Correlation between intensity fluctuations of polychromatic electromagnetic light waves on weak scattering

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In the analysis of the correlation between intensity fluctuations (CIF) of light waves on scattering from a medium, it is implicitly assumed that the incident field is monochromatic. However, under usual circumstances, the field always has a certain frequency width. We examine the CIF of polychromatic electromagnetic light waves on scattering. It is found, in general, that the frequency components of the CIF may change on scattering from a medium whose dielectric susceptibility is a random function of position. The critical angle at which no frequency shift arises is introduced and the corresponding analytic expression is derived. The result shows that the critical angle is dominated by the physical properties of the medium and the source. Finally, we propose a scaling law for the normalized CIF for the scattering of polychromatic electromagnetic light waves. Our theory can be regarded as the vectorial extension of the scalar theory of Wolf *et al.* [E. Wolf, J. T. Foley, and F. Gori, J. Opt. Soc. Am. A **6**, 1142 (1989)].

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I. INTRODUCTION

The correlation between intensity fluctuations (CIF), which reveals the high-order coherence characteristics of the field, is a topic of considerable importance. At first, the CIF was used to measure the angular diameter of radio stars in astronomy [1]. Subsequently, this method has been applied to various areas, such as high-energy physics, atom physics, and nuclear physics [2–4]. In particular, it has been used in the domain of ghost imaging strongly depending on intensity fluctuations as a tool for retrieving an unknown object's transmittance pattern [5,6]. The measurements of the CIF can be fulfilled through the Hanbury-Brown-Twiss (HBT) experiment, which is one of the celebrated experiments of modern physics that accommodates equally classical and quantum interpretations [7].

Due to the important applications in remote sensing, climate research, medical diagnosis, and so on, the weak scattering theory has attracted substantial attention [8–19] (for a review of this research, please see Ref. [20]). Recently, the CIF of light waves on scattering from a medium has been a popular topic. For instance, Xin *et al.* discussed the CIF of scalar light waves scattered by a quasihomogeneous medium. It is found that the CIF in the far zone is determined by the spatial Fourier transforms of both the strength function and the degree of spatial correlation of the scattering potential, and the normalized CIF equals the square of the modulus of the degree of spectral coherence of the scattered field [21]. Jacks *et al.* generalized from the plane wave to the CIF of the arbitrarily correlated incident field scattered by a random medium [22]. Li *et al.* presented the statistical properties of the CIF of an electromagnetic plane wave on scattering from a spatially quasihomogeneous, anisotropic medium. The effects of the polarization properties of the special polarization source on the CIF of the scattered field have been discussed [23]. In addition, determining the structure information of the scatterer from the CIF of the scattered field was also discussed in detail [24], and recently this method has been generalized from second-order CIF to a third-order one [25]. In this paper, we focus on the CIF of polychromatic electromagnetic light waves on scattering. We obtain the generalized analytic expression for the CIF of polychromatic electromagnetic light waves on scattering. An example is presented to show how the frequency components of the incident field, scattering directions, and the physical properties of the scatterer affect the far-zone CIF. Furthermore, we will put forward a so-called scaling law for the normalized CIF of the scattering of polychromatic electromagnetic light waves.

The whole paper is organized as follows: In Sec. II, we derive the generalized analytic formula for the CIF of polychromatic electromagnetic light waves on scattering. In Sec. III, as an example, we present the changes of the CIF of polychromatic electromagnetic light waves scattered by a quasihomogeneous medium, and the concept of the critical angle is introduced. In Sec. IV, we propose a scaling law for the normalized CIF of the scattering of polychromatic electromagnetic light waves. Finally, we present a brief summary and discuss the potential application of our results and the link to the Wolf effect in the Sec. V.

II. EXPRESSION FOR THE CIF OF POLYCHROMATIC ELECTROMAGNETIC LIGHT WAVES ON WEAK SCATTERING

Consider that a polychromatic electromagnetic plane wave, propagating in the direction of a unit vector $\mathbf{s}_0$ along the z axis, is incident on a statistically stationary random scatterer occupying a finite volume D (see Fig. 1). The property of the incident field at a pair of points $\mathbf{r}_1'$ and $\mathbf{r}_2'$ within the domain of the scatterer can be described by its cross-spectral density

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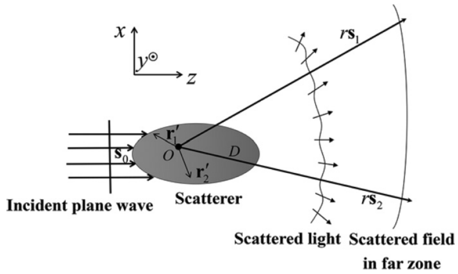

FIG. 1. Illustration of notations.

matrix, which is defined as [26]

$$\mathbf{W}^{(\mathrm{i})}(\mathbf{r}_1',\mathbf{r}_2',\mathbf{s}_0;\omega) \equiv \left[W_{ij}^{(\mathrm{i})}(\mathbf{r}_1',\mathbf{r}_2',\mathbf{s}_0;\omega)\right] = [\langle U_i^*(\mathbf{r}_1',\omega)U_j(\mathbf{r}_2',\omega)\rangle] \quad (i = x,y;\ j = x,y), \tag{1}$$

where the asterisk represents the complex conjugate, the angular brackets denote the ensemble average, $U_x(\mathbf{r}',\mathbf{s}_0;\omega_0)$ and $U_y(\mathbf{r}',\mathbf{s}_0;\omega_0)$ are the Cartesian components of $U(\mathbf{r}',\mathbf{s}_0;\omega_0)$ with respect to two mutually orthogonal x and y directions, respectively, which has the form [26]

$$U_i(\mathbf{r}',\mathbf{s}_0;\omega) = A_i a_i(\omega)\exp(ik\mathbf{s}_0\cdot\mathbf{r}'), \tag{2}$$

where A_i is a constant, representing the amplitude of the electric field along the ith axis, $a_i(\omega)$ is (generally complex) frequency-dependent random variables, and $k = \omega/c$ is the wave number with c being the speed of light in vacuum. For the sake of simplicity, we assume that the x and y components of the electric field in the source plane are uncorrelated, i.e., the nondiagonal elements in Eq. (1) are zero [26].

