Dynamical evolutions in non-Hermitian triple-well systems with a complex potential

Liping Guo,[1,*] Lei Du,[1] Chuanhao Yin,[2] Yunbo Zhang,[1] and Shu Chen[2]
[1]*Institute of Theoretical Physics, Shanxi University, Taiyuan 030006, People's Republic of China*
[2]*Institute of Physics, Chinese Academy of Sciences, Beijing 100080, People's Republic of China*

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We investigate the dynamical properties for non-Hermitian triple-well systems with a loss in the middle well. When chemical potentials in the two end wells are uniform and nonlinear interactions are neglected, there always exists a dark state whose eigenenergy becomes zero and the projections onto which do not change over time and the loss factor. The increasing of the loss factor only makes the damping form from the oscillating decay to the overdamping decay. However, when the nonlinear interaction is introduced, even interactions in the two end wells are also uniform, and the projection of the dark state will be obviously diminished. Simultaneously, the increasing of the loss factor will also aggravate the loss. In this process, the interaction in the middle well plays no role. When two chemical potentials or interactions in the two end wells are not uniform, all disappear with time. In addition, when we extend the triple-well system to a general $(2n + 1)$ well, the loss is greatly reduced by the factor $1/2n$ in the absence of the nonlinear interaction.

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I. INTRODUCTION

In quantum mechanical pictures, the Hamiltonian must be Hermitian to describe a physical system, which is sufficient to ensure that the system has real energy eigenvalues and the conservation of the number of particles. But this condition is too rigorous in real systems. In optics, a non-Hermitian Hamiltonian is used to describe the propagation of light in the medium with a complex refraction index [1–6]. Recently, the controlled removal of atoms from a Bose-Einstein condensate (BEC) was realized by a narrow electron beam or a narrow laser beam [7,8], which promotes simulations of the atomic system with dissipation. The systems with the dissipation process are described via the non-Hermitian Hamiltonians with negative imaginary chemical potential [9–17] and can be solved in terms of the master equations [18–24]. More importantly, in most cases, dissipation is considered as an undesirable destructing factor, and thus researchers make arduous efforts to avoid it if at all possible, by inverting the dissipation by means of an intrinsic mechanism to balance the losses [8,25,26], probing a quantum system with controlled dissipation [27], designing the effective dissipative process in an optical superlattice using the coupling between the system and the reservoir [18], etc. Massive efforts have been invested in the study of the dynamics of non-Hermitian systems in experiment and theory [18,28–31].

The study of few-well systems reveals a variety of interesting quantum phenomena. For example, condensates in double or triple wells have popularly been investigated both theoretically and experimentally [32–45]. In past years, the nonlinear Josephson oscillation and self-trapping phenomena have been two of many important findings for double wells. However, more attention has been focused on three-well systems [42–45], which has a more abundant physical picture by adjusting the tunneling and interaction parameters, as well as chemical potentials. For example, under periodic driving of this model, coherent destruction of tunneling and a dark Floquet state have been predicted in theory [46]. Thus, dark states can also be controlled and realized in the three-state (three-well) system. Even chaotic phenomena and the bifurcation mechanism causing self-trapping have been studied in the dynamics of three coupled condensate systems [47,48]. In addition, in the light propagation in waveguides, the Kerr nonlinear interactions induce a variety of interesting quantum phenomena [49]. Thus when dissipation and the nonlinear interaction together play a role in a triple-well system, novel features may be expected in the dynamical evolution of the system.

In the present paper, we mainly study the quantum dynamics of a non-Hermitian triple-well system. We focus on the time evolutions of modulus squared of coefficients in three local states without nonlinear interactions. The analytic solutions of the Schrödinger equations directly give the time-dependent information for uniform chemical potentials in two end wells. The finding is that the eigenstate is a dark state, whose eigenenergy is zero, the projection on which is not dependent of time and the loss factor. But when chemical potentials in two ends are not uniform, the dark state would no longer be the eigenstate. Moreover, when nonlinear terms in three wells are considered, the modulus squared in three wells will be quickly diminished. These results are still suitable for systems of odd wells with similar structure.

