Bound state and localization of excitation in many-body open systems

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(Received 1 December 2017; published 30 April 2018)

We study the exact bound state and time evolution for single excitations in one-dimensional XXZ spin chains within a non-Markovian reservoir. For the bound state, a common feature is the localization of single excitations, which means the spontaneous emission of excitations into the reservoir is prohibited. Exceptionally, the pseudo-bound state can be found, for which the single excitation has a finite probability of emission into the reservoir. In addition, a critical energy scale for bound states is also identified, below which only one bound state exists, and it is also the pseudo-bound state. The effect of quasirandom disorder in the spin chain is also discussed; such disorder induces the single excitation to locate at some spin sites. Furthermore, to display the effect of bound state and disorder on the preservation of quantum information, the time evolution of single excitations in spin chains is studied exactly. An interesting observation is that the excitation can stay at its initial location with high probability only when the bound state and disorder coexist. In contrast, when either one of them is absent, the information of the initial state can be erased completely or becomes mixed. This finding shows that the combination of bound state and disorder can provide an ideal mechanism for quantum memory.

DOI: 10.1103/PhysRevA.97.042129

I. INTRODUCTION

Bound state in open systems was first defined and studied in photonic materials, in which the level of the embedded atom as an impurity is dressed by a radiation field [1]. Physically, the energy of the atom-photon bound state lies in the photonic band gap, and thus the excited photon is exponentially localized in the vicinity of the atom. The existence of the atom-photon bound state is a universal feature of photonic material, independent of the fine structure of the atom. Recently this issue has been reconsidered from the general viewpoint of open systems. In this case, the bound state can still be defined when the spectral density vanishes [2,3] or a finite band occurs [4,5]. Similar to the case in photonic materials, the bound state is responsible for the vanishing of spontaneous emission and thus can be used to protect the system against decoherence. Experimentally, the bound state has been verified in photonic crystals [6], in which both inhibited and enhanced decay rates can be controlled by the crystal-lattice parameters.

Recently the bound state in two-qubit open quantum systems has received extensive interest focusing on the preservation of quantum information [3,5]. It is known now that the existence of bound states can be used to protect entanglement against decoherence. Furthermore, in topological two-band systems, the bound state can also be found when the system is coupled with the environment, which is responsible for the robustness of Hall conductance [7]. In addition, the bound state for cold atoms in optical lattices has been studied [8], which provides an alternative way to control the atomic state. A generalization of bound states into multiple levels is also discussed in Ref. [9].

However, a general understanding of bound states in open systems seems absent because the previous works focused mainly on small systems. Consequently, it is an interesting issue whether there exists bound states in the many-body case, and what the difference is. For this purpose the bound state in one-dimensional XXZ spin chains within a non-Markovian environment is discussed in this paper. For concreteness, only the single excitation in spin chains is discussed. As for the case of two or more excitations, the computational resource required to find the exact result numerically is much greater [9], which is beyond our computer performance. Despite the simplicity, a single-excitations in a spin chain has extensive applications in quantum information transfer [10]. Moreover, it is shown that universal quantum computation can be realized in single-excitation subspace [11]. Recently a mechanism for quantum spin lenses has been proposed in which the spin excitation can be focused on a definite site in a one-dimensional chain for storing quantum information [12]. With respect to these applications our consideration has extensive interest.

Our study shows that at most N bound states can be found, in which N is the spin number in a one-dimensional XXZ spin chain. In general the bound state displays the localization of a single excitation in a spin chain, which inhibits the spontaneous emission of the spin excitation into the environment. However, an exceptional case can be identified in which the excitation is inevitably absorbed with a finite probability by the environment. Furthermore we find that that the probability would tend to be 0.5 when $N \to \infty$, which corresponds to a balance between localization and spontaneous emission of spin excitations. Thus we argue that this state is not a true bound state, so it is named a pseudo-bound state in this paper.

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With respect to the recent interest in the localization of many-body systems, the effect of disorder in the XXZ model is studied. The appearance of disorder enhances greatly the ability of the bound state to protect excitations against spontaneous emission. It is attributed to strong localization in the bound state, for which the distribution of excitations in spin chains becomes more pronounced at some spin sites. We argue that the combination of the bound state and disorder provides an alternative mechanism for preservation of quantum information. This finding is counterintuitive since incorporation of the environment effectively induces additional long-range hopping, which is believed to destroy localization [13]. Moreover, it suggests that the bound state in open many-body systems would be localized, and furthermore that the localization can be strengthened by disorder in the system. Consequently the combination of a bound state and disorder provides an ideal platform for preserving information in quantum systems.

The discussion is divided into five sections. In Sec. II the model and definition of bound state are introduced. In Secs. III A and III B, bound states are evaluated explicitly up to $N = 12$ without disorder. The effect of disorder is studied solely in Sec. III C. Although of small N, some general features of the bound state can be found. To highlight the crucial role of disorder and of the bound state in preserving quantum information, the time evolution of a single excitation in a spin chain is discussed in Sec. IV. Finally, a conclusion and further discussion are presented in Sec. V.

II. MODEL

Consider the one-dimensional XXZ model coupled to a zero-temperature reservoir, of which the Hamiltonian is

$$H = \frac{J}{2}\sum_{i=1}(S_i^+S_{i+1}^- + S_i^-S_{i+1}^+) + \sum_{i=1}^{N} h_i S_i^z + U\sum_{i=1} S_i^z S_{i+1}^z + \sum_k \omega_k a_k^\dagger a_k + \sum_{k,i}(g_k a_k S_i^+ + g_k^* a_k^\dagger S_i^-), \tag{1}$$

with $\hbar \equiv 1$ and the spin number N. $S_i^+(S_i^-)$ and $a_k^\dagger(a_k)$ are the creation (annihilation) operators, respectively, for spin $1/2$ and the kth model with frequency ω_k in the reservoir. J is the tunneling strength between nearest-neighbor sites. U characterizes the Ising interaction. The on-site field h_i can be homogeneous or randomly distributed, which has distinct effect on bound states, as shown in the following discussion. Periodic and open boundary conditions in spin chains are considered. Note that, because of the existence of the Ising interaction, the complete spectrum of the one-dimensional XXZ model cannot be determined analytically, and one has to rely on numerical methods.

The bound state in open systems is defined as a discrete eigenstate with energy outside the continuum and thus is stable against decay because of the existence of an energy gap [14]. Physically, this feature can be attributed to a shift of an atomic level by the emitted photon that pushes the level beyond the cutoff [15]. Hence, in the present case the bound state can occur only for $E < 0$ since the continuum spectrum in the environment ranges from 0 to ∞. Then, by solving the eigenequation

$$H|\psi_E\rangle = E|\psi_E\rangle, \tag{2}$$

with $E < 0$, the bound state can be determined by solving Eq. (7). As for unbound states occurring when E coincides with some ω_k, spontaneous decaying is inevitable since there is no energy gap to protect the system against decoherence. Thus it can only be determined by the following Eq. (4a).

Actually the bound state characterizes a bipartite entanglement between the system and its environment. Thus, when the measure of entanglement is vanishing, the system is decoupled from the environment since the bound state is completely separable in this case. In the general case that entanglement of the bound state is finite, the coherence in the system could be protected partially against decoherence.