The second-order coherence and polarization properties of the scattered field at two points $r\mathbf{s}_1$ and $r\mathbf{s}_2$ can be characterized by a so-called 3×3 cross-spectral density matrix, which can be expressed as [26]

$$\mathbf{W}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega) \equiv \left[W_{ij}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right] = \left[\left\langle U_i^{(s)*}(r\mathbf{s}_1,\omega)U_j^{(s)}(r\mathbf{s}_2,\omega)\right\rangle\right] \quad (i = x,y,z;\ j = x,y,z), \tag{3}$$

where $U_i^{(s)}(r\mathbf{s}_1,\omega)$ is the Cartesian component of $\mathbf{U}^{(s)}(r\mathbf{s}_1,\omega)$ along the ith axis. Assume that the scatterer is so weak that the scattering can be analyzed within the accuracy of the first-order Born approximation. The three-dimensional distribution of the scattered field then takes the form [15]

$$U_x^{(s)}(r\mathbf{s},\omega) = \int_D F(\mathbf{r}',\omega)G(r\mathbf{s},\mathbf{r}';\omega) \times \left\{\left(1-s_x^2\right)U_x(\mathbf{r}',\mathbf{s}_0;\omega) - s_x s_y U_y(\mathbf{r}',\mathbf{s}_0;\omega)\right\}d^3r', \tag{4a}$$

$$U_y^{(s)}(r\mathbf{s},\omega) = \int_D F(\mathbf{r}',\omega)G(r\mathbf{s},\mathbf{r}';\omega) \times \left\{-s_x s_y U_x(\mathbf{r}',\mathbf{s}_0;\omega) + \left(1-s_y^2\right)U_y(\mathbf{r}',\mathbf{s}_0;\omega)\right\}d^3r', \tag{4b}$$

$$U_z^{(s)}(r\mathbf{s},\omega) = \int_D F(\mathbf{r}',\omega)G(r\mathbf{s},\mathbf{r}';\omega) \times \{-s_x s_z U_x(\mathbf{r}',\mathbf{s}_0;\omega) - s_y s_z U_y(\mathbf{r}',\mathbf{s}_0;\omega)\}d^3r'. \tag{4c}$$

Here, s_x,s_y, and s_z are the Cartesian components of the unit vector $\mathbf{s}$ along the x, y, and z axis, respectively. $F(\mathbf{r}',\omega) = k^2\eta(\mathbf{r}',\omega)$ is the scattering potential of the medium, with $\eta(\mathbf{r}',\omega)$ being dielectric susceptibility, and $G(r\mathbf{s},\mathbf{r}';\omega)$ is the outgoing free-space Green function, which can be expressed as

$$G(r\mathbf{s},\mathbf{r}';\omega) \sim \frac{e^{ikr}}{r}\exp(-ik\mathbf{s}\cdot\mathbf{r}'). \tag{5}$$

Now let us consider the CIF at two points, specified by the position vectors $r\mathbf{s}_1$ and $r\mathbf{s}_2$, in the scattered field, which is defined as

$$C^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega) \equiv \langle \Delta I^{(s)}(r\mathbf{s}_1,\omega)\Delta I^{(s)}(r\mathbf{s}_2,\omega)\rangle, \tag{6}$$

where $\Delta I^{(s)}(r\mathbf{s},\omega)$ represents the intensity fluctuation of the scattered field at a point, which is given by

$$\Delta I^{(s)}(r\mathbf{s},\omega) = I^{(s)}(r\mathbf{s},\omega) - \langle I^{(s)}(r\mathbf{s},\omega)\rangle. \tag{7}$$

Here, $I^{(s)}(r\mathbf{s},\omega)$ and $\langle I^{(s)}(r\mathbf{s},\omega)\rangle$ denote the instantaneous and the averaged intensities of the scattered field, respectively, and can be calculated from

$$I^{(s)}(r\mathbf{s},\omega) = U_x^{(s)*}(r\mathbf{s},\omega)U_x^{(s)}(r\mathbf{s},\omega) + U_y^{(s)*}(r\mathbf{s},\omega)U_y^{(s)}(r\mathbf{s},\omega) + U_z^{(s)*}(r\mathbf{s},\omega)U_z^{(s)}(r\mathbf{s},\omega), \tag{8}$$

$$\langle I^{(s)}(r\mathbf{s},\omega)\rangle = \left\langle U_x^{(s)*}(r\mathbf{s},\omega)U_x^{(s)}(r\mathbf{s},\omega)\right\rangle + \left\langle U_y^{(s)*}(r\mathbf{s},\omega)U_y^{(s)}(r\mathbf{s},\omega)\right\rangle + \left\langle U_z^{(s)*}(r\mathbf{s},\omega)U_z^{(s)}(r\mathbf{s},\omega)\right\rangle = \mathrm{Tr}\mathbf{W}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega), \tag{9}$$

with Tr denoting the trace.

