Quantum effects of Aharonov-Bohm type and noncommutative quantum mechanics

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Quantum mechanics in noncommutative space modifies the standard result of the Aharonov-Bohm effect for electrons and other recent quantum effects. Here we obtain the phase in noncommutative space for the Spavieri effect, a generalization of Aharonov-Bohm effect which involves a coherent superposition of particles with opposite charges moving along a single open interferometric path. By means of the experimental considerations a limit $\sqrt{\theta} \simeq (0.13\,\mathrm{TeV})^{-1}$ is achieved, improving by 10 orders of magnitude the results derived by Chaichian *et al.* [Phys. Lett. B **527**, 149 (2002)] for the Aharonov-Bohm effect. It is also shown that the noncommutative phases of the Aharonov-Casher and He-McKellar-Willkens effects are nullified in the current experimental tests.

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I. INTRODUCTION

Recently there has been a growing interest in studying quantum mechanics in noncommutative (NC) space [1–6]. Because quantum nature experiments are measured with high precision, these are feasible scenarios for setting limits on the experimental manifestation of NC space. The Aharonov-Bohm (AB) effect, in which two coherent beams of charged particles encircle an infinite solenoid [7], has been studied by Chaichian *et al.* [8] and Li and Dulat [9] in the NC space. The expression of the obtained phase includes an additional term dependent on the NC space parameter, θ [measured in units of $(length)^2$]. The limit on θ found in the AB effect is of the order of $\sqrt{\theta} \leqslant 10^6\,\mathrm{GeV}^{-1}$, which corresponds to a relatively large scale of 1 Å [8]. This same approach was extended to the Aharonov-Casher (AC) [10] effect by Li and Wang [11] and Mirza and Zarei [12]; in this effect two coherent beams of neutral particles encircle an infinite charged wire. Considering the reported experimental error of the AC effect ($\sim$25%) [13] a limit $\sqrt{\theta} \leqslant 10^7\,\mathrm{GeV}^{-1}$ is obtained [12,11]. The He-Mckellar-Wilkens (HMW) [14,15] effect, in which neutral particles with electric dipole moment interact with an magnetic field, has been studied in the NC context by Wang and Li [16,17] and Dayi [18], and in the context of the Anandan phase [19] by Passos [20]. There is no experimental report on the parameter limit θ in quantum effects for electric dipoles. We consider here an effect of the AB type, proposed by Spavieri in Ref. [21]. In this effect, two beams of particles with charges $+q$ and $-q$ move along a single side of an infinite solenoid, even though the beams do not enclose the solenoid (as in the ordinary AB effect). The advantage of this effect, called here "the S effect," is that the size of the solenoid has no limit, so that it can be considered to be very large, such as a cyclotron. The S effect has been studied by Spavieri and Rodriguez [22] in the context of massive electrodynamics (or photon mass). Under certain experimental considerations proposed and discussed in Ref. [22], Spavieri and Rodriguez envisage a limit on the mass of the photon of $m_\gamma \sim 10^{-51}$g, which is the best limit obtainable for the photon mass by means of a laboratory experiment with a quantum approach. Consequently, due to the success of the S effect in the photon mass scenario, we derive here the phase of the S effect in the context of the NC quantum mechanics as an application of the phase found by Chaichian *et al.* [8]. Keeping the experimental proposal of Ref. [22] we get a limit on θ in the context of the quantum effects of the AB type. In addition, recent advances in atomic interferometry have allowed obtaining measurements of the HWM phase [23], which allows exploring experimentally the manifestation of the NC space in the HWM effect by means of the phases found in Ref. [20] for these effects. This same analysis may be extended to the experimental configuration proposed by Sangster *et al.* [24] for the AC effect where the particles do not enclose the charged wire.