As for single-excitation, $|\psi_E\rangle$ can be written generally as

$$|\psi_E\rangle = \left(\sum_{i=1}^{N}\alpha_i|\uparrow\rangle_i|\downarrow\rangle^{\otimes(N-1)}\right)\otimes|0\rangle^{\otimes M} + |\downarrow\rangle^{\otimes N}\otimes\left(\sum_{k=1}^{M}\beta_k|1\rangle_k|0\rangle^{\otimes(M-1)}\right), \tag{3}$$

in which $|\uparrow\rangle_i$ ($|\downarrow\rangle_i$) is the eigenstate of S_i^z with the eigenvalue $1/2$ ($-1/2$), $|0\rangle_k$ is the vacuum state of a_k and $|1\rangle_k = a_k^\dagger|0\rangle_k$, and M the number of modes in the reservoir. Substituting the expression for $|\psi_E\rangle$ into Eq. (2), one obtains

$$\frac{J}{2}(\alpha_{i+1}+\alpha_{i-1}) + (h_i - U)\alpha_i + \sum_{k=1}^{M} g_k\beta_k = E\alpha_i, \tag{4a}$$

$$\omega_k\beta_k + g_k^*\sum_{i=1}^{N}\alpha_i = E\beta_k, \tag{4b}$$

in which the resulting constants have been incorporated into E. From Eq. (4b), we obtain

$$\beta_k = \frac{g_k^*}{E-\omega_k}\sum_{i=1}^{N}\alpha_i. \tag{5}$$

Substituting the relation above into Eq. (4a), one obtains

$$\frac{J}{2}(\alpha_{i+1}+\alpha_{i-1}) + (h_i - U)\alpha_i + \left(\sum_{k=1}^{M}\frac{|g_k|^2}{E-\omega_k}\right)\sum_{i=1}^{N}\alpha_i = E\alpha_i.$$

With respect to the continuum spectrum in reservoir,

$$\sum_{k=1}^{M}\frac{|g_k|^2}{E-\omega_k} = \int_0^\infty \frac{J(\omega)}{E-\omega}d\omega, \tag{6}$$

in which the spectral density $J(\omega) = \sum_{k=1}^{M}|g_k|^2\delta(\omega-\omega_k)$. Then

$$\frac{J}{2}(\alpha_{i+1}+\alpha_{i-1}) + (h_i - U)\alpha_i + \sum_{i=1}^{N}\alpha_i\int_0^\infty d\omega\frac{J(\omega)}{E-\omega} = E\alpha_i. \tag{7}$$

It is obvious that the integral (6) is divergent when $E > 0$. Thus the bound state can exist only when $E < 0$ (for complex E, the imaginary part of E means dissipation and thus the

bound state cannot exist in this case). For concreteness, the following spectral function of the environment is considered in this paper:

$$J(\omega) = \eta\omega\left(\frac{\omega}{\omega_c}\right)^{s-1} e^{-\omega/\omega_c}, \tag{8}$$

in which η characterizes the coupling strength between the system and environment. The environment can be classified as sub-ohmic ($s < 1$), ohmic ($s = 1$), and super-ohmic ($s > 1$), which characterizes different relaxation [16]. ω_c is the cutoff of the environment spectrum, beyond which the spectral density starts to fall off. Hence it determines a frequency regime in the environment, which is dominant for the decoherence of the system. In general the value of ω_c depends on the specific environment. So in the following discussion, $\omega_c = 3$ is chosen. This choice is arbitrary because our numerical evaluation shows that ω_c has no intrinsic effect on the bound state and the time evolution of the spin system.

It was known for single spin ($N = 1$) that the relation $\frac{\omega_0}{\omega_c} < \eta\Gamma(s)$ has to be satisfied if the bound state occurs by setting $E = 0$ in Eq. (7) [5], in which ω_0 is the fixed frequency of spin and $\Gamma(s)$ is the Gamma function. However, the situation is slightly different when $N \geqslant 2$. Obviously, the condition for the bound state to occur is determined by the linear system of equations (7). Thus, a general relation is difficult to find. Actually, one can find N solutions at most by solving Eq. (7) with properly chosen parameters, as shown in the following. However, not all solutions correspond to genuine bound states. We first point out in this article that there always exists a so-called *pseudo-bound-state* that satisfies Eq. (7) but whose spin excitation can be transferred to the environment with a finite probability.

III. BOUND STATE

The sufficient and necessary condition for a nontrivial solution to α_i in the linear system of equation (7) is that the determinant of the coefficient the matrix is zero. Thus one can obtain an equation for variable E with the maximal power of N. Generally, this equation is difficult to solve (noting that E is also involved in the integral), and one has to rely on numerical methods. Moreover, as for large N, we also note that there exists some point on which the equation of E shows strong oscillation. Thus, in this case a stable solution cannot be determined even with the highest computational precision. So our numerical evaluation has to be restricted to $N \leqslant 12$, except for some special case. Despite this deficiency, some interesting results can be found.

The crucial feature of a bound state is the vanishing spontaneous emission of excitations. To display this point, two distributions, defined as

$$c_i = |\alpha_i|^2,$$
$$d = \sum_k |\beta_k|^2 = \left|\sum_{i=1}^{N}\alpha_i\right|^2 \int_0^\infty \frac{J(\omega)}{(E-\omega)^2} d\omega, \tag{9}$$

are calculated exactly, in which c_i characterizes the distribution of excitation in the spin chain, and d is the distribution in the reservoir. In addition, the normalization $d + \sum_{i=1}^{N} c_i = 1$ is imposed. For brevity, the following discussion focuses only on the case of an ohmic environment ($s = 1$). As for sub- and super-ohmic cases, our calculations show no intrinsic difference from the ohmic case. Although the distribution of c_i is varied, the basic feature of localized spin excitation is unchanged.

A. Periodic boundary conditions

With respect to periodic boundary conditions in a spin chain, $S_{i+N} = S_i$ and $h_i = h_0$ are imposed. By proper choice of $h_0 - U$, one can find N solutions of E at most for definite N. In Fig. 1, c_i and d are plotted for different N when $s = 1$. A common feature is the periodic variation of c_i by spin site when $N \geqslant 4$, which can be attributed to the spin-site translational invariance in Eq. (1).

An interesting observation is the double degeneracy in bound energy levels E_n ($n \geqslant 2$) when $N \geqslant 3$. Furthermore an even-odd effect can be found. When N is even, there exists two nondegenerate levels: the ground state E_0 and some excited state. And the other levels are doubly degenerate. In contrast, when N is odd, there is only one nondegenerate level, the others being doubly degenerate.

Another observation is that there always exist a unique energy level with nonvanishing d, which means that the spin excitation would be emitted into the reservoir with a finite probability. For the other bound states, $d = 0$ exactly. For instance, as shown in Fig. 1, the level E_2 for $N = 3$, E_3 for $N = 4$, 6, 8, 10, 12, and E_4 for $N = 9$ have $d > 0.1$. Consequently, the occurrence of the state would be unfavorable for the protection of quantum information. However, since it also satisfies Eq. (7), we claim that it is not a true bound state since the excitation cannot be localized exactly in a spin chain, and in this place it is named a *pseudo-bound state*.

The pseudo-bound state can display three crucial features: First, our numerical evaluation shows that the pseudo-bound state is always nondegenerate. Second, we find that all α_i share the same value, proportional to $1/\sqrt{N}$ up to a coefficient determined by d. Third, up to exact evaluation of the ground bound state up to $N = 22$, it is found that there is only one bound state when $(U - h_0) + E < -1$ or $E < -2$ correspondingly as for the chosen parameters in Fig. 1. This bound state is also a pseudo-bound state, as shown in Table I. These facts imply that the ground bound state for $N \to \infty$ would be nondegenerate and a pseudo-bound state.

