Changes of phase structure of a paraxial beam due to spin-orbit coupling

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We analytically derive an expression for the optical field describing the spin Hall effect of a paraxial Gaussian beam in a lenslike inhomogeneous medium using the matrix optics method. Interestingly, we found changes in the phase structure and rotation of the polarization of the Gaussian beam induced by spin-orbit coupling, in addition to a polarization-dependent transverse deflection of the entire beam. This rotation of the polarization of the beam is not the result of the Berry phase. In addition, we show that the intrinsic optical angular momentum density of the beam is changed due to the spin-orbit coupling, and we also know that polarization-dependent rotation of the beam occurs when a circularly polarized nonaxisymmetric beam propagates in a lenslike inhomogeneous medium. Our results provide further potential applications of the spin Hall effect of light in optical signal processing.

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I. INTRODUCTION

The spin Hall effect of light produces a polarization-dependent transverse deflection of the propagation trajectory of light when circularly polarized light propagates in an inhomogeneous medium [1–10] or is reflected and refracted at a dielectric interface [11–16]. When circularly polarized light propagates along a helical trajectory, there is an additional polarization-dependent geometrical phase called the Berry phase [17–20]. Spin-orbit coupling is known to play a key role in the Berry phase and the spin Hall effect of light [10]. Because these polarization-dependent effects have strong potential for application in different fields, the spin Hall effect of light and Berry phase have attracted significant attention in recent years.

In the geometrical optics approximation, the propagation of a circularly polarized light wave in a smoothly inhomogeneous isotropic medium is equivalent to the evolution of a massless spin-1 particle (photon) in an external field. On the basis of the geometrical optics approximation, which can be considered a semiclassical approximation, the Berry phase and spin Hall effect of light have been investigated in the framework of semiclassical quantum theory, and some theoretical results in agreement with experimental results have been obtained [1–10]. The spin Hall effect of light can be generally described by the following expression [3,6,7]:

$$\dot{\mathbf{r}} = \frac{\mathbf{p}}{p} - \frac{\sigma}{k_0}\frac{\mathbf{p}\times\dot{\mathbf{p}}}{p^3}, \tag{1}$$

where the dot denotes the derivative with respect to the ray parameter s, which is related to the ray length l, as $dl = nds$; n is the refractive index of the optical medium; $\mathbf{p}$ is the momentum of photons, $\mathbf{p} = \nabla n$; k_0 is the wave number in vacuum; and $\sigma = \pm 1$ denotes the wave helicity corresponding to right and left circular polarization of light. Introducing a small geometrical optics parameter, $\mu = \lambda/L \ll 1$, where $\lambda = 2\pi/k_0$, and $L|\nabla n/n|^{-1}$ is the characteristic scale of the medium inhomogeneity, we see that the second term in Eq. (1) corresponds to the first-order approximation of the geometrical optics and describes the polarization-dependent transverse deflection of the propagation trajectory of light induced by spin-orbit coupling. Equation (1) gives a good description of the spin Hall effect of light both theoretically and experimentally [6,10]. However, Eq. (1) is clearly based on the dynamical evolution of the light ray (or photons) and describes only the motion of the center of gravity of the light. In other words, Eq. (1) cannot completely describe the evolution of a circularly polarized beam in an inhomogeneous medium because the optical beam cannot simply be replaced by a light ray, as it possesses a unique intrinsic structure. Although the spin Hall effect of the beam has been investigated in [5,8], and Eq. (1) can be used to describe the polarization-dependent transverse deflection of a circularly polarized beam propagating in an inhomogeneous medium, to the best of our knowledge, a complete analytical description of the evolution of the beam has not been presented in the literature.

