the function leaves flexibility for further optimization, which we shall address below.

A. Systematic error

We shall consider the optimization of STA, since there may exist systematic errors in both Rabi frequency and detuning, simultaneously, since they refer to the control error in the amplitude and frequency of the magnetic field, $\vec{B}(t)$. First, we describe the systematic error only in the Rabi frequency by the following form:

$$H'(t) = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{2}\delta\Omega \\ \sqrt{2}\delta\Omega & 0 \end{pmatrix}, \quad (47)$$

which means the amplitude of the magnetic field is changed as $\Omega \rightarrow \Omega(1 + \delta)$ due to the fluctuation. By using time-dependent perturbation theory, we have

$$\begin{aligned} |\Psi(T)\rangle &= |\Psi_+(T)\rangle - i \int_0^T dt U_0(T,t) H' |\Psi_+(t)\rangle - \int_0^T dt \\ &\times \int_0^t dt' U_0(T,t) H' U_0(t,t') H' |\Psi_+(t')\rangle + \dots, \end{aligned} \quad (48)$$

where the unperturbed time evolution operator $U_0(T,t) = |\Psi_+(T)\rangle\langle\Psi_+(t)| + |\Psi_-(T)\rangle\langle\Psi_-(t)|$ with $|\Psi_{\pm}(t)\rangle = e^{i\gamma_{\pm}(t)}|\phi_{\pm}(t)\rangle$.

Accordingly, we can calculate the estimated fidelity to be found at the final state $|\psi_{1,1}\rangle$ from initial state $|\psi_{1,0}\rangle$:

$$F \simeq 1 - \left| \int_0^T dt \langle\Psi_+(t)|H'|\Psi_-(t)\rangle \right|^2. \quad (49)$$

Similarly, by defining the error sensitivity [27],

$$q_{\Omega} = -\frac{1}{2} \frac{\partial^2 F}{\partial \delta^2} \Big|_{\delta=0}, \quad (50)$$

we will have

$$q_{\Omega} = \frac{1}{4} \left| \int_0^T dt \dot{\theta} \sin^2 \theta e^{im(t)} \right|^2, \quad (51)$$

with $m(t) = 2\gamma_- - \beta$. Thus the fidelity is approximated as $F \simeq 1 - q_{\Omega}\delta^2$, keeping the second order. In the special case, $m(t) = 0$ and $q_{\Omega} = \pi^2/4$, independent of T . With the choice of $\beta = -\pi/2$, the result will recover the flat π pulse.

Following [27,28], we can obtain $q_{\Omega} = \sin^2(n\pi)/(4n^2)$ by imposing $m(t) = n(2\theta - \sin 2\theta)$, so that $q_{\Omega} = 0$ is achieved, if $n = 1, 2, 3, \dots$. In the case of $n = 1$, we have

$$\beta = -\text{arccot}(4 \sin^3 \theta), \quad (52)$$

which results in $q_{\Omega} = 0$. When q_{Ω} is nullified, the minimum of q_{Ω} is achieved, which gives the maximal robustness with respect to amplitude variations of magnetic field. As an example, we choose smooth function

$$\theta = \frac{\pi}{2} \left[1 + \sin \frac{\pi(2t - T)}{2T} \right], \quad (53)$$

satisfying Eqs. (17) and (18). Notice that we use a different ansatz from the previous one, to show the versatility. As a consequence, we can solve for Ω and Δ through Eqs. (45) and (46).

Figure 4 illustrates the optimal design of the driving magnetic field with respect to amplitude error, and corresponding

