Demultiplexing of photonic temporal modes by a linear system

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(Received 6 November 2017; published 20 March 2018)

Temporally and spatially overlapping but field-orthogonal photonic temporal modes (TMs) that intrinsically span a high-dimensional Hilbert space are recently suggested as a promising means of encoding information on photons. Presently, the realization of photonic TM technology, particularly to retrieve the information it carries, i.e., demultiplexing of photonic TMs, is mostly dependent on nonlinear medium and frequency conversion. Meanwhile, its miniaturization, simplification, and optimization remain the focus of research. In this paper, we propose a scheme of TM demultiplexing using linear systems consisting of resonators with linear couplings. Specifically, we examine a unidirectional array of identical resonators with short environment correlations. For both situations with and without tunable couplers, propagation formulas are derived to demonstrate photonic TM demultiplexing capabilities. The proposed scheme, being entirely feasible with current technologies, might find potential applications in quantum information processing.

DOI: 10.1103/PhysRevA.97.033841

I. INTRODUCTION

Single photons are excellent carriers of quantum information, since their interaction with the environment and among themselves is very weak. There are numerous approaches to encode quantum information in photon states, including the use of transverse modes [1] and polarizations [2]. However, these methods can only store very limited information in each photon, usually not much more than one qubit due to their small Hilbert space. Recently, it is found that temporal modes (TMs) of single-photon states—field-orthogonal broadband wave-packet states—might possess infinite degrees of freedom in the continuum of the electromagnetic field even within the confinement of a one-dimensional (1D) optical fiber or waveguide. The creation [3], manipulation [4,5], storage [6,7], and demultiplexing [8–11] of the temporal modes [12–14] then become the key tasks toward its use in carrying fragile quantum information over transmission lines in a network.

As precisely explained by Bercht *et al.*, for a single photon, TM is a coherent superposition over many of its possible creation times [14], and arbitrarily large sets of orthogonal TMs can be defined. Apart from the apparent capability of increasing the channel bandwidth [16] and security [15,17], TM is also potentially applicable to quantum computation [18,19]. Contemporarily, the TM shaping of a single photon has been demonstrated in both optical [3] and microwave [5] regimes. And the established enabler for the extraction of information from photonic TM, i.e., demultiplexing or sorting of TM, is the quantum pulse gate (QPG).

QPG is based on frequency conversion. More specifically, it operates in the form of a parametrized sum frequency conversion (SFG). SFG is a process where two photons are transformed by nonlinear material into a photon of sum energy, which is the inversion of the well-known parametrical down-conversion (PDC). SFG is more easily controlled than PDC because it has two input channels. When the input of one channel is a carefully engineered gate pulse, the other channel can be made into a parametrical up-conversion of specific TM [20], whereas photons of orthogonal TMs can pass through the QPG unaffected [21].

In spite of the success that has been accomplished so far, it remains a focus of interest to improve upon the established QPG approach in TM demultiplexing. Multiple stages for the selection of every single mode had been a practical necessity [22,23], until a recent new idea that employs SFG devices with multiple pump wavelengths [24–26]. Alternatively, it has been shown that coherent optical storage systems possess TM-selective qualities [27–29], which are yet to be formulated as a demultiplexer implementation for a set of orthogonal TMs.

In this paper, we propose a demultiplexing scheme of photonic TM using neither nonlinear effect nor frequency conversion. By simple physical principles and under a couple of conditions that can be easily satisfied, we will show that a collection of resonator modes in an arbitrary linear system (Fig. 1) can be connected one by one to a set of orthogonal TMs from the input channel. Photons of a specific TM that enter the system will traverse other modes of the system but eventually arrive at its corresponding resonator. Therefore, a wide range of linear systems can be potentially capable of separating a corresponding set of orthogonal TMs, which would be much easier to miniaturize than the present schemes based on SFG. As an example, we will demonstrate TM demultiplexing with a unidirectional array of duplicative resonators, which is entirely based on presently available experimental techniques. Moreover, the method of calculating the corresponding set of TM for a unidirectional array is given. The effect of several errors on the demultiplexer fidelity is analyzed as well.

The remainder of this paper is organized as follows. In Sec. II, we formalize the dynamics of a single resonator, and

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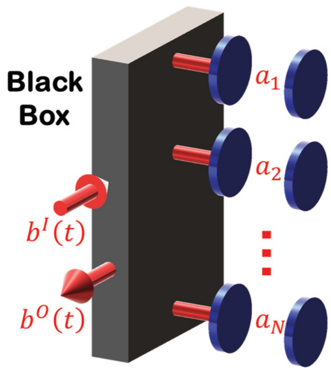
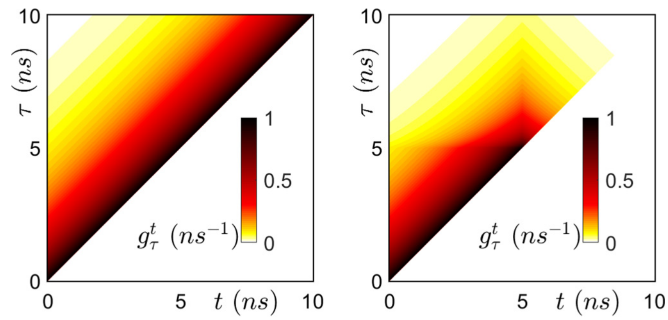


FIG. 1. General illustration of a linear system TM demultiplexer.



(a) (b)

FIG. 2. Examples of tensor g with $T = 10$ ns and $\kappa_0 = 1$ ns^{-1}. (a) Time-independent system. (b) Tunable coupling system in the form given by Eq. (27) with $T_c = 5$ ns.

deliver the theoretical basis of our proposal in a general framework. The key requirement of a linear TM demultiplexer—the vanishing condition—is introduced and explained. In Sec. III, we demonstrate the functioning of the proposed demultiplexer by calculating the dynamics of the unidirectional system. The vanishing condition is shown to hold for a broad range of parameters, and the operation of the TM demultiplexer with the time-independent linear system is illustrated both with and without the tunable coupler. In Sec. IV, we investigate the effect of a variety of errors, such as photon loss, on the fidelity of the proposed demultiplexer. In Sec. V, the constraint of uniform coupling is relaxed, and a method to get the coupling functions to demultiplex a given set of TMs is presented and demonstrated. Finally, we conclude in Sec. VI.

II. FORMALISM

A. Dynamics of a single resonator

For clarity, we begin by formalizing the model of a single resonator coupled to a half infinitely long dispersion-free waveguide at one side, which is a basic building block for a large set of photonic network systems.

Within rotating wave approximation and Markovian approximation, the Hamiltonian in the interaction picture reads [31]

$$V(t) = i\hbar \int d\omega \sqrt{\frac{\kappa(t)}{2\pi}} [e^{i[(\omega-\omega_0)t]} \bar{b}^\dagger(\omega) a - \text{H.c.}], \qquad (1)$$

where $\kappa(t)$ is the coupling coefficient between the resonator and the waveguide, ω_0 is the frequency of the resonator mode, a is the annihilation operator of the resonator, and $\bar{b}^\dagger(\omega)$ is the creation operator of the photon mode with frequency ω.

For simplicity, we assume that the coupling $\kappa(t)$ does not vary with the frequency of channel modes. The Markovian input-output relations of a single linear cavity thereby reads [32]

$$b^O(t) = b^I(t) + \sqrt{\kappa(t)} a(t), \qquad (2)$$

$$\dot{a}(t) = -\tfrac{1}{2}\kappa(t) a(t) - \sqrt{\kappa(t)} b^I(t), \qquad (3)$$

in which, within the rotating frame of resonator frequency, input and output photon modes in time representation [33–35] are given by their frequency counterpart $\bar{b}(\omega,-\mathcal{T})$ and $\bar{b}(\omega,\mathcal{T})$, respectively,

$$b^I(t) = \int \frac{d\omega}{\sqrt{2\pi}} \bar{b}(\omega,-\mathcal{T}) e^{-i[(\omega-\omega_0)t]}, \qquad (4)$$

$$b^O(t) = \int \frac{d\omega}{\sqrt{2\pi}} \bar{b}(\omega,\mathcal{T}) e^{-i[(\omega-\omega_0)t]}. \qquad (5)$$

In this definition, $\mathcal{T} \gg T$ is a time large enough that the photon at $t = -\mathcal{T}$ or $t = \mathcal{T}$ is considered not interacting with the system of interest.

