Erratum: Quantum imaging as an ancilla-assisted process tomography [Phys. Rev. A 94, 042102 (2016)]

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I. Explanation of the problem. In our paper we proposed a "mode mixing" (MM) process which we (incorrectly) considered as a "completely positive trace preserving (CPTP) quantum operation (quantum channel)" [1] described with the following relation:

$$\mathcal{F}_{\rm MM}[\varrho] = \frac{(\mathsf{P}_\Xi + \mathsf{Q})\varrho(\mathsf{P}_\Xi + \mathsf{Q})^\dagger}{\mathrm{Tr}[(\mathsf{P}_\Xi + \mathsf{Q})\varrho(\mathsf{P}_\Xi + \mathsf{Q})^\dagger]}, \tag{1}$$

where $\mathsf{P}_\Xi = |\Xi\rangle(\langle 01| + \langle 10|)$ and $\mathsf{Q} = (|00\rangle + |11\rangle)(\langle 00| + \langle 11|)$. The action of $\mathcal{F}_{\rm MM}$ is such that it transforms the two orthogonal states $|10\rangle$ and $|01\rangle$ to the same (arbitrary but fixed) state $|\Xi\rangle$,

$$\begin{aligned}\mathcal{F}_{\rm MM}[|10\rangle\langle 10|] &= |\Xi\rangle\langle\Xi| \\ \mathcal{F}_{\rm MM}[|01\rangle\langle 01|] &= |\Xi\rangle\langle\Xi|.\end{aligned} \tag{2}$$

However, this operation will lead to the possibility of a faster-than-light communication (signaling). To see this, consider that there is an "Alice" who can perform $\mathcal{F}_{\rm MM}$, and she shares the quantum state $|\psi_0\rangle_{AB} = |10\rangle_A \otimes |1\rangle_B + |01\rangle_A \otimes |0\rangle_B$ with a "Bob" who is in a spacelike distance from Alice. Now Alice applies a phase-shifter gate on her first photon. Hence $|\psi_0\rangle$ changes to $|\psi_1\rangle = e^{-i\phi}|10\rangle_A \otimes |1\rangle_B + |01\rangle_A \otimes |0\rangle_B$. Now she applies $\mathcal{F}_{\rm MM}$ on her pair of photons which yields

$$\begin{aligned}|\psi_2\rangle &= e^{-i\phi}|\Xi\rangle_A \otimes |1\rangle_B + |\Xi\rangle_A \otimes |0\rangle_B \\ &= |\Xi\rangle_A \otimes (e^{-i\phi}|1\rangle + |0\rangle)_B.\end{aligned} \tag{3}$$

That is, immediately after the application of $\mathcal{F}_{\rm MM}$ by Alice, Bob's state becomes $e^{-i\phi}|1\rangle_B + |0\rangle_B$. This state bears the full information of the phase ϕ, which Bob can obtain by suitable local measurement or operation *without any need for classical communication with Alice (despite their spacelike distance)* [2].

II. Fixing the signaling problem. Here we argue that the culprit is exactly related to the fact that $\mathcal{F}_{\rm MM}$ is not a CPTP quantum operation. In fact, since $(\mathsf{P}_\Xi + \mathsf{Q})^\dagger(\mathsf{P}_\Xi + \mathsf{Q}) \neq \mathbb{1}$, $\mathcal{F}_{\rm MM}$ is an incomplete part of a POVM or a generalized measurement [1]. Hence the effect of $\mathcal{F}_{\rm MM}$ can be considered as a postselection of the results of the following POVM: $\mathsf{M} = \{\mathsf{M}_1, \mathsf{M}_2\}$ in which

$$\begin{aligned}\mathsf{M}_1 &= \mathsf{P}_\Xi + \mathsf{Q}, \\ \mathsf{M}_2 &= \sqrt{\mathbb{1} - \mathsf{M}_1^\dagger \mathsf{M}_1} = \mathbb{1} - |\Xi\rangle\langle\Xi|.\end{aligned} \tag{4}$$

Note that, although applying M_1 aligns the two idler modes, i.e., transforms the two orthogonal states $|10\rangle$ and $|01\rangle$ to the same state $|\Xi\rangle$,

$$\mathsf{M}_1|10\rangle\langle 10|\mathsf{M}_1^\dagger = \mathsf{M}_1|01\rangle\langle 01|\mathsf{M}_1^\dagger = |\Xi\rangle\langle\Xi|. \tag{5}$$

This alignment does not occur by applying M_2,

$$\mathsf{M}_2|10\rangle\langle 10|\mathsf{M}_2^\dagger \neq \mathsf{M}_2|01\rangle\langle 01|\mathsf{M}_2^\dagger \neq |\Xi\rangle\langle\Xi|. \tag{6}$$

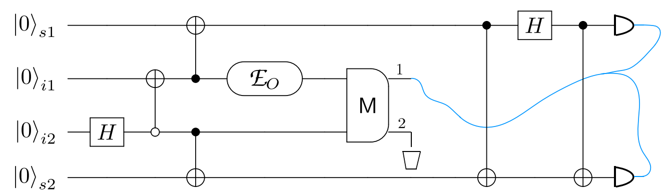

FIG. 1. Modified version of the proposed setup for quantum imaging. Here the blue lines indicate classical (one-way) communication.

This implies that the alignment is realized only if the measurement result is 1. Hence, for the setup to work, we need to postselect the measurement results corresponding to outcome 1 (see Fig. 1). This means that, to get an image, one needs to communicate the outcomes of the POVM measurements on the idlers with the detectors on the signal modes. The need for this communication (and hence success of imaging) removes the signaling problem. However, this modification leads the circuit proposed in Fig. 1 to be different from the experimental setup of Ref. [3] in which there is no communication between signals and idlers. In the next section, replacing the MM with a controlled-SWAP gate, we propose a quantum circuit in which quantum imaging with undetected photons becomes possible.

III. A quantum circuit for the "quantum imaging with undetected photons" experiment. Here we show that how quantum imaging with undetected photons becomes possible by replacing the MM with a controlled-SWAP gate. In the controlled-SWAP gate of Fig. 2, s_1 acts as the control qubit,

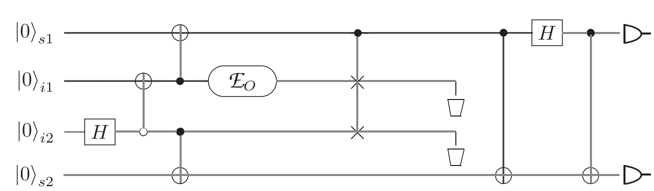

FIG. 2. Proposed quantum circuit for the quantum imaging experiment of Ref. [3].

and SWAP is applied on i_1 and i_2 when the state of s_1 is $|1\rangle_{s_1}$. With this replacement of the MM, there would be no need for measuring the idler photons to have an image. In addition, the final state of the signal photons as well as the probability of having an image will be in complete correspondence with the experimental result of Ref. [3] (an alternative circuit for this setup has been introduced in Ref. [4]). To show this, we apply the controlled-SWAP gate on the state of the system after passing through the object, i.e., on the state ϱ_3 given in Eq. (C2) of Appendix C of our paper which yields Eq. (C3) of Appendix C in which $|\Xi\rangle\langle\Xi|$ is replaced with $|0\rangle_{i_1}\langle 0| \otimes |1\rangle_{i_2}\langle 1|$. Hence the final state of the signal photons is the same as the one in Ref. [3].

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