Here we investigate the spin Hall effect of a circularly polarized paraxial beam in an inhomogeneous isotropic medium on the basis of matrix optics and analytically derive the expression for a circularly polarized Gaussian beam in a lenslike inhomogeneous medium. It is known that when the incident direction of a beam is in the meridian plane of a lenslike medium, there is no Berry phase during evolution of the light [7]. We show that there is a polarization-dependent phase correction induced by spin-orbit coupling, which leads to a change in the phase structure and rotation of the polarization of the circularly polarized Gaussian beam, in addition to the spin Hall effect that is a polarization-dependent transverse

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deflection of the entire beam. This polarization-dependent phase change is completely different from the Berry phase in previous works [6,7]. By analyzing the Poynting vector and angular momentum of the circularly polarized Gaussian beam, we know that there is a change of the intrinsic optical angular momentum density of the beam induced by the spin-orbit coupling which may lead to the change of intrinsic structure of the light in inhomogeneous media. We also show that polarization-dependent rotation of the beam occurs when a nonaxisymmetric beam propagates in a lenslike inhomogeneous medium. Here we present an analytical expression for a circularly polarized paraxial beam in an inhomogeneous medium that describes the spin Hall effect of light.

This paper is organized as follows. The theoretical model of the propagation of a polarized beam in inhomogeneous medium is presented in Sec. II. The numerical calculation results of the evolution properties of a circularly polarized Gaussian beam in a lenslike inhomogeneous medium is presented in Sec. III. This paper is concluded in Sec. IV.

II. THEORY MODEL

Matrix optics provides a connection between geometrical optics and the diffraction theory of light [21]. It can be used to investigate the propagation of various beams in various media [22–31]. To investigate the spin Hall effect of a polarized light beam using the matrix optics method, the ray matrix should first be obtained. Propagation of an electromagnetic wave in a smoothly inhomogeneous isotropic dielectric medium can be described by the following wave equation:

$$\nabla^2\mathbf{E} + k^2\mathbf{E} - \nabla(\nabla\cdot\mathbf{E}) = 0, \tag{2}$$

where $k = k_0 n$ is the wave number of light in the medium. The last term on the left-hand side of Eq. (2) includes a correction term for the light-matter interaction induced by the inhomogeneity of the medium [1]. In a smoothly inhomogeneous isotropic medium, the electric field of a light wave remains nearly transverse, $\nabla\cdot\mathbf{D} = \nabla\cdot(n^2\mathbf{E}) = 0$, and one obtains $\nabla\cdot\mathbf{E} = -2(\nabla\ln n)\cdot\mathbf{E}$. Then, Eq. (2) becomes

$$\nabla^2\mathbf{E} + k^2\mathbf{E} + 2[(\nabla\ln n)\times\nabla]\times\mathbf{E} = 0. \tag{3}$$

A ray-accompanying frame (η_1,η_2,s) is introduced with the unit vectors $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{t})$, where $\mathbf{t} = d\mathbf{r}/ds$ is the tangent direction of the ray. The electric field of light can be written in ray coordinates as $\mathbf{E} = \mathbf{E}_\perp + \mathbf{E}_\parallel$, where $\mathbf{E}_\perp = E_1\mathbf{e}_1 + E_2\mathbf{e}_2$ is the transverse component of the field and $\mathbf{E}_\parallel = E_3\mathbf{t}$ is the longitudinal component. It is known that $E_3 \ll E_1,E_2$, therefore, the wave equation (3) can be written as

$$\nabla^2\mathbf{E}_\perp + k^2\mathbf{E}_\perp + 2[(\nabla\ln n)\times\nabla]\times\mathbf{E}_\perp = 0. \tag{4}$$

Linearly polarized light can be considered as a superposition of right and left circularly polarized light, $\mathbf{E}_\perp = E^+\mathbf{e}^+ + E^-\mathbf{e}^-$, where $E^\pm = (E_1 \mp iE_2)/\sqrt{2}$ and $\mathbf{e}^\pm = (\mathbf{e}_1 \pm i\mathbf{e}_2)/\sqrt{2}$; then the wave equation (3) can be written in the form

$$k_0^{-2}\nabla^2E^\sigma + n^2E^\sigma - 2ik_0^{-2}\mathbf{t}\cdot[(\nabla\ln n)\times\nabla]E^\sigma = 0, \tag{5}$$