In the following, we first derive a set of equations that will be used in our theory, then we present our main theory in next subsection.

Substituting the input-output relations with operator $v(t) = \sqrt{\kappa(t)} a(t)$, which denotes the instantaneous emission of the resonator, we obtain a new dynamic equation that is easier to integrate [31],

$$\dot{v}(t) = f(t) v(t) - \kappa(t) b^I(t), \qquad (6)$$

$$f(t) \equiv \tfrac{1}{2}[\kappa^{-1}(t)\dot{\kappa}(t) - \kappa(t)]. \qquad (7)$$

Integrating Eq. (6), we have

$$v(t) = v(0)F(t) - \int_0^t d\tau \frac{F(t)}{F(\tau)} \kappa(\tau) b^I(\tau), \qquad (8)$$

$$F(t) \equiv e^{\int_0^t d\tau f(\tau)}. \qquad (9)$$

To better understand the linear feature of the dynamics of the single cavity in the time representation, we introduce an adapted Einstein notation, $\alpha_\mu \beta^\mu \equiv \int_0^T d\mu \alpha_\mu \beta^\mu$, and define

$$g^t_\tau \equiv \Theta(t-\tau) \frac{F(t)}{F(\tau)} \kappa(\tau). \qquad (10)$$

Here $\Theta(t-\tau)$ is the heaviside function. Examining the derivation from Eqs. (7)–(10), we find that the tensor g is solely determined by $\kappa(t)$, and it establishes the relation between the emission of the resonator and the incident photon. Two examples of the tensor g is illustrated in Fig. 2 with two given $\kappa(t)$ that will be employed in later discussions.

Comparing Eq. (8) with the input-output relations Eqs. (2) and (3), and defining $t = 0$ as the initial time and $t = T$ as the final time, an integration form of the input-output relation for the single resonator follows:

$$b_O^t = g_\tau^t \frac{\delta_0^\tau}{\sqrt{\kappa(0)}} a(0) + \left(\delta_\tau^t - g_\tau^t\right) b_I^\tau, \tag{11}$$

$$a(T) = \frac{\delta_t^T}{\sqrt{\kappa(T)}} g_\tau^t \frac{\delta_0^\tau}{\sqrt{\kappa(0)}} a(0) - \frac{\delta_t^T}{\sqrt{\kappa(T)}} g_\tau^t b_I^\tau, \tag{12}$$

with $b_O^t \equiv b^O(t)$, $b_I^t \equiv b^I(t)$, and $\delta_\tau^t \equiv \delta(t-\tau)$. Note that, in this integration form of input-output relation, $a(t)$ does not appear, except its form at the beginning and final time, i.e., $a(0)$ and $a(T)$. In the next subsection, we will use this new input-output relation to derive our main theory.

B. Main theory

With other devices like beam splitters, a wide range of network can be constructed by a group of resonators. We thereby begin our analysis for temporal mode demultiplexing by generally considering a linear system consisting of several boson modes and a single input channel (Fig. 1), in which the propagation equation takes the following form in the Heisenberg picture,

$$a_m(T) = \sum_n \alpha_{mn}(T,0) a_n(0) + \int_0^T dt\, B_m(t) b_I(t), \tag{13}$$

where $a_m(t)$ is the annihilation operator of bosonic mode m in the Heisenberg picture, $b_I(t)$ is the annihilation operator of an input photon at time t, $\alpha_{mn}(t,\tau)$ is the propagator from resonator n to m during the time between τ and t, and $B_m(t)$ are complex functions yet to be specified.

Apparently, the above equation is derived for single resonators but it is applicable for systems other than the single resonators. For simplicity, we rewrite Eq. (13) into the following form:

$$a_m(T) = \sum_n \alpha_{mn}(T,0) a_n(0) + \beta_m \tilde{b}_m,$$
$$\tilde{b}_m = \frac{1}{\beta_m} \int_0^T dt\, B_m(t) b_I(t), \tag{14}$$

where β_m is a normalization constant that guarantees $[\tilde{b}_m, \tilde{b}_m^\dagger] = 1$. The introduced operator $\tilde{b}_m$ thereby can be treated as an annihilation operator of a temporal mode, which is guaranteed solely by the condition that the system has only a single input channel.

In general, in a 1D optical or microwave channel with a given single transverse spacial mode, a set of bosonic mode operators can be defined by the coherent superposition of time or frequency eigenstates [14], provided that only photons propagating in one direction is considered,

$$A_n \equiv \int C_n(t) b(t) dt = \int \bar{C}_n(\omega) \bar{b}(\omega) d\omega. \tag{15}$$

Here $C_n(t)$ and $\bar{C}_n(\omega)$ are complex functions, each being the Fourier transformation of the other. $[b(t), b^\dagger(\tau)] = \delta(t-\tau)$, and $[\bar{b}(\omega), \bar{b}^\dagger(\omega')] = \delta(\omega - \omega')$. Therefore, A_n corresponds to a set of field-orthogonal bosonic modes, if they satisfy

$$[A_m, A_n{}^\dagger] = \int C_m(t) C_n^*(t) dt = \delta_{mn}. \tag{16}$$

This set of bosonic modes is referred to as orthogonal TMs.

It is worth addressing that a TM photon from the set $\tilde{b}_m$ is not yet guaranteed to be fully absorbed by the system, since β_m remains undetermined. Also, this set of modes so far is not necessarily field orthogonal. Nonetheless, orthogonal states should remain orthogonal in a unitary evolution. This suggests that orthogonal excitation states of resonator modes at the final time should come from orthogonal states in the past. With this observation, we conclude that a TM demultiplexer can be realized provided that all resonator modes at the final time come from states in the channel at the initial time.

Note that if the initial excited states in the resonators do not evolve to any other excited states of the resonators at the final time T, the resonator excited states would come only from the channel. For any pair of resonators m,n in the system, this requires

$$\alpha_{mn}(T,0) \to 0, \tag{17}$$

which we hereby refer to as *the vanishing condition*. As we will demonstrate later, this condition can be easily satisfied in a Markovian system provided the operation time T is long.

To show this point concretely, it is worth pointing out first that the resonator modes and the input field satisfy

$$[a_m(0), b_I(t)] = 0,$$
$$[a_m(\tau), a_n^\dagger(\tau)] = \delta_{mn}, \tag{18}$$
$$[b_I(\tau), b_I^\dagger(t)] = \delta(\tau - t).$$

Applying these equations into Eq. (14) yields [30]

$$[a_m(T), a_n^\dagger(T)] = \delta_{mn} = \sum_{m',n'} \alpha_{mm'}(T,0) \alpha_{nn'}^*(T,0) \delta_{m'n'}$$
$$+ \beta_m \beta_n^* [\tilde{b}_m, \tilde{b}_n^\dagger]. \tag{19}$$

When the vanishing condition is satisfied, the first term from the second line of the above equation would vanish, leading the TM operators defined in Eq. (14) to satisfy

$$[\tilde{b}_m, \tilde{b}_n^\dagger] \to \delta_{mn}, \tag{20}$$

which is the commutation requirement for a set of field-orthogonal bosonic modes. Likewise, $\beta_n \beta_n^* \to 1$, implying a full absorption, is also guaranteed.

The vanishing condition thereby ensures the linear system in question to deliver an incident photon of corresponding orthogonal TMs into different resonators. Since no assumption has been made on the particular configuration of this linear system other than the requirement of a single input channel, the present theory thereby is applicable to a wide range of systems.

For concrete systems discussed in the next section, we will show that the vanishing condition is indeed satisfied.

III. DEMONSTRATION

A. Unidirectional array

We hereby restrict the discussion to a particular configuration of resonator network—unidirectional array. In general, for both microwave resonator and optical cavity coupled to waveguides at both sides, excitations inside would relax in both directions. In order to construct a unidirectional array of

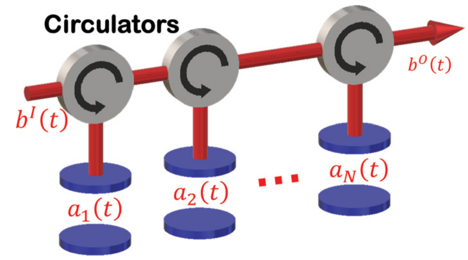


FIG. 3. Illustration of a unidirectional array of N resonators.

resonators, it is required that the resonators are coupled at only one side to a waveguide through a circulator [36].