If we assume that the fluctuations of the scattered field obey the Gaussian statistics, as is often the case, then this allows the calculation of the fourth-order correlation from the second-order correlation [26]. Thus, the CIF of the scattered field can be simplified as

$$C^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega) = \sum_{ij}\left|W_{ij}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right|^2. \tag{10}$$

On substituting from Eqs. (2), (4), (5), (8), and (9) into Eq. (10), after some tedious calculations, we obtain a compact expression for the CIF of the scattered field, with a form of

$$C^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega) = \frac{1}{r^4}\left(\frac{\omega}{c}\right)^8\Theta(\mathbf{s}_1,\mathbf{s}_2;\omega) \times \tilde{C}_\eta^2[-k(\mathbf{s}_1-\mathbf{s}_0),k(\mathbf{s}_2-\mathbf{s}_0);\omega], \tag{11}$$

where

$$\Theta(\mathbf{s}_1,\mathbf{s}_2;\omega) = \left(1 - s_{1x}^2\right)\left(1 - s_{2x}^2\right)A_x^4 S_x^2(\omega) + 2s_{1x}s_{1y}s_{2x}s_{2y}A_x^2A_y^2S_x(\omega)S_y(\omega) + \left(1 - s_{1y}^2\right)\left(1 - s_{2y}^2\right)A_y^4 S_y^2(\omega) \tag{12}$$

is governed by the polarization and spectrum properties of the incident field, with $S_i(\omega) = \langle a_i^*(\omega)a_i(\omega)\rangle$ being the spectrum of the incident field along the ith axis, and

$$\tilde{C}_\eta[\mathbf{K}_1,\mathbf{K}_2;\omega] = \int_D\int_D C_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega) \times \exp[-i(\mathbf{K}_2\cdot\mathbf{r}_2' + \mathbf{K}_1\cdot\mathbf{r}_1')]d^3r_1'd^3r_2' \tag{13}$$

is the six-dimensional spatial Fourier transform of the correlation function, with

$$C_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega) = \langle\eta^*(\mathbf{r}_1',\omega)\eta(\mathbf{r}_2',\omega)\rangle \tag{14}$$

representing the correlation function of the dielectric susceptibility of the scattering medium. $\mathbf{K}_1 = -k(\mathbf{s}_1 - \mathbf{s}_0)$ and $\mathbf{K}_2 = k(\mathbf{s}_2 - \mathbf{s}_0)$ are analogous to the momentum transfer vector of the quantum mechanical theory of potential scattering.

Equation (11) is a general formula by which the far-zone CIF of polychromatic electromagnetic light waves on scattering from a random medium can be discussed. This formula shows the dependence of the CIF of the scattered field on the frequency of the incident field, i.e., the incident field in different frequency ranges has different CIF on scattering from a random medium. Besides, the CIF is close to the physical properties of both the source $\Theta(\mathbf{s}_1,\mathbf{s}_2;\omega)$ and the medium $\tilde{C}_\eta[\mathbf{K}_1,\mathbf{K}_2;\omega]$, and these two parts also contain the influence of the scattering direction $\mathbf{s}$ on the CIF; hence, in general, the far-zone CIF will be different between two different directions of observation. On the other hand, we note that Eq. (11) also shows a remarkable property: it is independent of the component U_z of the incoming field. This is because we assume that the incoming plane wave is along the z axis, i.e., the scatterer is illuminated with the normal incidence plane wave. Under this circumstance, there is no component U_z of the incoming field. This choice for representation of the incoming field in terms of U_x and U_y is the most efficient one as it leads to the result that all physical quantities are expressed in terms of only U_x and U_y and not U_x, U_y, and U_z. Finally, we would also like to point out that when we do not consider the frequency width of the incident field, i.e., the incident field is monochromatic, Eq. (11) can reduce to be $C^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega) = \frac{S^2(\omega)}{r^4}\Phi(\mathbf{s}_1,\mathbf{s}_2;\omega)\tilde{C}_F^2[-k(\mathbf{s}_1-\mathbf{s}_0),k(\mathbf{s}_2-\mathbf{s}_0);\omega]$, where $\Phi(\mathbf{s}_1,\mathbf{s}_2;\omega) = (1-s_{1x}^2)(1-s_{2x}^2)A_x^4 + 2s_{1x}s_{1y}s_{2x}s_{2y}A_x^2A_y^2 + (1-s_{1y}^2)(1-s_{2y}^2)A_y^4$ represents the polarization properties of the source and $\tilde{C}_F[-k(\mathbf{s}_1-\mathbf{s}_0),k(\mathbf{s}_2-\mathbf{s}_0);\omega]$ is the Fourier transform of the correlation function of scattering potential $C_F(\mathbf{r}_1',\mathbf{r}_2';\omega)$. The formula clearly gives an insightful relationship between the CIF and the polarization properties of the source and the coherence properties of the scattering medium. We may therefore also say that Eq. (11) can be regarded as a generalized formula for the CIF of electromagnetic light waves on scattering.