where $\sigma = \pm1$ denotes the wave helicity of right and left circular polarization. The last term on the left-hand side of Eq. (4) corresponds to the spin-orbit coupling correction and is associated with the spin Hall effect of light. If we take E^σ as the wave function of photons and $-ik_0^{-1}\nabla$ as the momentum operator of photons [3,5,6,9], Eq. (4) can be written as the semiclassical Schrödinger-type equation $\hat{H}E^\sigma = 0$, and then the Hamiltonian of photons including the spin-orbit-coupling correction is obtained. Using the canonical equation, we can obtain the motion equation of photons describing the spin Hall effect of light, which has the same form as Eq. (1). However, the motion equation of photons just describes the evolution of the center of gravity of a circularly polarized beam, and it cannot provide a complete description of the evolution of the beam in an inhomogeneous medium. According to the definition of a circularly polarized light field, the wave function of photons can be written as $E^\sigma = A(\mathbf{r})\exp[ik_0\psi(\mathbf{r})]\exp(-i\sigma\pi/4)$, and the eikonal equation and amplitude transport equation are obtained in the first-order geometrical optics approximation as

$$(\nabla\psi)^2 - n^2 - \sigma\left(\frac{2\nabla n\times\nabla\psi}{k}\right)\cdot\mathbf{l} = 0, \tag{6}$$

$$2\nabla\psi\cdot\nabla A + A\nabla^2\psi - \sigma\left(\frac{2\nabla n\times\nabla A}{k}\right)\cdot\mathbf{l} = 0, \tag{7}$$

respectively. Obviously, the last terms in Eqs. (6) and (7) originate from the spin-orbit-coupling correction terms. If we ignore the last terms in Eqs. (6) and (7), these two equations return to the well-known eikonal equation and amplitude transport equation [30], respectively. From Eq. (6), we obtain

$$\nabla\psi = n\frac{d\mathbf{r}}{ds} + \sigma\frac{\nabla n\times\nabla\psi}{kn}. \tag{8}$$

The second term in Eq. (8) indicates that there is a polarization-dependent transverse deflection of the light propagation trajectory, and describes the spin Hall effect of light. Equation (8) can also be written in another form, $\nabla\psi = n^\sigma d\mathbf{r}/ds$, where $n^\sigma = n + \sigma(\nabla n\times)/k$ can be considered as the refractive indices of right and left circularly polarized light. From this viewpoint, a smoothly inhomogeneous isotropic medium becomes a weakly anisotropic medium for light with different circular polarizations [2]. Equation (7) describes the energy flow of the light and indicates that there will be a polarization-dependent transverse energy flow induced by spin-orbit coupling. According to the eikonal equation (8), the ray equation and ray matrix for different inhomogeneous media can be obtained easily. Because there is a polarization-dependent transverse deflection of the light ray trajectory, the ray matrix describing the spin Hall effect should be a 4×4 matrix for the two orthogonal directions. In the next section, we will investigate the spin Hall effect of a circularly polarized Gaussian beam in the lenslike inhomogeneous medium based on the ray matrix derived from Eq. (8).

III. NUMERICAL RESULTS AND DISCUSSION

For simplicity, we consider that a circularly polarized paraxial beam propagates in a lenslike inhomogeneous medium with the refractive index distribution $n(r) = n_0(1-\alpha r^2)$, where $r^2 = x^2 + y^2$, and n_0 is the refractive index along the optical axis, α is a small coefficient of gradient refractive index, and the incident direction of the beam is in the meridian plane of the medium. In this case, it is known that the ray matrix is a 2×2 matrix because the light ray is always in the meridian plane without the spin-orbit-coupling correction. Considering the spin-orbit coupling, a 2×2 ray matrix cannot describe the polarization-dependent transverse deflection of the light ray. Then, in the paraxial approximation, the ray matrix can be obtained using Eq. (8), and has a 4×4 matrix form,

$$T = \begin{bmatrix} A_x & B_x & E_x & F_x \\ C_x & D_x & G_x & H_x \\ E_y & F_y & A_y & B_y \\ G_y & H_y & C_y & D_y \end{bmatrix}, \tag{9}$$