Circulators are nonreciprocal multiport photonic devices in which an incident photon at one port is routed to exit at the next port. Here, three-port circulators [37] are employed in the unidirectional system under consideration; see Fig. 3.

Although this system consists of a waveguide with two input-output ports, incident photons at the designated exit port would not be routed through the resonators, separating the system into two isolated subsystems. The requirement of possessing a sole input channel is therefore satisfied for the one that involves the resonators, which is the system of our interest.

For simplicity, we consider the circulators to be ideal and ignore the channel dispersion. In this case, the output field of one resonator is the same as the input field of the next resonator. To be able to address frequency mismatch later, our formalism of unidirectional array begins as follows:

$$\bar{b}_n(\omega,\mathcal{T}) = \bar{b}_{n+1}(\omega,-\mathcal{T}), \tag{21}$$

which is independent of the rotating frame but within the interaction picture of the channel propagation. By using Eqs. (4) and (5), we have

$$b^I_{n+1}(t) = b^O_n(t)e^{i\Delta\omega_n t}, \tag{22}$$

in which $\Delta\omega_n = \omega_{n+1} - \omega_n$ is the frequency mismatch between two neighboring resonators. Also, a time realignment is assumed so that the phase effect from propagation between resonators vanishes.

In the absence of frequency mismatch, i.e., $\Delta\omega_n = 0$, by sequential iteration of Eq. (11) and using Eq. (12), we arrive at

$$a_m(T) = \sum_{n=1}^{m} \alpha_{mn}(T,0)a_n(0) - \frac{\delta^T}{\sqrt{\kappa(T)}}g(I-g)^{m-1}b^t_I,$$
$$\alpha_{mn}(T,0) = \begin{cases} \frac{\delta^T}{\sqrt{\kappa(T)}}g\frac{\delta_0}{\sqrt{\kappa(0)}}, & m=n; \\ -\frac{\delta^T}{\sqrt{\kappa(T)}}g(I-g)^{m-n-1}g\frac{\delta_0}{\sqrt{\kappa(0)}}, & m>n, \end{cases} \tag{23}$$

where indices that cannot cause confusion are omitted by assuming matrix product rules, and hence δ function becomes identity operator I if both indices are omitted. Apparently, this resonator array system conforms with the general formulation of Eq. (14), which enables it to potentially be a demultiplexer of a specific set of orthogonal TMs.

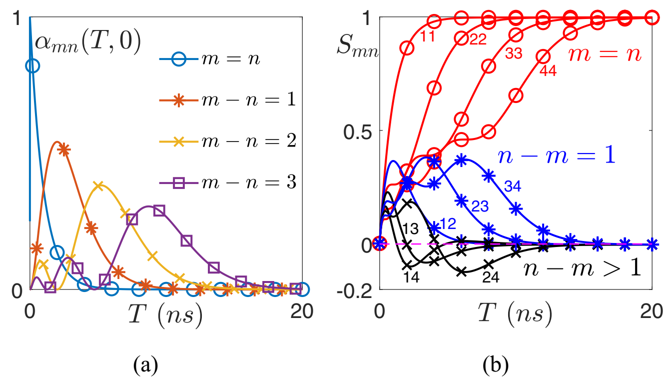


FIG. 4. Numerical demonstration of the vanishing condition for a time-independent array of four resonators with $\kappa_0 = 1$ ns^{-1}. (a) The vanishing initial excitation in the network. (b) The integrations $S_{mn} \equiv \int_0^T dt\, B_m(t)B^*_n(t)$.

B. Vanishing condition

From Eq. (23), the complex function characterizing photon absorption of each resonator in the array, as previously derived in Eq. (13), follows:

$$B_n(t) = \begin{cases} -\frac{\delta^T}{\sqrt{\kappa_0}}g_t, & n=1; \\ -\frac{\delta^T}{\sqrt{\kappa_0}}g(I-g)^{n-2}(I_t-g_t), & n>1. \end{cases} \tag{24}$$

It remains far from obvious or perhaps counterintuitive for a network of duplicative resonators with the same coupling coefficient at any moment. However, provided that the vanishing condition is satisfied, the general analysis in Sec. II tells us that these complex functions would be normalized and orthogonal. These functions then would become the rotating frame wave functions of a set of orthogonal TMs, and the array of resonators would therefore become a demultiplexer for this set of TMs.

Given the general formulation for the unidirectional array of duplicative resonators, we first examine the properties of a time-independent system defined by $\kappa_i = \kappa_0$, which gives

$$g^t_\tau = \Theta(t-\tau)e^{-\frac{\kappa_0}{2}(t-\tau)}\kappa_0. \tag{25}$$

As shown in Fig. 4(a), for an array of four resonators and under condition $\kappa_0 = 1$, the vanishing condition is roughly satisfied with $T > 20$ ns. And thereby, as expected, Fig. 4(b) shows that the complex functions $B_i(t)$ corresponding, respectively, to the resonators become normalized and orthogonal as the vanishing condition is met.

Of course, excitation in the array cannot be fully evacuated in a finite time, but the probability to remain in the array is very small and can be negligible. In fact, for the Markovian array, full evacuation of any resonator in the array can be achieved provided T is large.

C. TM demultiplexing

With the vanishing condition guaranteed, Fig. 5(a) shows the orthogonal TMs corresponding to the four resonators in the array, each is determined by a complex function of time,

$$\tilde{b}_n = \int_0^T dt\, B_n(t)b^I(t), \tag{26}$$

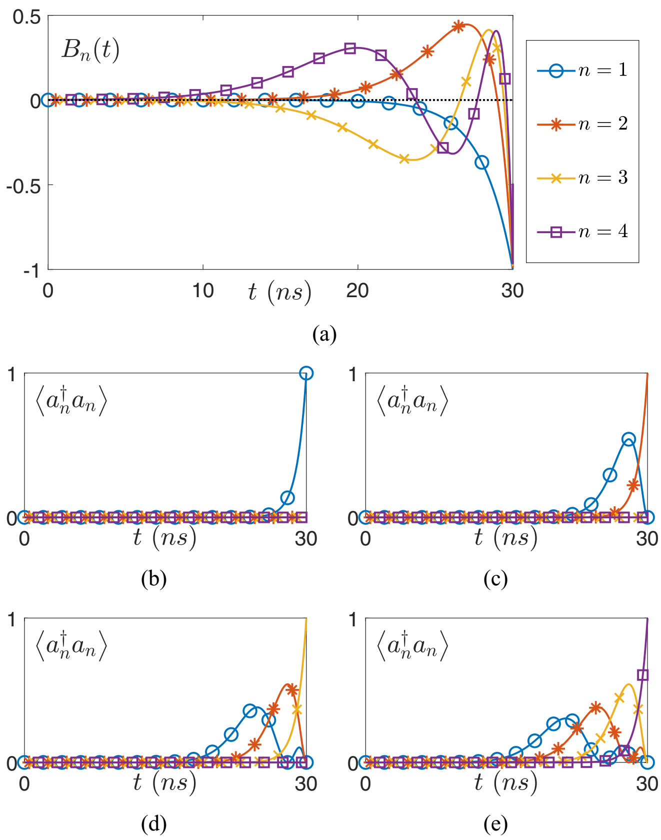


FIG. 5. Rotating frame wave function of orthogonal TMs (a) for a time-independent array of resonators with $\kappa_0 = 1\ \text{ns}^{-1}$, and the corresponding dynamics of the array upon their entry (b)–(e).

