Analysis of angular momentum properties of photons emitted in fundamental atomic processes

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Many atomic processes result in the emission of photons. Analysis of the properties of emitted photons, such as energy and angular distribution as well as polarization, is regarded as a powerful tool for gaining more insight into the physics of corresponding processes. Another characteristic of light is the projection of its angular momentum upon propagation direction. This property has attracted a special attention over the past decades due to studies of twisted (or vortex) light beams. Measurements being sensitive to this projection may provide valuable information about the role of angular momentum in the fundamental atomic processes. Here we describe a simple theoretical method for determination of the angular momentum properties of the photons emitted in various atomic processes. This method is based on the evaluation of expectation value of the total angular momentum projection operator. To illustrate the method, we apply it to the textbook examples of plane-wave, spherical-wave, and Bessel light. Moreover, we investigate the projection of angular momentum for the photons emitted in the process of the radiative recombination with ionic targets. It is found that the recombination photons do carry a nonzero projection of the orbital angular momentum.

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I. INTRODUCTION

In recent decades various experimental techniques have been developed to produce beams of light carrying a nonzero projection of the orbital angular momentum (OAM) onto the propagation direction [1–4]. These twisted (or vortex) beams possess helical phase wave front and nonhomogeneous intensity profile. Due to these distinguishing features the twisted photons have found extensive applications, e.g., in optical [5] and free-space [6–8] communications, metrology [9], and biophysics [10]. Many of these applications require a detailed description of the fundamental atomic processes.

During recent years numerous theoretical studies have been conducted to investigate the effects of the twisted light beams in absorption [11–19] and scattering [20,21] processes. Much less attention has been paid to the question of whether emitted light is twisted or not. The "twistedness" of the postinteraction photons has been estimated mainly in the processes being dedicated to their production. As an example, the OAM of the emitted light was evaluated in the Compton scattering [22–24] and in the process of the high harmonic generation [25–28]. The methods of these studies, however, are strongly related to the features of particular processes and cannot be extended to other situations. To the best of our knowledge, no effort has been done to provide a theoretical approach which would allow one to analyze the angular momentum properties of outgoing photons for arbitrary reaction.

In this contribution we describe a simple theoretical method for the analysis of the angular momentum properties of the photons emitted in fundamental atomic processes. This method is based on the calculation of the average value of the total angular momentum (TAM) projection operator of the outgoing photons. The averaged value can be naturally calculated within the framework of the density matrix formalism. This method allows one to find out whether the emitted photons are twisted or not for arbitrary reaction.

We apply our method to analyze the angular momentum properties of photon beams for several cases. First, the twistedness of the plane-wave, spherical-wave, and Bessel radiation has been reexplored. As the second example, we analyze the angular momentum properties of light emitted due to the radiative recombination (RR) of electrons with bare nuclei. We show that the RR photons, emitted along the electron beam direction, do carry a nonzero and well-defined projection of angular momentum.

Relativistic units ($m_e = \hbar = c = 1$) and the Heaviside charge unit ($e^2 = 4\pi\alpha$) are used in the paper.

II. BASIC FORMALISM

The main goal of the present paper is to formulate a theoretic method which will allow one to determine whether the photons emitted in basic atomic processes are twisted or not. For this purpose we start with the mathematical definition of the twisted light. Here and throughout we restrict ourselves to the case of the Bessel twisted photons.

A. Twisted photons

Let us consider the brief theoretical description of the Bessel-wave twisted photons. These waves are the solutions of the free-wave equation in an empty space with the well-defined energy ω, the helicity λ, and the projections of the momentum k_z and total angular momentum (TAM) m_γ onto the propagation direction. This direction is chosen along the z axis. Additionally, the absolute value of the transverse momentum $\varkappa_\gamma = (\omega^2 - k_z^2)^{1/2}$ is well defined. Such a twisted photon state $|\varkappa_\gamma m_\gamma k_z \lambda\rangle$ is described by the vector potential [22,24,29]

$$\mathbf{A}^{(\mathrm{tw})}_{\varkappa_\gamma m_\gamma k_z \lambda}(\mathbf{r}) = i^{\lambda - m_\gamma} \int \frac{e^{i m_\gamma \varphi_k}}{2\pi k_\perp} \delta(k_\parallel - k_z)\delta(k_\perp - \varkappa_\gamma) \times \mathbf{A}^{(\mathrm{pl})}_{\mathbf{k}\lambda}(\mathbf{r}) d\mathbf{k}, \tag{1}$$

where $k_\parallel$ and $k_\perp$ are the longitudinal and transversal components of momentum $\mathbf{k}$, respectively, and $\mathbf{A}^{(\mathrm{pl})}_{\mathbf{k}\lambda}$ is the vector potential of the plane-wave photon

$$\mathbf{A}^{(\mathrm{pl})}_{\mathbf{k}\lambda}(\mathbf{r}) = \frac{\boldsymbol{\epsilon}_\lambda(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{2\omega(2\pi)^3}}. \tag{2}$$

Equation (1) implies that the Bessel light can be "seen" as a coherent superposition of the plane-wave photons with the linear momenta $\mathbf{k}$ laying on the surface of a cone with the opening angle $\theta_\gamma = \arctan(\varkappa_\gamma / k_z)$.