II. NC QUANTUM MECHANICS

In NC quantum mechanics, the commutation relationships of the position operators satisfy the relation $[\hat{x}_i,\hat{x}_j] = \mathrm{i}\theta_{ij}$, where $\{\theta_{ij}\}$ is a fully antisymmetric real matrix representing the property noncommutativity of space and $\hat{x}_i$ represents the coordinate operator ($\hat{p}_i$ is the corresponding moment operator) in the NC space. In this scenario the product of two functions is replaced by the Moyal-Weyl product (or star "$*$") [25], so the ordinary Schrödinger equation, $H\psi = E\psi$, is written as

$$H(\hat{x}_i,\hat{p}_i) * \psi = E\psi. \tag{1}$$

The star product between two functions in an NC plane $(i,j = 1,2)$ is defined by

$$\begin{aligned}(f * g)(x) &= e^{\frac{\mathrm{i}}{2}\theta_{ij}\partial_{x_i}\partial_{x_j}} f(x_i)g(x_j)\\ &= f(x)g(x) + \frac{\mathrm{i}}{2}\theta_{ij}\partial_i f\partial_j g\Big|_{x_i=x_j} + O(\theta^2),\end{aligned} \tag{2}$$

where $f(x)$ and $g(x)$ are two arbitrary functions. Usually the NC operators are expressed by means of the formulation of the Bopp shift [26] [equivalent to (2)]. This formalism maps the NC problem in the usual commutative space using NC variables defined in terms of the commutative variables. That is to say,

$$\hat{x}_i = x_i - \frac{1}{2\hbar}\theta_{ij} p_j, \quad i,j = 1,2, \tag{3}$$

where the variables x_i and p_i satisfy the usual canonical commutation relations, $[x_i,x_j] = 0$, $[x_i,p_j] = i\hbar\delta_{ij}$, and $[p_i,p_j] = 0$. With these considerations the Hamiltonian undergoes a coordinate transformation, $H(\hat{x}_i,\hat{p}_i) = H(x_i - \frac{1}{2\hbar}\theta_{ij} p_j, p_i)$. Note that $\theta_{ij} \ll 1$, so that the effects of the NC space can always be treated as a disturbance. If we consider a particle of mass m and charge q in the presence of a magnetic field (or potential vector A_i), then the Hamiltonian in the space NC, $H(\hat{x}_i,\hat{p}_i,\hat{A}_i)$ undergoes a Bopp shift in both $\hat{x}_i$ and $\hat{A}_i$. Therefore, in the NC space and with a magnetic field the Schrödinger equation takes the form

$$\frac{\hbar^2}{2m}\left(p_i - qA_i - \frac{1}{2}q\theta_{lj}p_l\partial_j A_i\right)^2 \psi = E\psi, \tag{4}$$

whose solution is

$$\psi = \psi_0 \exp\left[\mathrm{i}\frac{q}{\hbar}\int_{x_0}^{x}\left(A_i + \frac{1}{2}\theta_{lj}p_l\partial_j A_i\right)dx_i\right], \tag{5}$$

where ψ_0 is the solution of (4) when $A_i = 0$.

III. PHASE OF THE S EFFECT IN NC QUANTUM MECHANICS AND LIMIT OVER θ

In Ref. [21] Spavieri pointed out that the amount observable in the AB effect is actually the phase difference

$$\Delta\varphi = \frac{e}{\hbar}\left[\int \mathbf{A}\cdot d\mathbf{l} - \int \mathbf{A}_0\cdot d\mathbf{l}\right], \tag{6}$$

where the integral can be taken over an open path integral. For the usual closed path C encircling the solenoid and limiting the surface S, the observable quantity is the phase-shift variation, $\Delta\phi \propto \oint_C \mathbf{A}\cdot d\mathbf{l} - \oint_C \mathbf{A}_0\cdot d\mathbf{l} = \oint_C \mathbf{B}\cdot d\mathbf{S} - \oint_C \mathbf{B}_0\cdot d\mathbf{S}$. In fact, in interferometric experiments involving the AB and AC effects [24,27] the direct measurement of the phase $\varphi \propto \int \mathbf{A}\cdot d\mathbf{l}$ or phase shift $\phi \propto \oint \mathbf{A}\cdot d\mathbf{l}$ is impossible in principle without the comparison of the actual interference pattern, due to $\mathbf{A}$, with an interference reference pattern, due to $\mathbf{A}_0$. Thus, φ o ϕ are not observable, but the variations $\Delta\varphi$ and $\Delta\phi$ are both gauge-invariant observable quantities [21]. Therefore, with these considerations introduced by Spavieri [21], it is possible consider an effect of the AB type without particles encircling a solenoid. In this case the particles must have opposite charges, $\pm e$, moving along one side of the solenoid, i.e., along path b. Thus, the phase of this effect called the Spavieri effect, S, is