where

$$\begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} = \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos(z\sqrt{2\alpha}) & -\sqrt{2\alpha}\sin(z\sqrt{2\alpha}) \\ \sin(z\sqrt{2\alpha})/\sqrt{2\alpha} & \cos(z\sqrt{2\alpha}) \end{bmatrix}, \tag{10}$$

$$\begin{bmatrix} E_x & F_x \\ G_x & H_x \end{bmatrix} = -\begin{bmatrix} E_y & F_y \\ G_y & H_y \end{bmatrix} = \begin{bmatrix} \beta(C + Az) & \beta(D + Bz) \\ \beta Cz & \beta Dz \end{bmatrix}, \tag{11}$$

and $\beta = 2\sigma\alpha/k$. According to the results in [32], the propagation of a circularly polarized paraxial beam in a lenslike inhomogeneous medium obeys the expression

$$E(x,y,z) = -\frac{ik}{2\pi C}\exp(ikz)\iint dx_0dy_0E(x_0,y_0,z=0) \times \exp\left\{\frac{ik}{2C}\left[D\left(x_0^2+y_0^2\right) + A(x^2+y^2) -2(x_0x + y_0y) + 2\beta z(x_0y - xy_0) + 2\beta Cxy\right]\right\}. \tag{12}$$

Obviously, if the polarization-dependent correction terms are neglected in Eq. (12), then Eq. (12) returns to the well-known Huygens-Fresnel integral. By using Eq. (12), the evolution of a paraxial beam in a lenslike inhomogeneous medium can be obtained. Let us consider that a circularly polarized Gaussian beam propagates in the lenslike inhomogeneous medium. The field distribution of the Gaussian beam on the input plane is

$$E(x_0,y_0,z=0) = \exp\left[\frac{ik\left(x_0^2+y_0^2\right)}{2q_0}\right], \tag{13}$$

where $q_0 = -iL$ and $L = kw_0^2/2$. By substituting Eq. (13) into Eq. (12), the analytical expression describing the evolution of a circularly polarized Gaussian beam in the lenslike inhomogeneous medium is obtained,

$$E(x,y,z) = \frac{w_0}{w_z}\exp\left[-\frac{(x-\delta_x)^2+(y-\delta_y)^2}{w_z^2}\right]\exp(ik\beta xy) \times \exp\left\{\frac{ik}{2R_z}[(x-\delta_x)^2+(y-\delta_y)^2]\right\} \times \exp\left\{i\left[kz - \arctan\left(\frac{C}{LD}\right)\right]\right\}, \tag{14}$$

where $\delta_x = \beta zy$, $\delta_y = -\beta zx$, $R_z = (C^2 + L^2D)/(AC + L^2BD)$, and $w^2(z) = w_0^2(C^2 + L^2D)/L^2$ correspond to the radius of the curvature and the size of the beam. Obviously, when the polarized light beam propagates in free space, $\beta = 0$, $A = D = 1$, $B = 0$, $C = z$, and Eq. (14) becomes the general expression of a Gaussian beam in free space. Equation (14) completely describes the evolution of a polarized Gaussian beam in a lenslike inhomogeneous medium, including the transverse shift of the center of gravity of the beam and the change of the beam phase front.

According to Eq. (14), the intensity of the Gaussian beam is

$$I = \left(\frac{w_0}{w_z}\right)^2 \exp\left\{-\frac{2[(x-\delta_x)^2+(y-\delta_y)^2]}{w_z^2}\right\}. \tag{15}$$

Evidently, there is a polarization-dependent transverse shift of the center of gravity of the Gaussian beam, which is responsible for the spin Hall effect of the beam [8]. Further, Eq. (14) also shows that there is a polarization-dependent shift of the entire phase front, which corresponds to a polarization-dependent shift of the amplitude. This means that there is a polarization-dependent shift of the entire Gaussian beam, which corresponds to the spin Hall effect of the beam and has been investigated in [8]. More interestingly, Eq. (14) shows a polarization-dependent phase $\exp(ik\beta xy)$. Considering that the incident direction of the beam is in the meridian plane of the medium, there is no Berry phase in our theoretical model. Thus, this polarization-dependent phase differs from the Berry phase. From Eq. (14), we also know that the Gouy phase shift is independent of the polarization of the beam. Because of the presence of the phase $\exp(ik\beta xy)$, the phase front of the Gaussian beam will be changed. For the sake of simplicity, as shown in Fig. 1, we present the phase front change of the beam using the combination of the general phase of the Gaussian beam and the polarization-dependent correction phase, $\exp[i(x^2+y^2)/R]\exp(i\beta xy)$, where R is a constant indicating the curvature radius of the phase front. In fact, according to Eq. (12), the phase front change $\exp(ik\beta xy)$ will be retained for the evolution of any paraxial beam in inhomogeneous medium. This is a different polarization-dependent effect due to the spin-orbit coupling.