II. MODEL AND ANALYTIC SOLUTIONS IN LINEAR CASE

We consider a coupled triple-well system with an imaginary chemical potential in the middle well. In general, the wave function $|\psi(t)\rangle$ of the system is a superposition of states at

*guolp@sxu.edu.cn

three local sites, i.e.,

$$|\psi(t)\rangle = c_1(t)|1\rangle + c_2(t)|2\rangle + c_3(t)|3\rangle, \tag{1}$$

where $c_i(t)$ are the amplitudes for three states $|i\rangle$ ($i = 1,2,3$). In this local site space, where the spatial dependence of the states will not be considered, the dynamic equation of the system [15–17] reads ($\hbar = 1$)

$$i\frac{\partial}{\partial t}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = H\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \tag{2}$$

with the Hamiltonian

$$H = \begin{pmatrix} \mu_1 + g_1|c_1|^2 & -J & 0 \\ -J & \mu_2 + g_2|c_2|^2 & -J \\ 0 & -J & \mu_3 + g_3|c_3|^2 \end{pmatrix}. \tag{3}$$

The chemical potentials μ_1 and μ_3 are real and $\mu_2 = \eta - i\alpha$ is a complex number, which denotes an effective loss ($\alpha > 0$) or a gain ($\alpha < 0$) at the state $|2\rangle$ [13,14]. g_i is the strength of the Kerr nonlinearity in state $|i\rangle$ and J is the coupling strength [50]. We set $J = 1$ so that all energies are in units of J.

We first focus on the simplest case that the chemical potentials are symmetrically distributed ($\mu_1 = \mu_3$), and the interactions are neglected, $g_i = 0$. The Schrödinger equation (2) can be solved by a substitution, $c_i(t) = c_i^0 \exp(-i\lambda_i t)$, and one has the eigenvalues λ_i for the Hamiltonian (3),

$$\begin{aligned} \lambda_1 &= \mu_1, \\ \lambda_{2,3} &= \tfrac{1}{2}(\mu_1 + \mu_2 \pm \Theta), \end{aligned} \tag{4}$$

and the corresponding ket space is spanned by three eigenvectors,

$$|\psi_1\rangle = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \ |\psi_2\rangle = \begin{pmatrix} 1 \\ \lambda_3 - \mu_2 \\ 1 \end{pmatrix}, \ |\psi_3\rangle = \begin{pmatrix} 1 \\ \lambda_2 - \mu_2 \\ 1 \end{pmatrix}, \tag{5}$$

where $\Theta = \sqrt{(\mu_1 - \mu_2)^2 + 8}$. We do not bother to normalize them because the normalization factor will not affect the final result. The dual, bra space with eigenvector, e.g., $\langle\psi_2| = (1,\lambda_3^* - \mu_2^*,1)$, is not orthogonal to the ket space. It is thus necessary to define the Hilbert space of $H^\dagger$, $|\tilde{\psi}_i\rangle = |\psi_i^*\rangle$, with the bra vectors being $\langle\tilde{\psi}_i| = \langle\psi_i^*|$. Here the symbol $*$ means the complex conjugate for all complex numbers. These eigenvectors together form a biorthogonal basis, i.e., the completeness relation reads [51]

$$\sum_k \frac{|\psi_k\rangle\langle\tilde{\psi}_k|}{\langle\tilde{\psi}_k|\psi_k\rangle} = 1, \tag{6}$$

and the orthogonality means

$$\frac{\langle\psi_k|\tilde{\psi}_{k'}\rangle}{\langle\psi_k|\tilde{\psi}_k\rangle} = \delta_{kk'}. \tag{7}$$

Note that the eigenvector ψ_1 is a dark state, which is the superposition of two local states in the left and right wells. The completeness dictates that an arbitrary normalized initial state $|\psi(0)\rangle = (c_1^0,c_2^0,c_3^0)^T$ can be expressed in the eigenvector space (5),

$$|\psi(0)\rangle = A_1|\psi_1\rangle + A_2|\psi_2\rangle + A_3|\psi_3\rangle, \tag{8}$$

where the coefficients A_i are suitable combinations of c_i^0, for example, $A_1 = (c_3^0 - c_1^0)/\sqrt{2}$ for a normalized $|\psi_1\rangle$. At time t, the wave function evolves according to

$$|\psi(t)\rangle = A_1 e^{-i\lambda_1 t}|\psi_1\rangle + A_2 e^{-i\lambda_2 t}|\psi_2\rangle + A_3 e^{-i\lambda_3 t}|\psi_3\rangle. \tag{9}$$

