Nonlinearities in reservoir engineering: Enhancing quantum correlations

Xiangming Hu,[1,*] Qingping Hu,[1,2] Lingchao Li,[1] Chen Huang,[1] and Shi Rao[1]
[1]*College of Physical Science and Technology, Central China Normal University, Wuhan 430079, People's Republic of China*
[2]*Institute of Technical Physics, SEEE, Wuhan Textile University, Wuhan 430073, People's Republic of China*

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There are two decisive factors for quantum correlations in reservoir engineering, but they are strongly reversely dependent on the atom-field nonlinearities. One is the squeezing parameter for the Bogoliubov modes-mediated collective interactions, while the other is the dissipative rates for the engineered collective dissipations. Exemplifying two-level atomic ensembles, we show that the moderate nonlinearities can compromise these two factors and thus enhance remarkably two-mode squeezing and entanglement of different spin atomic ensembles or different optical fields. This suggests that the moderate nonlinearities of the two-level systems are more advantageous for applications in quantum networks associated with reservoir engineering.

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I. INTRODUCTION

The squeezed and entangled states of atomic ensembles and optical fields are of considerable theoretical and experimental interest to the quantum optics and quantum information communities [1–4]. The atomic ensembles, which are ideal nodes in quantum networks for storing and processing local quantum information, have collectively enhanced interactions with electromagnetic fields, provide one with efficient and controllable coupling to nonclassical light fields [5–10]. The optical fields are known to be the most suitable carriers to transmit quantum information from one node to another. Of particular interest is the creation of squeezed and entangled states, which behave as indispensable resources in quantum information and quantum communication networks.

Reservoir dissipation is normally regarded as the worst enemy for preparation and preservation of quantum coherence. However, an appropriately engineered reservoir dissipation is a very efficient and robust way to make a system into a desired state. On one hand, vacuum fields mediate Raman transitions between stable atomic ground states and induce collective dissipation of the ground state spin atomic ensembles [11–14]. Krauter *et al.* [15] reported the experimental realization of one such scheme. On the other hand, atoms act as an engineered dissipative reservoir in high-Q resonators for driving two optical fields to squeezed and entangled states [16–19]. In addition, a great number of related schemes have been proposed, which involve squeezed states of atomic ensembles [20–23], the engineered dissipation for entanglement of single atoms [24–28], engineered dissipation for other quantum states of atoms [29–32], different ways for entanglement of distant atomic ensembles [33–35], entanglement of distant mechanical oscillators [36–41], and entanglement of Bose-Einstein condensates [42].

In the reservoir engineering for two-mode squeezing and entanglement, two factors combine to determine the two-mode quantum correlations and depend strongly on nonlinearities or saturations. One is the squeezing parameter r by which the two individual modes $\sigma_{1,2}$ (e.g., spins) combine into the Bogoliubov-like modes and interact collectively with the vacuum modes [43]

$$\pi_1 = \sigma_1 \cosh r - \sigma_2^+ \sinh r,$$
$$\pi_2 = \sigma_2 \cosh r - \sigma_1^+ \sinh r. \quad (1)$$

The other is the dissipative rates $A_{1,2}$, by which these Bogoliubov modes relax under adiabatic condition according to the master equation of the density operator ρ_a like [44]

$$\dot{\rho}_a = \sum_{l=1}^{2} \frac{A_l}{2}(2\pi_l \rho_a \pi_l^+ - \pi_l^+ \pi_l \rho_a - \rho_a \pi_l^+ \pi_l). \quad (2)$$

The ideal case is $r \to \infty$ and $A_{1,2} \gg \Gamma_{1,2}$, where $\Gamma_{1,2}$ are the damping rates for the atoms due to the environment vacuum. When these conditions are satisfied, the two-mode variance tends to vanish, that is, the Einstein-Podolsky-Rosen entangled state is obtainable [45]. If the squeezing parameter r is not so large, the best achievable two-mode variance below the standard quantum level is e^{-2r}. The squeezing degree is finally limited by the squeezing parameter r. The larger the squeezing parameter the more the squeezing degree, and vice versa. The e^{-2r} squeezing is only possible when the $\pi_{1,2}$ modes can rapidly relax to the vacuum state. This also requires that the engineered dissipation dominates over the vacuum dissipation [11–19].