Furthermore, d for a pseudo-bound state increases with spin number N, shown in Table I. Thus, it is interesting to find the nontrivial upper bound for d. With respect that the α_i in the pseudo-bound state are the same, it is found that $d \to 0.5$ when $N \to \infty$, as shown in Table II. This means that the relaxation and localization of the spin excitation becomes balanced in this case. Moreover, our exact calculation shows that the ground state is the unique pseudo-bound state for larger N. The appearance of the pseudo-bound state in the excited level for small N shown in Fig. 1 can be attributed to the finite-number effect.

Conclusively, two distinct bound states can be found in this case: the true bound state with $d = 0$ exactly and the pseudo-bound state with finite d. A critical case can be identified as $(U - h_0) + E = -1$, which separates the true bound

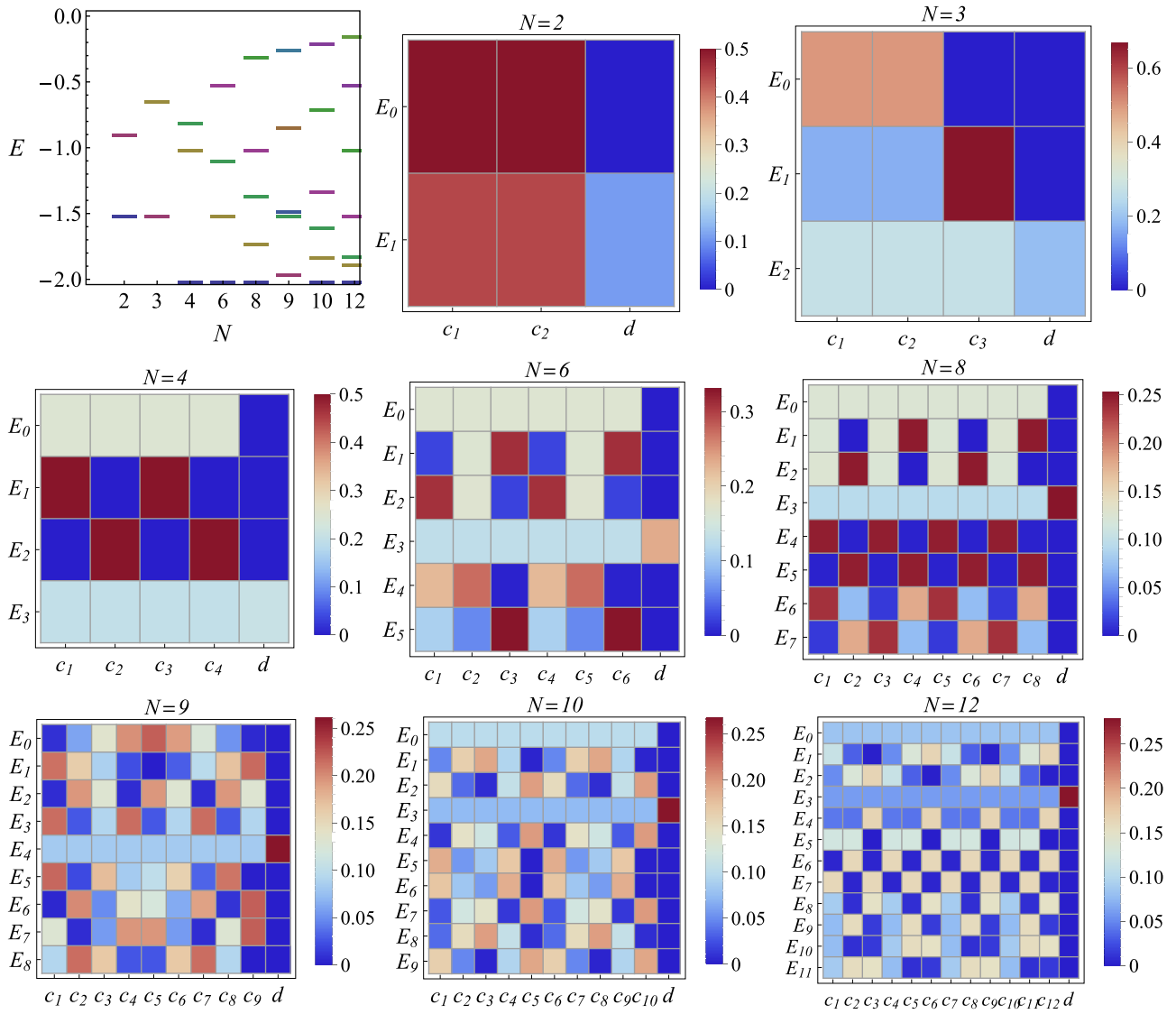

FIG. 1. Plot of bound-state energy level E (top-left panel) and density plots of normalized distribution c_i and d of bound states for $N = 2$ to $N = 12$ with periodic boundary conditions. $h_0 - U = -1, s = 1, \eta = 0.1, \omega_c = 3$ in units of J have been chosen for this plot. The eigenenergy E_n ($n = 0,1,2,\ldots$) is arranged in increasing order.

state from the pseudo-bound state. When $(U - h_0) + E < -1$, there is only one bound state, which is also a pseudo-bound state. For $(U - h_0) + E > -1$, all energy levels are true bound states with $d = 0$ exactly under $N \to \infty$. Interestingly, we find for even N that the state, if it exists at the critical level $(U - h_0) + E = -1$ or $E = -2$, is a true and nondegenerate state and, moreover, the spin state in the first term in Eq. (3) has the form

$$\frac{1}{\sqrt{N}} \sum_{i=1}^{N} (-1)^i |\underbrace{0\cdots0}_{i-1} 1 \underbrace{0\cdots0}_{N-1}\rangle. \tag{10}$$

B. Open boundary conditions

For open boundary conditions, translational invariance in the spin chain is broken. Figure 2 presents the energy level E_n and the density plot for the distribution c_i and d with different N. The obvious feature is the broken degeneracy of the bound-state level, which means that the double degeneracy is protected by translational invariance in the spin chain.

In contrast to the periodic boundary, the difference between the pseudo-bound state and the true bound state becomes ambiguous because d could be very small but not vanish exactly. However, the critical energy $(U - h_0) + E_n = -1$ or $E_n = -2$ can still be identified, below which there is only one bound state, which is also a pseudo-bound state. In Table III the ground-state energy level with $E < -2$ is listed up to $N = 22$, in which d increases with spin number N. Moreover, our calculation shows that the corresponding c_i tends to be homogenous with the increment of N, as shown in Fig. 3.

C. Effect of disorder in XXZ model

An interesting issue is whether disorder in spin systems would affect the bound state. This consideration comes from the recent interest in localization of many-body systems [17], which characterizes a nonergodic behavior in the statistical

TABLE I. The ground bound-state level E_0 for $N =$ 14 to 22 and the distribution d with periodic boundary conditions. The other parameters are the same as in Fig. 1.

	$s=1$	
N	E_0	d
14	−2.01932	0.289168
15	−2.12051	0.293435
16	−2.21916	0.297389
17	−2.31547	0.301069
18	−2.40961	0.304187
19	−2.50172	0.30773
20	−2.59159	0.310762
21	−2.68041	0.313621
22	−2.7672	0.316325

mechanics of isolated quantum systems. A general feature for localization in many-body systems is the disorder-induced nonequilibrium properties, such as the localization of the electronic wave function [18] or the deviation of statistical properties from the expectation of thermodynamical equilibrium [19]. As shown in the previous discussion, the true bound state displays an obvious localization of spin excitation with respect to the probability of vanishing spontaneous emission. From this point the open spin system can also display nonergodic features. Hence it is an interesting issue whether the disorder in the spin chain could enhance the localization of excitations.