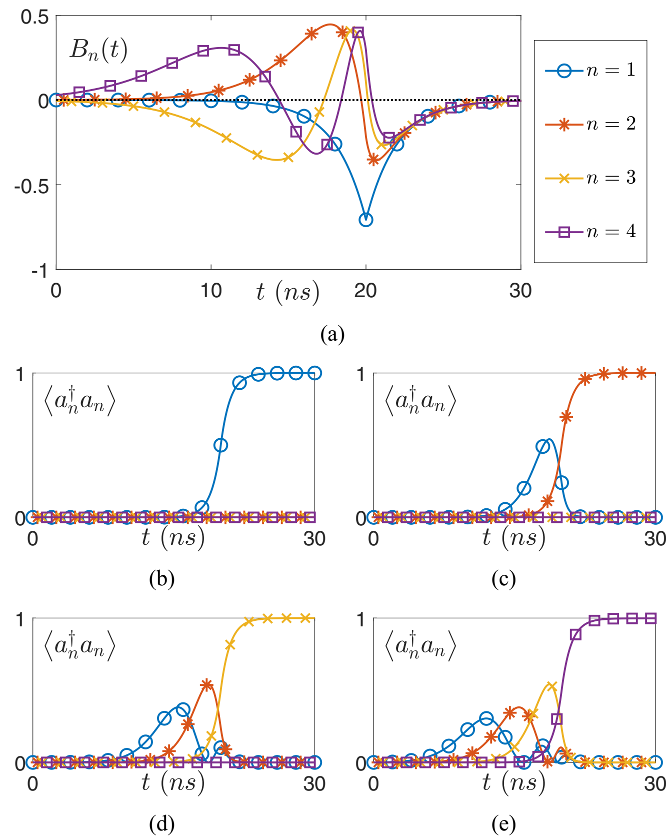


FIG. 6. Rotating frame wave function of orthogonal TMs (a) for an array of resonators with tunable coupling under $\kappa_0 = 1\ \text{ns}^{-1}$ and $T_c = 20$ ns, and the corresponding dynamics of the array upon their entry (b)–(e).

in which $b^I(t)$ is the annihilation operator of an input photon of the array at time t in the rotating frame defined by the resonators in the array. For later discussions, note that $b^I(t) = b_1^I(t)$ holds only if the first resonator is not detuned from other resonators.

Figures 5(b)–5(e) show the dynamics of the array with a single photon of each TM entering the array. Apparently, one resonator eventually absorbs the corresponding incident photon entirely. Also, as shown most evidently in Fig. 5(e), although resonators in the way are temporarily excited during the operation, a TM photon will not end up in the wrong resonator at the terminal time.

One can also see in Fig. 5(a) that the first resonator corresponds to a TM of inverting pulse [38,39]. As expected, all the rest of these resonators are the same as the first, whereas at final time they only capture the incident photon of their corresponding TMs, which are all orthogonal to each other.

This can be understood as follows. As a photon being reflected by a resonator, its TM is expected to be transformed. These orthogonal TMs are to be transformed into inverting pulse in a specific number of encounters with resonators.

Despite the apparent capability of TM demultiplexing in principle, the design stated above is still prevented from being practically usable. The figures clearly indicate that the efficiency of TM photon absorption depends highly on the final time T, which is an issue to resolve in the following.

D. Employing tunable coupler

Since a transfer or measurement on the information carrier would need time, retaining the TM modes in resonators is therefore highly desired. This task can be implemented by the proposed scheme via modulating the couplings. In superconducting systems, a tunable coupler can be constructed using two fixed inductors and one superconducting quantum interference device (SQUID) [40], which can be applied to the connection between a resonator and a transmission line [41–43].

As an illustration, we assume that all resonators are the same and they're coupled to the waveguide with a uniform coupling function that reads

$$\kappa(t) = \begin{cases} \kappa_0, & t < T_c; \\ \kappa_0 \cdot e^{\kappa_0 (T_c - t)} [2 - e^{\kappa_0 (T_c - t)}]^{-1}, & t > T_c, \end{cases} \tag{27}$$

which leads to a cutoff of couplings asymptotically in the long time limit, $T \gg T_c$, and subsequently the resonator retains the TM photon. For $T_c < 20$ ns, the system remains time independent, and as previously guaranteed by Fig. 4(a), any initial excitation would roughly vanish within this time period. Tedious but straightforward analysis shows that

$$F(t) = \begin{cases} e^{-\frac{\kappa_0}{2} t}, & t < T_c; \\ e^{\frac{\kappa_0}{2} t} [2 e^{\kappa_0 t} - e^{\kappa_0 T_c}]^{-1}, & t > T_c. \end{cases} \tag{28}$$

The corresponding TMs of this design is then determined and shown in Fig. 6(a), and so is their corresponding absorption process in subsequent figures. Apparently, for $t', t'' \gg T_c$, due to $\kappa_n \to 0$ we have $a_n(t') \to a_n(t'')$, for which the absorption of TM photons is no longer susceptible to the final time of operation.

IV. FIDELITIES

The presented scheme may find potential applications in quantum information processing. Here we formalize and calculate errors introduced by a variety of means and demonstrate their effect on the fidelity of the proposed TM demultiplexer within the chosen paradigm.

A. Average fidelity

Consider a scenario where a quantum digit (qudit) of a given state, which is allowed to have a Hilbert space of more than two dimensions, is processed by a subject system. The output qudit is subsequently measured in an eigenbasis that includes the ideally expected final state. Suppose the possibility of a given qubit is evenly distributed on the unite sphere in its Hilbert space; the expectation of the measurement yielding the result of ideal expectation is referred to as the average fidelity of the subject system [44], which reads

$$\mathcal{F} = \int |\langle\psi|U_T^\dagger U|\psi\rangle|^2 dV(\psi), \tag{29}$$

in which U_T is the intended evolution operator, U is the evolution operator of the subject system, and V describes the possibility distribution of input qudits.

Given $U_T^\dagger U$ being a linear operator, it has been shown that [45]

$$\mathcal{F} = \frac{Tr(UU^\dagger) + |Tr(U_T^\dagger U)|^2}{d(d+1)}, \tag{30}$$

in which d is the dimension of the qudit.

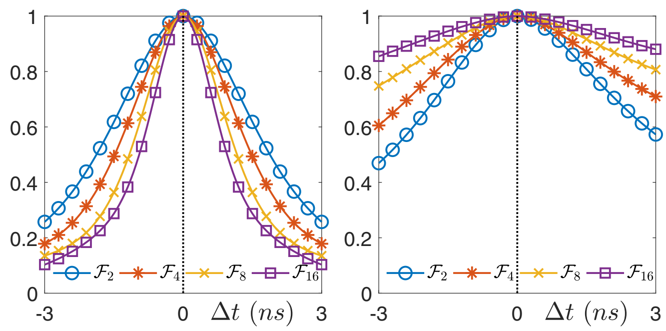
In the context of a TM demultiplexer, U_T is simply the identity operator I. We consider an ideal incident single-photon TM qudit, and U describes the transfer from the qudit state to a single excitation state of the resonator array. U is thereby linear, but nonunitary [46] in the case of photon loss.

B. Time mismatch with the sender

Time synchronization across long distance is known to be challenging, hence we begin our discussions of error scenarios with a simple time mismatch between the sender of a TM photon and the party who controls the demultiplexer.

Consider an ideal single photon qudit enters a TM demultiplexing array equipped with tunable coupler and satisfying $\kappa(t) \to 0$ as $t \to T$, and suppose the qudit arrives later than expected by a small delay Δt. This situation can be simply treated as a uniform time displacement between an ideal TM photon and the actual incident TM photon. We thereby straightforwardly obtain the transfer matrix from photonic TM qudit to the resonator array single excitation mode,

$$U_{mn} = \iint dt d\tau B_n^*(t) B_m(\tau)\delta(t-\tau-\Delta t) = B_t^m D_\tau^t(\Delta t) B_n^\tau, \tag{31}$$



(a) (b)

FIG. 7. Effect of time error on the average fidelities $\mathcal{F}_d$ of the demultiplexer, where d is the dimension of the TM qudit. Tunable coupling in the previously given form is used with $\kappa_0 = 1\ \text{ns}^{-1}$. (a) The time mismatch between the demultiplexer and an ideal sender. (b) Delay of operation by the tunable coupler of the first resonator.

in which the single-photon TM wave function is formalized by vectors $B_t^n \equiv B_n(t)$ and $B_n^t \equiv B_n{}^*(t)$, the time shift is introduced as $D_\tau^t(\Delta t) \equiv \delta_\tau^{t-\Delta t} = \delta_{\tau+\Delta t}^t$, and the global phase is ignored.