In the literature one may find many definitions of the twisted light. Here we will term photons as twisted, in the sense of pure Bessel beams, if they possess a well-defined TAM projection and a well-defined opening angle differing from 0°. Therefore, in order to determine the OAM properties of the photon one needs to calculate its TAM projection and the opening angle. Instead, of the evaluation of the opening angle one can calculate its sine or cosine. For the monochromatic photon beam, the evaluation of the opening angle cosine simplifies to the calculation of the longitudinal momentum. In the framework of the present investigation we restrict our consideration to this type of beam.

B. Evaluation of TAM projection and opening angle of light

As described above, the twisted light is characterized by the TAM projection and by the opening angle. Below we consider a method of the evaluation of the mean values of these two quantities. In the previous section it was assumed that the propagation direction of the twisted light coincides with the z axis. But this is not always the case for the atomic processes. We analyze, therefore, the TAM projection onto the propagation direction of the photons emitted in some arbitrary $\hat{\mathbf{n}}_0$ direction. The mean values of the TAM projection operator and the opening angle are conveniently evaluated within the framework of the density matrix formalism. In this approach, the average value of the projection of the TAM operator $\mathbf{J}$ onto some arbitrary $\hat{\mathbf{n}}_0$ axis, defining the propagation direction of the emitted photons, is given by

$$\langle \mathbf{J}\cdot\hat{\mathbf{n}}_0\rangle = \frac{\mathrm{Tr}\left[\rho^{(\mathrm{ph})}\rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0}(\mathbf{J}\cdot\hat{\mathbf{n}}_0)\right]}{\mathrm{Tr}\left[\rho^{(\mathrm{ph})}\rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0}\right]}, \tag{3}$$

where $\rho^{(\mathrm{ph})}$ is the density operator of the photon and the operator $\rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0}$ describes the detector. The form of the detector operator depends on a particular experiment. In our study we consider so large a detector that it can be approximated by a plane-wave detector located perpendicular to the $\hat{\mathbf{n}}_0$ direction.

The right-hand side of Eq. (3) is written in the operator form. For practical applications it is more convenient to rewrite this expression in the matrix form, which requires choosing the basis representation of photon states. Here we use the helicity basis of plane-wave solutions, $|\mathbf{k}\lambda\rangle$, where $\mathbf{k}$ is the wave vector and λ is the helicity, in which the expression (3) is given by

$$\langle \mathbf{J}\cdot\hat{\mathbf{n}}_0\rangle = \frac{\sum_{\lambda\lambda'\lambda''}\int d\mathbf{k}\, d\mathbf{k}' d\mathbf{k}''(8\omega\omega'\omega'')\langle\mathbf{k}\lambda|\rho^{(\mathrm{ph})}|\mathbf{k}'\lambda'\rangle\langle\mathbf{k}'\lambda'|\rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0}|\mathbf{k}''\lambda''\rangle\langle\mathbf{k}''\lambda''|(\mathbf{J}\cdot\hat{\mathbf{n}}_0)|\mathbf{k}\lambda\rangle}{\sum_{\lambda\lambda'}\int d\mathbf{k}\, d\mathbf{k}'(4\omega\omega')\langle\mathbf{k}\lambda|\rho^{(\mathrm{ph})}|\mathbf{k}'\lambda'\rangle\langle\mathbf{k}'\lambda'|\rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0}|\mathbf{k}\lambda\rangle}. \tag{4}$$

The states $|\mathbf{k}\lambda\rangle$ are described by the vector potential (2) and satisfy the following completeness condition:

$$\sum_\lambda \int d\mathbf{k}(2\omega)|\mathbf{k}\lambda\rangle\langle\mathbf{k}\lambda| = I, \tag{5}$$

with I being the unity operator. In the helicity basis of plane-wave solutions the matrix element of the detector operator expresses as [30]

$$\langle\mathbf{k}'\lambda'|\rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0}|\mathbf{k}\lambda\rangle = \frac{1}{2\omega}\delta(\mathbf{k}-\mathbf{k}')\theta(\mathbf{k}\cdot\hat{\mathbf{n}}_0)\delta_{\lambda\lambda'}, \tag{6}$$

where $\theta(x)$ is the Heaviside function. Substituting Eq. (6) into Eq. (4) one obtains the following expression for the average value of the TAM projection operator:

$$\langle \mathbf{J}\cdot\hat{\mathbf{n}}_0\rangle = \frac{\sum_{\lambda\lambda'}\int d\mathbf{k}\, d\mathbf{k}'(4\omega\omega')\langle\mathbf{k}\lambda|\rho^{(\mathrm{ph})}|\mathbf{k}'\lambda'\rangle\langle\mathbf{k}'\lambda'|(\mathbf{J}\cdot\hat{\mathbf{n}}_0)|\mathbf{k}\lambda\rangle\theta(\mathbf{k}\cdot\hat{\mathbf{n}}_0)}{\sum_\lambda\int d\mathbf{k}(2\omega)\langle\mathbf{k}\lambda|\rho^{(\mathrm{ph})}|\mathbf{k}\lambda\rangle\theta(\mathbf{k}\cdot\hat{\mathbf{n}}_0)}. \tag{7}$$

The explicit form of the photon density matrix $\langle\mathbf{k}\lambda|\rho^{(\mathrm{ph})}|\mathbf{k}'\lambda'\rangle$ depends on the particular "scenario" under investigation. In the present paper we consider the cases of plane-wave, spherical-wave, and Bessel radiation as well as of RR photons.

While the evaluation of the photon density matrix requires the knowledge about a process under consideration, the matrix element of the operator $(\mathbf{J}\cdot\hat{\mathbf{n}}_0)$, which also enters into Eq. (4), is independent on the particular scenario. It is conveniently calculated in the momentum representation for the TAM operator [31]

$$\mathbf{J}_p = -i[\mathbf{p}\times\nabla_p] + \mathbf{S}, \tag{8}$$

where $\mathbf{S}$ is the spin-1 operator. Apart from the operator $\mathbf{J}$, the vector potential of the plane-wave photon has also to be written in the momentum representation:

$$\mathbf{f}^{(\mathrm{pl})}_{\mathbf{k}\lambda}(\mathbf{p}) = \frac{\boldsymbol{\epsilon}_\lambda(\mathbf{k})}{\sqrt{2\omega}}\delta(\mathbf{p}-\mathbf{k}), \tag{9}$$

which is related to the vector potential in the coordinate representation (2) by the following simple relation:

$$\mathbf{A}^{(\mathrm{pl})}_{\mathbf{k}\lambda}(\mathbf{r}) = \frac{1}{\sqrt{(2\pi)^3}}\int d\mathbf{p}\, \mathbf{f}^{(\mathrm{pl})}_{\mathbf{k}\lambda}(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{r}}. \tag{10}$$

Utilizing Eqs. (8) and (9), one can derive the explicit expression for the matrix element of the TAM projection operator

$$\begin{aligned}\langle\mathbf{k}'\lambda'|(\mathbf{J}\cdot\hat{\mathbf{n}}_0)|\mathbf{k}\lambda\rangle &= \int d\mathbf{p}\, \mathbf{f}^{(\mathrm{pl})\dagger}_{\mathbf{k}'\lambda'}(\mathbf{p})(\mathbf{J}_p\cdot\hat{\mathbf{n}}_0)\mathbf{f}^{(\mathrm{pl})}_{\mathbf{k}\lambda}(\mathbf{p})\\ &= \frac{1}{4\pi k^2}\delta(k-k')\frac{\delta_{\lambda\lambda'}}{2\omega}\sum_\mu(\hat{\mathbf{n}}_0)^\mu\\ &\quad\times\sum_{jm_jm'_j}(2j+1)\sqrt{j(j+1)}C^{jm'_j}_{jm_j\,1\mu}\\ &\quad\times D^j_{m_j\lambda}(\varphi_k,\theta_k,0)D^{j*}_{m'_j\lambda}(\varphi'_k,\theta'_k,0).\end{aligned} \tag{11}$$

Here $(\hat{\mathbf{n}}_0)^\mu$ are the contravariant vector components, $C^{JM}_{j_1m_1\,j_2m_2}$ is the Clebsch-Gordan coefficient, $D^J_{MM'}$ is the Wigner matrix [32,33], (k,θ_k,φ_k) are the spherical coordinates of $\mathbf{k}$, and $(k',\theta'_k,\varphi'_k)$ are those of $\mathbf{k}'$.