To our knowledge, two assumptions have been made generally in previous work. One assumption, as a primary one, is that the squeezing parameter r and the engineered reservoir dissipation rates $A_{1,2}$ can both be large [11–19]. Perhaps this is possible for some specific situations. However, we notice that the squeezing parameter r and the dissipative rates $A_{1,2}$ depend on the nonlinearities of the atom-field interactions in drastically different ways. There exist two limiting cases. For weak nonlinearity, the dissipation rates can be large while the squeezing parameter is considerably small. For too strong nonlinearity, the squeezing parameter is large while the dissipation rates cannot be enough to overcome the vacuum dissipation. In both limiting cases, the squeezing parameter and the engineered reservoir dissipation rates cannot be simultaneously large, and thus the squeezing is either negligibly weak or even vanishing. The other assumption, as a secondary one, is that adiabatic conditions are satisfied [11–19]. For interacting atomic ensembles and optical fields,

*Corresponding author: xmhu@mail.ccnu.edu.cn

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when one is considered as a system, the other as a reservoir is assumed to relax much more rapidly and is allowed to be eliminated adiabatically. More often than not, however, the interacting parties have comparable relaxation times [46,47], and the adiabatic elimination no longer holds.

Here we would like to examine the effects of the atom-field nonlinearities on the squeezing parameter and the dissipation rates and to optimize the conditions for the reservoir engineering. In particular, we consider the simultaneous interactions of two dressed two-level atomic ensembles with two cavity quantum fields. The important purpose is to see whether it is possible to make the squeezing parameter and the dissipative rates as compatibly large as possible in the regime of moderate nonlinearities. Our analysis is performed without adiabatic elimination. As expected, we find that, with moderate nonlinearities, the squeezing parameter is remarkably enhanced at a proper cost of decreasing the dissipative rates. The balance between them by the moderate nonlinearities remarkably enhances the two-mode squeezing and entanglement of separated spin atomic ensembles or different optical fields. The enhanced efficiency with the moderate nonlinearities is interesting in applications in quantum nodes based on the reservoir engineering.

The remaining part of the present paper is organized as follows. Given in Sec. II are the model and nonlinearities. Section III describes the nonlinearity-induced dissipation, and Sec. IV presents the two-mode spin squeezing and entanglement. In Sec. V we compare with the two-mode field squeezing and entanglement. Finally, the conclusion is given in Sec. VI.

II. MODEL AND NONLINEARITY

Our purpose is to study the simultaneous interactions of two atomic ensembles with two cavity quantum fields. For this purpose, we propose a possible setup as shown in Fig. 1(a). In this setup, two single-ended folded cavities cross at two crossing sections, in which are placed two atomic ensembles of different but close resonance frequencies. One common coherent field drives the atomic ensembles and induces the Rabi resonances with quantized cavity fields. The separated cavities are advantageous for choosing different but close frequencies and for interacting simultaneously with the two atomic ensembles. The folded cavities are often used in correlated-emission lases [17] and a system of two or more mechanical oscillators coupling different cavity fields [36]. An alternative setup can be based on separate, cascaded cavities [11,24], which are coupled through the unidirectional coupling from one cavity to the other cavity [48]. In this case the coupling efficiency between the two cavities should be taken into account. Here we consider the folded and crossed cavities for the sake of simplicity. The frequencies of the involved elements are shown in Fig. 1(b). The circular frequency of the dressing field ω_0 locates at the center, the circular frequencies of the atoms $\omega_{1,2}$ lie at the inner positions, and the circular frequencies of the cavity modes $\nu_{1,2}$ are seated at the outer positions.

The master equation for the density operator ρ of the atom-field system is derived in the dipole approximation and in an appropriate rotating frame as [44]

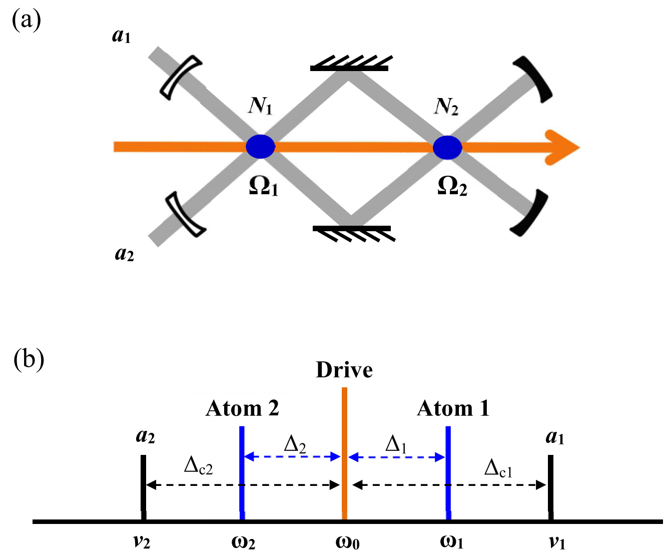

FIG. 1. (a) Possible double-cavity setup. Two folded cavities cross at crossing sections, where two atomic ensembles $N_{1,2}$ are driven by a strong field with Rabi frequencies $\Omega_{1,2}$ and are coupled to each of two cavity modes $a_{1,2}$. (b) The circular frequency of the dressing field ω_0 locates between the circular frequencies of the atoms $\omega_{1,2}$, and the circular frequencies of the cavity modes $\nu_{1,2}$ lie at the two ends.

