Collective behavior of light in vacuum

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Under the action of light-by-light scattering, light beams show collective behaviors in vacuum. For instance, in the case of two counterpropagating laser beams with specific initial helicity, the polarization of each beam oscillates periodically between the left and right helicity. Furthermore, the amplitudes and the corresponding intensities of each polarization propagate like waves. Such polarization waves might be observationally accessible in future laser experiments, in a physical regime complementary to those explored by particle accelerators.

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I. INTRODUCTION

Light self-interaction is a purely quantum effect, since the classical Maxwell equations are linear, and this forbids processes such as light-by-light scattering ($\gamma\gamma \to \gamma\gamma$) that are allowed in quantum electrodynamics. Indirect evidence of such processes has been found in particle accelerators [1–8], while the search for signatures of light-by-light scattering in optics is still in progress [9–51]. However, this situation might be overcome in the near future. In fact, it has been shown [49] that, despite their weakness, quantum corrections due to light-by-light scattering can change dramatically the dynamics of the electromagnetic field, inducing effects that can be tested experimentally.

Quantum corrections to Maxwell equations have been calculated a long time ago by Heisenberg and Euler [52], and extensively studied by other authors [53–56]. The effective Lagrangian of the electromagnetic field, obtained retaining only the dominant one-electron-loop corrections,[1] is [56]

$$L = \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \epsilon^2\big[(F_{\mu\nu}F^{\mu\nu})^2 - \tfrac{7}{16}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\big], \qquad (1)$$

where $F^{\mu\nu} = A^{\mu,\nu} - A^{\nu,\mu}$ is the electromagnetic field,[2] A^μ is the electromagnetic four-potential, $\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$, $\epsilon^2 = \alpha^2(\hbar/m_e c)^3/90 m_e c^2$, $\alpha = e^2/4\pi\epsilon_0\hbar c \simeq 1/137$ is the fine structure constant, ϵ_0 the dielectric permeability of vacuum, m_e the electron mass, and c the speed of light.

In this paper we analyze the effect of light-by-light scattering on the dynamics of the electromagnetic field in vacuum. Hereafter, we consider low energetic photons with energies $\ll m_e c^2$, so that particle creation is inhibited, and light-by-light scattering is the only process involving photons. Under these hypothesis, the Lagrangian (1) is fit for our purpose.

The terms $\sim\epsilon^2$ in (1) account for light-by-light scattering, introducing cubic corrections in the equations for the four-potential A^μ. Since $\epsilon^2 \approx 4 \times 10^{-31}$ m^3/J, one has $\epsilon^2 F_{\mu\nu}F^{\mu\nu} \ll 1$ and $\epsilon^2 F_{\mu\nu}\tilde{F}^{\mu\nu} \ll 1$ in realistic physical conditions, so that nonlinear corrections to Maxwell equations are usually negligible.[3] However, this is not the case for some specific configurations of the electromagnetic field that become unstable due to the action of hidden resonances. In fact, in [49] it has been shown that such tiny nonlinearities affect heavily the polarization of the electromagnetic waves in vacuum; indeed, their polarization oscillates periodically in time between right and left helicity states.

Here we extend this result, which has been obtained for counterpropagating homogeneous (in space) plane waves, to more general configurations. Such extension is mathematically straightforward, but its physical implications are relevant. We show that the polarization oscillations occur both in space and time. It is found that the amplitudes of the different polarizations, and the corresponding intensities, propagate as plane waves. The occurrence of superluminal polarization waves is considered, and possible contradictions with special relativity, and their solution, are discussed. Finally, we discuss the possibility of observing polarization waves in laser experiments and argue how the recurrence time of the polarization oscillations can be reduced in order to favor their detection.

The importance of these results is in the fact that they show that light exhibit collective behaviors in vacuum, which are triggered by light-by-light scattering. Such collective modes are represented by polarization waves, whose properties have been completely characterized. Remarkably, this phenomenology has been obtained analytically through a simple multiscale approach, as described below.