With respect to recent experiments on localization of many-body systems [20–22], a quasirandom disorder is introduced in the spin chain with open boundary conditions,

$$h_i = \Delta \cos(2\pi\beta i + \phi), \tag{11}$$

in which $\beta = \frac{532}{738} \approx 0.721$, $\phi = 1/0.6188333$, and Δ characterizes the strength of disorder. Although h_i does not denote a genuine disorder, the intrinsic effect of disorder can be simulated and demonstrated in this system [21,22]. Consequently, the distribution c_i and d of bound states for $N = 12$ are plotted in Fig. 4. It is obvious that some c_i become so pronounced with the increment of disorder strength Δ, which means that the spin excitation cannot be transported freely in spin chains, too. Furthermore, with respect that the spin chain is embedded in the environment, it means that localization of the excitation is

TABLE II. The ground bound-state level E_0 and the distribution d for huge N with periodic boundary conditions. The other parameters are the same to those in Fig. 1.

	$s=1$	
N	E_0	d
100	−7.33804	0.391086
200	−11.0897	0.416524
400	−16.4958	0.436926
800	−24.2306	0.45294
1600	−35.2459	0.46527
3200	−50.9722	0.474613

TABLE III. The ground bound-state level E_0 and the distribution d in open boundary conditions. The other parameters are the same as those in Fig. 2.

	$s=1$	
N	E	d
13	−2.01274	0.209038
14	−2.09872	0.265101
15	−2.19033	0.277008
16	−2.28187	0.284867
17	−2.37255	0.290923
18	−2.46206	0.295991
19	−2.55026	0.300411
20	−2.63714	0.304362
21	−2.72269	0.30795
22	−2.80692	0.311247

stable against decoherence induced by the environment. This picture is very different from the knowledge that the reservoir would always destroy localization, as shown in Refs. [22–24]. We argue that the existence of the bound state is responsible for the stability of localization.

As for d, the disorder shows limited effect. Generally, our calculation shows that d is insensitive to the disorder in the spin chain, as shown in Fig. 4. However, some exceptions can be found; for instance, the bound-state levels E_8 and E_{10} in Fig. 4 in which d shows a relative variation upon incrementing Δ. But in most cases, d does not change obviously. This feature implies that disorder in spin systems has small effect on the spontaneous emission of excitation. Finally, we point out that our exact examination shows similar behavior for $N < 12$, which is not presented here for brevity.

Importantly, one should note that, for the single-excitation case, Eq. (7) actually characterizes a free-particle system within the reservoir since the interaction U only contributes to the diagonal elements. Thus, the Hamiltonian of the spin chain can be diagonalized in this case by using the Jordan–Wigner transformation; for example, as done in Ref. [25]. Thus, the disorder-induced localization is actually Anderson like [18,20].

IV. TIME EVOLUTION

Localization in quantum systems is of interest as a possible quantum memory since some local details of the system's initial state can be preserved. Thus, an interesting issue is the time evolution of system when both bound state and disorder occur simultaneously. Thus, by the Schrödinger equation, one can obtain the equation of time evolution:

$$i\frac{\partial}{\partial t}\alpha_i(t) = \frac{J}{2}[\alpha_{i+1}(t) + \alpha_{i-1}(t)] + (h_i - U)\alpha_i(t) - i\sum_{i=1}^{N}\int_0^t d\tau \alpha_i(\tau) f(t-\tau), \tag{12}$$

in which

$$f(t-\tau) = \frac{\eta}{\omega_c^{s-1}} \frac{\Gamma(s+1)}{[i(t-\tau)+1/\omega_c]^{s+1}}$$

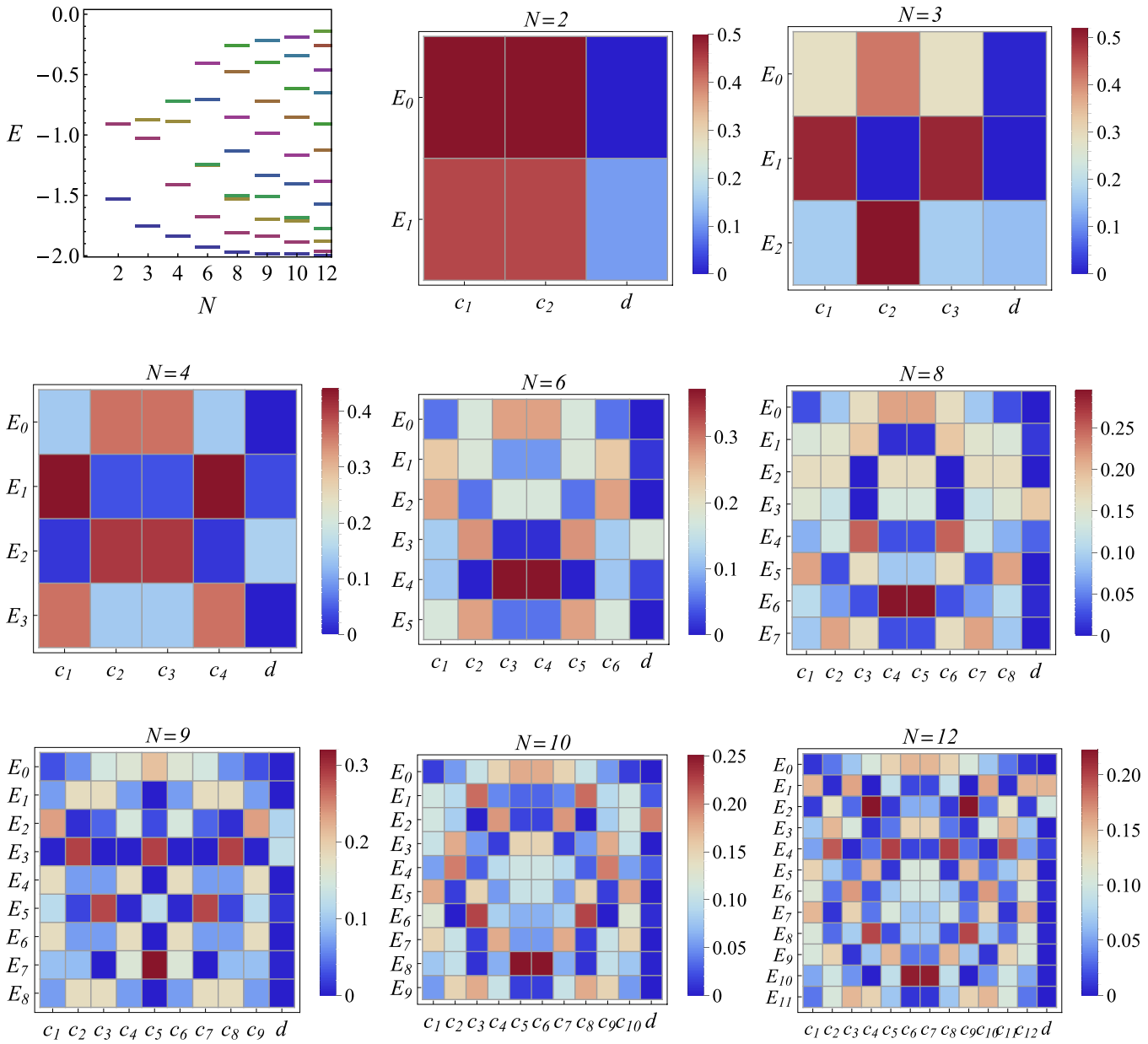

FIG. 2. A plot for bound-state energy level E (top-left panel) and density plot of c_i and d for $s = 1$ with open boundary conditions. The other parameters are the same as those in Fig. 1.

and i is the imaginary unit. The influence of the reservoir is displayed by the last term, which also characterizes the non-Markovian memory structure. With respect to the disorder occurring ($\Delta \neq 0$) or not ($\Delta = 0$), the following discussion is divided into two sections. As shown below, the dynamics of the spin system presents two distinct behaviors.