Substituting the explicit form of $\langle\mathbf{k}\lambda|\rho^{(\mathrm{ph})}|\mathbf{k}'\lambda'\rangle$ and Eq. (11) into Eq. (4), one can evaluate the average value of the TAM projection. But the mean value of the TAM projection operator cannot solely describe the twistedness of light. Indeed, in accordance with the definition (see Sec. II A), the light is called twisted if its TAM projection onto the propagation direction is well defined. Therefore, one needs to know not only the mean value but also the dispersion of TAM

$$\Delta_J = \sqrt{\langle(\mathbf{J}\cdot\hat{\mathbf{n}}_0)^2\rangle - \langle\mathbf{J}\cdot\hat{\mathbf{n}}_0\rangle^2}. \tag{12}$$

As is seen from this expression, the evaluation of Δ_J requires the knowledge of not only $\langle\mathbf{J}\cdot\hat{\mathbf{n}}_0\rangle$ given by Eq. (11) but also of $\langle(\mathbf{J}\cdot\hat{\mathbf{n}}_0)^2\rangle$. By using Eqs. (8) and (9) and performing some tedious but straightforward calculations, one obtains the explicit expression for the matrix element of the $(\mathbf{J}\cdot\hat{\mathbf{n}}_0)^2$ operator. For the sake of brevity we will omit details of these calculations here and just present the final result:

$$\begin{aligned}&\langle\mathbf{k}'\lambda'|(\mathbf{J}\cdot\hat{\mathbf{n}}_0)^2|\mathbf{k}\lambda\rangle\\ &= \frac{1}{\sqrt{4\pi}k^2}\delta(k-k')\frac{\delta_{\lambda\lambda'}}{2\omega}\sum_{J_nM_n}C^{J_n0}_{10\,10}Y^*_{J_nM_n}(\hat{\mathbf{n}}_0)\\ &\quad\times\sum_{jm_jm'_j}j(j+1)(2j+1)^{3/2}C^{jm'_j}_{jm_j\,J_nM_n}\begin{Bmatrix}1 & 1 & J_n\\ j & j & j\end{Bmatrix}\\ &\quad\times D^j_{m_j\lambda}(\varphi_k,\theta_k,0)D^{j*}_{m'_j\lambda}(\varphi'_k,\theta'_k,0).\end{aligned} \tag{13}$$

Here $\{\cdots\}$ denotes the Wigner $6j$ symbol [33] and $Y_{lm}(\theta,\varphi)$ is the spherical harmonic.

Up to now we have discussed the evaluation of the mean value and the dispersion of the TAM projection of light. As was already mentioned, in order to determine whether the emitted photon is twisted or not one needs also to evaluate the opening angle θ_γ or its cosine. In the case of the monochromatic photon beam

$$\cos\theta_\gamma = \frac{1}{\omega}\langle\mathbf{p}\cdot\hat{\mathbf{n}}_0\rangle, \tag{14}$$

where $\mathbf{p}$ is the momentum operator. Rewriting the expression (14) in the form similar to Eq. (7) and utilizing the explicit form of the matrix elements,

$$\langle\mathbf{k}'\lambda'|(\mathbf{p}\cdot\hat{\mathbf{n}}_0)|\mathbf{k}\lambda\rangle = \frac{\delta_{\lambda\lambda'}}{2\omega}\delta(\mathbf{k}-\mathbf{k}')(\mathbf{k}\cdot\hat{\mathbf{n}}_0), \tag{15}$$

$$\langle\mathbf{k}'\lambda'|(\mathbf{p}\cdot\hat{\mathbf{n}}_0)^2|\mathbf{k}\lambda\rangle = \frac{\delta_{\lambda\lambda'}}{2\omega}\delta(\mathbf{k}-\mathbf{k}')(\mathbf{k}\cdot\hat{\mathbf{n}}_0)^2, \tag{16}$$

one can evaluate the mean value and the opening angle cosine $\cos\theta_\gamma$. The dispersion Δ_p is defined analogous to the dispersion Δ_J.

III. RESULTS AND DISCUSSIONS

A. TAM and its dispersion for plane-wave, spherical-wave, and Bessel photons

In order to demonstrate the method which is described above let us evaluate the TAM projection, momentum projection, which is directly related to the opening angle cosine (14), and their dispersions for the plane-wave, spherical-wave, and twisted photons.

1. Plane-wave photons

As was discussed in Sec. II B, in order to find the average value of the TAM and its dispersion it is sufficient to calculate the trace of the density matrix with TAM and squared TAM projection operators. The density operator for the plane-wave photon with the momentum $\mathbf{k}$ and polarization $\boldsymbol{\epsilon}_\lambda$ is given by

$$\rho^{(\mathrm{pl})}_{\mathbf{k}\lambda} = |\mathbf{k}\lambda\rangle\langle\mathbf{k}\lambda|. \tag{17}$$

In the present study we restrict ourselves to the case of TAM projection onto the photon propagation direction, i.e., $\hat{\mathbf{n}}_0 = \hat{\mathbf{k}} \equiv \mathbf{k}/|\mathbf{k}|$. For this case one obtains

$$\langle\mathbf{J}\cdot\hat{\mathbf{k}}\rangle_{\mathrm{pl}} = \lambda,\quad \langle(\mathbf{J}\cdot\hat{\mathbf{k}})^2\rangle_{\mathrm{pl}} = 1,\quad \Delta^{(\mathrm{pl})}_J = 0, \tag{18}$$

$$\langle\mathbf{p}\cdot\hat{\mathbf{k}}\rangle_{\mathrm{pl}} = \omega,\quad \langle(\mathbf{p}\cdot\hat{\mathbf{k}})^2\rangle_{\mathrm{pl}} = \omega^2,\quad \Delta^{(\mathrm{pl})}_p = 0. \tag{19}$$

The formulas (18) represent the well-known fact that the TAM projection of the plane-wave photon on its propagation direction is given by the helicity λ. The expressions (19) indicate that the plane-wave photon is the eigenfunction of the $\mathbf{p}$ operator.