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[1]We are neglecting other contributions to light-by-light scattering, such as those due to the μ and τ loops, which are suppressed by a factor $\sim (m_e/m_\mu)^4\,\epsilon^2 F_{\mu\nu}F^{\mu\nu}$ and $\sim (m_e/m_\tau)^4\,\epsilon^2 F_{\mu\nu}F^{\mu\nu}$, respectively.

[2]We use the covariant formalism, so that the zeroth coordinate is defined as $x^0 = c\,t$.

[3]We note that the next-to-leading terms in the expansion of the Heisenberg-Euler Lagrangian are suppressed by a factor $\sim(\epsilon^2 F_{\mu\nu}F^{\mu\nu})^2$; indeed, they are negligible to any extent in realistic laboratory conditions for low energetic ($E \ll m_e c^2$) photons. What is more, they are subdominant with respect to contributions due to other channels in light-by-light scattering, e.g., the μ and τ loops.

II. MULTISCALE EQUATIONS

Starting from the Lagrangian (1) it is easy to show that the modified Maxwell equations for the electromagnetic four-potential A^α in the Lorentz gauge ($\partial_\alpha A^\alpha = 0$) are

$$\Box A^\alpha(1 + 8\,\epsilon^2\, F_{\mu\nu}F^{\mu\nu}) + \epsilon^2\, B^\alpha = 0\,, \tag{2}$$

where $\Box$ is the d'Alembertian operator and

$$B^\alpha \equiv 8\,[F^{\alpha\beta}\partial_\beta(F_{\mu\nu}F^{\mu\nu}) - \tfrac{7}{16}\tilde{F}^{\alpha\beta}\partial_\beta(F_{\mu\nu}\tilde{F}^{\mu\nu})]\,. \tag{3}$$

At zeroth order in ϵ Eq. (2) reduces to the Maxwell equations in vacuum $\Box A^\alpha = 0$, which can be solved exactly. Let us consider a zeroth-order solution $A^{(0)\alpha}$ corresponding to a system of two plane electromagnetic waves propagating in the x^3 direction. With a proper gauge choice we set $A^{(0)0} = A^{(0)3} = 0$, so that we can write the four-potential in a more convenient vector form as

$$\vec{A}^{(0)} = \vec{a} + \vec{b} + \text{c.c.}\,, \tag{4}$$

(where c.c. stands for complex conjugate), i.e., as the superposition of the two plane waves a and b defined as

$$\vec{a} = (a_L\,\hat{e}_L + a_R\,\hat{e}_R)\,e^{ikx}, \quad \vec{b} = (b_L\,\hat{e}_L + b_R\,\hat{e}_R)e^{ihx}\,, \tag{5}$$

with the two wave vectors $k = (k_0,0,0,k_3)$ and $h = (h_0,0,0,h_3)$ satisfying the dispersion relation $|k_0/k_3| = |h_0/h_3| = 1$. Here $\hat{e}_L = (1,i,0)/\sqrt{2}$ and $\hat{e}_R = (1,-i,0)/\sqrt{2}$ are the left and right polarization vectors. Therefore, the coefficients a_L, a_R, b_L, and b_R are the complex amplitudes of the left and right polarizations of the plane waves a and b, and their squared modules are proportional to the corresponding intensities.