The matrix for the time-evolution operator C in the local site space defined as [8]

$$|\psi(t)\rangle = \begin{pmatrix} C_{11}(t) & C_{12}(t) & C_{13}(t) \\ C_{21}(t) & C_{22}(t) & C_{23}(t) \\ C_{31}(t) & C_{32}(t) & C_{33}(t) \end{pmatrix}|\psi(0)\rangle \tag{10}$$

can be calculated as

$$C = S^\dagger \mathrm{Diag}\left(e^{-i\lambda_1 t}, e^{-i\lambda_2 t}, e^{-i\lambda_3 t}\right)\tilde{S}, \tag{11}$$

where S is transformation matrix between the site space and the eigenvector space with the adjoint matrix $\tilde{S}$. These operators are necessarily not unitary. Actually, formed by merging the eigenvectors $|\psi_i\rangle$ or $|\tilde{\psi}_i\rangle$ into a square matrix row by row, the operators $S^\dagger$ and $\tilde{S}^\dagger$ satisfy $S^\dagger\tilde{S} = \tilde{S}S^\dagger = 1$ and $\tilde{S}^\dagger S = S\tilde{S}^\dagger = 1$. In this way, the time-dependent matrix elements $C_{ij}(t)$ in Eq. (10) can be determined as

$$\begin{aligned} C_{11}(t) &= C_{33}(t) = \frac{1}{2}e^{-i\lambda_1 t} + \frac{1}{2}f_+(t), \\ C_{31}(t) &= C_{13}(t) = -\frac{1}{2}e^{-i\lambda_1 t} + \frac{1}{2}f_+(t), \\ C_{22}(t) &= f_-(t), \\ C_{12}(t) &= C_{21}(t) = C_{23}(t) = C_{32}(t) \\ &= \frac{1}{\Theta}\left(e^{-i\lambda_3 t} - e^{-i\lambda_2 t}\right), \end{aligned} \tag{12}$$

where

$$\begin{aligned} f_+(t) &= \frac{\lambda_2 - \mu_2}{\Theta}e^{-i\lambda_2 t} - \frac{\lambda_3 - \mu_2}{\Theta}e^{-i\lambda_3 t}, \\ f_-(t) &= \frac{\lambda_2 - \mu_2}{\Theta}e^{-i\lambda_3 t} - \frac{\lambda_3 - \mu_2}{\Theta}e^{-i\lambda_2 t}. \end{aligned} \tag{13}$$

The symmetry in the coefficients C_{ij} directly reflects that of the Hamiltonian in the local site space. For an arbitrary initial state $|\psi(0)\rangle$, we can infer the state evolution $|\psi(t)\rangle$ by the coefficients $C_{ij}(t)$, i.e., the amplitude on the ith local state reads $c_i(t) = C_{i1}(t)c_1^0 + C_{i2}(t)c_2^0 + C_{i3}(t)c_3^0$. Correspondingly, the modulus squared of the coefficient in each local state is

$$P_i(t) = |c_i(t)|^2, \tag{14}$$

and the sum is

$$P_{all}(t) = \sum_i P_i(t). \tag{15}$$

For simplicity, now we assume that the chemical potentials in the left and right states vanish and that in local state $|2\rangle$ are purely imaginary, i.e., $\mu_1 = \mu_3 = 0$ and $\mu_2 = -i\alpha$. We focus on the case of dissipation in the rest of the paper, i.e., $\alpha > 0$. It is easy to see that while Θ in the eigenvalues $\lambda_{2,3}$ is positive and real for $\alpha^2 < 8$, it becomes purely imaginary for $\alpha^2 > 8$. This will greatly change the time dependence of the functions $f_\pm$. When $\alpha^2 < 8$, we see $\Theta = |\Theta|$ and the functions $f_\pm(t)$