$$\Delta\varphi_S = \frac{e}{\hbar}\left[\int_b \mathbf{A}\cdot d\mathbf{l} - \int_b \mathbf{A}\cdot d\mathbf{l}\right] = \frac{2e}{\hbar}\int_b \mathbf{A}\cdot d\mathbf{l}. \tag{7}$$

Now, we interesting in the phase (7) in the context of NC quantum mechanics. Substituting the phase (5) in (7), and retaining only the term related with the parameter θ, we obtain the correction due to NC space for the S effect, thus:

$$\Delta\varphi_S^{\mathrm{NC}} = \frac{e}{\hbar}\int_b \theta_{lj}p_l\partial_j A_i\, dx_i. \tag{8}$$

Writing (8) in Cartesian coordinates, we obtaine the phase shift of the S effect in NC space:

$$\Delta\phi_S^{\mathrm{NC}} = -\frac{em}{4\hbar^2}\vec{\theta}\cdot\int\left[(\mathbf{v}\times\nabla A_i) - \frac{e}{m}(\mathbf{A}\times\nabla A_i)\right]dx_i, \tag{9}$$

where $i = 1,2$ are Cartesian components x and y, m is the mass of the electron, and $\mathbf{v}$ is the velocity of particles. Although the effect for $\pm q$ charged particles is viable [21], the technology and interferometry for testing this effect need improvements. It is worth recalling that not long ago the technology and interferometry for beams of particles with opposite magnetic $\pm\mathbf{m}$ or electric $\pm\mathbf{d}$ dipole moments were likewise unavailable but are today a reality [24,28]. Discussions of this subject may act as a stimulating catalyst for further studies and technological advances that will lead to experimental tests of this quantum effect. An important step in this direction has already been made [21] by showing that, at least in principle and as far as gauge invariance requirements are concerned, this effect is physically feasible.

In the experimental setups detecting the traditional AB effect limitations are imposed by the suitable type of interferometer related to the electron wavelength, the corresponding convenient size of the solenoid or toroid, and the maximum achievable size ρ of the coherent electron beam encircling the magnetic flux [29]. In the analysis made by Boulware and Deser [29] in the context of the limit of mass photon, the radius of the solenoid is $a = 0.1$cm, and ρ is taken to be about 10 cm, implying that the electron beam keeps its state of coherence up to a size $\rho = 10^2 a$, i.e., 50 times the solenoid diameter. The advantage of the approach for the $\pm q$ beam of particles is that the dimension of the solenoid has no upper limits and is conditioned only by practical limits of the experimental setup, while the size of the coherent beam of particles plays no important role. Due to these advantages of the approach introduced by Spavieri and its success in the exploration of the limit of the mass of the photon, one may ask which limit could be reached for the parameter θ of the NC quantum mechanics. To answer this question, we consider that the electrons move along the straight line $y = y_0$ from $x = -x_0$ to $x = x_0$ [open path b in (8)], with a velocity $\mathbf{v} = v\mathbf{i}$, thus $i = x$ in (9). In addition, as in Ref. [8], here we consider that $\vec{\theta} = \theta\mathbf{z}$. To complete the calculation it is necessary to know the component A_x of the external vector potential to an infinite solenoid,

$$A_x = -B_0\frac{a^2}{2}\left(\frac{y}{x^2+y^2}\right),$$

thus,

$$\left[\nabla A_x = B_0\frac{a^2}{2}\left(\frac{2xy}{(x^2+y^2)^2}\mathbf{i} - \frac{(x^2-y^2)}{(x^2+y^2)^2}\mathbf{j}\right)\right].$$