Let us study how the dynamics or the plane waves (4) are modified due to the effect of quantum corrections. We anticipate that such dynamics entails slow variations of the complex amplitudes in space and time. The smallness of ϵ^2 suggests that a complete solution of (2) might be obtained through a standard perturbative expansion of the four-vector in powers of ϵ^2. However, such a naive approach fails when the two waves a and b are counterpropagating, e.g., when $k_0/k_3 = -h_0/h_3 = 1$, due to the occurrence of secular divergences of small perturbations. In fact, in such a configuration, evaluating B^α over the solution (4) one has [49]

$$\begin{aligned}\vec{B} &= 32\,k_0^2\,h_0^2\{[(-3\,a_L(|b_L|^2 + |b_R|^2) + 22\,a_R b_L \bar{b}_R)\hat{e}_L \\ &\quad + (-3\,a_R(|b_L|^2 + |b_R|^2) + 22\,a_L b_R \bar{b}_L)\hat{e}_R]e^{ikx} \\ &\quad \times [(-3\,b_L(|a_L|^2 + |a_R|^2) + 22\,b_R a_L \bar{a}_R)\hat{e}_L \\ &\quad + (-3\,b_R(|a_L|^2 + |a_R|^2) + 22\,b_L a_R \bar{a}_L)\hat{e}_R]e^{ihx}\} \\ &\quad + \text{c.c.} + \text{nonresonant terms}.\end{aligned} \tag{6}$$

Therefore $\vec{B}$ contains resonant terms $\sim e^{ikx}$ and $\sim e^{ihx}$ that make any small perturbation of (4) diverge as $\delta A \approx \epsilon^2\, t\, c\, k^3 (A^{(0)})^3\, e^{ikx}$, where we have assumed for simplicity that $k_0 \approx h_0 \approx k$. See [49] for a discussion of the secular divergence of small perturbations of (4) in this configuration.

The emergence of secularities and the consequent failure of perturbative power expansions is quite common in physics. Usually, this happens in problems in which the solutions depend simultaneously on widely different scales. In such cases, the divergences can be handled through a multiscale expansion, introducing suitable slow variables; see [57] for an introduction to the multiscale perturbative method. As we will see, the multiscale approach provides an approximate solution of (2) that captures all the essential features of the problem under analysis.

We introduce the slow variable y^0 as

$$y^0 \equiv \epsilon^2(n_0\, x^0 + n_3\, x^3)\,, \tag{7}$$

where $n_0, n_3 \in \mathbb{R}$ are the covariant components of a four-vector $n = (n^0,0,0,n^3)$, so that the relativistic covariance of (7), as well as that of the multiscale approximate solutions, is preserved. We choose n dimensionless, so that y^0 is measured in m^4/J. The multiscale treatment requires that n_0 and n_3 are such that $|n_0| + |n_3| \approx 1$. Moreover, to have meaningful multiscale equations, it will be necessary to impose the condition $n_0 \neq \pm n_3$. Using (7) one has $\partial_{x^0} \to \partial_{x^0} + \epsilon^2\, n_0\, \partial_{y^0}$ and $\partial_{x^3} \to \partial_{x^3} + \epsilon^2\, n_3\, \partial_{y^0}$, which finally gives the d'Alembertian in terms of the derivatives with respect to slow and fast variables as

$$\Box \to \Box + 2\epsilon^2(n_0\,\partial_{x^0} - n_3\,\partial_{x^3})\partial_{y^0} + o(\epsilon^4)\,. \tag{8}$$

We split the dependence of the four-potential into slow and fast variables, assuming that the amplitudes a_L, a_R, b_L, and b_R depend only on the slow variable y^0. We search the solutions of the Eq. (2) in the form $\vec{A} = \vec{A}^{(0)} + \epsilon^2\delta\vec{A}$, so that at order $\sim\epsilon^2$ Eq. (2) gives

$$2(n_0\,\partial_{x^0} - n_3\,\partial_{x^3})\partial_{y^0}\vec{A}^{(0)} + \Box\delta\vec{A} + \vec{B} = 0. \tag{9}$$