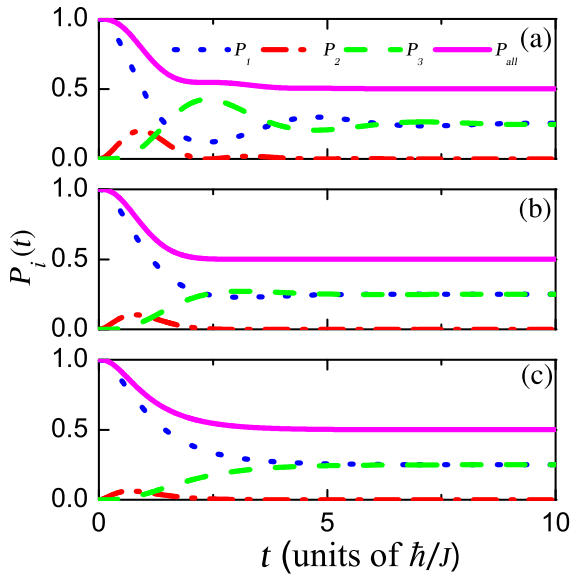

FIG. 1. Time evolution of the modulus squared of the coefficient in each well $P_i(t)$ and the sum P_{all} (pink solid line) with the loss factor (a) $\alpha = 1$, (b) $\alpha = 2$, and (c) $\alpha = 3$, starting from the initial state $c_1^0 = 1$, $c_2^0 = c_3^0 = 0$. Here, $\mu_1 = \mu_3 = \eta = 0$.

reduce to

$$f_{\pm}(t) = \pm e^{-\frac{1}{2}\alpha t}\frac{\sin\left(\frac{1}{2}|\Theta|t \pm \beta\right)}{\sin\beta}, \tag{16}$$

where $\beta = \arcsin\left(|\Theta|/\sqrt{8}\right)$. Due to the sinusoidal functions in Eq. (16), it describes an oscillating decay process starting from $f_{\pm}(0) = 1$. We find that the critical damping occurs at $\alpha^2 = 8$. In the other case when $\alpha^2 > 8$, $\Theta = i|\Theta|$, the system enters the overdamping region,

$$f_{\pm}(t) = \pm e^{-\frac{1}{2}\alpha t}\frac{\sinh\left(\frac{1}{2}|\Theta|t \pm \beta'\right)}{\sinh\beta'}, \tag{17}$$

where $\beta' = \text{arcsinh}(|\Theta|/\sqrt{8})$. Since $|\alpha| > |\Theta|$, the decaying term $e^{-\alpha t/2}$ of $f_+(t)$ in Eq. (17) will be compensated by the monotonically increasing hyperbolic sine function, which leads to overdamping, i.e., a relatively slow decay compared with Eq. (16) in the chemical parameter region $\alpha^2 < 8$. As an example, starting from the initial state $c_1^0 = 1$ and $c_2^0 = c_3^0 = 0$, the distributions in three states are, respectively,

$$\begin{aligned} P_1(t) &= |C_{11}(t)|^2 = \tfrac{1}{4}|f_+(t) + 1|^2, \\ P_3(t) &= |C_{31}(t)|^2 = \tfrac{1}{4}|f_+(t) - 1|^2, \end{aligned} \tag{18}$$

and

$$\begin{aligned} P_2(t) &= |C_{21}(t)|^2 \\ &= \frac{2}{|\Theta|^2}e^{-\alpha t}\begin{cases} 1 - \cos\left(|\Theta|t\right), & \alpha^2 < 8 \\ \cosh\left(|\Theta|t\right) - 1, & \alpha^2 > 8. \end{cases} \end{aligned} \tag{19}$$

The time evolution of the modes is shown in Fig. 1 for three different values of α. For $\alpha = 1$, we find $P_1(t)$ and $P_3(t)$ undergo explicit oscillations around the same equilibrium value of 0.25. These modulus squared values return rapidly to their equilibrium value for $\alpha = 2$, with an obvious damping oscillation. For $\alpha = 3$, however, the decrease of $P_1(t)$ and the increase of $P_3(t)$ are much slower, which shows the typical behavior of overdamping. In both cases, $P_2(t)$ quickly oscillates to a vanishingly small value, which means the leakage of the mode from the middle well. In addition, we find that the equilibrium values for P_i in the limit $t \to +\infty$ are independent of α . In this limit, $e^{-i\lambda_{2,3}t} \to 0$ and $f_{\pm}(t) \to 0$; for the arbitrary initial state, the wave function $|\psi(t)\rangle$ reduces to