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}\rho, \tag{3}$$

with the total Hamiltonian

$$H = H_0 + H_I, \tag{4}$$

where

$$H_0 = \hbar \sum_{l=1}^{2} \left[\Delta_l \sigma_l^+ \sigma_l + \frac{1}{2}(\Omega_l \sigma_l^+ + \Omega_l^* \sigma_l) \right] \tag{5}$$

describes the free terms of atoms and the interactions between the atoms and the dressing field, and

$$H_I = \hbar \sum_{k=1}^{2} \Delta_{c_k} a_k^\dagger a_k + \hbar \sum_{k,l=1}^{2} g_{kl}(a_k \sigma_l^+ + \sigma_l a_k^\dagger) \tag{6}$$

denotes the free terms of two cavity modes and the interactions of cavity fields with the atoms. Of the above formulas, $\hbar$ is the Planck constant. $\sigma_l = \sum_{\mu=1}^{N_l} \sigma_{l\mu}$ ($\sigma_{l\mu} = |1_{l\mu}\rangle\langle 2_{l\mu}|$) are the collective spin-flip operators of the lth ensemble ($l = 1,2$), $|1_l\rangle$ and $|2_l\rangle$ are the ground and excited states respectively, and N_l are the numbers of lth atomic ensemble. a_l and $a_l^\dagger$ are the annihilation and creation operators for the cavity modes. $\Delta_l = \omega_l - \omega_0$ and $\Delta_{c_l} = \nu_l - \omega_0$ are the detunings of the atomic transition circular frequencies ω_l and the cavity mode resonance circular frequencies ν_l, respectively, with respect to the dressing field circular frequency ω_0. $\Omega_l = \mu_l E_l/\hbar$ are Rabi frequencies associated with the lth atomic ensemble, where μ_l are the electric dipole moments and E_l are the electric fields of the dressing fields. g_{kl} are the coupling strengths between the kth cavity field and the lth atomic ensemble. The damping term in the master equation (3) takes the form [44]

$$\mathcal{L}\rho = \sum_{l=1}^{2}\left(\frac{\gamma_l}{2}\sum_{\mu=1}^{N_l}\mathcal{L}_{\sigma_{l\mu}}\rho + \frac{\kappa_l}{2}\mathcal{L}_{a_l}\rho\right), \quad (7)$$

where $\mathcal{L}_{\sigma_{l\mu}}\rho$ and $\mathcal{L}_{a_l}\rho$ describe the atomic and cavity relaxations with rates γ_l and κ_l, respectively, and

$$\mathcal{L}_o\rho = 2o\rho o^\dagger - o^\dagger o\rho - \rho o^\dagger o, \quad o = \sigma_{1\mu},\sigma_{2\mu},a_{1,2}. \quad (8)$$

Throughout this article we assume that the cavity quantum fields are negligibly weak compared with the classical dressing field. It is the case because the dressed atoms are in wave-mixing interactions with the cavity quantum fields [49]. In principle, the cavity quantum fields can be in different vacuum states. The susceptibility can be obtained by neglecting the quantum cavity fields and calculating the polarizations $P_l = \mu_l\langle\sigma_l\rangle$ for the lth atomic ensemble. The susceptibilities induced by the dressing fields are $\chi_l = P_l/(\varepsilon_0 E_l)$, which are derived as [49]

$$\chi_l = \frac{|\mu_l|^2 N_l}{\varepsilon_0\hbar}\frac{-2\Delta_l + i\gamma_l}{4\Delta_l^2 + \gamma_l^2 + 2|\Omega_l|^2}, \quad l = 1,2, \quad (9)$$

where ε_0 is the permittivity of vacuum. The imaginary and real parts of χ_l describe the absorption and the dispersion, respectively, of the atomic ensembles. The $|\Omega_l|^2$ term in the denominator represents the saturation or the nonlinearity [49]. Usually, in order to avoid the absorption and the spontaneous emission one resorts to the far-off resonance case $|\Delta_l| \gg (\gamma_l,|\Omega_l|)$, in which the dispersion dominates over the absorption and the nonlinearity is negligibly small. It is easy to imagine that it is difficult to induce the interaction and correlation between the cavity quantum fields a_1 and a_2 for the present two-level atomic system, unless we go beyond the weak nonlinearity.