The multiscale equations are obtained by imposing that the first term in (9) cancels the resonant terms in $\vec{B}$, while $\Box\delta\vec{A}$ equals the remaining nonresonant terms so that the small perturbation $\delta\vec{A}$ is stable. In that way, we obtain the dynamical equations for the complex amplitudes as

$$\begin{aligned} &i a_L' + 16\frac{k_0 h_0^2}{n_0 - n_3}[-3\,a_L(|b_L|^2 + |b_R|^2) + 22\,a_R b_L \bar{b}_R] = 0,\\ &i a_R' + 16\frac{k_0 h_0^2}{n_0 - n_3}[-3\,a_R(|b_L|^2 + |b_R|^2) + 22\,a_L b_R \bar{b}_L] = 0,\\ &i b_L' + 16\frac{k_0^2 h_0}{n_0 + n_3}[-3\,b_L(|a_L|^2 + |a_R|^2) + 22\,b_R a_L \bar{a}_R] = 0,\\ &i b_R' + 16\frac{k_0^2 h_0}{n_0 + n_3}[-3\,b_R(|a_L|^2 + |a_R|^2) + 22\,b_L a_R \bar{a}_L] = 0\,,\end{aligned} \tag{10}$$

where $f' \equiv df/dy^0$. It is now evident why the condition $n_0 \neq \pm n_3$ is necessary in order to have meaningful multiscale equations.

III. RESULTS

Let us study (10) in detail. First of all, it is quite immediate to recognize that the energy densities $\langle\rho_a\rangle = k_0^2(|a_L|^2 + |a_R|^2)$ and $\langle\rho_b\rangle = h_0^2(|b_L|^2 + |b_R|^2)$ of the two beams a and b are constant; therefore, the intensities of the two plane waves a and b are conserved separately. Furthermore, the quantity $S = k_0(n_0 - n_3)(|a_L|^2 - |a_R|^2) + h_0(n_0 - n_3)(|b_L|^2 - |b_R|^2)$, that in the case $n_0 = 1$ and $n_3 = 0$ corresponds to the spin density, is also conserved.[4] Exploiting these relations, the system (10) can be simplified and then resolved exactly [49], showing that the evolution of the modules of the complex amplitudes is periodic.

We can now estimate the period Δy^0 of the polarization oscillations. This is roughly given by the scale at which the amplitudes change significantly. Assuming that $a_L \approx a_R \approx a_0$ and $b_L \approx b_R \approx b_0$, such a scale is given by the conditions $\Delta a_{L,R}/a_{L,R} \approx 1$ and $\Delta b_{L,R}/b_{L,R} \approx 1$, which, using (10), gives the two scales $\Delta y_a^0 \approx \frac{|n_0+n_3|}{k_0 h_0^2 |b_0|^2}$ and $\Delta y_a^0 \approx \frac{|n_0-n_3|}{k_0^2 h_0 |a_0|^2}$. Since the solutions of (10) are periodic in y^0, Δy^0 will be the minimum between Δy_a^0 and Δy_b^0, i.e.,

$$\Delta y^0 \approx inf\left\{\frac{|n_0+n_3|}{k_0 h_0^2 |b_0|^2}, \frac{|n_0-n_3|}{k_0^2 h_0 |a_0|^2}\right\}. \tag{11}$$

At this point we can characterize the dynamics of the system. Some of the solutions of (10) are easily found. In fact, if $a_L\, a_R = 0$ and $b_L\, b_R = 0$ at $y^0 = 0$ so that the waves a and b have circular polarization, such products remain always zero and the solutions of (10) are complex exponentials $\sim e^{i\omega y^0}$. This is true also in the case of two linearly polarized waves with $a_L = \pm a_R$ and $b_L = \pm b_R$. However, such solutions are not interesting, since the modules of the complex amplitudes remains constant, and the effect of light-by-light scattering is just a negligibly small $\sim\epsilon^2$ correction to the dispersion relation of the plane waves a and b. (See [49] for possible implications for quantum gravity phenomenology [58].)