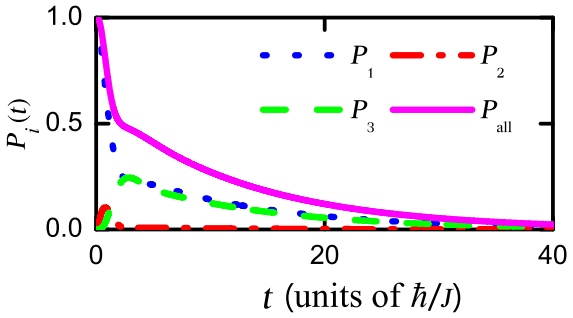

FIG. 2. Time evolution of the modulus squared of the coefficient in each well $P_i(t)$ and the sum P_{all} (pink solid line) for nonequal distribution of the chemical potentials starting from the initial state $c_1^0 = 1$, $c_2^0 = c_3^0 = 0$. Here, $\mu_1 = 0.1$, $\mu_3 = 0.5$, and $\alpha = 2$.

$$|\psi(t \to +\infty)\rangle = \frac{c_3^0 - c_1^0}{2}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \tag{20}$$

and the associated norms are

$$\begin{aligned} P_{1,3}(t \to +\infty) &= \frac{\left(c_1^0 - c_3^0\right)^2}{4}, \\ P_2(t \to +\infty) &= 0. \end{aligned} \tag{21}$$

This indicates that the steady state is the dark state $|\psi_1\rangle$, the projection on which would be stored forever. For the initial state $c_1^0 = 1$ and $c_2^0 = c_3^0 = 0$, it is easy to show that the total norm $P_{all}(t \to +\infty) = 0.5$, which does not vary with α, as depicted in Fig. 1. These results show the apparent suppression of dissipation because the projection on the dark state does not change with time. Similar results are found for other initial states, even for the dark state ψ_1 itself [10,14,52–54].

The linear non-Hermitian system with nonzero chemical potentials can be readily solved by $|\psi(t)\rangle = \exp\left(-iHt\right)|\psi(0)\rangle$ and the three modulus squared parameters P_i are given by (14). For nonequal chemical potentials in the left and right wells $\mu_1 \neq \mu_3$, the dark state $|\psi_1\rangle$ is no longer the eigenstate of the system [42,55]. An immediate result is that P_i in all three states will be lost in the limit $t \to +\infty$. We show this full leakage in Fig. 2 for $\alpha = 2$, $\mu_1 = 0.1$, and $\mu_3 = 0.5$. Clearly, the probability $P_1 = 1$ in the initial state $(1,0,0)^T$ decays from 1 to 0, at the same time $P_{2,3}$ reduce to zero after a temporary increase.

Under the balanced condition $\mu_1 = \mu_3$, it is interesting to study the influence of the real part η of μ_2 on the evolution of each state. As an example, we set $\mu_1 = \mu_3 = 0.5$ and $\alpha = 2$ and increase the real part η from 0 to 6 as shown in Fig. 3. The modulus squared $P_{1,3}$ are found to oscillate in longer and longer time, which effectively slows down the process for the sum of P_i to reach the equilibrium. However, it has no effect on the distribution of the steady state in the limit $t \to +\infty$.

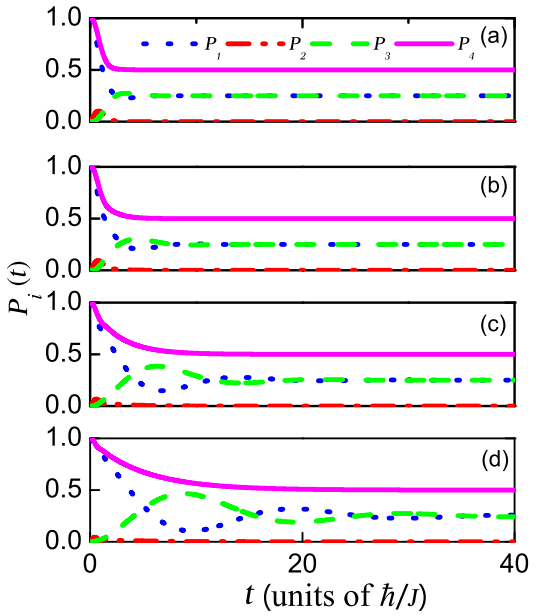

FIG. 3. The effect of the real part of μ_2 on the evolution of $P_i(t)$ with (a) $\eta = 0$, (b) 2, (c) 4, and (d) 6. Here, $\mu_1 = \mu_3 = 0.5$ and $\alpha = 2$.