III. AN EXAMPLE: THE CIF OF THE FIELD SCATTERED BY STATISTICALLY QUASIHOMOGENEOUS MEDIA

Next let us consider that the medium is statistically quasihomogeneous. A medium can be regarded as a quasihomogeneous medium when its correlation function of the dielectric susceptibility can be characterized by a product that contains a slow function I_η and a fast function μ_η at each frequency ω [26], i.e.,

$$C_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega) = I_\eta\left(\frac{\mathbf{r}_1'+\mathbf{r}_2'}{2},\omega\right)\mu_\eta(\mathbf{r}_2'-\mathbf{r}_1',\omega), \tag{15}$$

where I_η and μ_η are the strength function and the normalized correlation coefficient of the dielectric susceptibility of the scattering medium, respectively. On substituting from Eq. (15) first into Eq. (13), and making use of variable transforms as $\mathbf{r}_S = (\mathbf{r}_1'+\mathbf{r}_2')/2$ and $\mathbf{r}_D = \mathbf{r}_2'-\mathbf{r}_1'$, after manipulating the Fourier transform, then substituting into Eq. (11), we can obtain the CIF of polychromatic electromagnetic light waves on scattering from a quasihomogeneous medium, which is expressed by the form

$$C^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega) = \frac{1}{r^4}\left(\frac{\omega}{c}\right)^8\Theta(\mathbf{s}_1,\mathbf{s}_2;\omega) \times \tilde{I}_\eta^2(\mathbf{K}_1+\mathbf{K}_2,\omega) \times \tilde{\mu}_\eta^2\left(\frac{\mathbf{K}_2-\mathbf{K}_1}{2},\omega\right). \tag{16}$$

To better illustrate how the relative CIF behaves on scattering from a quasihomogeneous medium, it will be useful to introduce the normalized CIF, referring to the treatment about the spectral changes on propagation [26], using the formula

$$C_n^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega) = \frac{C^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)}{\int_0^\infty C^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)d\omega}. \tag{17}$$

On substituting from Eq. (16) into Eq. (17), we obtain the following expression:

$$C_n^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega) = \frac{\omega^8\Theta(\mathbf{s}_1,\mathbf{s}_2;\omega)\tilde{I}_\eta^2(\mathbf{K}_1+\mathbf{K}_2,\omega)\tilde{\mu}_\eta^2\left(\frac{\mathbf{K}_2-\mathbf{K}_1}{2},\omega\right)}{\int_0^\infty \omega^8\Theta(\mathbf{s}_1,\mathbf{s}_2;\omega)\tilde{I}_\eta^2(\mathbf{K}_1+\mathbf{K}_2,\omega)\tilde{\mu}_\eta^2\left(\frac{\mathbf{K}_2-\mathbf{K}_1}{2},\omega\right)d\omega}. \tag{18}$$

Assume now that $I_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega)$ and $\mu_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega)$ have Gaussian forms, viz.,

$$I_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega) = C_0\exp\left[-\frac{(\mathbf{r}_1'+\mathbf{r}'_2)^2}{8\sigma_s^2}\right], \tag{19a}$$

$$\mu_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega) = \exp\left[-\frac{(\mathbf{r}_2'-\mathbf{r}_1')^2}{2\sigma_g^2}\right], \tag{19b}$$

where C_0 is a positive real constant, and σ_s and σ_g denote the effective width and the effective correlation width of the distribution function, respectively. For a quasihomogeneous medium, it must meet $\sigma_s \gg \sigma_g$. Furthermore, assume that the spectrum of the incident field along x and y have the same distribution of Gaussian function, with a form of

$$S_x(\omega) = S_y(\omega) = B_0 \exp\left[-\frac{(\omega-\omega_0)^2}{2\Gamma_0^2}\right], \tag{20}$$

where ω_0 denotes the central frequency, Γ_0 stands for the linewidth of the spectrum, and B_0 is a positive real constant.

On substituting from Eqs. (13), (15), (16), (19), and (20) into Eq. (18), with the help of a product theorem for Gaussian functions [8], the exponent part of the numerator and denominator in Eq. (18) can then be expressed as

$$\exp\left\{\frac{(\omega-\omega_0)^2}{\Gamma_0} + \left[\frac{\sigma_s^2(2-2\cos\theta)}{c^2} + \frac{\sigma_g^2(2-2\cos\theta)}{4c^2}\right]\omega^2\right\} = \exp\left\{\frac{(\omega-\omega_0')^2}{\Gamma^2} + \frac{(\Gamma_0^2-\Gamma^2)\omega_0^2}{\Gamma_0^4}\right\}. \tag{21}$$

Here, $1/\Gamma^2 = 1/\Gamma_0^2 + \sigma_s^2(2-2\cos\theta)/c^2 + \sigma_g^2(2-2\cos\theta)/4c^2$, $\omega_0' = \omega_0\Gamma^2/\Gamma_0^2$, and θ denotes the angle between incident direction $\mathbf{s}_0$ and scattering direction $\mathbf{s}_1$. In the process of solving the scattering problem, in general, we choose $\mathbf{s}_2$ along the incident direction. Thus, the normalized CIF of the scattered field can be obtained as

$$C_n^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2;\omega) = \frac{\omega^8 \exp\left[-\frac{(\omega-\omega_0')^2}{\Gamma^2} + \frac{(\Gamma_0^2-\Gamma^2)\omega_0^2}{\Gamma_0^4}\right]}{\int_0^\infty \omega^8 \exp\left[-\frac{(\omega-\omega_0')^2}{\Gamma^2} + \frac{(\Gamma_0^2-\Gamma^2)\omega_0^2}{\Gamma_0^4}\right]d\omega} = \frac{\omega^8 \exp\left[-\frac{(\omega-\omega_0')^2}{\Gamma^2}\right]}{\Gamma\sqrt{\pi}\left(\omega_0'^8 + 14\Gamma^2\omega_0'^6 + 210\Gamma^4\omega_0'^4 + 1680\Gamma^6\omega_0'^2 + 420\Gamma^8\right)}. \tag{22}$$

It should be pointed out that on the above integration calculations, we have assumed that $\Gamma_0 \ll \omega_0$, which means that the spectral width of the incident light is so small compared with ω_0. Thus, the low limit of the integration can be extended to negative infinity, at a good approximation.