Other solutions show a behavior much more interesting, since the polarizations of the light beams change dramatically during the evolution of the system. These solutions correspond to initial conditions such that at least one of the products $a_L\, a_R$ or $b_L\, b_R$ is different from zero. This is due to the fact that when nonzero, the last terms in Eqs. (10) are responsible for the oscillatory behavior that we describe below. The system (10) can be solved analytically [49]; however, for our purposes it will be sufficient to discuss numerical solutions. Solving (10) numerically, it is possible to see that the polarizations of the two counterpropagating waves oscillate periodically between left and right configurations.

This effect is particularly evident when only one of the waves a and b is initially polarized circularly, e.g., $a_L\, a_R = 0$ and $b_L\, b_R \neq 0$. For instance, we solve (10) for $k_0 = h_0 = 0.1$ and initial values $a_R^0 = 0$, $a_L^0 = 1$, $b_L^0 = 1$, $b_R^0 = i$, $n_0 = 10^{-3}$, and $n_3 = 1$. From Fig. 1 we see that $|a_R|$ is initially zero but it grows to $|a_R| = |a_L^0|$, while $|a_L|$ goes to zero. Thus, the beam a is initially in the left-handed polarization, but then it switches to the right-handed polarization. It remains in this state until it jumps back to its initial left-handed configuration after the first period. The behavior of the beam b is similar. In fact, $|b_L|$ goes to zero, while $|b_R|$ goes to $\sqrt{|b_L^0|^2 + |b_R^0|^2}$,

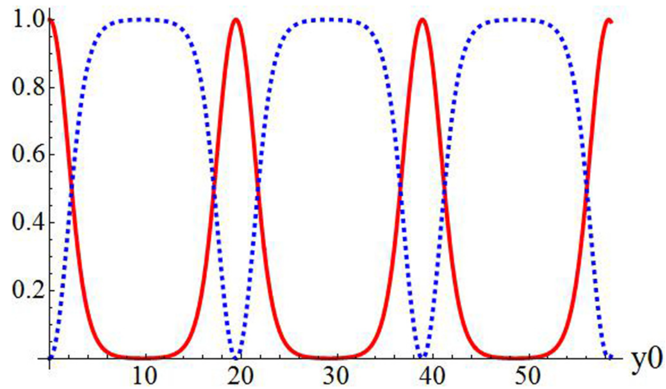

FIG. 1. We plot the evolution of $|a_L|^2/|a_L^0|^2 + |a_R^0|^2$ (solid red line) and $|a_R|^2/|a_L^0|^2 + |a_R^0|^2$ (dashed blue line) against y^0 (in units of m^4/J). The plot shows the oscillatory behavior of the polarization of the light beam a.

and after the first period $|b_L|$ and $|b_R|$ go back to their initial values. The difference with respect the beam a is that $|b_L|$ never reaches the zero. From Figs. 1 and 2 it is also evident that the evolution of the modules of the complex amplitudes is periodic.

Numerical investigation of (10) shows that the oscillatory behavior of the system is not affected (qualitatively) by the choice of the parameters in (10), while the period of the oscillations depends on such parameters (in order magnitude) as in (11). For instance, for the solution plotted in Figs. 1 and 2, Eq. (11) gives a period $\Delta y^0 \approx 3.9$, which is a good estimation (as an order of magnitude) of the actual period $\simeq 20$, as seen in the plots.

At that point it becomes necessary to discuss the physical meaning of the two parameters n_0 and n_3. Such parameters are not fixed by the multiscale, except for the conditions $|n_0| + |n_3| \approx 1$ and $n_0 \neq \pm n_3$. The freedom in their choice reflects the fact that our multiscale solution is not the general solution of (2), but it still contains some residual freedom in the choice of the initial values of the derivatives of the complex amplitudes of a and b.