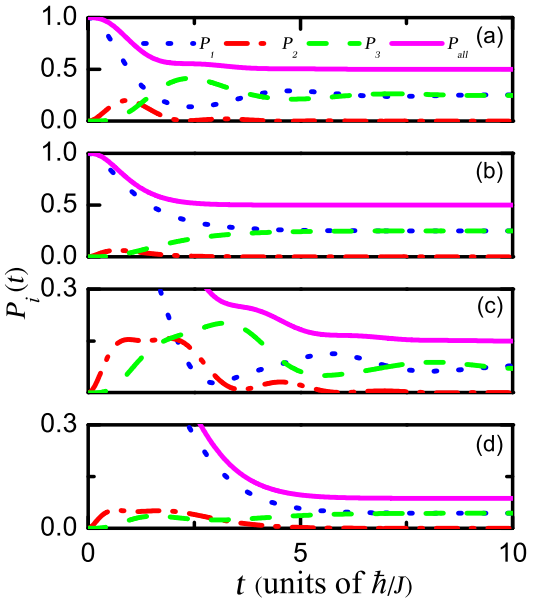

FIG. 4. A numerical investigation of evolutions for different nonlinear parameters: when $g_1 = g_3 = 0$, $g_2 = 3$, (a) $\alpha = 1$ and (b) $\alpha = 3$; when $g_1 = g_3 = g_2 = 3$, (c) $\alpha = 1$ and (d) $\alpha = 3$. Here, $\mu_1 = \mu_3 = 0$ and initial conditions $c_1^0 = 1$ and $c_2^0 = c_3^0 = 0$.

III. NUMERICAL SCHEME FOR NONLINEAR INTERACTION CASE

The analytical solution in the above section is not available when the nonlinear interaction is introduced in the Hamiltonian (3). In looking for the similar variational ansatz solution (1), we need to take several approximations into account: (a) First of all, the time evolution of the wave function (1) is described as the superposition of three local states [41]. The nonlinear terms in the dynamic equations (2), on the other hand, would destroy such a superposition. When the probability in the tunneling region of the adjacent wells is small enough such that the nonlinear interaction in these regions is negligible, the superposition ansatz (1) is applicable. (b) In the meantime, we decompose the time and the spatial dependence of the wave function $|\psi(t)\rangle$, which has been verified numerically in the study of BEC trapped in a double-well potential [56]. (c) The spatial dependence of the local states will not be considered here, despite the fact that the overlap of the states determines the tunneling strength J and the interaction parameters g_i [57]. To investigate the dynamics of the system with nonlinear terms, we deal with the time-dependent Hamiltonian in the site space by means of the successive iteration, i.e., starting from an arbitrarily normalized initial state $|\psi(0)\rangle$, the wave function at time $t + \delta t$ is evolved from previous time t through

$$|\psi(t+\delta t)\rangle = \exp(-iH[t]\delta t)|\psi(t)\rangle, \tag{22}$$

while the time dependence of the Hamiltonian $H[t]$ is described by the interaction terms $|c_i(t)|^2$ in Eq. (3). Accordingly, we numerically split the evolution time t into many small intervals, with the time step δt being small enough to admit a solution with good precision. We note that unlike in the case of time-dependent Gross-Pitaevskii equations for dissipative BEC in a double well [14,54] or the barrier transmission of BEC in a waveguide [53], the absence of the kinetic term makes it much easier for the convergence of the solution.

We now discuss the typical numerical results with different nonlinear parameters. For nonzero interaction existing only in the middle well, i.e., $g_1 = g_3 = 0$ and $g_2 = 3$, the stationary solutions for different loss $\alpha = 1$ and 3 are shown in Figs. 4(a) and 4(b). The stationary solutions are identical to the results of the noninteracting case in Fig. 1(b) and 1(c), which shows that g_2 does not affect the evolution of $P_i(t)$ in the limit $t \to \infty$. For three identical interaction parameters $g_1 = g_3 = g_2 = 3$, on the other hand, we observe quite different behavior. The nonlinear terms $g_{1,3}$ obviously diminish the projection of the dark state to a very low level and, moreover, P_i also decrease with the increase of α. Unequal $g_1 \neq g_3$ would destroy the coherent character completely, leading to a full leakage of the wave packet.