In contrast, the essence of the reservoir engineering is to introduce the induced absorption to the collective or Bogoliubov modes. In order to examine the effects of the atom-field nonlinearities on the squeezing parameter and the dissipation rates and to optimize the conditions for the reservoir engineering, we have to go beyond the weak nonlinearity. As $|\Omega_l|^2$ increases, the saturation becomes deep and the nonlinearity is enhanced. The nonlinearities are in divergent series of indefinitely high orders if we expand the susceptibility in terms of $(|\Omega_l|/\Delta_l)^2$ when $|\Omega_l/\Delta_l| \gtrsim 1$ and $\Delta_l \gg \gamma_l$. For the strong nonlinearities, the perturbative expansions are no longer valid, and we have to keep all terms of the high orders in $(|\Omega_l|/\Delta_l)^2$. Therefore the nonlinearity of the present two-level system due to the dressing field is determined by the ratio of the Rabi frequency to the detuning

$$\eta_l = \frac{|\Omega_l|}{\Delta_l}, \quad (10)$$

which we call the nonlinear parameter for convenience. In what follows we will consider the effects of the moderate and even considerably strong nonlinearity induced by the dressing field,

$$|\eta_l| \gtrsim 1. \quad (11)$$

III. NONLINEARITY-INDUCED DISSIPATION

Now we turn to discussing the effects of the nonlinearities on the interactions between the dressed atoms and the cavity quantum fields. The dressed state picture is best suitable for this purpose. By diagonalizing the Hamiltonian H_0, we obtain the dressed states that are expressed in terms of the bare atomic states as [50]

$$|+_l\rangle = \sin\theta_l e^{-i\phi_l}|1_l\rangle + \cos\theta_l|2_l\rangle,$$
$$|-_l\rangle = \cos\theta_l|1_l\rangle - \sin\theta_l e^{i\phi_l}|2_l\rangle, \quad (12)$$

with $\cos^2\theta_l = \frac{1}{2} + \frac{\Delta_l}{2\bar{\Omega}_l}$, $\sin^2\theta_l = \frac{1}{2} - \frac{\Delta_l}{2\bar{\Omega}_l}$, $\bar{\Omega}_l = \sqrt{\Delta_l^2 + |\Omega_l|^2}$, and $\phi_l = \arg\Omega_l$. Because of the simultaneous interactions of the two atomic ensembles with the two fields, the phases can be unequal, $\phi_1 \neq \phi_2$. The relative weights of the components $|1_l\rangle$ and $|2_l\rangle$ in the superposition states $|\pm_l\rangle$ are simply determined by the ratios $\eta_{1,2}$ of the Rabi frequencies to the detunings. As will be shown below, it is the parameters $\eta_{1,2}$ that determine the nonlinearity-induced dissipative atom-field interactions.

In the dressed states representation the Hamiltonian of the dressed atoms H_0 becomes the free form

$$H_0 = \sum_{l=1}^{2}\hbar(\lambda_l^+\sigma_{++}^{(l)} + \lambda_l^-\sigma_{--}^{(l)}), \quad (13)$$

where $\sigma_{\pm\pm}^{(l)} = \sum_{\mu=1}^{N_l}|\pm_{l\mu}\rangle\langle\pm_{l\mu}|$. These dressed states $|\pm_l\rangle$ correspond respectively to the eigenvalues $\lambda_l^\pm = \hbar(\Delta_l \pm \bar{\Omega}_l)/2$, which give the Rabi resonances at sidebands $\omega_{l\pm} = \omega_0 \pm \bar{\Omega}_l$. We assume that the following conditions are well satisfied, i.e.,

$$\bar{\Omega}_{1,2} \gg (\gamma_{1,2},\kappa_{1,2}), \quad (14)$$

which guarantee that the dressed states are well separated from each other. Applying the dressed states transformation to the atomic relaxation terms (i.e., the γ's terms) and assuming that the cavity fields to be in their vacuum states temporarily (as shown below, it will be the case), we obtain the steady-state populations $N_l^\pm = \langle\sigma_{\pm\pm}^{(l)}\rangle$ as

$$N_l^+ = \frac{N_l\sin^4\theta_l}{\cos^4\theta_l + \sin^4\theta_l}, \quad N_l^- = \frac{N_l\cos^4\theta_l}{\cos^4\theta_l + \sin^4\theta_l}. \quad (15)$$