From Eq. (22), it can be seen that different from the normalized CIF of the incident light, $(\sqrt{\pi}\Gamma_0)^{-1}\exp[-(\omega-\omega_0)^2/\Gamma_0^2]$, which is independent of positions, $C_n^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2;\omega)$ depends strongly on the scattering direction and its distribution width is narrower ($\Gamma < \Gamma_0$). Only when the scattering direction $\mathbf{s}_1$ is also along the incident direction, i.e., $\theta = 0$, $\omega_0' = \omega_0$ can be established. Otherwise, due to the physical properties of the medium, as being presented by physical parameters σ_s and σ_g, the distribution center of the Gaussian function in Eq. (22) moves towards lower frequency with respect to the center of the Gaussian line of the normalized CIF of the incident light (i.e., is redshifted). On the other hand, the factor ω^8 in Eq. (22) is an increasing function of the frequency, and hence will produce a shift towards the higher frequency (i.e., a blueshift). Consequently, the normalized CIF of the scattered field will be either a redshift or blueshift with respect to the normalized CIF of the incident field, depending on the magnitudes of these two contributions.

The behaviors of the normalized CIF of polychromatic light waves on scattering from different quasihomogeneous media are plotted in Fig. 2. Figure 2(a) displays the behaviors of the normalized CIF of the scattered field for different effective width σ_s. It is indeed shown that the frequency components of the CIF of the scattered field will produce shift, for polychromatic light waves on scattering from a quasihomogeneous medium. It is also shown that different effective width σ_s leads to different CIF shifts at a certain scattering angle. Moreover, the larger is the effective width σ_s, the more is the redshift. Figure 2(b) depicts the behaviors of the normalized CIF of the scattered field for different effective correlation width σ_g. It is shown that the influence of the effective correlation width σ_g on correlation shifts can be negligible. These results can be viewed as the reciprocity theorem for the CIF of polychromatic light waves scattered by quasihomogeneous media.

In the following, we discuss a special case in which two contributions accounting for redshift or blueshift achieve balance. That is to say, to find a special scattering angle at which no frequency shift appears is important and we call this angle the critical angle. To this end, we must first determine the magnitude of the central frequency of the shifted line, which can be derived by $\partial C_n^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2;\omega)/\partial\omega = 0$. Obviously, only one root of this equation is legitimate, which has the following form:

$$\omega(\theta) = \frac{\omega_0'}{2}\left[1 + \sqrt{1 + \left(\frac{4\Gamma}{\omega_0'}\right)^2}\right] \approx \omega_0\left(\frac{\Gamma^2}{\Gamma_0^2} + \frac{4\Gamma_0^2}{\omega_0^2}\right). \tag{23}$$

It should be emphasized that on above derivations, only the first two terms are taken into account in a Taylor expansion.

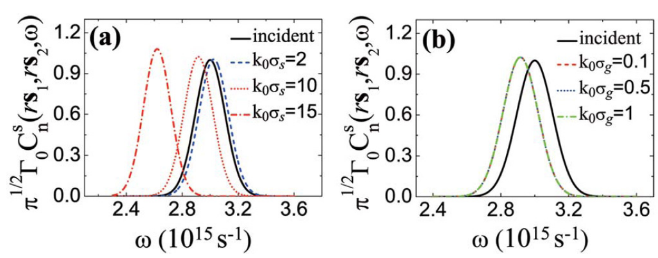

FIG. 2. Behaviors of the normalized CIF of polychromatic light waves on scattering from different quasihomogeneous media. The parameters for calculations are $c = 3\times10^8$, $\omega_0 = 3\times10^{15}\ \mathrm{s}^{-1}$, $k_0 = \omega_0/c$, $\Gamma_0 = 0.05\omega_0$, $\theta = 0.4$, (a) $k_0\sigma_g = 0.1$, and (b) $k_0\sigma_s = 10$.

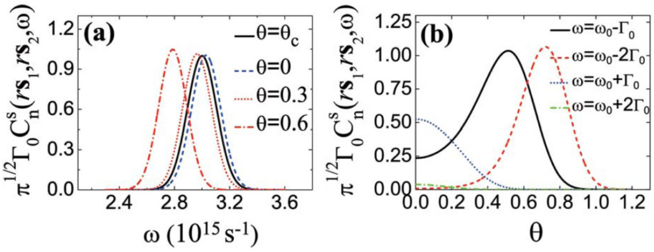

FIG. 3. (a) Illustrating the influence of the scattering angle on the normalized CIF of the scattered field. (b) Normalized CIF of the scattered field as a function of θ for several selected frequencies. The parameters for calculations are $k_0\sigma_s = 10$ and $k_0\sigma_g = 1$. The other parameters are the same as Fig. 2 and the corresponding critical angle is $\theta_c = 0.2011$.