2. Spherical-wave photons

The density operator for the spherical-wave photon with energy ω, TAM j, and TAM projection onto the z axis m_γ is given by

$$\rho^{(\mathrm{sph})}_{\omega j m_\gamma \pi} = |\omega j m_\gamma \pi\rangle\langle\omega j m_\gamma \pi|, \tag{20}$$

with $\pi = 0$ for the magnetic and $\pi = 1$ for the electric photon. The explicit form of the vector potential of the spherical photon in the momentum space expresses as follows [34]:

$$\mathbf{f}^{(\mathrm{sph})}_{\omega j m_\gamma \pi}(\mathbf{p}) = \frac{4\pi^2}{\omega^{3/2}}\delta(|\mathbf{p}| - \omega)\mathbf{Y}^{(\pi)}_{j m_\gamma}(\hat{\mathbf{p}}), \tag{21}$$

where $\mathbf{Y}^{(\pi)}_{j m_\gamma}$ is the spherical harmonic vectors [33]. Utilizing the formalism described in Sec. II B and Eqs. (20) and (21), one can calculate the average value of the projection of the TAM operator onto some arbitrary $\hat{\mathbf{n}}_0$ axis. Here we focus on the situation when $\hat{\mathbf{n}}_0$ coincides with the quantization z axis, i.e., $\hat{\mathbf{n}}_0 = \hat{\mathbf{e}}_z$ with $\hat{\mathbf{e}}_z$ being the unit vector directed along the z axis. In this case,

$$\langle J_z\rangle_{\mathrm{sph}} = m_\gamma, \quad \langle J_z^2\rangle_{\mathrm{sph}} = m_\gamma^2, \quad \Delta^{(\mathrm{sph})}_J = 0. \tag{22}$$

From these equations one can see that the spherical-wave photon is the eigenfunction of the J_z operator with the eigenvalue m_γ. It is worth mentioning that the average value of the opening angle cosine and its' dispersion are both dependent on the j and m_γ. And since these dependencies cannot be expressed by a compact formula we omit them for the sake of brevity.

3. Twisted photons

Let us now consider the case of the Bessel-wave twisted photon propagating along the z axis. The corresponding density operator is given by

$$\rho^{(\mathrm{tw})}_{\varkappa_\gamma m_\gamma k_z \lambda} = |\varkappa_\gamma m_\gamma k_z \lambda\rangle\langle\varkappa_\gamma m_\gamma p_z \lambda|, \tag{23}$$

where $\varkappa_\gamma$ and k_z are the transversal and longitudinal momenta, λ is the helicity, and m_γ is the TAM projection onto the propagation direction. As in the case of the plane- and spherical-wave photons, we restrict ourselves to evaluation of TAM projection and its dispersion for the particular direction of $\hat{\mathbf{n}}_0$. Namely, we study the situation when $\hat{\mathbf{n}}_0$ is directed along the propagation direction, i.e., $\hat{\mathbf{n}}_0 = \hat{\mathbf{e}}_z$. In this case one obtains

$$\langle J_z\rangle_{\mathrm{tw}} = m_\gamma, \quad \langle J_z^2\rangle_{\mathrm{tw}} = m_\gamma^2, \quad \Delta^{(\mathrm{tw})}_J = 0, \tag{24}$$

$$\langle p_z\rangle_{\mathrm{tw}} = k_z, \quad \langle p_z^2\rangle_{\mathrm{tw}} = k_z^2, \quad \Delta^{(\mathrm{tw})}_p = 0. \tag{25}$$

As is expected from the form of the density operator (23), the mean value of the TAM projection onto the propagation direction of the twisted photon equals m_γ with the zero dispersion. The formulas (25) denote that the Bessel-wave twisted photon is the eigenfunction of the p_z operator.

B. Radiative recombination of electrons with bare nuclei

Until now we have applied our approach to study the twistedness of light to the textbook examples of the plane-wave, spherical-wave, and Bessel-wave twisted radiation. Let us now turn to the analysis of the photons emitted in one of the fundamental processes of light-matter interaction, namely the radiative recombination (RR) of electrons with bare nuclei. Despite a large number of studies devoted to this process (for a review see Ref. [35]) no attention has been paid so far to the angular momentum properties of the RR photons. Below we analyze these properties.