Periodic beam self-focusing and self-defocusing are known to occur when a Gaussian beam propagates in a lenslike inhomogeneous medium, and the waist of the beam is minimum and maximum at some specific locations $z = N\pi/\sqrt{2\alpha}$ and $z = (N + 1/2)\pi/\sqrt{2\alpha}$, respectively, where $N = 1,2,3,\ldots$. Consequently, the phase front of the Gaussian beam also changes periodically, and the phase front is a plane at those spe-

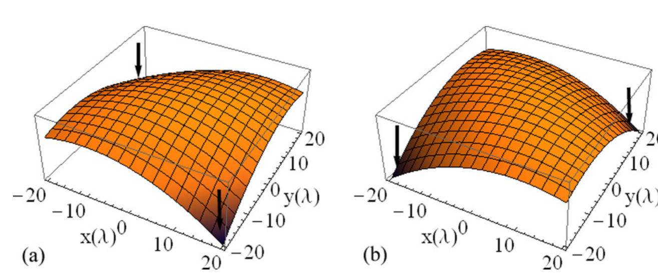

FIG. 1. Schematic diagram of phase front change of (a) right and (b) left circularly polarized Gaussian beam due to the spin-orbit coupling.

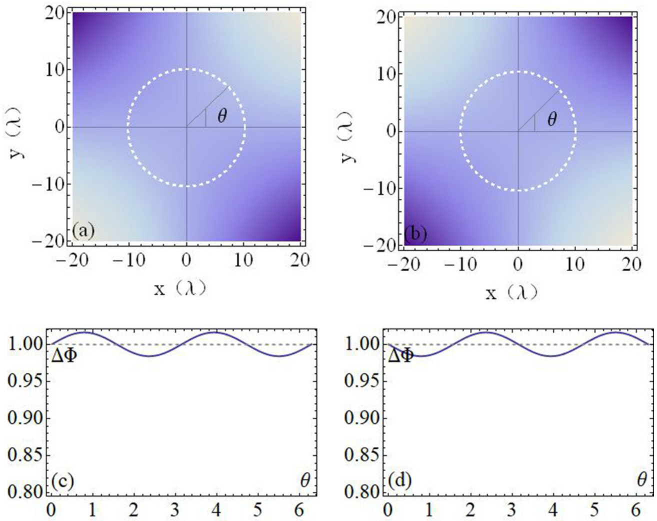

FIG. 2. Phase distribution of (a) right and (b) left circularly polarized Gaussian beam at the position $z = N\pi/\sqrt{2\alpha}$; the corresponding phase fluctuations around the center of gravity of the beam are shown in (c) and (d), respectively.