In general, as for the bound state occurring, the time evolution of the state $|\psi(t)\rangle$ of the spin chain can be decomposed into two parts:

$$|\psi(t)\rangle = \sum_{E_b} \alpha_b |\psi_b\rangle e^{-iE_b t} + \int dE_c \alpha(E_c) e^{-iE_c t} |\psi_c\rangle, \quad (13)$$

in which the first term comes from the contribution of bound states, denoted by $|\psi_b\rangle$ with bound-state energy E_b, while the second term is the contribution from the continuum spectrum E_c. Then the first term depicts a unitary dynamics in system, while in contrast the second term means dissipation. Consequently, when there is no bound state a complete dissipation of system is inevitable. In the following discussion one can find that, for many-body systems, the bound state itself cannot completely protect the system against decoherence. Instead, the combination of disorder and bound state provides a promising possibility of preserving faithfully the quantum information in system.

Some comments have to be presented in advance. First, we chose $s = 1$ in the following discussion. Admittedly, the dynamics of c_i is inevitably dependent on the value s. However, since the current focus is on the effect of the bound state and disorder for the preservation of quantum information in spin systems, our numerical evaluation shows that different s does not induce intrinsic changing of this feature. Hence

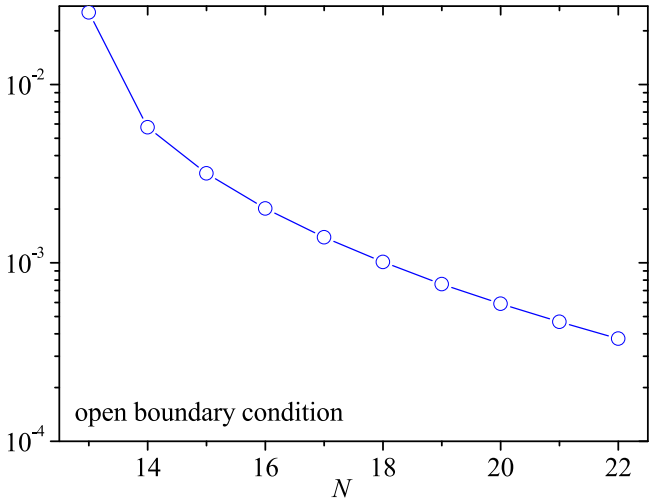

FIG. 3. The isotropy of c_i, measured by $\sum_{i=1}^{N}[\sqrt{c_i/(1-d)} - 1/\sqrt{N}]^2/N$, which $c_i/(1-d)$ corresponds to the reduced distribution of excitation in the spin system. The parameters are the same as those in Fig. 1.

the following discussion is focused only on $s = 1$ for brevity. Second, to highlight the effect of the bound state, two situations are considered in the following discussion: all bound states occurring or no bound states occurring. This choice can more clearly demonstrate the different dynamics of c_i if the bound state appears or not, and avoids the conclusion being dependent on some special bound states. Third, to display the preservation of quantum information, it is necessary to find the fidelity $F = \langle\psi(t)|\psi(0)\rangle$. As for a single excitation located initially on a special spin site i, F simply reduces to

$$F = \left|\sum_i \alpha_i(t)\alpha_i(0) + \sum_k \beta_k(t)\beta_k(0)\right| = |\alpha_i(t)| = \sqrt{c_i}.$$

It thus suffices to study the dynamics of c_i.

A. $\Delta \neq 0$

By exact numerical evaluation of Eq. (12), the time evolution of distribution c_i ($i = 1,2,\ldots,N$) for $N = 12$ is presented in Figs. 5 and 6 with $\Delta = 1, 2, 4, 6$ and different initial states. The two situations (i.e., all bound states appear or none appear are considered, respectively, and are shown in the left and right columns in Figs. 5 and 6. In addition we also check c_i for two different initial states by setting that spin excitation initially at $i = 1$ (Fig. 5) or $i = 11$ (Fig. 6). Two distinct behaviors of c_i can be found, which can be understood properly by the localization in bound states.

First, when the spin excitation is located initially at $i = 1$, an oscillation between c_1 and c_8 is developed with the increment of Δ when bound states appear. This feature can be understood by noting that c_1 and c_8 are always pronounced simultaneously, as shown in the bound-state levels E_{10} and E_{11} in Fig. 4. By Eq. (13), the two bound states would become dominant for the time evolution of c_i, which is responsible for the coherent transfer of spin excitation in the initial state between the two spin sites. In contrast, when there is no bound state, c_1 and c_8 become stable simultaneously at a value close to 0.25. In the absence of the bound state, the feature could be attributed to the influence of disorder (11). This picture implies that the disorder in the system cannot alone protect the system against decoherence induced by the environment. Moreover, the appearance of the bound state is a key ingredient for the preservation of quantum information.

Recently, the open quantum dynamics in localized systems has been studied extensively, for which two nonergodic features can be found. In Ref. [24], a localized steady state is disclosed in open Anderson-localized system, achieved by a proper dissipation. This finding means that the environment could play a constructive role in localization. Whereas in Ref. [23], a stretched exponential decay is discovered in open many-body systems with a general consideration of the environment, for which the nonergodic character of the system persists for a long time. The underlying mechanism for the exotic dynamics can be attributed to the existence of integrals of motion in localized systems [26], which prevents not only complete thermalization of any given subsystem, but also transport over macroscopic scales. Thus a typical localization length can be defined, which means that the system can be considered as localized when the system scale is much lager than the localization length. Consequently, for the present discussion, some similar feature can also be found, as illustrated by Fig. 7. Consider the case when there is no bound state, shown by the right column in Fig. 7. It is obvious that c_1 tends to be vanishing with time evolution if $N \leqslant 6$. In contrast, when $N \geqslant 8$, c_1 is finally steady at 0.25 and, moreover, c_8 is developed and also steady at 0.25, as shown by the bottom-right panels in Fig. 5 and 7. This feature means that the information of the initial state could be partially stored on the spin system only if $N \geqslant 8$. Thus, it seems that there would exist a localization length l_c. When the length of the spin chain is smaller than l_c, the excitation is inevitably decaying into the environment. In contrast, when the spin chain is larger than l_c, the excitation can be partially localized in the spin chain. Moreover, l_c can still be in function when bound states occur, as shown by the left column in Figs. 7 and 5. It is obvious that a strong oscillation between c_1 and c_8 is developed when $N \geqslant 8$.