We tune the cavity fields resonant with the Rabi sidebands, $\Delta_{c_1} = -\Delta_{c_2} = \bar{\Omega}_1 = \bar{\Omega}_2$. Making the further unitary transformation and neglecting rapidly oscillating terms, we obtain the atom-field interaction Hamiltonian

$$\begin{aligned} H_I = {}& \hbar a_1^\dagger(\tilde{g}_{11}\cos^2\theta_1\tilde{\sigma}_1 + \tilde{g}_{12}\cos^2\theta_2\tilde{\sigma}_2^+) \\ & - \hbar a_2^\dagger(\tilde{g}_{22}\sin^2\theta_2 e^{2i\phi_2}\tilde{\sigma}_2 + \tilde{g}_{21}\sin^2\theta_1 e^{2i\phi_1}\tilde{\sigma}_1^+) \\ & + \text{H.c.}, \end{aligned} \quad (16)$$

where we have used H.c. to denote the Hermitian conjugate of the terms before it. We have also defined the effective atom-field coupling strengths $\tilde{g}_{kl} = g_{kl}\sqrt{J_l}$, and the spin down-flip operators $\tilde{\sigma}_1 = \sum_{\mu=1}^{N_1}|-_{1\mu}\rangle\langle+_{1\mu}|/\sqrt{J_1}$, $\tilde{\sigma}_2 = \sum_{\mu=1}^{N_2}|+_{2\mu}\rangle\langle-_{2\mu}|/\sqrt{J_2}$ for $\Delta_{1,2} > 0$, respectively. $J_l = |N_l^+ - N_l^-|$ are the total spins, which are assumed to be large. Here we are dealing with the atomic ensemble spins which are considerably excited and deviate from the Holstein-Primakoff approximation [51]. The fluctuations due to the deviation,

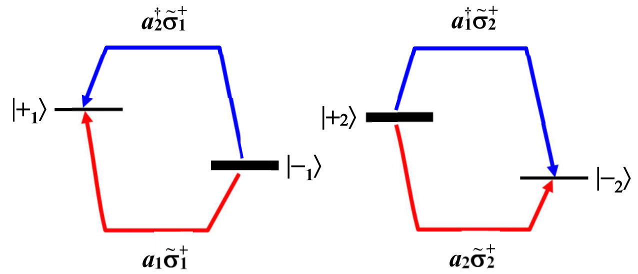

FIG. 2. Atom-field interactions in the dressed state picture (i.e., in a rotating frame). Each spin flip-up of the dressed states is companied with both the annihilation of one photon (blue lines) and the creation of the other photon (red lines).

which are included in the following two sections for the analytic and numerical calculations, are neglected temporarily for the present qualitative analysis. For $\Delta_{1,2} < 0$ we exchange $\tilde{\sigma}_l$ and $\tilde{\sigma}_l^+$. The new spin operators have commutation relations $[\tilde{\sigma}_l,\tilde{\sigma}_l^+] \doteq 1$ $(l = 1,2)$ and $[\tilde{\sigma}_1,\tilde{\sigma}_2] = [\tilde{\sigma}_1,\tilde{\sigma}_2^+] = 0$.

We see from Hamiltonian (16) that the nonlinearity by the dressing field causes the characteristic features such as four aspects as follows.

(i) *The spin operators $\tilde{\sigma}_{1,2}$ are just those orthogonal to the total spins.* For the spins, to be useful for noise consideration in actual experiments and applications, one generally needs to rotate the original frame of reference to make the total spin be in one of the coordinate axes. The fluctuations to limit the measurement precision are those of the quadratures in a plane orthogonal to the direction of the total spin. The mean values of the spin components in such a plane are zero. For the present case, the dressed transform has twofold roles. One is to describe the direct interactions of the fields with the dressed atoms, and the other is to perform a frame rotation and to define the total spins $J_l = |N_l^+ - N_l^-|$, $l = 1,2$ and the quadratures orthogonal to the total spins. These quadratures simply are the combinations of the operators $\tilde{\sigma}_{1,2}$.