The density matrix of the photons emitted in the course of the RR of the asymptotic plane-wave electron with the momentum $\mathbf{p}$ and the helicity μ into the final bound f state with the TAM projection m_f has the following form:

$$\langle\mathbf{k}\lambda|\rho_{\mathbf{p}\mu;fm_f}|\mathbf{k}'\lambda'\rangle = \tau_{\mathbf{p}\mu;fm_f,\mathbf{k}\lambda}\tau^*_{\mathbf{p}\mu;fm_f,\mathbf{k}'\lambda'}, \tag{26}$$

with the amplitude

$$\tau_{\mathbf{p}\mu;fm_f,\mathbf{k}\lambda} = \int d\mathbf{r}\,\Psi^\dagger_{fm_f}(\mathbf{r})R^\dagger_{\mathbf{k}\lambda}(\mathbf{r})\Psi^{(+)}_{\mathbf{p}\mu}(\mathbf{r}). \tag{27}$$

Here Ψ_{fm_f} is the wave function of the electron in the final state, $R_{\mathbf{k}\lambda}$ designates the transition operator which has the following form in the Coulomb gauge:

$$R_{\mathbf{k}\lambda}(\mathbf{r}) = -\sqrt{\frac{\alpha}{\omega(2\pi)^2}}\boldsymbol{\alpha}\cdot\boldsymbol{\epsilon}_\lambda e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{28}$$

with $\boldsymbol{\alpha}$ being the vector of Dirac matrices, and $\Psi^{(+)}_{\mathbf{p}\mu}$ is the wave function of the electron in the initial state given by [36–38]

$$\Psi^{(+)}_{\mathbf{p}\mu}(\mathbf{r}) = \frac{1}{\sqrt{4\pi\varepsilon p}}\sum_{\kappa m_j} C^{j\mu}_{l0\,1/2\mu} i^l\sqrt{2l+1}e^{i\delta_\kappa} \times D^j_{m_j\mu}(\varphi_p,\theta_p,0)\Psi_{\varepsilon\kappa m_j}(\mathbf{r}). \tag{29}$$

Here $\kappa = (-1)^{l+j+1/2}(j+1/2)$ is the Dirac quantum number with j and l being the angular momentum and parity, respectively, and δ_κ is the phase shift corresponding to the potential of the extended nucleus.

Above we have presented the density matrix (26) of the photon emitted in the course of the RR of a plane-wave electron with a bare nucleus. Now we turn to the evaluation of twistedness of this radiation. Let us fix the z axis along the propagation direction of the incoming electron. For such choice of the coordinate system the TAM projection onto the z axis, i.e., $\hat{\mathbf{n}}_0 = \hat{\mathbf{e}}_z$, and its dispersion equal, respectively,

$$\langle J_z\rangle = \mu - m_f, \quad \Delta_J = 0. \tag{30}$$

This equation indicates that the photons being emitted in the course of the RR of the polarized plane-wave electron and propagating in the forward direction do possess the well-defined projection of TAM onto their propagation direction. This means that the RR photons can carry the nonzero projection of the OAM onto the propagation direction which is determined solely by the helicity μ of the incident electron and by the magnetic quantum number of the residual ion m_f.

Above we analyzed the angular momentum properties of the photons emitted along the z axis. Let us recall here that the z axis is fixed along the propagation direction of the incoming electron. Now let us consider the angular momentum properties of the RR photons emitted into some arbitrary direction $\hat{\mathbf{n}}_0 \neq \hat{\mathbf{e}}_z$. This case is represented in Fig. 1 for the RR with the bare argon nuclei. On the left panel of this figure it is seen that for the forward and backward emission angles the TAM projection takes the well-defined values. This fact is predicted by the relation (30). From the right panel of Fig. 1