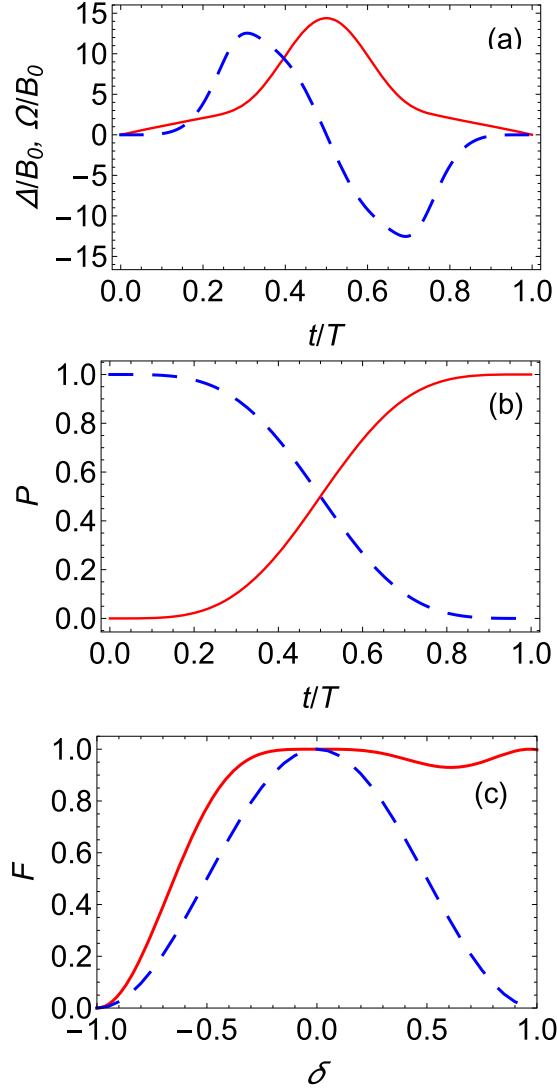


FIG. 4. Optimal designed protocol for fast generation of the entangled Bell state in two Ising-interacting spins with driving magnetic fields, by taking into account the amplitude error. (a) Optimal protocol of time dependence Ω (red, solid line) and Δ (blue, dashed line), in the unit of B_0 . (b) Dynamics of spin, described by the probability of state $|\psi_{1,1}\rangle$ (blue, dashed line) and $|\psi_{1,0}\rangle$ (red, solid line). (c) Fidelity vs the systematic error δ , comparing the optimal shortcut (red, solid line) with the flat π pulse (blue, dashed line). Parameter: $T = 1$.

spin evolution from $|\psi_{1,1}\rangle$ to $|\psi_{1,0}\rangle$. The dynamics and fidelity are calculated numerically by solving the time-dependent Schrödinger equation with Hamiltonian (40). The comparison between the shortcut protocol and flat π pulse is made to show the maximum robustness. Besides, we can also optimize the shortcut with respect to the systematic error in detuning, as in [29]. We will discuss a similar situation below.

B. Dzyaloshinskii-Moriya interaction

We turn to consider the effect of DM interaction on the Bell state generation and further reduce the control error by optimizing STA. After phase transformation, the total

Hamiltonian, with the DM term, is written as

$$\tilde{H}(t) = \begin{pmatrix} B_z & \frac{\Omega}{\sqrt{2}}e^{-i\omega t} & 0 & 0 \\ \frac{\Omega}{\sqrt{2}}e^{i\omega t} & -J/2 & -iD & \frac{\Omega}{\sqrt{2}}e^{-i\omega t} \\ 0 & iD & -J/2 & 0 \\ 0 & \frac{\Omega}{\sqrt{2}}e^{i\omega t} & 0 & -B_z \end{pmatrix}, \quad (54)$$

from which we can further obtain the effective Hamiltonian for the three-level system, by adiabatic elimination [20]:

$$\tilde{H}(t) = \begin{pmatrix} B_z & \frac{\Omega}{\sqrt{2}}e^{-i\omega t} & 0 \\ \frac{\Omega}{\sqrt{2}}e^{i\omega t} & -J/2 + \delta & \frac{\Omega}{\sqrt{2}}e^{-i\omega t} \\ 0 & \frac{\Omega}{\sqrt{2}}e^{i\omega t} & -B_z \end{pmatrix}, \quad (55)$$

with $\delta = 2D^2/J$. By choosing $B_x = \Omega \cos \omega t$, $B_y = \Omega \sin \omega t$, and $B_z = At$ as before, the Hamiltonian, after the phase transformation, becomes

$$\tilde{H}(t) = \begin{pmatrix} -\omega + At & \frac{1}{\sqrt{2}}\Omega & 0 \\ \frac{1}{\sqrt{2}}\Omega & -\frac{J}{2} + \delta & \frac{1}{\sqrt{2}}\Omega \\ 0 & \frac{1}{\sqrt{2}}\Omega & \omega - At \end{pmatrix}, \quad (56)$$