Therefore, the terms in parentheses of (9) are

$$\mathbf{v}\times\nabla A_x = -B_0\frac{a^2 v}{2}\frac{(x^2-y^2)}{(x^2+y^2)^2}\mathbf{z} \tag{10}$$

and

$$\mathbf{A}\times\nabla A_x = -\frac{1}{4}\frac{B_0^2 a^4 y}{(x^2+y^2)^2}\mathbf{z}, \tag{11}$$

where a is the radius of solenoid and B_0 is the magnetic field enclosed within the solenoid. Substituting (10) and (11) in (9) we obtain

$$\delta\phi_{\theta}^{\rm NC} = \frac{em}{2\hbar^2}\vec{\theta}\cdot\int\left[\begin{array}{c} -B_0\frac{a^2 v}{2}\frac{(x^2-y^2)}{(x^2+y^2)^2}\mathbf{z} \\ +\frac{e}{m}\frac{1}{4}\frac{B_0^2 a^4 y}{(x^2+y^2)^2}\mathbf{z}\end{array}\right]dx,$$

and performing the integration of 0 to $-x_0$ (because the integrand is even) and remembering that $\vec{\theta} = \theta\mathbf{z}$, the correction NC to the phase of the S effect is calculated by means of the following expression:

$$\delta\phi_{\theta}^{\rm NC} = \frac{em}{2\pi\hbar^2}\theta\Phi\left[\begin{array}{c}\left(\frac{e}{m}\frac{\Phi y}{2\pi}+2vy^2\right)\int_0^x\frac{dx}{(x^2+y^2)^2} \\ -v\int_0^x\frac{dx}{(x^2+y^2)}\end{array}\right] \tag{12}$$

where $\Phi = \pi a^2 B_0$ is the magnetic flux enclosed within the solenoid, and now the integrals in (12) are

$$\int_0^x\frac{dx}{(x^2+y^2)^2} = \frac{1}{2y^3}\left[\arctan\left(\frac{x}{y}\right)+\frac{xy}{x^2+y^2}\right]$$

and

$$\int_0^x\frac{dx}{x^2+y^2} = \frac{1}{y}\arctan\left(\frac{x}{y}\right);$$

thus, the final expression of the phase of the S effect in NC quantum mechanics is

$$\Delta\phi_S^{\rm NC} = \frac{1}{8}\theta\left(\frac{\Phi}{\Phi_0}\right)^2\left\{\begin{array}{c}\frac{\arctan\left(\frac{x}{y}\right)}{y^2}+\frac{x/y}{x^2+y^2} \\ +\frac{8\pi}{\lambda_e}\frac{\Phi_0}{\Phi}\frac{v}{c}\frac{x}{x^2+y^2}\end{array}\right\}, \tag{13}$$

where $\Phi_0 = \frac{h}{2e} = 2{,}06\times10^{-15}\,\mathrm{Tm}^2$ is the quantum flux elemental, $\lambda_e = \frac{h}{mc} = 2{,}42\times10^{-12}\,\mathrm{m}$ is the Compton wavelength of the electron, and c is the speed of light. To estimate a limit on θ here, we consider the same experimental parameters introduced and discussed in Ref. [22] for the study of the mass of the photon in the context of the S effect, which are $a = 5\,\mathrm{m}$, $x = 5a = 30\,\mathrm{m}$, $y = 8a/5 = 8\,\mathrm{m}$, and $B_0 = 10\,\mathrm{T}$. With these parameters it can be demonstrated that the order of magnitude (in units m^{-2}) of the terms in square brackets are the following:

$$\frac{\arctan\left(\frac{x}{y}\right)}{y^2} \sim 10^{-2},$$

$$\frac{x/y}{x^2+y^2} \sim 10^{-3},$$

and

$$\frac{8\pi}{\lambda_e}\frac{\Phi_0}{\Phi}\frac{v}{c}\frac{x}{x^2+y^2} \sim 3{,}3\times10^{-15}v.$$