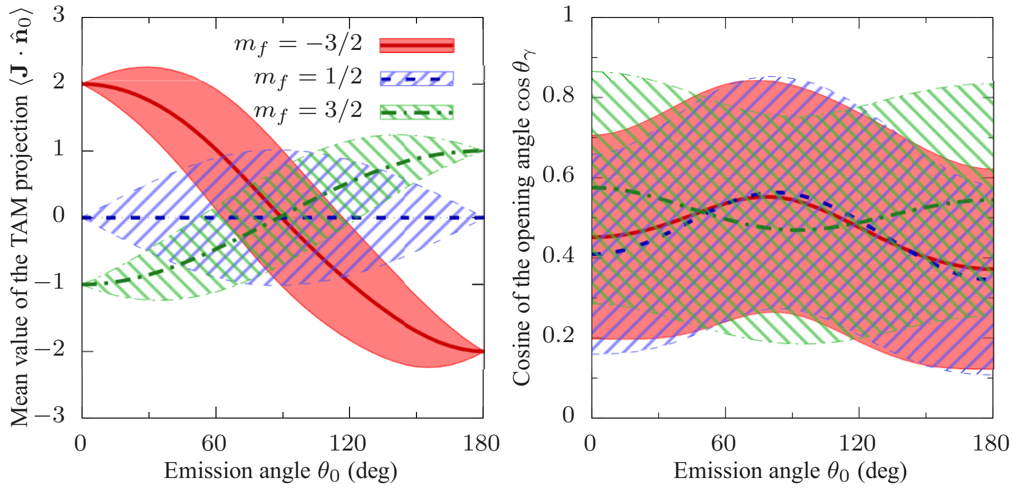

FIG. 1. Mean value of the operator of the TAM projection on the direction of photon emission $\hat{\mathbf{n}}_0 = (\sin\theta_0, 0, \cos\theta_0)$ (left panel) and the cosine of the opening angle (right panel). The recombination of the 2 keV plane-wave electron with $\mu = 1/2$ into the $2p_{3/2}(m_f)$ state of the H-like Ar ($Z = 18$) ion is considered. The shadowed areas designate the dispersions.

one can conclude that for all propagation directions the emitted photons do not have the well-defined opening angle θ_γ and consequently transversal momentum. This can be explained as follows. In the external field of the nucleus the momentum does not conserve and, as a result, the distribution of the momentum occurs. In accordance with the definition given in Sec. II A, the RR photons cannot be regarded as twisted. But, these photons can neither be regarded as the plane or the spherical wave since the cosine of the opening angle always differs from 1 and 0, respectively (see the right panel of Fig. 1). Therefore, one can say that the RR photons emitted in the forward or backward directions are, in some sense, twisted.

Up to now we discussed the RR of the polarized electron into a particular magnetic sublevel of the ion. The scenarios in which the incident electron is unpolarized or (and) the population of the magnetic sublevels of the residual ion that remains unobservable are also worth investigation. First, let us consider the angular momentum properties of the photons emitted in the course of the RR of an *unpolarized* electron with a bare nucleus. This case is presented in Fig. 2.

From the left panel of this figure it is seen that the most interesting situation is expected for the forward and backward emission directions and the population of $m_f = \pm 3/2$ magnetic sublevels. For this case the TAM projection of the RR photon equals ∓ 1 with almost zero dispersion that makes these photons twisted in some sense. This result can be explained as follows. As an example, let us consider the capture into $m_f = 3/2$ magnetic sublevel of the $2p_{3/2}$ state. Since the incident electron is unpolarized, this magnetic sublevel is populated via the RR of the electrons with the spin projection $\mu = 1/2$ and $\mu = -1/2$. These two captures are related to the pure non-spin-flip and pure spin-flip channels, respectively. It is a well-known fact that in the recombination processes the spin-flip channel is strongly suppressed with respect to the non-spin-flip one (see, e.g., Refs. [35,39]). The population of $m_f = 3/2$ magnetic sublevel is therefore strongly dominated by the capture of the

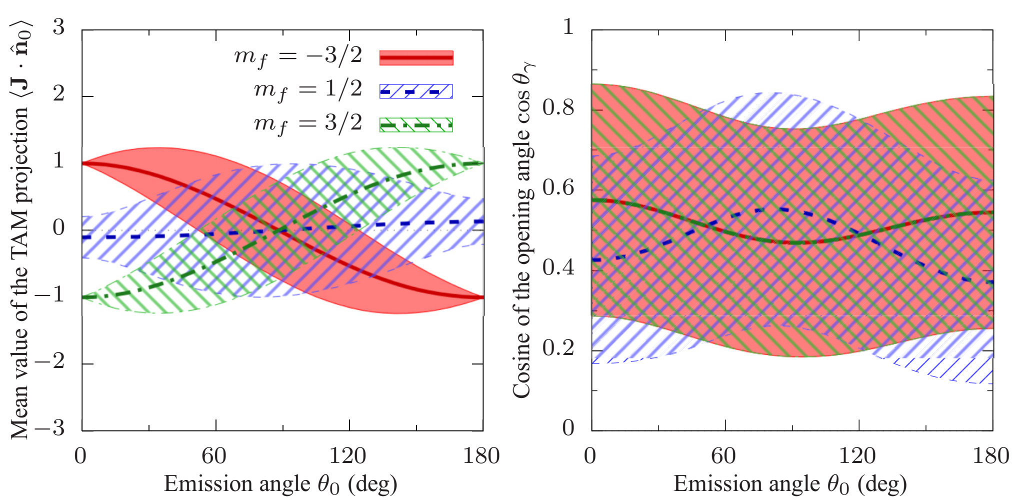

FIG. 2. Mean value of the operator of the TAM projection on the direction of photon emission $\hat{\mathbf{n}}_0 = (\sin\theta_0, 0, \cos\theta_0)$ (left panel) and the cosine of the opening angle (right panel). The recombination of the unpolarized 2 keV plane-wave electron into the $2p_{3/2}(m_f)$ state of the H-like Ar ($Z = 18$) ion is considered. The shadowed areas designate the dispersions.