from which we learn that level crossing happens at three different times, $t_{12} = (\omega - \frac{J}{2} + \delta)/A$, $t_{13} = \omega/A$, and $t_{23} = (\omega + \frac{J}{2} - \delta)/A$. By comparing the Hamiltonian (56) with (39), we finally find the Hamiltonian, involving only two states of $|\psi_{1,1}\rangle$ and $|\psi_{1,0}\rangle$, as follows:

$$\tilde{H}(t) = \frac{1}{2} \begin{pmatrix} \Delta - \delta & \sqrt{2}\Omega \\ \sqrt{2}\Omega & -\Delta + \delta \end{pmatrix}, \quad (57)$$

where Δ is the same as before. As a result, the DM term in this case can be considered as the shift of detuning, and described by the perturbative Hamiltonian

$$H'(t) = \frac{1}{2} \begin{pmatrix} -\delta & 0 \\ 0 & \delta \end{pmatrix}. \quad (58)$$

Similar to Eqs. (49) and (51), we can finally obtain [29]

$$q_\Delta = \frac{1}{4} \left| \int_0^T dt \sin \theta e^{im(t)} \right|^2. \quad (59)$$

Again, we may assume $m(t)$ in Eq. (24) to nullify q_Δ , with free parameter α . After some straightforward calculations, we finally obtain $q_\Delta = 0$ with the parameter $\alpha = -0.206$. Figure 5 demonstrates the optimal protocol and corresponding spin dynamics designed by STA, in which the flat π pulse, $q_\Delta = (T/\pi)^2$, is also compared. Since $\delta = 2D^2/J$, the error induced by DM could be negative and positive depending on antiferromagnetic coupling $J > 0$ or ferromagnetic coupling $J < 0$. When $\delta = 4$, we have $D = 14.4$ in the unit of B_0 , with the parameter $J = 10$. Thus, it is proved, by solving the time-dependent Schrödinger equation with Hamiltonian (40), that our designed protocol is insensitive to the DM interaction [see Fig. 5(c)], when D is less than the magnitude of the

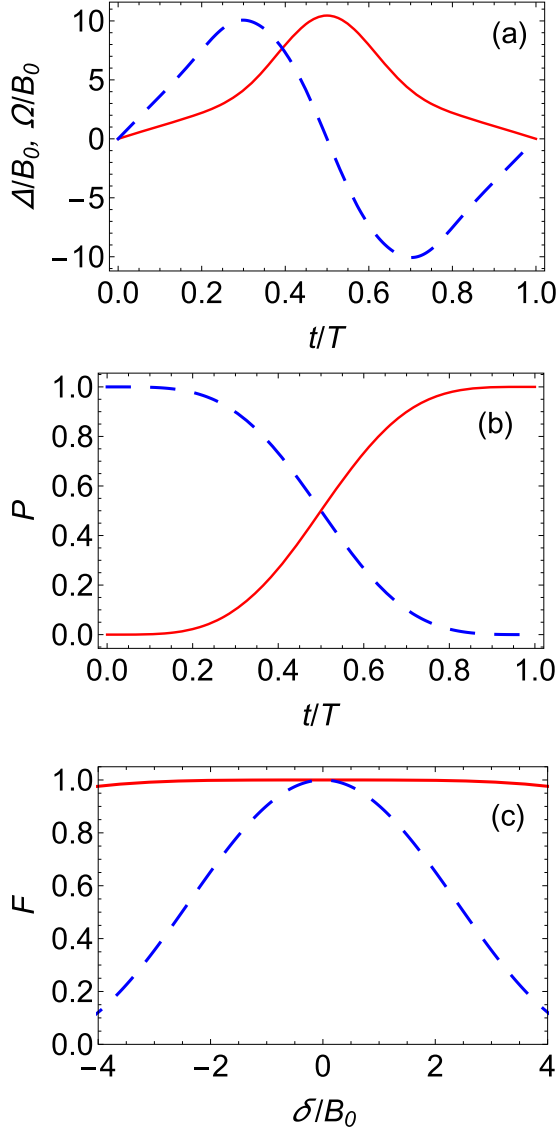


FIG. 5. Optimal designed protocol for fast generation of the entangled Bell state in two Ising-interacting spins with driving magnetic fields, by taking into account the DM interaction. (a) Optimal protocol of time dependence Ω (red, solid line) and Δ (blue, dashed line), in the unit of B_0 . (b) Dynamics of spin, described by the probability of state $|\psi_{1,1}\rangle$ (blue, dashed line) and $|\psi_{1,0}\rangle$ (red, solid line). (c) Fidelity vs the systematic error δ , relevant to the DM term, comparing the optimal shortcut (red, solid line) with the flat π pulse (blue, dashed line). Parameters: $J/B_0 = 10$, $D/B_0 = 1$, $T = 1$, and $\alpha = -0.206$.

driving magnetic field. This is extremely useful to generate the entangled Bell state in two coupled spin systems.