Ideally, we should have $D_\tau^t(-\Delta t) D_{t'}^\tau(\Delta t) = \delta_{t'}^t$ and $D(-\Delta t) = D^\dagger(\Delta t)$ as the matrix. However, as a result of numerical cutoff at $t = 0$ and $t = T$, time shift causes an issue of premature cutoff, which is made obvious by

$$D_\tau^t(-\Delta t) D_{t'}^\tau(\Delta t) = \delta_{t'}^t \Theta(T - \Delta t - t)\Theta(t - \Delta t). \tag{32}$$

Fortunately, this issue can be resolved by expanding the operation cutoff time and secure a large border area with no dynamics. Note that for our tunable coupling model, the TMs are fixed in temporal space by parameter T_c under $0 \ll T_c \ll T$.

With transfer matrix determined, fidelity can therefore be calculated. As shown in Fig. 7(a), the numerical result with greatly expanded operation time is produced for up to an array of 16 duplicative resonators. As qudit dimensions increase, resonators in the unidirectional array are sequentially employed. As expected, greater demand on accuracy of synchronization is imposed for greater qudit dimensions, while for minor time mismatch, the effect on fidelity appears tolerable.

C. Delay within the array

Another obvious source of error is the inaccuracy in the operation within the demultiplexer network. The possibilities of this kind of error are numerous; we therefore investigate a situation that also involves time, where one resonator suffers a delay of control by Δt. This scenario can be formalized as follows:

$$\kappa_n(t) \to \kappa_n(t - \Delta t), \tag{33}$$

which is most easily characterized by a time displacement in the output-to-input connection between resonators, which we formulate with

$$g_n \to D(\Delta t) g_n D(-\Delta t). \tag{34}$$

We then restrict our discussion to the delay of the first resonator in an array of duplicative resonators. The transfer matrix thereby reads

$$U_{mn} = \begin{cases} B^1 D(-\Delta t) B_n, & m = 1; \\ B^{m-1} D(\Delta t)(I - g) D(-\Delta t) B_n, & m > 1, \end{cases} \tag{35}$$

in which all time indices are omitted by assuming matrix product rules.

Strikingly, with our choice of tunable coupling, as indicated in Fig. 7(b), qudits of higher dimensions are averagely less sensitive to the delaying or expediting of the first resonator. Also, delaying appears to be relatively less disruptive than expediting. We can explain this as follows. Forms of TMs corresponding with each resonator rely differently on the first resonator in terms of the coupling coefficient of different moments, which change differently in delaying and expediting. However, note that no general assertion can be made for all forms of tunable coupling.

D. Frequency mismatch within the array

As an example of formalizing the effect of frequency mismatch within the interaction picture, we introduce the frequency mismatch of a single resonator from the array in relative terms. Suppose the first resonator is detuned from the second one by $\Delta\omega$; Eq. (22) leads to the following substitution:

$$g_\tau^t \rightarrow \tilde{D}_{t'}^t(-\Delta\omega) g_{\tau'}^{t'} \tilde{D}_\tau^{\tau'}(\Delta\omega), \tag{36}$$

in which tensor $\tilde{D}_\tau^t(\Delta\omega) \equiv \delta_\tau^t e^{-i\Delta\omega(t-T)}$ describes frequency mismatch. Note that by matrix rules, it satisfies $\tilde{D}(-\Delta\omega) = \tilde{D}^\dagger(\Delta\omega)$ and $\tilde{D}(-\Delta\omega)\tilde{D}(\Delta\omega) = I$.

Similarly, the transfer matrix with nonzero detuning of the first resonator reads

$$U_{mn} = \begin{cases} B^1 \tilde{D}(\Delta\omega) B_n, & m = 1; \\ B^{m-1} \tilde{D}(-\Delta\omega)(I - g)\tilde{D}(\Delta\omega) B_n, & m > 1. \end{cases} \tag{37}$$

With the transfer matrix calculated, we can derive fidelity for any given incident qudit state, and two noteworthy observations can be made.

First, as shown in Fig. 8(a), average fidelity at the final time oscillates as detuning increases. Detuning of the first to other resonators results in an instability in the case where coherent superposition with the first eigenstate appears in the system, which is clearly visible in the transfer matrix; see Fig. 8(c). Also, the detrimental effect of frequency mismatch does not end with the demultiplexing process of the qudit; phase shift would continue to be produced during the storage time. However, demultiplexing of TM eigenstates, which is all that matters with an implementation in classical communication, would not suffer from phase instability.

Second, comparing Fig. 8(a) with Fig. 8(b), we find that fidelities of eigenstate demultiplexing vanish while average fidelity does not. As detuning becomes significantly large, the first resonator is effectively removed, leading to a photon of the nth TM to be captured by the $(n+1)$th resonator. Such an error would make the fidelity of eigenstate transfer vanish. Whereas for their superposition state, overlap between the actual and resulting state may still exist.

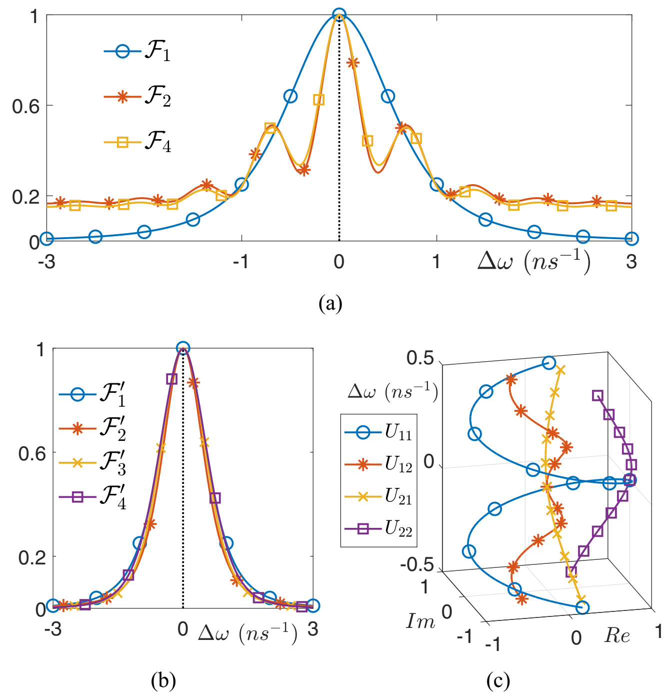


FIG. 8. Numerical results with frequency mismatch of the first resonator. Tunable coupling in the previously given form is used with $\kappa_0 = 1\ \text{ns}^{-1}$. (a) Average fidelities $\mathcal{F}_d$ of the demultiplexer, where d is the dimension of the TM qudit. (b) Fidelities $\mathcal{F}'_n$ for the transfer of a single photon in a TM eigenstate associated with resonator n. (c) Four transfer matrix elements as functions of detuning.

E. Photon loss

In this subsection, we address the effect of photon loss on the paradigm demultiplexer. To such end, we introduce additional input-output channels for each resonator. In general, the revised input-output relations read

$$\dot{a} = -\frac{1}{2}\left(\sum_l \kappa_l\right) a - \left(\sum_l \sqrt{\kappa_l} b_l^I\right), \tag{38}$$

$$b_l^O(t) - b_l^I(t) = \sqrt{\kappa_l} a(t). \tag{39}$$

These equations can be rewritten in the following form:

$$\dot{a} = -\frac{1}{2}\tilde{\kappa} a - \sqrt{\tilde{\kappa}} \tilde{b}^I, \tag{40}$$

$$b_l^O(t) - b_l^I(t) = r_l(t)\sqrt{\tilde{\kappa}(t)} a(t), \tag{41}$$

which is similar to that for a single channel. Here we have defined $\tilde{\kappa}(t) = \sum_l \kappa_l(t)$, and the effective input photon field reads $\tilde{b}^I(t) = \sum_l r_l(t) b_l^I(t)$, with

$$r_l(t) = \sqrt{\frac{\kappa_l(t)}{\tilde{\kappa}(t)}}. \tag{42}$$

As we consider only the photon loss, the environment has no contribution to the photon input, we thereby have $\tilde{b}^I(t) = r(t) b^I(t)$ and $\tilde{\kappa}(t) = \kappa(t) + \gamma$, where γ denotes the decay rate of the resonator. Following the derivation from Eq. (4) to Eq. (9), we arrive at a set of equations that takes the photon loss into account,

$$a(T)=\frac{\delta_t^T}{\sqrt{\tilde{\kappa}(T)}}\tilde{g}_\tau^t\frac{\delta_0^\tau}{\sqrt{\tilde{\kappa}(0)}}a(0)-\frac{\delta_t^T}{\sqrt{\tilde{\kappa}(T)}}\tilde{g}_\tau^t r(\tau)b_I^\tau, \tag{43}$$

$$b_O^t=\delta_\tau^t b_I^\tau+r(t)\tilde{g}_\tau^t\frac{\delta_0^\tau}{\sqrt{\tilde{\kappa}(0)}}a(0)-r(t)\tilde{g}_\tau^t r(\tau)b_I^\tau, \tag{44}$$

in which $\tilde{g}$ is determined by $\tilde{\kappa}$ by the same chain of equations as their lossless counterparts g and κ.