If the velocity of electrons is $v = 2\times10^8\,\mathrm{m/s}$ as in the Tonomura *et al.* [30] experiment for the Aharonov-Bohm effect, then the order of magnitude of the kinetic term is 10^{-7}. This analysis shows that the kinetic term is up to five times smaller than the geometric terms. This contrasts with the analysis made by Chiachian *et al.* [8] for the AB effect where the kinetic term is five orders greater than the geometric term. Consequently, for the estimation of the limit on θ we consider only the first term in brackets of expression (13):

$$\Delta\phi_S^{\rm NC} \simeq \frac{1}{8}\theta\left(\frac{\Phi}{\Phi_0}\right)^2\frac{\arctan\left(\frac{x}{y}\right)}{y^2}.$$

As the NC correction is a very small, its effect must be masked within experimental error, ϵ, so $\Delta\phi_S^{\rm NC} \leqslant \epsilon$. This same argument is followed in the works related to the estimation of the mass of the photon [22,29,31,32]. According to recent advances in atomic interferometry [33,34], the experimental error that can be reached in the measurement of the quantum phases is of the order of 10^{-4} rad; this can be seen in the measurement of the AC [24], where the phase has been measured with an experimental error of $0.11\,\mathrm{mrad} = 1.1\times10^{-4}\,\mathrm{rad}$. Even Zhout *et al.* [35], by means of simulation, provided for the measurement of the AC phase with a relative error of 10^{-5} rad. Consequently, in this work, to be conservative, it is considered that $\epsilon = 10^{-4}$ rad. Therefore the estimated limit on θ in the context of the S effect is

$$\sqrt{\theta} \leqslant \left[\frac{1}{8y}\left(\frac{\Phi}{\Phi_0}\right)\sqrt{\frac{\arctan\left(\frac{x}{y}\right)}{\epsilon}}\right]^{-1} \simeq [0.13\times\mathrm{TeV}]^{-1},$$

which is 10 orders of magnitude smaller than the value of Chiachian *et al.* [8] for the AB effect.

IV. QUANTUM EFFECT FOR ELECTRIC AND MAGNETIC DIPOLES IN NC QUANTUM MECHANICS

The phase to magnetic dipoles, $\mathbf{m}$ (AC effect) and for electric dipoles, $\mathbf{d}$ (HMW effect) in NC space also has been calculated by Passos *et al.* [20]. The expressions are as follows:

$$\phi_{\rm AC} = \mathrm{i}\oint(\mathbf{m}\times\mathbf{E})\cdot d\mathbf{r} + \frac{\mathrm{i}}{2}m\oint\vec{\theta}\cdot[\mathbf{v}\times\nabla\cdot(\mathbf{m}\times\mathbf{E})]\cdot d\mathbf{r}$$
$$-\frac{\mathrm{i}}{2}m\oint\vec{\theta}\cdot[(\mathbf{m}\times\mathbf{E})\times\nabla\cdot(\mathbf{m}\times\mathbf{E})]\cdot d\mathbf{r}, \tag{14}$$

$$\phi_{\rm HMW} = -\mathrm{i}\oint(\mathbf{d}\times\mathbf{B})\cdot d\mathbf{r} - \frac{\mathrm{i}}{2}m\oint\vec{\theta}\cdot[\mathbf{v}\times\nabla\cdot(\mathbf{d}\times\mathbf{B})]\cdot d\mathbf{r}$$
$$+\frac{\mathrm{i}}{2}m\oint\vec{\theta}\cdot[(\mathbf{d}\times\mathbf{B})\times\nabla\cdot(\mathbf{d}\times\mathbf{B})]\cdot d\mathbf{r}, \tag{15}$$