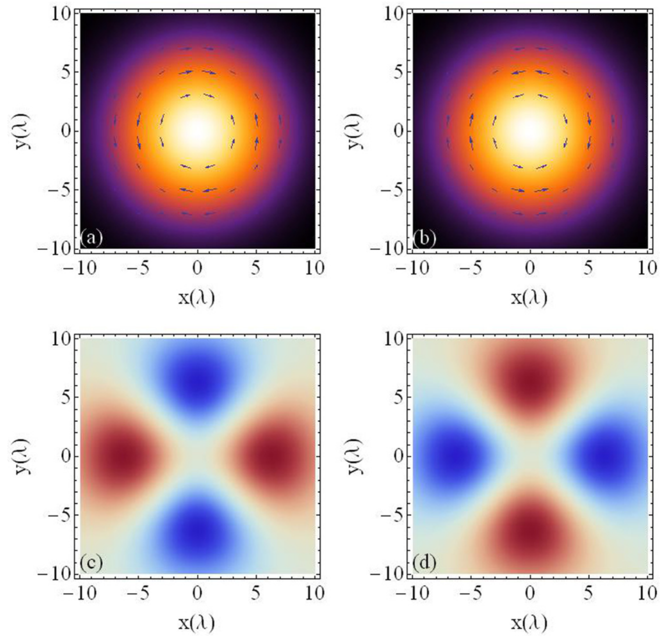

FIG. 3. Numerical azimuthal energy flow $\omega kr\beta \cos 2\theta |u|^2$ induced by the spin-orbit coupling for a (a) right and (b) left circularly polarized Gaussian beam in lenslike inhomogeneous medium when $z = N\pi/\sqrt{2\alpha}$, where the background of (a) and (b) is the intensity of the beam. (c),(d) The corresponding change of the angular momentum density.

cific positions if spin-orbit coupling is neglected. According to Eq. (14), we know that the phase correction term $\exp(ik\beta xy)$ is independent of the propagation distance and can be written as $\exp(ik\beta x'y')$ if we ignore the terms of the order β^2, where $x' = x - \delta_x$ and $y' = y - \delta_y$. Then, in order to clearly display the change of the phase structure of the Gaussian beam induced by spin-orbit coupling, we can directly use the correction term $\exp(ik\beta xy)$ to describe the phase distribution of right and left circularly polarized Gaussian beams at the specific locations $Z = N\pi/\sqrt{2\alpha}$, just as shown in Fig. 2. Obviously, the phase front is no longer a plane, and the phase distribution of the Gaussian beam demonstrates a polarization-dependent asymmetrical distribution. Figures 2(c) and 2(d) show that there is a periodic azimuthal phase fluctuation around the center of gravity of the beam. We also know there is a phase difference $\pi/2$ of the azimuthal phase fluctuation for the right and left circularly polarized Gaussian beam. It is known that a linearly polarized light can be considered as a superposition of right and left circularly polarized light. Then, when a linearly polarized Gaussian beam propagates in the meridian plane of a lenslike inhomogeneous medium, there will be a polarization rotation due to spin-orbit coupling, which is completely different from the polarization rotation corresponding to the Berry phase [6,7].

More interestingly, these changes of the phase structure lead to the change of the complex amplitude of the beam and will induce a polarization-dependent azimuthal energy flow which is closely related to the intrinsic orbital angular momentum of the beam. The time-averaged Poynting vector of the circularly polarized beam can be written as [33]

$$\frac{c}{8\pi}(\mathbf{E}^* \times \mathbf{B} + \mathbf{E} \times \mathbf{B}^*) = \frac{c}{8\pi}\left[i\omega(u\nabla_\perp u^* - u^*\nabla_\perp u) + 2\omega k|u|^2\mathbf{e}_z + \omega\sigma\frac{\partial |u|^2}{\partial r}\mathbf{e}_\theta\right], \tag{16}$$

where u is the complex amplitude describing the field distribution of the paraxial beam. The first term is a transverse component of energy flow which is a radial component if we ignore the spin-orbit-coupling correction; the second term is a longitudinal component of energy flow in the propagation direction of the beam; the last term denotes the azimuthal energy flow component relating to the spin angular momentum of the beam. Considering the propagation of a circularly polarized paraxial beam in an inhomogeneous medium, there will be an azimuthal energy flow induced by the spin-orbit coupling. By substituting the field distribution given by Eq. (14) into the first term of Eq. (16), we obtain

$$\begin{aligned} &i\omega(u\nabla_\perp u^* - u^*\nabla_\perp u) \\ &= \omega kr\left\{\beta\sin(2\theta) + \frac{1}{R_z}[(\cos\theta - \beta z\sin\theta)^2 \right. \\ &\quad \left. + (\sin\theta + \beta z\cos\theta)^2]\right\}|u|^2\mathbf{e}_r + \omega kr\beta\cos 2\theta|u|^2\mathbf{e}_\theta. \end{aligned} \tag{17}$$