(ii) *Each spin ensemble is in a down-conversion-like interaction with one different field and in a beam-splitter-like interaction with the other field.* As shown in Fig. 2, the spin flip-up $\tilde{\sigma}_1^+$ of the atomic ensemble 1 happens with the annihilation of photon a_1 and the creation of photon a_2, while the spin flip-up $\tilde{\sigma}_2^+$ of the atomic ensemble 2 goes with the creation of photon a_1 and the annihilation of photon a_2. Each spin flip-up is accompanied with the annihilation of one different photon and the creation of the other photon. Usually, the down-conversion-like interaction leads to two-mode squeezing, while the beam-splitter-like interaction transfers the quantum state from one to the other. When the cavity fields as two engineered reservoir components decay much more rapidly than the atoms, the two atomic spin ensembles as two system modes will undergo the engineered dissipation effects. The cavity mode a_1 (a_2) entangles with the spin $\tilde{\sigma}_2$ ($\tilde{\sigma}_1$) and transfers immediately its state to $\tilde{\sigma}_1$ ($\tilde{\sigma}_2$). In this way the cavity fields will possibly make the two atomic ensembles $\tilde{\sigma}_{1,2}$ evolve into the two-mode squeezed and entangled state.

(iii) *In addition to the above two aspects, the nonlinearities are also merged into the effective atom-field coupling strengths.* In principle, such nonlinearities in $g_{1l}\cos^2\theta_l$ and $g_{2l}\sin^2\theta_l$ $(l = 1,2)$ will determine whether the squeezing parameter and the engineered reservoir dissipation rates are compatibly large. If $g_{12}\cos^2\theta_2$ and $g_{21}\sin^2\theta_1$ are too small, then the squeezing cannot be established due to the absence of the down-conversion-like interactions. This corresponds to the weak nonlinearity case. Therefore, it is necessary to go to moderate nonlinearity for enhancing quantum correlations.

(iv) *The squeezing parameter and the engineered reservoir dissipation rates have opposite dependences on the nonlinearity, and an optimization for large cross correlation exists for the moderate nonlinearity.* This can be explicitly shown for the symmetric case: $\Delta_1 = -\Delta_2 = \Delta$, $\Omega_{1,2} = \Omega$ $(\theta_1 = \frac{\pi}{2} - \theta_2 = \theta)$, $\phi_1 = \phi_2$, $g_{kl} = g$, $\gamma_{1,2} = \gamma$, $\kappa_{1,2} = \kappa$, $N_{1,2} = N$ $(J_{1,2} = J)$. Under these conditions, we rewrite the Hamiltonian (16) as

$$H_I = \sum_{l=1}^{2} \hbar G_l (a_l \pi_l^+ + \pi_l a_l^\dagger), \tag{17}$$

where we have introduced the Bogoliubov spin modes as in Eq. (1) for $\Delta > 0$ as

$$\pi_1 = \tilde{\sigma}_1 \cosh r + \tilde{\sigma}_2^+ \sinh r,$$
$$\pi_2 = \tilde{\sigma}_2 \cosh r + \tilde{\sigma}_1^+ \sinh r, \tag{18}$$

with the squeezing parameter $\tanh r = \tan^2\theta$ and the effective coupling constants $G_l = (-1)^{l-1} g\sqrt{J|\cos(2\theta)|}$. Without confusion, now we refer to r, $\tanh r$, or $\sinh(2r)$ as the squeezing parameter for convenience. The new operators satisfy the commutation relations $[\pi_l,\pi_l^+] \doteq 1$ $(l = 1,2)$ and $[\pi_1,\pi_2] = [\pi_1,\pi_2^+] = 0$. For $\Delta < 0$, we have the similar formulas by exchanging $\tilde{\sigma}_l$ with $\tilde{\sigma}_l^+$ and exchanging $\sin\theta$ with $\cos\theta$.

Hamiltonian (17) indicates that the spin Bogoliubov modes $\pi_{1,2}$ are in dissipative interactions with the cavity fields $a_{1,2}$. As the cavity fields relax to the vacuum state, the collective spin modes evolve into their steady state. Under the adiabatic conditions ($\kappa \gg \gamma$), we can eliminate the cavity modes $a_{1,2}$ and derive the master equation for reduced density operator $\rho_a = \mathrm{Tr}_{\mathrm{fields}}\rho$ of the spin atomic ensembles [44] in the same form as Eq. (2)

$$\dot{\rho}_a = \frac{A}{2}\sum_{l=1}^{2}(2\pi_l\rho_a\pi_l^+ - \pi_l^+\pi_l\rho_a - \rho_a\pi_l^+\pi_l), \tag{19}$$

where

$$A = \frac{2g^2}{\kappa} J|\cos(2\theta)| \tag{20}$$

is the common dissipation rate by the engineered reservoirs. This master equation indicates the dissipation of the Bogoliubov operators $\pi_{1,2}$ by the cavity fields in the usual form [44].