Then the magnitude of the frequency shift can be derived from

$$\Delta\omega(\theta) \equiv \omega(\theta) - \omega_0 = \omega_0\left(\frac{\Gamma^2}{\Gamma_0^2} + \frac{4\Gamma_0^2}{\omega_0^2}\right) - \omega_0. \tag{24}$$

The critical angle naturally gives $\Delta\omega(\theta) = 0$, and thus the critical angle can be obtained by solving the corresponding equation as

$$\theta_c = \arccos\left[1 - \frac{8c^2}{(4\sigma_s^2 + \sigma_g^2)(\omega_0 - 4\Gamma_0^2)}\right]. \tag{25}$$

From Eq. (25), we can clearly see that the critical angle is only dominated by two parts. One is the physical properties of the medium; the other is the optical properties of the incident light. For the given parameters of the medium and the source, the critical angle θ_c can be determined. In the case of $\theta > \theta_c$, $\Delta\omega(\theta) < 0$, it implies that redshift occurs, and when $\theta < \theta_c$, $\Delta\omega(\theta) > 0$, blueshift is produced.

Figure 3(a) plots the normalized CIF as a function of ω for different scattering angles. It can be seen that different scattering angles can induce different correlation shift, namely, for the larger scattering angle, the central frequency of the normalized CIF moves towards a smaller value, reflecting the obvious redshift. For the case of $\theta = 0$, the central frequency takes a larger value, i.e., blueshift appears, while the distribution with the critical angle remains equal to the initial central frequency ω_0. The results are in accordance with the phenomena we expected. To further investigate the effect of the scattering angle, the behaviors of the normalized CIF for several selected frequencies are displayed in Fig. 3(b). For the two frequencies $\omega = \omega_0 + 2\Gamma_0$ and $\omega = \omega_0 - 2\Gamma_0$, they have the same distribution of the CIF of the source. However, as shown in Fig. 3(b), when $\theta < \theta_c$, the high-frequency scattered field has stronger CIF compared with the low-frequency one, and vice versa. Another pair of frequencies also represents the same regulations. Hence, these results indicate that low-frequency scattered fields have a stronger CIF with the increase of scattering angle, i.e., we can say that there exist more low-frequency scattered fields at a large scattering angle.

IV. SCALING LAW FOR THE CIF OF POLYCHROMATIC ELECTROMAGNETIC LIGHT WAVES ON WEAK SCATTERING

Based on the previous discussions, it is shown that the CIF of polychromatic light on scattering is distinctly different from that of monochromatic light. The results show the key influence of the frequency on CIF. Meanwhile, the results also demonstrate that the frequency components of the CIF indeed change on scattering from a random medium. In particular, the scattering angle is an important factor that induces the CIF variances. In the following, we will derive a sufficient condition under which the normalized CIF of the scattered field is independent of the scattering angle, i.e., the normalized CIF of between arbitrary two points in the whole scattering space remains the same.

From Eqs. (11) and (17), we can see that for the normalized CIF, the scattered field will be irrelevant to the scattering direction if the correlation function of the medium and the spectrum distribution of the incident field satisfy the following relations, respectively:

$$\tilde{C}_\eta(\mathbf{K}_1,\mathbf{K}_2;\omega) = F(\omega)\tilde{H}(\mathbf{s}_1 - \mathbf{s_0})\tilde{H}(\mathbf{s}_2 - \mathbf{s_0}), \tag{26a}$$

$$S_x(\omega) = \kappa S_y(\omega), \tag{26b}$$

where κ is a frequency-independent constant, for in that case Eq. (17) reduces to

$$C_n^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega) = \frac{[S_x(\omega)\omega^4 F(\omega)]^2}{\int_0^\infty [S_x(\omega)\omega^4 F(\omega)]^2 d\omega}, \tag{27}$$

which is indeed independent of scattering direction $\mathbf{s}$.

Now we will further discuss Eq. (26a), with the choice $\mathbf{s}_1 = \mathbf{s}_2 = \mathbf{s_0}$, which reduces to

$$\tilde{C}_\eta(0,\omega) = F(\omega)\tilde{H}^2(0). \tag{28}$$

Thus, on substituting from Eq. (28) into Eq. (26a), after some arrangements, Eq. (26a) can be expressed as

$$\tilde{C}_\eta(\mathbf{K}_1,\mathbf{K}_2;\omega) = \frac{\tilde{C}_\eta(0,\omega)}{\tilde{H}^2(0)}\tilde{H}\left(-\frac{\mathbf{K}_1}{k}\right)\tilde{H}\left(\frac{\mathbf{K}_2}{k}\right). \tag{29}$$

An important consequence can be obtained if we take the inverse Fourier transform of Eq. (29), namely,

$$C_\eta(\mathbf{r_1'},\mathbf{r_2'};\omega) = \frac{\tilde{C}_\eta(0,\omega)}{(2\pi)^6\tilde{H}^2(0)} \times \int_D\int_D \tilde{H}\left(-\frac{\mathbf{K}_1}{k}\right)\tilde{H}\left(\frac{\mathbf{K}_2}{k}\right) \times \exp[i(\mathbf{K}_2\cdot\mathbf{r_2'} + \mathbf{K}_1\cdot\mathbf{r_1'})]d^3K_1 d^3K_2. \tag{30}$$