where m is the mass of electric or magnetic dipoles. In the experimental setup proposed by Sangster *et al.* [24], magnetic dipoles with opposing dipole moments are moving on the same interferometric path. With this configuration of magnetic moments the beams do not need to enclose a charged wire (as in the ordinary AC effect), but are moving in the presence of a homogeneous electric field produced by a capacitor of parallel plates. In the configuration of Sansgter [24] the magnetic dipole moments, $\mathbf{m}$, are perpendicular to the electric field, $\mathbf{E}$; thus the terms $\nabla\cdot(\mathbf{m}\times\mathbf{E})$ in (14) vanish, and the effects of the NC space cannot be observed in this configuration. In the same sense, in a recent experiment carried out to observe the HMW phase [23], the electric dipole moments, $\mathbf{d}$, of the beams are perpendicular to the magnetic field, $\mathbf{B}$, involved in the effect. Thus, the terms $\nabla\cdot(\mathbf{d}\times\mathbf{B})$ in the expression (15) vanish, and the NC effect, as a function of the expression (15) derived by Passos *et al.* (15), is not evidenced in this configuration.

Another proposed configuration for observing the AB effect for electric dipoles is known as the Takchuk effect [36]. In this configuration, we consider two infinite wires with an opposite magnetic polarization dependent on the length of the wire, $M(z) = -qz$, where q can be treated as a linear magnetic charge density. If the wires are sufficiently long, the magnetic vector potential can be written as $\mathbf{A}_T = z\mathbf{A}_{\mathrm{AB}}$, $\mathbf{A}_{\mathrm{AB}}$ being the ordinary vector potential of the AB effect. Thus, the magnetic field is $\mathbf{B} = \nabla \times \mathbf{A}_T = z(\nabla \times \mathbf{A}_{\mathrm{AB}}) - \mathbf{A}_{\mathrm{AB}} \times \mathbf{z}$. In this effect the beams of magnetic dipoles move in the middle plane of the wires, that is, $z = 0$, with their polarization, $\mathbf{d}$, parallel to the wire axis. The term of interest in the NC context according to (15) is $\mathbf{d} \times \mathbf{B}$. Therefore, $\mathbf{d} \times \mathbf{B} = d\mathbf{z} \times [z(\nabla \times \mathbf{A}_{\mathrm{AB}}) - \mathbf{A}_{\mathrm{AB}} \times \mathbf{z}]$, evaluated at $z = 0$, we obtain that $\mathbf{d} \times \mathbf{B} = d\mathbf{A}_{\mathrm{AB}}$, but $\nabla \cdot (\mathbf{d} \times \mathbf{B}) = d\nabla \cdot \mathbf{A}_{\mathrm{AB}} = 0$ for the ordinary AB effect (Coulomb's gauge). This implies that in the scenario presented by Takchuk [36], the NC effect cannot be observed.

V. CONCLUSIONS

We have derived the AB phase for open path, or S effect, in the context of NC quantum mechanics [Eq. (13)]. Considering the experimental parameters of the S effect to obtain a limit on mass photon, here we obtained an upper limit on the NC parameter, $\sqrt{\theta} \leqslant (0.13\,\mathrm{TeV})^{-1}$, 10 orders of magnitude smaller than in previous scenarios of the AB type, and three orders of magnitude smaller that the limit derived by Moumni *et al.* [37] $\sqrt{\theta} \leqslant (0.16\,\mathrm{GeV})^{-1}$, in the context of the energy lines of the hydrogen atom, which is also a quantum scenario. It can also be observed that the kinetic term in our result, which includes the speed and mass of the particle, is not relevant for θ calculation since it is several orders of magnitude smaller than the geometric terms, which is opposite to the result found by Chaichian *et al.* [8]. Also we have shown that the NC effects are not manifested in the experimental configurations (for an interferometric open path) proposed by Sangster *et al.* [24] for the AC effect and the proposal of Lepoutre *et al.* [23,38] and Takchuk [36] for the HMW effect from the point of view of the phases derived by Passos *et al.* [20] for these purposes. Finally, it is important to mention that these results can be improved in the future due to the development of atomic interferometers, with more precision and longer interferometric paths [33].