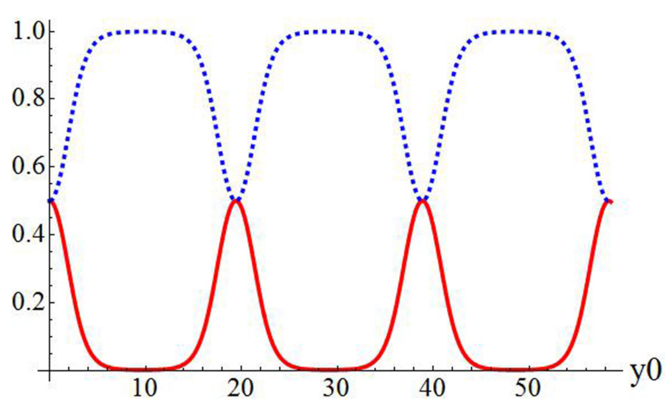

FIG. 2. We plot the evolution of $|b_L|^2/(|b_L^0|^2 + |b_R^0|^2)$ (solid red line) and $|b_R|^2/(|b_L^0|^2 + |b_R^0|^2)$ (dashed blue line) against y^0 (in units of m^4/J). The plot shows the oscillatory behavior of the polarization of the light beam b.

[4]It is worth mentioning that $\langle\rho_a\rangle$, $\langle\rho_b\rangle$, and S are the zeroth-order approximations of the energy and spin densities in the nonlinear classical theory [44], and they coincide with the corresponding quantities in perturbative quantum field theory. This is reasonable, since Eqs. (10) have been obtained in perturbation theory, and the quantum corrections have been calculated in perturbative quantum field theory.

For instance, imposing $n_0 = 1$ and $n_3 = 0$, we have $y^0 = \epsilon^2 x^0$, so that the amplitudes are homogeneous in space and periodic in the slow variable $y^0 = \epsilon^2 c\,t$, indeed, periodic in time. This class of solutions has been discussed extensively in [49]. On the contrary, the choice $n_0 = 0$ and $n_3 = 1$ corresponds to static solutions that are periodic in space, since in this case $y^0 = \epsilon^2 x^3$.

In general, the complex amplitudes are periodic functions of the slow variable $y^0 = \epsilon^2(n_0 x^0 + n_3 x^3)$; therefore they propagate as plane waves with speed $v_p = |n_0/n_3|$. A first remark is that the superposition principle is not valid for these waves, since the system (10) is nonlinear. Furthermore, it must be emphasized that that such "polarization waves" cannot travel at the speed of light, since it must be $n_0 \neq \pm n_3$, while they can—at least in principle—travel faster than light when $|n_0/n_3| > 1$.

It is not evident that the existence of superluminal polarization waves is in contradiction with special relativity. For instance, there is no manner to control the polarization wave form and therefore encode information that can travel faster than light. Instead, light beams self-organize in such a way that their polarizations evolve as waves which might propagate faster than light. What is more, since the partial intensities of the two beams are conserved separately, polarization waves do not carry energy. Moreover, it is know that the group velocity of a light beam, i.e., the velocity of its envelope, exceeds the speed of light in some circumstances [59]; but also in this case there is no contradiction with special relativity, since there is no propagation of signals or energy with a velocity above c.

However, it might result that the existence of superluminal polarization waves contradicts special relativity. In such eventuality, these superluminal polarization waves must be considered unphysical, and we must impose the condition $|n_0/n_3| < 0$; the meaning of these conditions would be that we should avoid unphysical initial conditions.

We mention that the search for the effects of light self-interactions in optics is already under study [9–51]. A review of solutions in nonlinear QED is given in [37]. Moreover, the self-interactions of magnetic and electric moments have been studied in [38], linear and nonlinear responses of constant background to electric charge have been studied in [39–41], the linear response in the form of a magnetic monopole has been studied in [42,43], and the finiteness of the self-energy of the pointlike charge has been analyzed in [44–47].