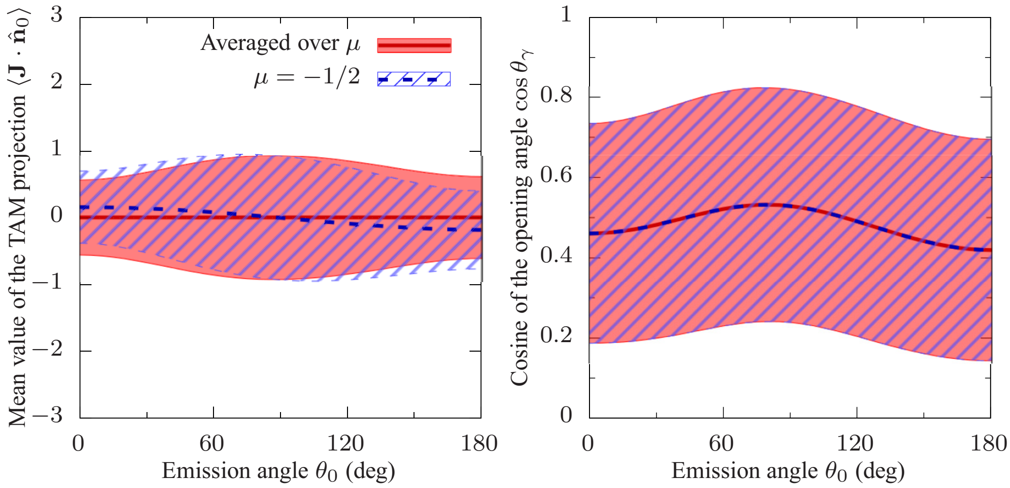

FIG. 3. Mean value of the operator of the TAM projection on the direction of photon emission $\hat{\mathbf{n}}_0 = (\sin\theta_0, 0, \cos\theta_0)$ (left panel) and the cosine of the opening angle (right panel). The recombination of the 2 keV plane-wave electron into the $2p_{3/2}$ state of the H-like Ar ($Z = 18$) ion is considered. It is assumed that the population of the magnetic sublevels of the residual ion remains unobservable. The shadowed areas designate the dispersions.

electron with $\mu = 1/2$. And, as a result, from Eq. (30) it follows that the TAM projection in this case equals -1. The situation is more involved for the population of, e.g., $m_f = 1/2$ magnetic sublevel. In this case, the RR of the electrons with both spin projections $\mu = \pm 1/2$ can proceed via the non-spin-flip channel, which leads to the growth of the dispersion.

The scenario in which the population of the magnetic sublevels of the residual ion remains unobservable is depicted in Fig. 3. In this case, the emitted photon, unfortunately, has neither a well-defined TAM projection nor a well-defined opening angle.

Let us now briefly discuss the feasibility of the experimental measurement of the angular momentum properties of the RR photons. As already mentioned, the most interesting situation occurs when the particular magnetic sublevels are populated in the course of this process (see Figs. 1 and 2). This requires the coincidence measurement of the radiative emission and the population of the magnetic sublevels of the residual ion, which is a rather challenging task. Instead, one can think of the RR with the polarized ionic (or atomic) targets with a single vacancy in closed shells. In this case, the emission following the recombination into this vacancy will carry away the polarization of the initial target since the TAM projection of the residual ion will be zero. One can expect that these RR photons emitted in the forward and backward directions can be, in some sense, twisted. The polarized atomic targets with a single vacancy can be routinely obtained with the present day experimental techniques. The creation of the analogous highly charged targets is a more difficult task. Nevertheless, such targets can be, in principle, obtained at the GSI and FAIR facilities (Darmstadt, Germany) with the usage of the techniques described in Refs. [40,41].

IV. CONCLUSION

In the present work, we described the simple theoretical method for the evaluation of the angular momentum properties of the photons emitted in basic atomic processes. As the applications of the proposed method, we evaluated the TAM projection and its dispersion for the plane-wave, spherical-wave, and twisted photons. We have also analyzed the projection of the angular momentum of the photons emitted in the radiative recombination of electrons with the bare argon nuclei. It was found that the RR photons emitted in the forward or backward directions have the well-defined TAM projection onto this direction. For these photons the TAM projection is determined solely by the polarizations of the incident plane-wave electron and by the magnetic quantum number of the residual ion. And, although the emitted photons do not have well-defined opening angles, we believe that the RR photons for the forward or backward emission directions are, in some sense, twisted.

To summarize, the developed method allows one to find out whether the emitted photons are twisted without going into details of the process. This method can be readily extended for the analysis of the angular momentum properties of other particles.

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