IV. GENERALIZATION TO ANY ODD SITE NUMBER

We have dealt with the three-well model with a loss in the middle site and found there is a dark state when $\mu_1 = \mu_3$ and $g_i = 0$. Then, based on this, we also discussed the time evolution for different parameters and even different interactions. These results can be generalized to a general $(2n+1)$-well system where only the middle site has a loss and is coupled with other wells. For simplicity, first we consider the five-well model. The Hamiltonian is

$$H_5 = \begin{pmatrix} \mu_1 & 0 & -1 & 0 & 0 \\ 0 & \mu_2 & -1 & 0 & 0 \\ -1 & -1 & \mu_3 & -1 & -1 \\ 0 & 0 & -1 & \mu_4 & 0 \\ 0 & 0 & -1 & 0 & \mu_5 \end{pmatrix}, \tag{23}$$

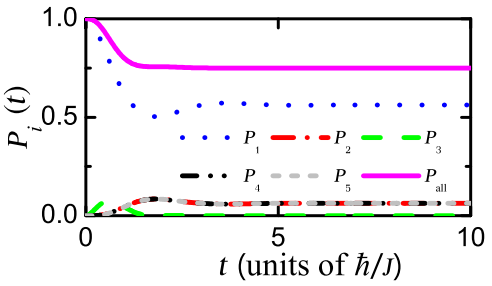

FIG. 5. Time evolutions of modulus squared of the coefficients in five wells for initial states $|1,0,0,0,0\rangle$. Here, nonlinear terms $g_i = 0$ and the loss factor $\alpha = 2$.

where $\mu_3 = -i\alpha$ and $\mu_{i\neq 3} = 0$ and only $|3\rangle$ is coupled with the rest of the wells. When dealing with $|H_5 - EI| = 0$, we can obtain

$$E^3(E^2 + i\alpha E - 4) = 0.$$

For eigenenergy $E = 0$, we can get

$$\begin{aligned} c_3 &= 0, \\ c_1 + c_2 + c_4 + c_5 &= 0, \end{aligned} \tag{24}$$

where c_i are the coefficients of $|i\rangle$. The solutions of (24) are not unique and correspond to triplet dark states with a node structure in the middle well and other coefficients summed up to zero. Hence we analyze the dynamics of this model by the numerical method, just as in Sec. III. But from (24), we can find that the eigenvectors with $E = 0$ do not have projection in $|3\rangle$ because of $c_3 = 0$, which is independent of any parameter. So the projection in these eigenvectors would not vary over time. In order to further explain it, using numerical method (22), we can find when $|\psi(0)\rangle = (1,0,0,0,0)^T$,

$$\lim_{t\to\infty} P_{\text{all}}(t) = 3/4,$$

which is the projection in the eigenvectors with $E = 0$ and is much larger than in the three-well system, as shown in Fig. 5. Accordingly, we study an arbitrary $(2n+1)$-well system and set $\mu_{n+1} = -i\alpha$ and $\mu_{j\neq n+1} = 0$. With the same numerical method, we find the law: $\lim_{t\to\infty} P_{\text{all}}(t) = 1 - 1/2n$ with $|\psi(0)\rangle = (1,0,0,\cdots,0)^T$, which coincides with the above-mentioned results. This gives a good application that more wells may be used to construct sophisticated dark states, the projection onto which is kept on a much higher level in the steady state of dynamical evolution.

V. CONCLUSIONS

We have presented a detailed analysis of dynamical evolutions of the three-well system with a loss in the middle well. When the chemical potentials in the two end wells are real and uniform, there is always a dark state, the projection on which does not change over time and the loss factor α. But there exists a critical value, where the norms at the two end sites evolute from damping oscillation to overdamping. When the chemical potentials in the two end wells are not uniform, the dark state is no longer the eigenstate of the system and three norms will decay to zero. In addition, when the nonlinear interactions are introduced and uniform in the two end wells, the projection of the dark state will be obviously diminished, but does not disappear. And the projection also decreases with the increase of the loss factor. However, the interaction at the middle site plays no role and the dark state is proven to be the key to the suppression of the dissipation. In addition, the other two interaction intensities would promote the loss. When extending the triple-well system to a general $(2n+1)$ well, we found that the total norm follows the law $1 - 1/2n$ in the absence of interactions, which can be used to enhance the antileakage capability in signal propagation in a certain medium with dissipation.

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