Expanding Eq. (19) in terms of $\tilde{\sigma}_{1,2}$ we have the spin interaction terms induced by the cavity fields

$$A\sinh(2r)(\tilde{\sigma}_1\rho_a\tilde{\sigma}_2 + \tilde{\sigma}_2\rho_a\tilde{\sigma}_1 - \tilde{\sigma}_2\tilde{\sigma}_1\rho_a - \rho_a\tilde{\sigma}_2\tilde{\sigma}_1) + \mathrm{H.c.} \tag{21}$$

It is obvious that the cross coupling strength $A\sinh(2r)$ depends simultaneously on the collective dissipation rates A

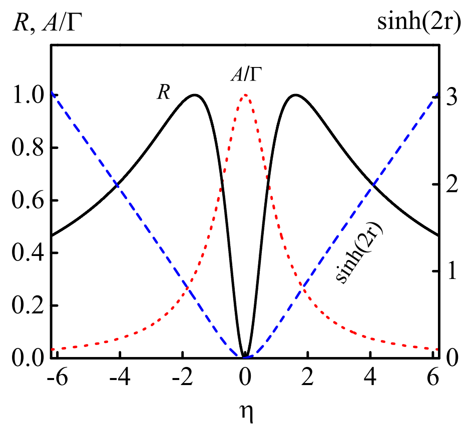

FIG. 3. The cross correlation R, the relative dissipation rate A/Γ, and the squeezing parameter $\sinh(2r)$ versus the nonlinear parameter $\eta = |\Omega|/\Delta$. Here R and A/Γ are scaled to unity.

and the squeezing parameter $\sinh(2r)$. The cross correlation is obtained as [44,48]

$$R = \frac{A}{\Gamma}\sinh(2r), \tag{22}$$

where $\Gamma = \gamma + \frac{\gamma}{2}\sin^2(2\theta)$ is the decay rates of dressed spins $\tilde{\sigma}_{1,2}$. We can give the dependence R on the nonlinear parameter $\eta = |\Omega|/\Delta$ in an explicit form

$$R = \frac{4g^2N}{\kappa\gamma}\frac{\eta^2(1+\eta^2)^{1/2}}{(2+\eta^2)(2+3\eta^2)}. \tag{23}$$

Clearly, the cross correlation depends strongly on the nonlinear parameter η. When the nonlinear parameter $|\eta| \to 0,\infty$, the cross correlation tends to vanish $R \to 0$. Shown in Fig. 3 are the cross correlation R, the engineered reservoir dissipation rate relative to the environment dissipation rate A/Γ, and the squeezing parameter $\sinh(2r)$ versus the nonlinear parameter η. This figure clearly shows that A/Γ and $\sinh(2r)$ have the opposite dependences on η. A/Γ falls monotonously while $\sinh(2r)$ rises as $|\eta|$ increases. Ideally, in order to have good squeezing, one needs both large squeezing parameter $\sinh(2r)$ and large dissipation rate A/Γ. In practice, when r is small, squeezing is weak even if A/Γ is large; and conversely, when A/Γ is small, squeezing is weak or nonexistent even if r is large. Therefore one needs to compromise these two factors. The two wide wings are a consequence of the optimization of the two competing factors. The two wings indicate that the cross correlation R takes a remarkably larger value within a large range of the moderate nonlinearity η.

IV. TWO-MODE SPIN SQUEEZING

Since the squeezing parameter r and the ratio A/Γ of engineered to environmental dissipation rates have opposite dependences on the nonlinear parameter η, they have competing effects on the quantum correlations. We are now in a position to examine the competing effects due to the nonlinearities. In the following discussion, an analytical description is given for the symmetrical case while numerical verification is presented for the general case. At the same time we include the nonadiabatic conditions.