It is clear that system modes come from the outside (not within any of the resonators), as long as the vanishing condition holds. However, with photon loss, the revised propagation function then took the form $a_n(T)=\tilde{b}_n+\tilde{b}_n'$, where operator $\tilde{b}_n'$ are on the loss channels. In this revised system, the vanishing condition gives

$$[\tilde{b}_m+\tilde{b}_m',\tilde{b}_n^\dagger+\tilde{b}_n'^\dagger]=\delta_{mn}. \tag{45}$$

Since loss channels don't overlap with the input channel, i.e., $[\tilde{b}_m',\tilde{b}_n^\dagger]=0$, we then have $[\tilde{b}_m,\tilde{b}_n^\dagger]=\delta_{mn}-[\tilde{b}_m',\tilde{b}_n'^\dagger]$, which indicates that photon loss cannot introduce crosstalk.

In practice, the decay rate of each resonator can be measured relatively easily, we therefore assume that the input TM photon wave functions are engineered to adapt. We then use the normalization of the input channel part of the transfer function as the input photon wave function. Under uniform coupling functions, they read

$$\tilde{B}_n(t)=\begin{cases}-\frac{\delta^T}{\beta_n\sqrt{\tilde{\kappa}(T)}}gr_t, & n=1;\\ -\frac{\delta^T}{\beta_n\sqrt{\tilde{\kappa}(T)}}g_t(I-r\tilde{g}r)^{n-2}(I_t-\tilde{g}r_t), & n>1,\end{cases} \tag{46}$$

in which $r_\tau^t=\delta_\tau^t r(t)$. The transfer matrix of these TM photons therefore reads

$$U_{mn}=\beta_n\tilde{B}_t^m\tilde{B}_n^t, \tag{47}$$

where the normalization factor β_n is defined as a positive real number.

To gain some simple insight in the effect of photon loss, we first analytically address the situation of time-independent coupling function. Denoting the decay rate of each resonator as γ, we then have a time-independent channel weight. Also, an ideal demultiplexer using $\kappa+\gamma$ as a coupling rate certainly satisfies the vanishing condition provided with sufficient operation time. By comparing with such an ideal demultiplexer and identifying added analytical items, the fidelity of the transfer of each mode therefore simply reads

$$\mathcal{F}_n'=\beta_n=\left(\frac{\kappa}{\kappa+\gamma}\right)^{n-\frac{1}{2}}. \tag{48}$$

A more complicated situation is then addressed numerically. Figure 9 presents the average fidelity of TM demultiplexing, with qudits of multiple dimensions, using a uniform coupling function of Eq. (27), and photon loss is introduced by $\tilde{\kappa}(t)=\kappa(t)+\gamma$. Apparently, despite that the input TM photons are adapted in shape to the effect of photon loss on the transfer equation, TM demultiplexing is very sensitive to the photon loss of each resonator.

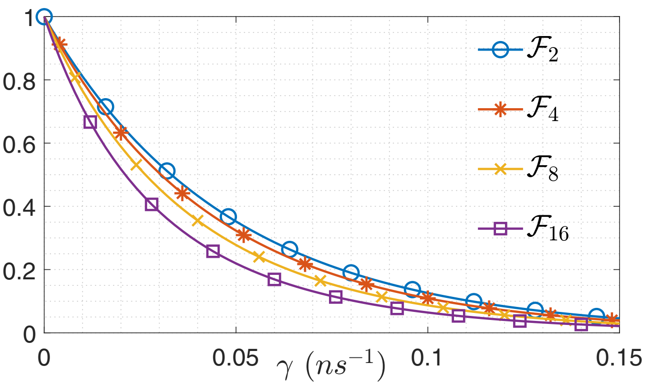


FIG. 9. Effect of photon loss on the average fidelities $\mathcal{F}_d$ of the demultiplexer, where d is the dimension of the TM qudit. Tunable coupling in previously given form is used with $\kappa_0=1\ \mathrm{ns}^{-1}$. TM photons are assumed as designed with photon loss considered.

V. ARBITRARY MODES

For simplicity, the discussion so far has been under the constraint of uniform coupling functions. However, if this artificial constraint is relaxed, the proposed demultiplexer configuration can be customized to work for a wide range of orthogonal TMs.

For unidirectional array and with a given set of orthogonal TM wave functions, an established approach can be employed to find the tunable coupling functions required, that is, a sequential iteration of the so-called zero-dynamic principle [47,48] on each resonator under the single excitation scenario.

A. Zero-dynamic principle

Here we briefly introduce the zero-dynamic principle. When it is applied on a storage subsystem, it is assumed that the subsystem in question produces no outgoing light throughout the absorption process, i.e., $b^O(t)=0$. Also, within the single excitation situation, a perfect absorption naturally gives

$$\langle a(t)a^\dagger(t)\rangle=\int_{-\infty}^t|B(\tau)|^2d\tau. \tag{49}$$

With the above formulas, Eq. (2) shall then lead to an explicit equation of $\kappa(t)$,

$$\kappa(t)=\frac{|B(t)|^2}{\langle a(0)a^\dagger(0)\rangle+\int_0^t|B(\tau)|^2d\tau}. \tag{50}$$

In practice, the initial population of the resonator $\langle a(0)a^\dagger(0)\rangle$ is most conveniently a small arbitrary offset to eliminate singularities at $t=0$. Also, note that this formalism assumes the input photon wave function is real and positive; phase shifting of the tunable coupler would be required to handle more the complicated input photon.

Treating each resonator in the unidirectional array as a storage subsystem, the coupling functions of each resonator in the array can then be obtained iteratively as shown in Fig. 10.

B. Schmidt modes

As an example, we demonstrate the demultiplexing of temporal Schmidt modes [49,50], the wave functions of which

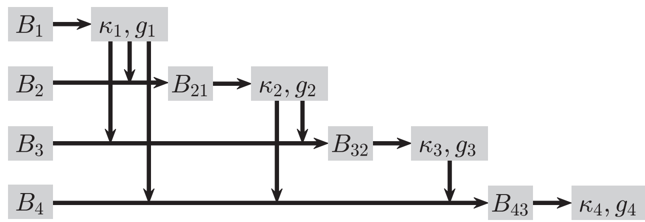


FIG. 10. Flow chart for solving the coupling functions to the demultiplexing of arbitrary orthogonal TMs.

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$$B_n(t) = \lambda^{-1}\varphi_{n-1}(\lambda^{-2}t),$$
$$\varphi_n(x) = (2^n n!\sqrt{\pi})^{-\frac{1}{2}} e^{-\frac{x^2}{2}} H_n(x), \quad (51)$$

in which H_n are the Hermite polynomials, and the parameter λ controls the scale of the Schmidt photon wave pack length, which makes $\lambda^{-2}t$ dimensionless.

For simplicity, we limit the demonstration to the first four Schmidt photons and address their demultiplexing task numerically. As shown in Fig. 11, under carefully designed nonuniform coupling functions, four orthogonal Schmidt mode photons are near perfectly absorbed by corresponding resonators. Note that we have manually truncated some fluctuations that came out of Eq. (50) by overwriting the early part of these coupling functions with zeros.