IV. DISCUSSION

In this section, we shall discuss the robustness of designed shortcut protocols, by comparing with the flat π pulse and a composite pulse $\frac{\pi}{2}(x)\pi(y)\frac{\pi}{2}(x)$. Especially, the techniques of composite pulses, originally proposed in NMR [6], are popular in robust high-fidelity quantum control [7,62]. First, we consider the spin-flip case when the Heisenberg interaction is present, in Sec. II. When the initial state is $|\psi_{1,1}\rangle$, the state

evolution can be represented as $|\psi(t)\rangle = U(t,0)|\psi_{1,1}\rangle$, where $U(t,0)$ is the propagator, connecting the initial state with final state. For instance, when the Hamiltonian $H = \Omega J_i$ ($i = x, y$), the propagator, $U_i(T,0) = \exp(-iHt)$, is calculated as

$$U_i(t,0) = I - i \sin(\Omega t) J_i + [\cos(\Omega t) - 1] J_i^2, \quad (60)$$

where J_i ($i = x, y$) are generator matrices of spin 1, as defined before. Considering the flat π pulse with the systematic error δ , the Hamiltonian is described by $H = \Omega(1 + \delta)J_x$ with $\Omega T = \pi$. By using the propagator in Eq. (60), the fidelity, $F = |\langle \psi_{1,-1} | U(T,0) | \psi_{1,1} \rangle|^2$, to be found in the desired state $|\psi_{1,-1}\rangle$, is thus analytically solved as

$$F = \cos^4\left(\frac{\pi\delta}{2}\right) \simeq 1 - \frac{\pi^2\delta^2}{2}. \quad (61)$$

This recovers that $q_S = \pi^2/2$, as discussed in Sec. II. For a composite pulse, we consider the typical one, $\frac{\pi}{2}(x)\pi(y)\frac{\pi}{2}(x)$, in order to minimize resonance offset effects, where the combination x, y refers to the two components of magnetic field and spin-1 operators, J_x and J_y , involved [6]. Similarly, we can calculate the propagator by using Eq. (60), and finally obtain the fidelity

$$F = \cos^8\left(\frac{\pi\delta}{2}\right) + \sin^2(\pi\delta)\cos^2\left(\frac{\pi\delta}{2}\right) \simeq 1 - \frac{\pi^4\delta^4}{8}. \quad (62)$$

Interestingly, we notice that for the composite pulse the error sensitivity, defined in Eq. (23), is null. This suggests that the composite pulse can improve the robustness, but more sequences require more time, as compared to the optimally robust shortcut protocol with single-shot pulse. Figure 6(a) demonstrates that the optimal shortcut with respect to the systematic error, as shown in Fig. 2, is more robust, as compared with the flat π pulse and composite pulse.

Regarding the generation of the entangled Bell state in two Ising-interaction spins, we consider the effective two-level Hamiltonian. In this case, we calculate the fidelities

$$F = \cos^2\left(\frac{\pi\delta}{2}\right) \simeq 1 - \frac{\pi^2\delta^2}{4}, \quad (63)$$

for the flat π pulse with $\sqrt{2}\Omega T = \pi$ and

$$F = 1 - \sin^4\left(\frac{\pi\delta}{2}\right) \simeq 1 - \frac{\pi^4\delta^4}{16}, \quad (64)$$

for composite pulse $\frac{\pi}{2}(x)\pi(y)\frac{\pi}{2}(x)$, with Pauli matrix, σ_x and σ_y , involved instead. For comparison, we calculate that $q_\Omega = \pi^2/4$ for the flat π pulse and $q_\Omega = 0$ for the composite pulse, based on Eq. (51), which are consistent with the discussion in Sec. III. Figure 6(b) confirms that the optimally shortcut protocol, as shown in Fig. 4, is ultrarobust against the systematic error, and its stability is better than the flat π pulse and composite pulse.