It is also worth mentioning that the interaction of two counterpropagating plane waves under the action of light-by-light scattering was already studied in [48]. In that paper the authors used the standard perturbation theory to study the evolution of initially small perturbations ΔE over a background electromagnetic field $E^{(0)}$. They found that at some finite time the perturbations ΔE become dominant over the background, i.e., they incidentally find the divergence of perturbations due to secularities discussed in the Introduction and outlined in [49]. However, when the (initially small) perturbations overcome the background, the perturbative solution is no longer valid. Indeed, such solution does not uncover the oscillatory behavior of polarizations, since this effect appears only on long time scales, when standard perturbation theory is unapplicable and one must recur to multiscale perturbation expansion.

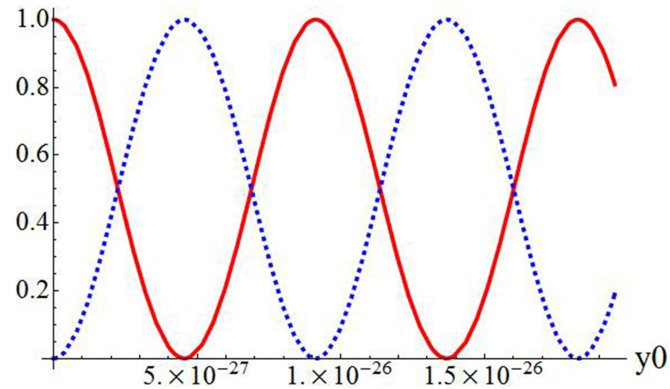

FIG. 3. We plot the evolution of $|a_L|^2/|a_L^0|^2 + |a_R^0|^2$ (solid red line) and $|a_R|^2/|a_L^0|^2 + |a_R^0|^2$ (dashed blue line) against y^0 (in units of m^4/J) for $n_0 = 1, n_3 = 2$, and $|a_L^0|^2 = 10^3$ J/m, $a_R^0 = 0$, $|b_L^0|^2 = |b_R^0|^2 = 10^3$ J/m, $k_0 = h_0 = 10^7$ m^{-1}.

The novelty of the results reported here and in [49] is that, by means of multiscale analysis, we have obtained precise analytical results enlightening the most important features of the collective behavior of light in vacuum induced by light-by-light scattering. Moreover, we have understood that we have to look at the polarization rather than at light intensity, and we know that the case of two counterpropagating laser beams is the best configuration to observe polarization oscillations.

Let us analyze the observational aspects of the polarization waves. To have an idea of the observation time required to reveal the polarization waves, we estimate their recurrence time T for light beams produced in petawatt-class lasers. The intensities attainable in these lasers reaches $I \approx 10^{23}$ W/cm^2 [60,61]. Thus, according to (11) the recurrence time $T \approx \Delta y^0/c$ will be

$$\begin{aligned} T &\approx inf\{|n_0+n_3|,|n_0-n_3|\}(\epsilon^2 k_0 I)^{-1} \\ &\approx 4\times10^2 \times inf\{|n_0+n_3|,|n_0-n_3|\}(\lambda/\mathrm{m})\,\mathrm{s}, \end{aligned} \quad (12)$$

where λ/m is the laser wavelength in meters (we used $k_0 \approx h_0 \approx 2\pi/\lambda$ and $k_0^2 a^2 \approx k_0^2 b^2 \approx \langle\rho\rangle \sim I/c$). Therefore, for $|n_0+n_3| \approx |n_0-n_3| \approx 1$ and $\lambda \approx 1\,\mu$m [60,61], observation time is of the order of 4×10^{-4} s.

This estimation is confirmed numerically. For instance, in Fig. 3 we plot $|a_L|^2/|a_L^0|^2 + |a_R^0|^2$ and $|a_R|^2/|a_L^0|^2 + |a_R^0|^2$ for the solution of (10) with $n_0 = 1, n_3 = 2$, and $|a_L^0|^2 = 10^3$ J/m, $a_R^0 = 0$, $|b_L^0|^2 = |b_R^0|^2 = 10^3$ J/m, $k_0 = h_0 = 10^7$ m^{-1}, corresponding to $I \simeq 10^{23}$ W/cm^2 and $\lambda \approx 1\,\mu$m. The value of $\Delta y^0 \simeq 10^{-26}$ m^4/J that can be read from the plot corresponds to a period $T = \Delta y^0/\epsilon^2 c \simeq 10^{-4}$ s, which is in good agreement with (12).