On changing the variables of integration from $\mathbf{K}_1$ to $\mathbf{K}_1^{'}/k$, and $\mathbf{K}_2$ to $\mathbf{K}_2^{'}/k$, we obtain from Eq. (30) the expression

$$C_\eta(\mathbf{r_1'},\mathbf{r_2'};\omega) = \frac{k^6\tilde{C}_\eta(0,\omega)}{\tilde{H}^2(0)}H(-k\mathbf{r_1'})H(k\mathbf{r_2'}). \tag{31}$$

The degree of spatial coherence of the dielectric susceptibility of the scatterer can be defined as

$$\mu_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega) = \frac{C_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega)}{C_\eta(0,\omega)}. \tag{32}$$

On substituting from Eq. (31) into Eq. (32), we arrive at

$$\mu_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega) = h(-k\mathbf{r}_1',k\mathbf{r}_2'), \tag{33}$$

with

$$h(-k\mathbf{r}_1',k\mathbf{r}_2') = \frac{H(-k\mathbf{r}_1')H(k\mathbf{r}_2')}{\tilde{H}^2(0)}. \tag{34}$$

Thus, on combining Eq. (26b) with Eq. (33), we can rewrite the sufficient condition in the following forms:

$$\mu_\eta(\mathbf{r}_1',\mathbf{r}_2';\omega) = h(-k\mathbf{r}_1',k\mathbf{r}_2'), \tag{35a}$$

$$S_y(\omega) = \kappa S_x(\omega). \tag{35b}$$

Equation (35a) can guarantee that the normalized CIF of the two arbitrary points remain the same for scalar light waves on scattering from random media. Equation (35b) requires that the spectrum of the incident field has the same distribution along two perpendicular directions. We can see that as long as Eq. (35) is satisfied, for the scattering of polychromatic electromagnetic light waves by random media, the normalized CIF of the two arbitrary points in the whole scattering space will remain the same. Hence, we refer to Eq. (35) as a scaling law for the CIF of polychromatic electromagnetic light waves on scattering.

V. SUMMARY AND DISCUSSION

In summary, the CIF of polychromatic electromagnetic light waves on weak scattering has been examined in detail. It is found that the frequency components of the CIF change on scattering from random media, and the frequency shift is close to the scattering angles and the physical properties of the medium. An extreme case in which no redshift or blueshift takes place has been investigated and the corresponding analytic expression for the critical angle has been derived. Finally, we have proposed a so-called scaling law, under which the normalized CIF of arbitrary two points in the whole scattering space remain the same, for the normalized CIF of the scattering of polychromatic electromagnetic light waves.

Our results have shown that the CIF of polychromatic light waves on scattering is distinctly different from that of monochromatic light, showing the significant influence of the frequency on the CIF of the scattered field, and the results may provide certain guidance for the application of correlation between intensity fluctuations in ghost imaging in scattering media. In addition, our results also contribute to determine the structure information of a scattering medium in terms of the critical angle. For instance, we can approximately measure the effective width of the correlation function of scattering potential of a quasihomogeneous medium. As the previous derivation in Eq. (25), the relationship between the structure parameters of a quasihomogeneous medium and the critical angle is expressed as $4\sigma_s^2 + \sigma_g^2 = 8c^2/(1-\cos\theta_c)(\omega_0 - 4\Gamma_0^2)$. Because the medium satisfies $\sigma_s \gg \sigma_g$, we have the following approximation: $\sigma_s = \sqrt{2c^2/(1-\cos\theta_c)(\omega_0 - 4\Gamma_0^2)}$. We can see that σ_s is determined provided that the critical angle and the optical parameters of the source are known.

Finally, it is worthwhile to mention that the analysis of our results also leads to the introduction of redshifts and blueshifts. This phenomenon has already been discussed by Wolf *et al.*, in the case of the emitted spectrum of surface sources, and is known as the "Wolf effect" [28,29] (for a review of this research, please see Ref. [30]), and this effect has been verified by experiment [31]. The problem of analogous effects in scattering has been addressed in detail by Wolf [8] and James [32]. The results show that the scattering spectral line displays redshifts or blueshifts with respect to the incident one, depending on the scattering angle and correlation properties of the medium. For the CIF of polychromatic light scattered by a random medium in our analysis, the central frequency of the CIF of the scattered field also produce redshifts or blueshifts, which also depends on the scattering angle and the physical properties of the medium. We would like to point out that although they have a similar physical mechanism leading to the frequency shift, there is no certain relationship between their frequency shifts (we can see this point from their expressions, respectively, in Eq. (22) and Eq. (4.7) in [8]); for example, when the spectral line of the scattered field is blueshift, the frequency components of the CIF may not be blueshift, but may display redshift. On the other hand, note that in our results, the effect of the correlation length of the medium on the shift of the frequency components of the CIF can be negligible; this is due to the reciprocity relationship in the case of the quasihomogeneous medium. If we consider the more general random medium, the correlation length of the medium also induces the shift of the frequency components of CIF.