A different picture can be found for the spin excitation located initially at $i = 11$, as shown in Fig. 6. When bound state and disorder occur together, as shown by the left column in Fig. 6, c_{11} settles rapidly on a value close to 0.9 with the increment of Δ. At the same time, the other c_i with $i \neq 11$ tend to vanish instead. This picture can also be understood by noting that c_{11} in the bound-state level E_6 becomes more pronounced upon incrementing Δ, as shown in Fig. 4. Thus, by Eq. (13) this bound state becomes dominant in the time evolution. In contrast, when there is no bound state, c_{11} tends to vanish rapidly with rescaled time Jt, as shown by the right column in Fig. 6. Since the contribution of bound states disappears, the dissipative term becomes dominant. In addition, one can find that the decaying of c_{11} becomes slower with the increment of Δ, which means that disorder has limited ability to preserve quantum information. Thus the disorder of the spin chain itself is not enough for the preservation of quantum information in open quantum systems.

Conclusively these two distinct pictures disclose that not all quantum information in the system can be preserved over time.

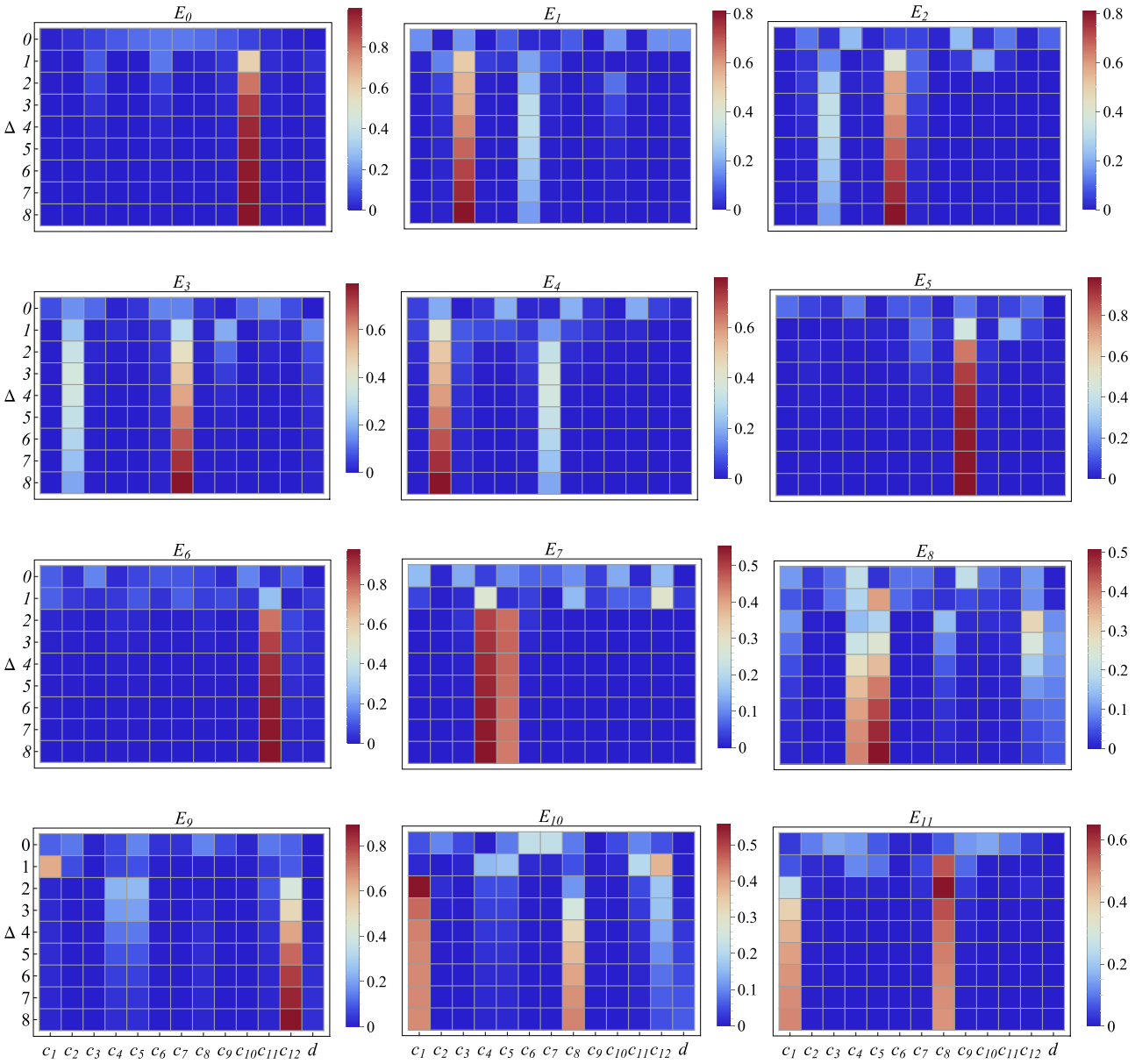

FIG. 4. Density plots of distribution c_i and d of bound-state level E_i ($i = 1,2,\ldots,12$) of $N = 12$ for $\Delta = 1$ to 8. For this plot, $s = 1$ and the open boundary conditions of the spin chain are imposed. The other parameters are $\eta = 0.1$, $\omega_c = 3$ in units of J. For $\Delta = 1$ to 8, $U = 1$, 1.5, 2.5, 3.2, 4.2, 5.2, 6.2, 7.2, 8.2 are used in order to find all bound states.

As shown in the previous discussion, the quantum information only for some special initial state can be retained. It can be understood by the fact that the underlying integrals of motion just determine a finite-energy manifold in Hilbert space: only the state in this manifold can show localization [26]. At another point the long-time behavior of $|\psi(t)\rangle$ is obviously determined by the bound states, which have the similar feature of localization as that in the initial state. Thus it could imply that this special energy manifold would have an intrinsic connection with the space supported by bound states.

B. $\Delta = 0$

In comparison, the situation that there is no disorder ($\Delta = 0$) is discussed in this section. First consider when all bound states occur, as shown by the left column in Fig. 8. Obviously, an oscillation happens between c_1 and c_2 when $N = 2$, as shown by the top-left panel in Fig. 8. However, with the increment of N, the difference between c_1 and the other c_i becomes ambiguous with the increment of Jt. Similar to the discussion in the previous section, this picture can be understood by the structure of the bound state, shown in Fig. 2. When $N = 2$, there are only two bound states, E_0 and E_1, in which one can find $c_1 = c_2$. Thus, by Eq. (13), the oscillation is attributed to the coherent superposition of two bound states. With the increment of N, there are more bound states, which tend to have equal contribution to the evolution of $|\psi(t)\rangle$. Consequently, the information of the initial state is diluted with time evolution. Second, when there is no bound state, a slight difference in the evolution of c_i can be found. For $N = 2$, shown by the top-right panel in Fig. 8, c_1 and c_2 evolve simultaneously to a steady value close to 0.25. However, when