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[1] K. Ma and J.-H. Wang, Ann. Phys. **383**, 120 (2017).
[2] K. Ma, J.-H. Wang, and H.-X. Yang, Phys. Lett. B **759**, 306 (2016).
[3] A. Benchikha, M. Merad, and T. Birkandan, Mod. Phys. Lett. A **32**, 1750106 (2017).
[4] O. Bertolami and P. Leal, Phys. Lett. B **750**, 6 (2015).
[5] S. Kováčikand and P. Prešnajder, J. Math. Phys. **58**, 012101 (2017).
[6] M. Ababekri, A. Abduwali, H. Mamatabdulla, and R. Reyima, Front. Phys. **4**, 1 (2016).
[7] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
[8] M. Chaichian, P. Prešnajder, M. Sheikh-Jabbari, and A. Tureanu, Phys. Lett. B **527**, 149 (2002).
[9] K. Li and S. Dulat, Eur. Phys. J. C **46**, 825 (2006).
[10] Y. Aharonov and A. Casher, Phys. Rev. Lett. **53**, 319 (1984).
[11] K. Li and J. Wang, Eur. Phys. J. C **50**, 1007 (2007).
[12] B. Mirza and M. Zarei, Eur. Phys. J. C **32**, 583 (2004).
[13] A. Cimmino, G. I. Opat, A. G. Klein, H. Kaiser, S. A. Werner, M. Arif, and R. Clothier, Phys. Rev. Lett. **63**, 380 (1989).
[14] X.-G. He and B. H. J. McKellar, Phys. Rev. A **47**, 3424 (1993).
[15] M. Wilkens, Phys. Rev. Lett. **72**, 5 (1994).
[16] J. Wang and K. Li, J. Phys. A: Math. Theor. **40**, 2197 (2007).
[17] J.-H. Wang and K. Li, Chin. Phys. Lett. **24**, 5 (2007).
[18] O. F. Dayi, Europhys. Lett. **85**, 41002 (2009).
[19] J. Anandan, Phys. Rev. Lett. **85**, 1354 (2000).
[20] E. Passos, L. R. Ribeiro, C. Furtado, and J. R. Nascimento, Phys. Rev. A **76**, 012113 (2007).
[21] G. Spavieri, Eur. Phys. J. D **37**, 327 (2006).
[22] G. Spavieri and M. Rodriguez, Phys. Rev. A **75**, 052113 (2007).
[23] S. Lepoutre, J. Gillot, A. Gauguet, M. Büchner, and J. Vigué, Phys. Rev. A **88**, 043627 (2013).
[24] K. Sangster, E. A. Hinds, S. M. Barnett, and E. Riis, Phys. Rev. Lett. **71**, 3641 (1993).
[25] O. Man'ko, V. Man'ko, G. Marmo, and P. Vitale, Phys. Lett. A **360**, 522 (2007).
[26] S. Dulat and K. Li, Mod. Phys. Lett. A **21**, 2971 (2006).
[27] R. G. Chambers, Phys. Rev. Lett. **5**, 3 (1960).
[28] J. P. Dowling, C. P. Williams, and J. D. Franson, Phys. Rev. Lett. **83**, 2486 (1999).
[29] D. G. Boulware and S. Deser, Phys. Rev. Lett. **63**, 2319 (1989).
[30] A. Tonomura, Rev. Mod. Phys. **59**, 639 (1987).
[31] B. Neyenhuis, D. Christensen, and D. S. Durfee, Phys. Rev. Lett. **99**, 200401 (2007).
[32] M. Rodriguez, Rev. Mex. Fis. **55**, 373 (2009).
[33] W. Jin, Chin. Phys. B **24**, 053702 (2015).
[34] T. L. Gustavson, A. Landragin, and M. A. Kasevich, Class. Quantum Grav. **17**, 2385 (2000).
[35] M.-K. Zhou, K. Zhang, X.-C. Duan, Y. Ke, C.-G. Shao, and Z.-K. Hu, Phys. Rev. A **93**, 023641 (2016).
[36] V. M. Tkachuk, Phys. Rev. A **62**, 052112 (2000).
[37] M. Moumni, A. BenSlama, and S. Zaim, J. Geom. Phys. **61**, 151 (2011).
[38] S. Lepoutre, A. Gauguet, G. Trénec, M. Büchner, and J. Vigué, Phys. Rev. Lett. **109**, 120404 (2012).