We stress that the recurrence time can be lowered further, choosing n_0 and n_3 in a proper way. In fact, one can make T smaller while preserving the validity of the multiscale treatment, e.g., taking $n_0 + n_3 = \eta$ and $n_0 - n_3 = 1$. From (12) it is then evident that we can reduce the recurrence time T choosing $\eta \ll 1$. This fact is confirmed numerically by solving (10) for different values of η. For instance, in Fig. 4 we plot $|a_L|/a_L^0$ as a function of y^0 for $\eta = 0.3, 0.15, 0.05$ and $k_0 = h_0 = 0.1$, $a_R^0 = 0$, $a_L^0 = 1$, $b_L^0 = 1$, $b_R^0 = i$, showing

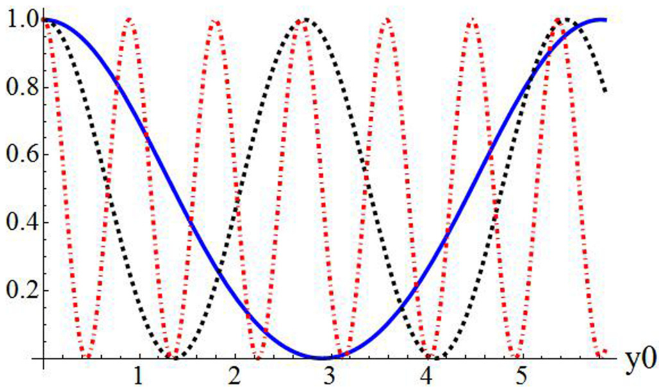

FIG. 4. We plot the evolution of $|a_L|^2/|a_L^0|^2 + |a_R^0|^2$ against y^0 (in units of m^4/J) for $\eta = 0.3$ (solid blue line), $\eta = 0.1$ (dashed black line), $\eta = 0.05$ (dashed-dotted red line), and $k_0 = h_0 = 0.1$, $a_R^0 = 0$, $a_L^0 = 1$, $b_L^0 = 1$, $b_R^0 = i$. The plot shows that the period of the oscillations decreases for decreasing η.

that the period of the oscillations decreases for decreasing η. Indeed, T is considerably reduced for $\eta \ll 1$, corresponding to polarization waves traveling nearly at the speed of light. However, the practical issue of preparing the system in the proper initial conditions corresponding to a specific choice of n_0 and n_3 remains.

Finally, we mention that polarization oscillations cannot be detected in the cosmic microwave background (CMB) radiation [62], since its energy density $\sim 10^{-14}$ J/m^3 gives extremely small corrections to the linear dynamics, see [49]. Moreover, polarization waves can be of interest in astrophysics, for instance, they can play a role in the behavior of magnetized neutron stars [63–69] and in astrophysical electromagnetic shocks [70,71]; however we will discuss these issues elsewhere.

IV. CONCLUSIONS

It has been shown that the extremely weak light-by-light interaction can induce unexpectedly strong deviations from the free dynamics of light. In particular, it is responsible for the generation of polarization waves that, in principle, can propagate faster than light. The phenomenology described above is quite surprising for different reasons. First, it is a notable example of how an extremely thin correction, such as that arising from light-by-light scattering, can produce a strong deviation from the free dynamics of a system. Furthermore, it is remarkable that photons have such an ordered behavior, showing a collective response to the quantum-induced nonlinear effects considered here rather than behaving in a chaotic way. Last but not least, polarization waves might be observationally accessible in laser experiments, and light-by-light scattering tested in a physical regime complementary to that explored by particle accelerators.

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