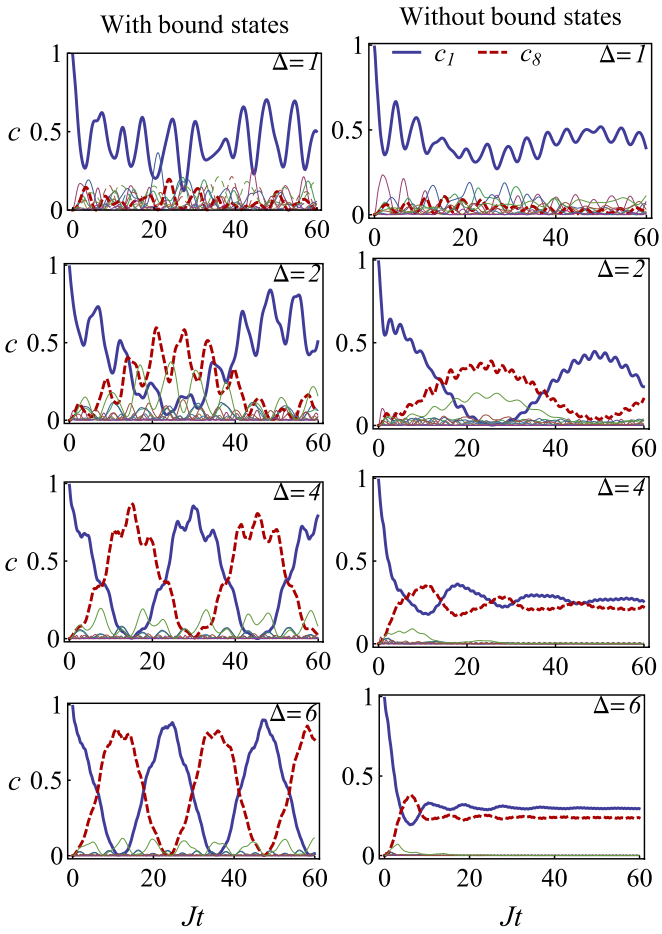

FIG. 5. A plot for c_i ($i = 1,2,\ldots,12$) varying with rescaled time Jt for $\Delta = 1$, 2, 4, 6 when $N = 12$ and $s = 1$. This initial state is chosen as the single excitation located on spin site $i = 1$. For clarity, the varying of c_1 and c_8 are highlighted by a thick blue solid line and a thick red dashed line to distinguish their features from the others. The varying of the other c_i are also shown by thin solid lines for convenience of comparison, which are not marked in this plot. When all bound states occur, shown by the left column, we choose $U = 1.5$ for $\Delta = 1$, $U = 2.5$ for $\Delta = 2$, $U = 4.2$ for $\Delta = 4$, and $U = 6.2$ for $\Delta = 6$ in order to include all 12 bound states. As for the absence of the bound state, shown by the right column, $U = -7$ is chosen for different Δv. In addition, $\eta = 0.1$, $\omega_c = 3$ are also chosen for this plot.

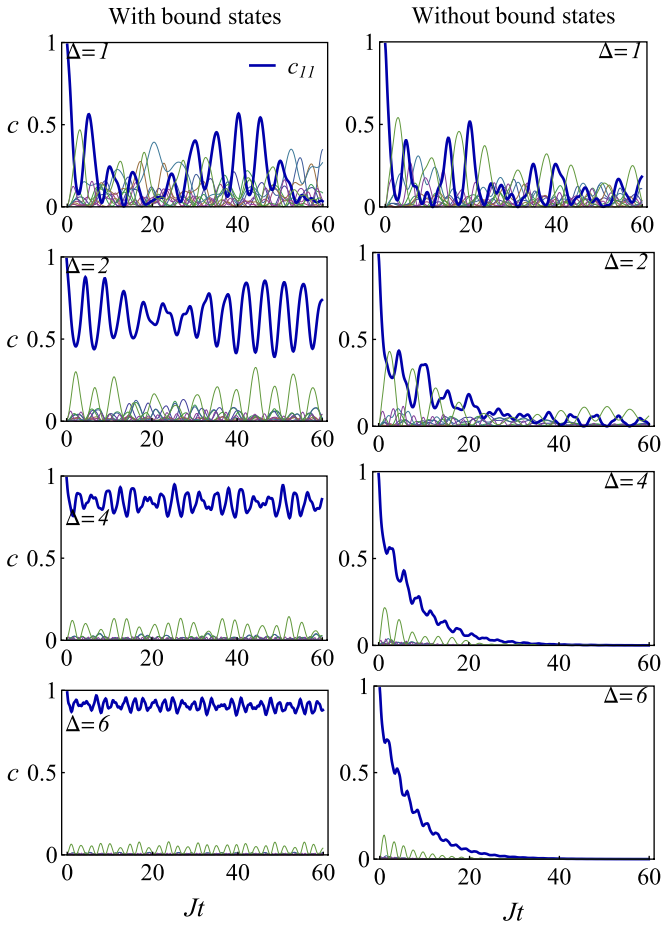

FIG. 6. A plot for varying of c_i ($i = 1,2,\ldots,12$) with rescaled time Jt when $N = 12$ and $s = 1$. Except that the excitation is initially located at $i = 11$, the parameters are the same as those in Fig. 5. c_{11} is highlighted by a thick blue solid line in this plot. We also present the other c_i by a thin solid line for convenience of comparison.

N is large, the evolution of c_i does not differ intrinsically from the previous case, except for a smaller amplitude of oscillation. In addition, with the increment of N, the difference in evolution becomes ambiguous; for instance, see the two bottom panels of Fig. 8 for $N = 12$.

C. Effect of on-site potential

Some comments are in order about Δ and U. As shown in the previous discussion, to find all bound states, different U have to be chosen upon varying Δ. For instance, when $N = 12$ and $\Delta = 6$, $U = 6.2$ is chosen. This point inevitably would induce a very high on-site potential for some spin sites, as shown by $h_i - U$ in Eq. (12). Thus it induces the natural question of whether the localization of information in the initial states is intrigued by a high on-site potential in comparison with the small system-environment coupling strength η; for example, as studied in Ref. [27]. Our answer is that this is not the case, as can be illustrated by the following two points:

First, phenomenally as shown by the right column in Fig. 6, the information of the initial state eventually vanishes when there is no bound state, even if the on-site potential is high; for example, $\Delta = 6$ and $U = -7$. This picture shows that the value of the on-site potential is irrelevant to the preservation of information in the initial state. SA similar feature can also be found in Fig. 7. Second, physically different from the case in Ref. [27], in which the qubit is coupled with the spin environment by a point contact, all spins are coupled homogeneously with the environment. Thus, because of the hopping term $\frac{J}{2}\sum_{i=1}(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+)$ in Eq. (1), the excitation can be transferred from one site, which would have high on-site potential, to another one with low on-site potential. In this case, the excitation may be absorbed by the environment. We thus argue in this place that the localization of the excitation induced by the bound state and disorder in the system would be a universal phenomenon, independent of the details of the system.