In this closing section, we shall emphasize the extension of the present optimal shortcuts to the many-spin system. For instance, we can consider the Hamiltonian

$$H(t) = J \sum_{i=1}^3 \sum_{j \neq i} S_i^z S_j^z + \vec{B}(t) \cdot \left(\sum_{i=1}^3 \vec{S}_i \right), \quad (65)$$

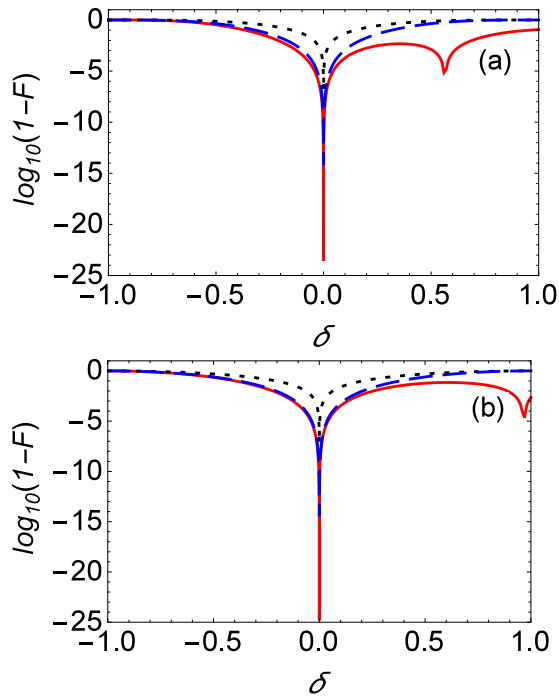


FIG. 6. Robustness of designed shortcut protocols (red, solid line), by comparing with flat π pulse (black, dotted line) and composite pulse (blue, dashed line), where (a) Heisenberg interaction and (b) Ising interaction are considered. Parameters: $T = 1$ and others are the same as those in Figs. 2 and 4.

describing three spins at the vertex of an equilateral triangle. By imposing the three components of the magnetic field, $B_x = \Omega \cos \omega t$, $B_y = \Omega \sin \omega t$, and $B_z = At$, we can use the similar approach in [52,53], and eventually write down the effective two-level Hamiltonian as follows:

$$H(t) = \frac{1}{2} \begin{pmatrix} \Delta & \sqrt{3}\Omega \\ \sqrt{3}\Omega & -\Delta \end{pmatrix}, \quad (66)$$

with $\Delta = At + \omega + J$, in the basis of

$$|\psi_{3/2,3/2}\rangle = |\uparrow\uparrow\uparrow\rangle, \quad (67)$$

$$|\psi_{3/2,1/2}\rangle = \frac{1}{\sqrt{3}}(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle). \quad (68)$$

This allows us to construct the dynamical invariant as before, and design inversely the components of the magnetic field to achieve fast and robust generation of the W entangled state $|\psi_{3/2,1/2}\rangle$ from the initial state $|\psi_{3/2,3/2}\rangle$. More interestingly, the same approach can be further applied to manipulate the larger number of coupled spins, when the Rabi frequency is replaced by $\sqrt{n}\Omega$ (see also [43]).

Also, one can generalize the shortcut protocol to the Heisenberg interaction with many spins, of which the Hamiltonian is

$$H(t) = J \sum_{(i,j)} \vec{S}_i \cdot \vec{S}_j + \vec{B}(t) \cdot \left(\sum_{i=1}^N \vec{S}_i \right), \quad (69)$$

where N is the total spin number and (i,j) stands for the summation process that we only take account of the nearest-neighbor spins. The dynamical invariant in this situation is constructed in [63], and STA technique will be utilized as well. But if the Heisenberg interaction is anisotropic [64], the dynamical invariant does not exist. Therefore, the above semianalytical analysis is not applicable, and more complicated numerical analysis is required.

V. CONCLUSION

In summary, we have studied the fast and robust control of spin states in two interacting spins with Heisenberg and Ising interaction by using STA technique. We first apply inverse engineering to design the time-dependent magnetic fields for spin flip, and optimize the spin dynamics with respect to control error and fluctuation. In particular, the optimal shortcut protocol provides an efficient way to suppress the unwanted transition or systematic error induced by DM, due to anisotropic antisymmetric exchange. Furthermore, the anisotropic Ising interaction enables us to generate the entangled Bell state, and the fast and stable process is interesting for quantum information processing.