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APPENDIX: DERIVATION OF EQS. (11) AND (25)

Now let us show how to obtain Eq. (11). Since Eq. (10) contains the sum of nine terms, it is of great difficulty to calculate them directly. With the aid of some skills, the solution to Eq. (10) will be simplified. To conveniently illustrate the processes of calculation, we assign

$$W_1 = \left[W_{xx}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right]^2 + \left[W_{xy}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right]^2 + \left[W_{xz}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right]^2, \tag{A1}$$

$$W_2 = \left[W_{yx}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right]^2 + \left[W_{yy}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right]^2 + \left[W_{yz}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right]^2, \tag{A2}$$

$$W_3 = \left[W_{zx}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right]^2 + \left[W_{zy}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right]^2 + \left[W_{zz}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)\right]^2. \quad \text{(A3)}$$

On substituting from Eqs. (2), (4), (5), (8), and (9) into Eq. (10), we first calculate W_1, and, making use of $s_{1x}^2+s_{1y}^2+s_{1z}^2=1$ and $s_{2x}^2+s_{2y}^2+s_{2z}^2=1$, the corresponding result can be simplified as

$$W_1 = \frac{1}{r^4}\left(\frac{\omega}{c}\right)^8 \{\left(1-s_{1x}^2\right)^2\left(1-s_{2x}^2\right)A_x^4S_x^2(\omega) + 2s_{1x}s_{1y}s_{2x}s_{2y}\left(1-s_{1x}^2\right)A_x^2A_y^2S_x(\omega)S_y(\omega) + s_{1x}^2s_{1y}^2\left(1-s_{2y}^2\right)A_y^4S_y^2(\omega)\} \times \tilde{C}_\eta^2[-k(\mathbf{s}_1-\mathbf{s}_0),k(\mathbf{s}_2-\mathbf{s}_0);\omega]. \quad \text{(A4)}$$

Similarly, we can obtain the expressions for W_2 and W_3, respectively, with forms of

$$W_2 = \frac{1}{r^4}\left(\frac{\omega}{c}\right)^8 \{s_{1x}^2s_{1y}^2\left(1-s_{2x}^2\right)A_x^4S_x^2(\omega) + 2s_{1x}s_{1y}s_{2x}s_{2y}\left(1-s_{1y}^2\right)A_x^2A_y^2S_x(\omega)S_y(\omega) + \left(1-s_{1y}^2\right)^2\left(1-s_{2y}^2\right)A_y^4S_y^2(\omega)\} \times \tilde{C}_\eta^2[-k(\mathbf{s}_1-\mathbf{s}_0),k(\mathbf{s}_2-\mathbf{s}_0);\omega], \quad \text{(A5)}$$

$$W_3 = \frac{1}{r^4}\left(\frac{\omega}{c}\right)^8 \{s_{1x}^2\left(1-s_{2x}^2\right)\left(1-s_{1x}^2-s_{1y}^2\right)A_x^4S_x^2(\omega) - 2s_{1x}s_{1y}s_{2x}s_{2y}\left(1-s_{1x}^2-s_{1y}^2\right)A_x^2A_y^2S_x(\omega)S_y(\omega) + s_{1y}^2\left(1-s_{1x}^2-s_{1y}^2\right)\left(1-s_{2y}^2\right)A_y^4S_y^2(\omega)\} \times \tilde{C}_\eta^2[-k(\mathbf{s}_1-\mathbf{s}_0),k(\mathbf{s}_2-\mathbf{s}_0);\omega]. \quad \text{(A6)}$$

Thus, Eq. (11) can finally be obtained in a pretty compact form, on summing $W_1+W_2+W_3$,

$$C^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega) = \frac{1}{r^4}\left(\frac{\omega}{c}\right)^8\Theta(\mathbf{s}_1,\mathbf{s}_2;\omega) \times \tilde{C}_\eta^2[-k(\mathbf{s}_1-\mathbf{s}_0),k(\mathbf{s}_2-\mathbf{s}_0);\omega]. \quad \text{(A7)}$$

In order to derive Eq. (25), we first need to determine the magnitude of the central frequency of the shifted line, which can be obtained by differentiating Eq. (22) with respect to ω, and then assign the corresponding result equal to zero. After some arrangements, we can arrive at

$$\omega^7\exp\left[-\frac{(\omega-\omega_0')^2}{\Gamma^2}\right]\left[8-\frac{2}{\Gamma^2}\omega^2+\frac{2}{\Gamma^2}\omega\omega_0'\right]=0. \quad \text{(A8)}$$

From Eq. (A8), we can see that it equals zero, which is legitimately established if and only if

$$8-\frac{2}{\Gamma^2}\omega^2+\frac{2}{\Gamma^2}\omega\omega_0'=0. \quad \text{(A9)}$$

On solving this equation, we can obtain

$$\omega(\theta)=\frac{\omega_0'}{2}\left[1\pm\sqrt{1+\left(\frac{4\Gamma}{\omega_0'}\right)^2}\right]. \quad \text{(A10)}$$

Both of the roots may be frequencies at which $C_n^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2;\omega)$ reach maximum. Only the one with the positive sign is permissible due to our use of the complex analytic signal representation of the field [27]. Hence, we arrive at

$$\omega(\theta)=\frac{\omega_0'}{2}\left[1+\sqrt{1+\left(\frac{4\Gamma}{\omega_0'}\right)^2}\right]. \quad \text{(A11)}$$

Assuming that $\Gamma_0 \ll \omega_0$, Eq. (A11) can be approximated as

$$\omega(\theta)=\frac{\omega_0'}{2}\left[1+\sqrt{1+\left(\frac{4\Gamma}{\omega_0'}\right)^2}\right]\approx\omega_0\left(\frac{\Gamma^2}{\Gamma_0^2}+\frac{4\Gamma_0^2}{\omega_0^2}\right). \quad \text{(A12)}$$

From Eqs. (24) and (A12), we can finally obtain Eq. (25).

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