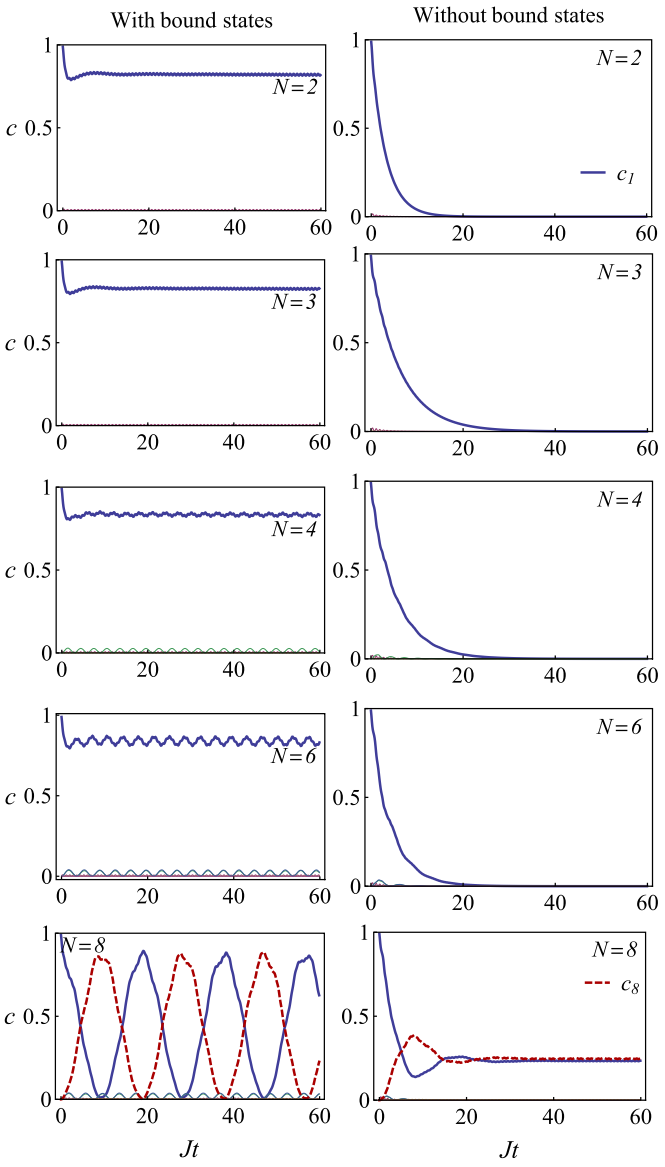

FIG. 7. Plot of c_i $(i = 1,2,\ldots,N)$ with rescaled time Jt for different N when $\Delta = 6$. The spin excitation is initially located at $i = 1$ in this plot. In addition, $U = 6.2$ is chosen when bound states occur. When there is no bound state, $U = -3$ for $N = 2$, $U = -6$ for $N = 3, 4, 6$ and $U = -7$ for $N = 8$. The other parameters are the same to those in Fig. 5. For clarity, the varying of c_1 or c_8 are highlighted by a thick blue solid line and a thick red dashed line, respectively, to distinguish them from the others. The other c_i are presented by thin solid lines for the convenience of comparison.

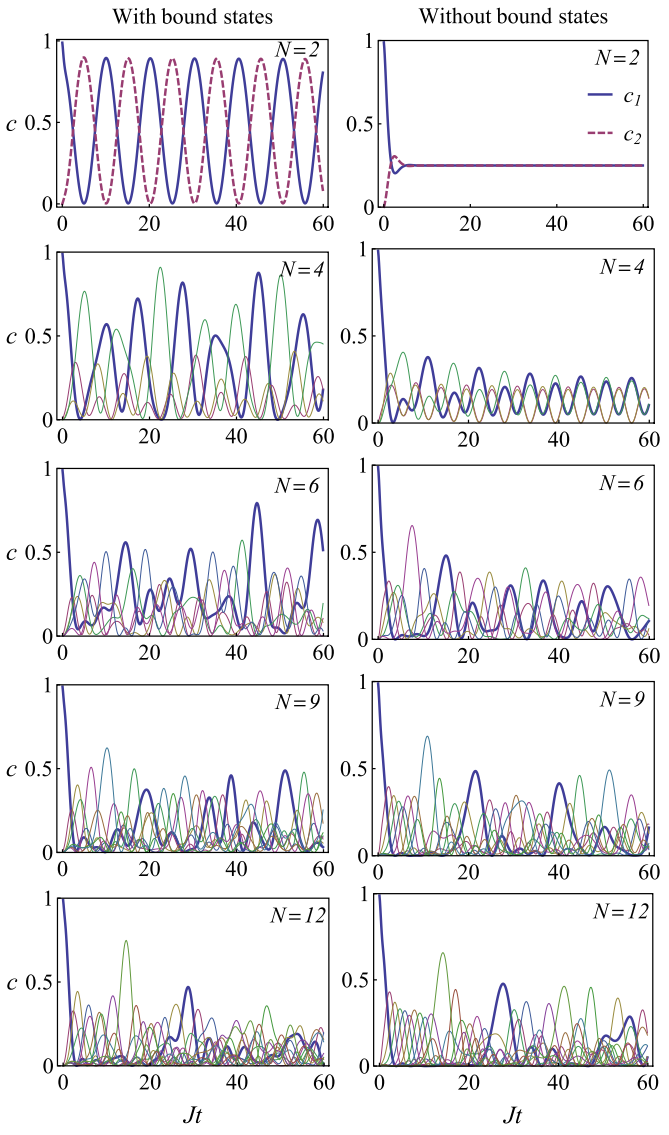

FIG. 8. Plot of c_i $(i = 1,2,\ldots,N)$ with rescaled time Jt when $\Delta = 0$ and $s = 1$. The open boundary conditions are impose for the spin chain. The initial state is chosen as the single excitation placed at spin site $i = 1$. When all bound states occur, $U = 1$ is chosen; when there is no bound state, we choose $u = -1$ for $N = 2$, $u = -1.5$ for $N = 4$, $u = -2.5$ for $N = 6$, $u = -4$ for $N = 9$, and $u = -5.5$ for $N = 12$. $\eta = 0.1$, $\omega_c = 3$ are also chosen for this plot. The varyiation in c_1 is highlighted by the thick blue solid line. The other c_i are presented by thin solid lines for convenience of comparison.

V. DISCUSSION AND CONCLUSION

In conclusion, the bound state and time evolution for single-excitation in a one-dimensional XXZ spin chain within the reservoir are studied exactly in this article. As for the bound state, four crucial observations are found: First, the true bound state is actually a excitation-localized state for which the single excitation is preserved in the spin system and spontaneous emission into the reservoir is prohibited exactly. Second, we point out first the existence of the pseudo-bound state, which is defined as the state satisfying the bound-state equation (7) but having a finite probability of spontaneous emission for a single excitation. Thus, in this case the excitation on the spin chain would be relaxed, probably into the reservoir. In addition, when the periodic boundary conditions are imposed in the spin chain, the pseudo-bound state shows some special properties. One is that the α_i are proportional to $1/\sqrt{N}$. But, for open boundary in the spin chain, this point disappears, which means that it is protected by the translational invariance of the spin chain. The other is that, for large N, the pseudo-bound state corresponds to the minimal energy level and d tends to 0.5, as shown in Table II. Third, a critical energy $(U - h_0) + E = -1$ can be identified, below which there is only one bound state. For a periodic boundary, the bound state is a pseudo-bound state

exactly. As for an open boundary, we find that the values of all c_i in the bound state tend to be isotropic, as shown in Fig. 3, and the value of d increases upon incrementing the spin number N, as shown in Table III. Fourth, when the quasirandom disorder (11) occurs, c_i in the bound state becomes more pronounced at some spin sites. This feature implies that single excitations can be localized in some spin sites despite the existence of the environment. Thus, the spin chain in this case can be used as a quantum memory.

To display the potential applications in quantum memory, time evolution of a single excitation is evaluated exactly by numerics. A crucial observation is that the information of the initial state can be faithfully preserved only if the bound state and disorder occur together. Moreover, the long-time behavior of the spin chain is determined completely by the structure of the bound state. When there is no bound state, the information of the initial state could be erased completely or become mixed, even if the disorder occurs. Thus, the combination of bound state and disorder provides a ideal mechanism for quantum memory.

ACKNOWLEDGMENTS

The author H.T.C. acknowledges the support of the NSF of China (Grant No. 11005002) and of NCET of the Ministry of Education of China (Grant No. NCET-11-0937). H.Z.S. acknowledges the support of the National Natural Science Foundation of China (NSFC) under Grant No. 11705025, the China Postdoctoral Science Foundation under Grants No. 2016M600223 and No. 2017T100192, and the Fundamental Research Funds for the Central Universities under Grant No. 2412017QD005. X.X.Y. acknowledges the support of the NSFC under Grants No. 11534002, No. 61475033, and No. 11775048.

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