We emphasize that the optimal shortcuts designed here are different from counterdiabatic driving [51,52] and fast-forward scaling [50], since the inverse engineering based on the LR invariant provides the explicit dynamics of spins and the flexibility for further optimization. A more robust protocol can be further designed by minimizing or nullifying the high order of the approximate fidelity, calculated from time-dependent perturbation theory. In addition, the smooth single-shot pulses are suitable for the applications, and could be connected to the composite pulse-sequence technique with a time-dependent phase [28]. Here we consider the special cases of the isotropic Heisenberg XXX and simple transverse Ising models with the semianalytical solutions, due to the symmetry. However, a more general anisotropic Heisenberg XYZ model requires numerical analysis and optimization [64], since the inverse engineering based on the dynamical invariant does not work. Therefore, open questions left for future work include comparing the present protocol with other methods, such as universal broadband composite pulse sequences [62], incorporating the recipe of optimal control [57], or extending to the anisotropic interacting many-spin systems for quantum annealing [47]. Last but not least, the results can be readily transposed to other systems, i.e., NMR, quantum dots, superconducting circuits, optomechanical systems, etc.

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APPENDIX

In this appendix, we would like to show how to derive the effective four-level and reduced Hamiltonians for two interacting spins. Generally speaking, the Hamiltonian of a two coupled-spin system with anisotropic Heisenberg interaction can be written as

$$H(t) = \sum_i J_i S_1^i S_2^i + \vec{B}(t) \cdot (\vec{S}_1 + \vec{S}_2), \quad (\text{A1})$$

in which J_i ($i = x, y, z$) are the coupling coefficients, and $\vec{B}(t)$ is the rotating magnetic field with three components, B_i . In the basis of $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, and $|\downarrow\downarrow\rangle$, the Hamiltonian can be rewritten as

$$H = \begin{pmatrix} J_z/4 + B_z & \frac{B_x - iB_y}{2} & \frac{B_x - iB_y}{2} & \frac{J_x - J_y}{4} \\ \frac{B_x + iB_y}{2} & -J_z/4 & \frac{J_x + J_y}{4} & \frac{B_x - iB_y}{2} \\ \frac{B_x + iB_y}{2} & \frac{J_x + J_y}{4} & -J_z/4 & \frac{B_x - iB_y}{2} \\ \frac{J_x - J_y}{4} & \frac{B_x + iB_y}{2} & \frac{B_x + iB_y}{2} & J_z/4 - B_z \end{pmatrix}. \quad (\text{A2})$$

To obtain the Hamiltonian, which can be reduced to the effective three-level system in Eq. (8), we represent the above Hamiltonian in the basis of singlet and triplet states, described

by Eqs. (2)–(5). Therefore, we have the following expression:

$$H(t) = \begin{pmatrix} \frac{J_z}{4} + B_z & \frac{B_x - iB_y}{\sqrt{2}} & 0 & 0 \\ \frac{B_x + iB_y}{\sqrt{2}} & \frac{J_x + J_y - J_z}{4} & 0 & \frac{B_x - iB_y}{\sqrt{2}} \\ 0 & 0 & -\frac{J_x + J_y + J_z}{4} & 0 \\ 0 & \frac{B_x + iB_y}{\sqrt{2}} & 0 & \frac{J_z}{4} - B_z \end{pmatrix}. \quad (\text{A3})$$

Since the state $|\psi_{0,0}\rangle$ is decoupled to others, the population does not change, up to the phase fact. Finally, we can write down the three-level system with Hamiltonian

$$H(t) = \begin{pmatrix} \frac{J_z}{4} + B_z & \frac{B_x - iB_y}{\sqrt{2}} & 0 \\ \frac{B_x + iB_y}{\sqrt{2}} & \frac{J_x + J_y - J_z}{4} & \frac{B_x - iB_y}{\sqrt{2}} \\ 0 & \frac{B_x + iB_y}{\sqrt{2}} & \frac{J_z}{4} - B_z \end{pmatrix}. \quad (\text{A4})$$

Obviously, we eventually obtain the simple Hamiltonian in Eq. (8), satisfying SU(2) Lie algebra, with the corresponding dynamical invariant, just by shifting the energy $J/4$, and setting $J_x = J_y = J_z$. Of course, we can perform a similar calculation for the Ising-interaction case, as well.

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