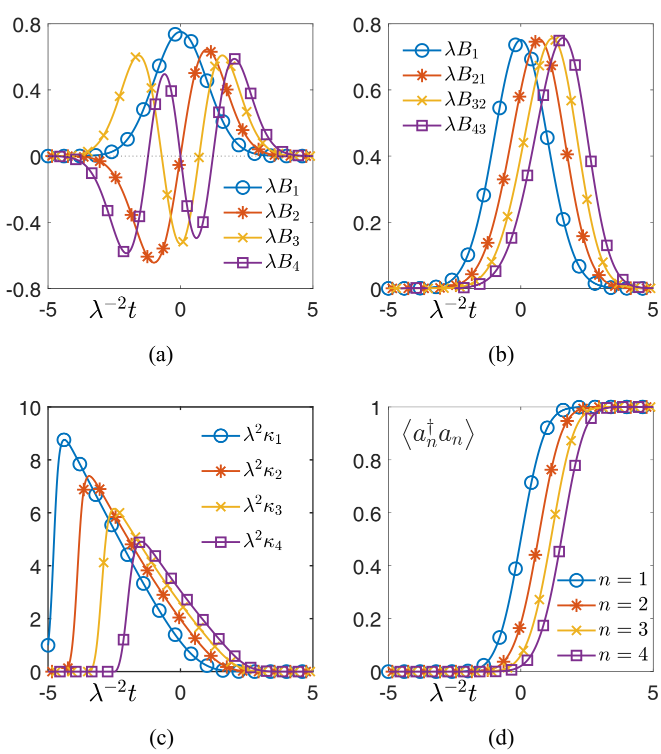

FIG. 11. Demultiplexing of Schmidt modes with nonuniform coupling functions. Parameter λ controls the pulse length of Schmidt modes. (a) First four Schmidt modes in dimensionless time representation. (b) The rotating frame wave function of each Schmidt mode photon upon arrival of the corresponding resonator. B_{nm} denotes the output photon wave function of the mth resonator when the input photon is the nth Schmidt mode. (c) Coupling function of each resonator. (d) Population of each resonator in the absorption of its corresponding Schmidt single photon.

Strikingly, each of these photons was transformed into roughly the same positive shape by the previous resonators it had traversed when it arrived at its destination, which means that phase shifting of the tunable coupler wouldn't be necessary. Also note that a resonator fully absorbing a TM photon automatically means it won't absorb its orthogonal TMs, because $a_m(T) \to \tilde{b}_m$ and $[\tilde{b}_m, \tilde{b}_n^\dagger] = \delta_{mn}$ gives $[a_m(T), \tilde{b}_n^\dagger] \to \delta_{mn}$.

VI. DISCUSSION AND CONCLUSION

As the orthogonal TM photons are being absorbed and secured, separately but coherently, by the resonators, subsequent measurement, transfer, or another kind of operation on them can then be easily carried out. In the unidirectional design, since resonators are only coupled at one end within the demultiplexer, the other end of each resonator is then available to be coupled with another system. For it to not affect the demultiplexing process, the other coupling has to be turned off during the demultiplexing process, which means tunable coupling has to be employed as well. For an implementation on the superconducting chip, which is the one physical system where the tunable coupler has been realized, this other system could preferably be a small-scale quantum processing circuitry within the same chip.

Of course, additional coupling, even a switched-off tunable coupler, would contribute to the photon loss of the resonators within the demultiplexer, the effect of which has been discussed.

In conclusion, we have presented a proposal to demultiplex photonic TMs in a microwave system. Instead of relying on frequency conversion, we have shown that a linear system, e.g., an array of identical resonators, is intrinsically capable of separating photons of different orthogonal TMs, provided that the system has only one input channel and that it guarantees all pre-existing excitations in the resonators would vanish at the final time.

A specific demultiplexer configuration, a unidirectional array of resonators, is studied in detail. In this system, circulator and tunable couplers are employed, both of which are currently within grasp of experimental technology due to recent progress in microwave systems. The demultiplexing capability is demonstrated first in the time-independent scenario, then with a uniform time-dependent coupling function.

We also examined the effect of several errors on the average fidelity of the proposed demultiplexer. As expected, precision in time and frequency, as well as the quality factor of the resonators, are important to the performance of the purposed TM demultiplexer.

Finally, with the constraint on constant coupling lifted, a general approach was presented to demultiplex a given set of orthogonal TM photons, which we demonstrated numerically with Schmidt modes.

ACKNOWLEDGMENT

This work is supported by National Natural Science Foundation of China (NSFC) under Grants No. 11534002, No. 61475033, No. 11775048, and No. 11705025.