Obviously, the last term in Eq. (17) represents an azimuthal component of energy flow induced by the spin-orbit coupling, which is shown in Fig. 3. We find that the direction of the azimuthal energy flow changes with the different azimuth angle. It is known that the azimuthal energy flow is closely related to intrinsic optical angular momentum of the beam [33]. Based on Eqs. (16) and (17), we can obtain the total intrinsic angular momentum density in the z direction,

$$J_z = \frac{\sigma r}{2\omega}\frac{\partial |u|^2}{\partial r} + \frac{\sigma\alpha r^2}{\omega}\cos 2\theta|u|^2. \tag{18}$$

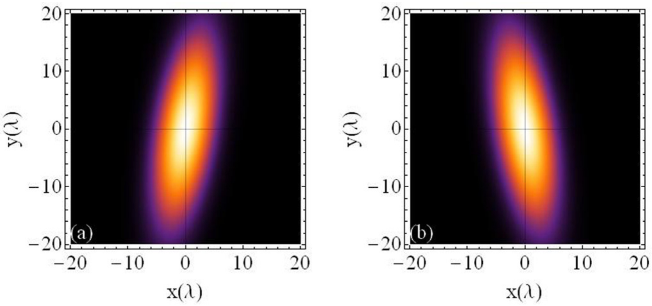

FIG. 4. Rotation of (a) right and (b) left circularly polarized elliptical Gaussian beam in lenslike inhomogeneous medium.

The first term in Eq. (18) is the spin angular momentum density component; the second term in Eq. (18) is a correction of intrinsic angular momentum density induced by the spin-orbit coupling, which is shown in Figs. 3(c) and 3(d). Yet, this correction does not really change the total intrinsic angular momentum of the beam because the integral of $(\sigma\alpha r^2/\omega)\cos 2\theta|u|^2$ is zero. Namely, the optical angular momentum of the beam is conserved. According to Eqs. (14) and (15), the transverse deflection of the beam is $\delta\mathbf{r} = \delta x\mathbf{e}_x + \delta y\mathbf{e}_y = -\beta z r\mathbf{e}_\theta$. It means that there is a tiny azimuthal rotation of the center of gravity of the beam, which leads to a periodic change of the extrinsic orbital angular momentum density of the beam. The synchronous changes of intrinsic and extrinsic optical angular momentum of the beam can maintain the conservation of the total optical angular momentum of the beam. Then, the polarization-dependent angular momentum density also can be considered as a manifestation of the spin Hall effect of the beam.

When the field distribution of the incident beam is non-axisymmetric, for example, for a circularly polarized elliptical Gaussian beam, there will be a polarization-dependent rotation of the beam, just as shown in Fig. 4. The intrinsic structure of the beam is deformed if a beam possessing intrinsic orbital angular momentum propagates in a bending ring-core fiber [34]. This rotation also can be considered as a joint effect of the polarization, propagation trajectory, and spin-orbit coupling of the beam [35]. A similar rotation has been investigated for a circularly polarized Airy beam propagating in an inhomogeneous medium [36].

IV. CONCLUSION

In summary, we analytically derived the expression for a circularly polarized Gaussian beam in a lenslike inhomogeneous medium using the matrix optics method. We found that there is a change of the phase structure of the beam and rotation of the polarization induced by spin-orbit coupling, besides the polarization-dependent transverse deflection of the entire beam which is called the spin Hall effect of the beam. In addition, we noted that the spin-orbit coupling leads to a change of the angular momentum density of the beam. And we also found that when an asymmetrical circularly polarized beam propagates in a lenslike inhomogeneous medium, the spin-orbit coupling induces a polarization-dependent rotation of the beam. The research technique in this paper can be extended to the investigation of the evolution of various paraxial beams, such as the Airy beam [37,38], the Laguerre-Gaussian beam [33], and the Ince-Gaussian beam [39]. These polarization-dependent effects indicate more potential applications of spin-orbit coupling in optical manipulation and signal processing based on spin optics.

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