[1] S. G. Leon-Saval, N. K. Fontaine, J. R. Salazar-Gil, B. Ercan, R. Ryf, and J. Bland-Hawthorn, Mode-selective photonic lanterns for space-division multiplexing, Opt. Express **22**, 1036 (2014).
[2] C. Ottaviani, D. Vitali, M. Artoni, F. Cataliotti, and P. Tombesi, Polarization Qubit Phase Gate in Driven Atomic Media, Phys. Rev. Lett. **90**, 197902 (2003).
[3] R. L. Lecamwasam, M. R. Hush, M. R. James, and A. R. Carvalho, Measurement-based generation of shaped single photons and coherent state superpositions in optical cavities, Phys. Rev. A **95**, 013828 (2017).
[4] H. F. Hofmann and H. Nishitani, Pulse-shape effects on photon-photon interactions in nonlinear optical quantum gates, Phys. Rev. A **80**, 013822 (2009).
[5] R. W. Andrews, A. P. Reed, K. Cicak, J. D. Teufel, and K. W. Lehnert, Quantum-enabled temporal and spectral mode conversion of microwave signals, Nat. Commun. **6**, 10021 (2015).
[6] Z. Zheng, O. Mishina, N. Treps, and C. Fabre, Atomic quantum memory for multimode frequency combs, Phys. Rev. A **91**, 031802 (2015).
[7] A. Tiranov, P. C. Strassmann, J. Lavoie, N. Brunner, M. Huber, V. B. Verma, S. W. Nam, R. P. Mirin, A. E. Lita, F. Marsili, M. Afzelius, F. Bussières, and N. Gisin, Temporal Multimode Storage of Entangled Photon Pairs, Phys. Rev. Lett. **117**, 240506 (2016).
[8] Y. P. Huang and P. Kumar, Mode-resolved photon counting via cascaded quantum frequency conversion, Opt. Lett. **38**, 468 (2013).
[9] D. V. Reddy, M. G. Raymer, and C. J. McKinstrie, Efficient sorting of quantum-optical wave packets by temporal-mode interferometry, Opt. Lett. **39**, 2924 (2014).
[10] D. V. Reddy and M. G. Raymer, Photonic temporal-mode multiplexing by quantum frequency conversion in a dichroic-finesse cavity, arXiv:1708.1705.
[11] V. Ansari, G. Harder, M. Allgaier, B. Brecht, and C. Silberhorn, Temporal-mode measurement tomography of a quantum pulse gate, Phys. Rev. A **96**, 063817 (2017).
[12] A. Hayat, X. Xing, A. Feizpour, and A. M. Steinberg, Multidimensional quantum information based on single-photon temporal wavepackets, Opt. Express **20**, 29174 (2012).
[13] O. Morin, C. Fabre, and J. Laurat, Experimentally Accessing the Optimal Temporal Mode of Traveling Quantum Light States, Phys. Rev. Lett. **111**, 213602 (2013).
[14] B. Brecht, D. V. Reddy, C. Silberhorn, and M. G. Raymer, Photon Temporal Modes: A Complete Framework for Quantum Information Science, Phys. Rev. X **5**, 041017 (2015).
[15] S. Gröblacher, T. Jennewein, A. Vaziri, G. Weihs, and A. Zeilinger, Experimental quantum cryptography with qutrits, New J. Phys. **8**, 75 (2006).
[16] J. T. Barreiro, T. C. Wei, and P. G. Kwiat, Beating the channel capacity limit for linear photonic superdense coding, Nat. Phys. **4**, 282 (2008).
[17] J. Leach, E. Bolduc, D. J. Gauthier, and R. W. Boyd, Secure information capacity of photons entangled in many dimensions, Phys. Rev. A **85**, 060304 (2012).
[18] N. C. Menicucci, X. Ma, and T. C. Ralph, Arbitrarily Large Continuous-Variable Cluster States from a Single Quantum Nondemolition Gate, Phys. Rev. Lett. **104**, 250503 (2010).
[19] N. C. Menicucci, Temporal-mode continuous-variable cluster states using linear optics, Phys. Rev. A **83**, 062314 (2011).
[20] B. Brecht, A. Eckstein, A. Christ, H. Suche, and C. Silberhorn, From quantum pulse gate to quantum pulse shaper—Engineered frequency conversion in nonlinear optical waveguides, New J. Phys. **13**, 065029 (2011).
[21] A. Eckstein, B. Brecht, and C. Silberhorn, A quantum pulse gate based on spectrally engineered sum frequency generation, Opt. Express **19**, 13770 (2011).
[22] D. V. Reddy, M. G. Raymer, and C. J. McKinstrie, Sorting photon wave packets using temporal-mode interferometry based on multiple-stage quantum frequency conversion, Phys. Rev. A **91**, 012323 (2015).
[23] J. B. Christensen, D. V. Reddy, C. J. McKinstrie, K. Rottwitt, and M. G. Raymer, Temporal mode sorting using dual-stage quantum frequency conversion by asymmetric Bragg scattering, Opt. Express **23**, 23287 (2015).
[24] V. G. Velev, C. Langrock, P. Kumar, M. M. Fejer, and Y. P. Huang, Selective manipulation of overlapping quantum modes, IEEE Photon. Soc. Summer Top. Meet. Ser. **1**, 138 (2014).
[25] P. Manurkar, N. Jain, M. Silver, Y. P. Huang, C. Langrock, M. M. Fejer, P. Kumar, and G. S. Kanter, Multidimensional mode-separable frequency conversion for high-speed quantum communication, Optica **3**, 1300 (2016).
[26] M. Silver, P. Manurkar, Y. P. Huang, C. Langrock, M. M. Fejer, P. Kumar, and G. S. Kanter, Spectrally multiplexed upconversion detection with C-band pump and signal wavelengths, IEEE Photon. Technol. Lett. **29**, 1097 (2017).
[27] J. I. Cirac, L. Duan, and P. Zoller, Quantum optical implementation of quantum information processing, arXiv:quant-ph/0405030.
[28] X. H. Bao, A. Reingruber, P. Dietrich, J. Rui, A. Dück, T. Strassel, L. Li, N. L. Liu, B. Zhao, and J. W. Pan, Efficient and long-lived quantum memory with cold atoms inside a ring cavity, Nat. Phys. **8**, 517 (2012).
[29] J. Nunn, J. H. Munns, S. Thomas, K. T. Kaczmarek, C. Qiu, A. Feizpour, E. Poem, B. Brecht, D. J. Saunders, P. M. Ledingham, D. V. Reddy, M. G. Raymer, and I. A. Walmsley, Theory of noise suppression in Λ-type quantum memories by means of a cavity, Phys. Rev. A **96**, 012338 (2017).
[30] Z. L. Xiang, M. Zhang, L. Jiang, and P. Rabl, Intracity Quantum Communication via Thermal Microwave Networks, Phys. Rev. X **7**, 011035 (2017).
[31] B. Vermersch, P. O. Guimond, H. Pichler, and P. Zoller, Quantum State Transfer via Noisy Photonic and Phononic Waveguides, Phys. Rev. Lett. **118**, 133601 (2017).
[32] J. Zhang, Y. X. Liu, R. B. Wu, K. Jacobs, and F. Nori, Non-Markovian quantum input-output networks, Phys. Rev. A **87**, 032117 (2013).
[33] C. M. Caves and D. D. Crouch, Quantum wideband traveling-wave analysis of a degenerate parametric amplifier, J. Opt. Soc. Am. B **4**, 1535 (1987).
[34] I. P. Christov, "Temporal modes" representation of propagating light pulses, Opt. Commun. **69**, 101 (1988).
[35] Y. Ben-Aryeh and S. Serulnik, The quantum treatment of propagation in non-linear optical media by the use of temporal modes, Phys. Lett. A **155**, 473 (1991).
[36] B. J. Chapman, E. I. Rosenthal, J. Kerckhoff, B. A. Moores, L. R. Vale, J. A. B. Mates, G. C. Hilton, K. Lalumière, A. Blais, and K. W. Lehnert, Widely Tunable On-Chip Microwave Circulator for Superconducting Quantum Circuits, Phys. Rev. X **7**, 041043 (2017).

[37] A. C. Mahoney, J. I. Colless, S. J. Pauka, J. M. Hornibrook, J. D. Watson, G. C. Gardner, M. J. Manfra, A. C. Doherty, and D. J. Reilly, On-Chip Microwave Quantum Hall Circulator, Phys. Rev. X **7**, 011007 (2017).

[38] E. Rephaeli, J. T. Shen, and S. Fan, Full inversion of a two-level atom with a single-photon pulse in one-dimensional geometries, Phys. Rev. A **82**, 033804 (2010).

[39] S. J. Srinivasan, N. M. Sundaresan, D. Sadri, Y. Liu, J. M. Gambetta, T. Yu, S. M. Girvin, and A. A. Houck, Time-reversal symmetrization of spontaneous emission for quantum state transfer, Phys. Rev. A **89**, 033857 (2014).

[40] R. C. Bialczak, M. Ansmann, M. Hofheinz, M. Lenander, E. Lucero, M. Neeley, A. D. O'Connell, D. Sank, H. Wang, M. Weides, J. Wenner, T. Yamamoto, A. N. Cleland, and J. M. Martinis, Fast Tunable Coupler for Superconducting Qubits, Phys. Rev. Lett. **106**, 060501 (2011).

[41] Y. Yin, Y. Chen, D. Sank, P. J. J. O'Malley, T. C. White, R. Barends, J. Kelly, E. Lucero, M. Mariantoni, A. Megrant, C. Neill, A. Vainsencher, J. Wenner, A. N. Korotkov, A. N. Cleland, and J. M. Martinis, Catch and Release of Microwave Photon States, Phys. Rev. Lett. **110**, 107001 (2013).

[42] J. Wenner, Y. Yin, Y. Chen, R. Barends, B. Chiaro, E. Jeffrey, J. Kelly, A. Megrant, J. Y. Mutus, C. Neill, P. J. J. O'Malley, P. Roushan, D. Sank, A. Vainsencher, T. C. White, A. N. Korotkov, A. N. Cleland, and J. M. Martinis, Catching Time-Reversed Microwave Coherent State Photons with 99.4% Absorption Efficiency, Phys. Rev. Lett. **112**, 210501 (2014).

[43] E. A. Sete, E. Mlinar, and A. N. Korotkov, Robust quantum state transfer using tunable couplers, Phys. Rev. B **91**, 144509 (2015).

[44] L. H. Pedersen, N. M. Møller, and K. Mølmer, Fidelity of quantum operations, Phys. Lett. A **367**, 47 (2007).

[45] P. Zanardi and D. A. Lidar, Purity and state fidelity of quantum channels, Phys. Rev. A **70**, 012315 (2004).

[46] J. Ghosh, A note on the measures of process fidelity for non-unitary quantum operations, arXiv:1111.2478.

[47] N. Yamamoto and M. R. James, Zero-dynamics principle for perfect quantum memory in linear networks, New J. Phys. **16**, 073032 (2014).

[48] S. Xu, H. Z. Shen, and X. X. Yi, Single-photon transistor based on tunable coupling in a cavity quantum electrodynamics system, J. Opt. Soc. Am. B **33**, 1600 (2016).

[49] M. V. Fedorov, Y. M. Mikhailova, and P. A. Volkov, Gaussian modeling and Schmidt modes of SPDS biphoton states, J. Phys. B **42**, 175503 (2009).

[50] D. V. Reddy, M. G. Raymer, C. J. McKinstrie, L. Mejling, and K. Rottwitt, Temporal mode selectivity by frequency conversion in second-order nonlinear optical waveguides, Opt. Express **21**, 013840 (2013).