We define the spin quadratures orthogonal to the total spins

$$x_o = \frac{o+o^\dagger}{\sqrt{2}}, \quad p_o = \frac{o-o^\dagger}{i\sqrt{2}}, \quad o = \tilde{\sigma}_{1,2}. \tag{24}$$

Then we express the two-mode quadrature operators as

$$\delta X_\beta = \delta x_{\beta_1} - \delta x_{\beta_2}, \quad \delta P_\beta = \delta p_{\beta_1} + \delta p_{\beta_2}. \tag{25}$$

When the variance of any quadrature is less than unity $\delta^2 X_\beta < 1$ or $\delta^2 P_\beta < 1$, the two-mode squeezing occurs [43,44]. For simplicity, we have defined the expression $\delta^2 o \equiv \langle(\delta o)^2\rangle$ for the variances. According to the criterion of Ramyer *et al.* [52], entanglement for spin atomic ensembles occurs when the inequality for the fluctuations satisfies

$$V_\beta = \delta^2 X_\beta + \delta^2 P_\beta < 2. \tag{26}$$

In order to include the dependence of quantum correlations on various parameters, we derive the Langevin equations from Hamiltonian (16). Following the standard technique [48], selecting the operator order $a_1^\dagger, a_2^\dagger, \tilde{\sigma}_1^+, \tilde{\sigma}_2^+, \tilde{\sigma}_2, \tilde{\sigma}_1, a_2, a_1$, and defining the corresponding c numbers $\alpha_1^*, \alpha_2^* e^{-2i\phi_2}, \beta_1^*, \beta_2^*, \beta_2, \beta_1, \alpha_2 e^{2i\phi_2}, \alpha_1$, we derive the Heisenberg-Langevin equations for $\Delta_1 > 0$ and $\Delta_2 < 0$ as follows [48]:

$$\begin{aligned}
\dot{\alpha}_1 &= -\kappa_1\alpha_1/2 - \tilde{g}_{11}\beta_1\cos^2\theta_1 + \tilde{g}_{12}\beta_2^\dagger\cos^2\theta_2 + F_{\alpha_1},\\
\dot{\alpha}_2 &= -\kappa_2\alpha_2/2 + \tilde{g}_{22}\beta_2\sin^2\theta_2 - \tilde{g}_{21}\beta_1^\dagger\sin^2\theta_1 e^{i\phi} + F_{\alpha_2},\\
\dot{\beta}_1 &= -\Gamma_1\beta_1/2 + \tilde{g}_{11}\alpha_1\cos^2\theta_1 - \tilde{g}_{21}\alpha_2^\dagger\sin^2\theta_1 e^{i\phi} + F_{\beta_1},\\
\dot{\beta}_2 &= -\Gamma_2\beta_2/2 - \tilde{g}_{22}\alpha_2\sin^2\theta_2 + \tilde{g}_{12}\alpha_1^\dagger\cos^2\theta_1 + F_{\beta_2},
\end{aligned} \tag{27}$$

where we have defined the relative phase $\phi = 2(\phi_1 - \phi_2)$. The F terms are noises with zero means and correlations $\langle F_x(t)F_y(t')\rangle = D_{xy}\delta(t-t')$, $D_{xy} = D_{yx}$ and $D_{x^*y^*} = D_{xy}^*$. The nonzero diffusion coefficients are $D_{\alpha_1^*\beta_2^*} = \tilde{g}_{12}\cos^2\theta_2$, $D_{\alpha_2^*\beta_1^*} = -\tilde{g}_{21}\sin^2\theta_1 e^{-i\phi}$, $D_{\beta_1^*\beta_1} = \Gamma_1 N_1^+/J_1$, and $D_{\beta_2^*\beta_2} = \Gamma_2 N_2^-/J_2$.

The above equations are used to calculate the quantum correlations without the adiabatic elimination [46,47]. Our calculations and results hold for arbitrary rates of the atomic and cavity relaxations. The characteristic features are presented as follows.

(i) *Weak squeezing for weak nonlinearity and no squeezing for too strong nonlinearity*. Using the above Langevin equations we can calculate the variances for various operators. At steady state $\langle\tilde{\sigma}_{1,2}\rangle = \langle a_{1,2}\rangle = 0$, then $\delta\tilde{\sigma}_{1,2} = \tilde{\sigma}_{1,2}$ and $\delta a_{1,2} = a_{1,2}$. For the symmetric case as in Sec. III, we can derive analytic expressions of the variances from Eq. (27) as

$$\delta^2 X_\beta = \delta^2 P_\beta = 1 - \frac{\kappa(1-e^{-2r}) - 2\Pi\Gamma\left(1+\frac{\kappa+\Gamma}{C\Gamma}\right)}{(\kappa+\Gamma)(1+C^{-1})}, \tag{28}$$

where the unity next to the equality sign represents the standard quantum level [43,44] and the second term is due to the fluctuations under the selected operator ordering. We have defined the cooperativity parameter $C = 4g^2J|\cos(2\theta)|/(\kappa\Gamma)$ and the additional parameter $\Pi = N_1^\pm/J$ ("+" for $\Delta > 0$ and "−" for $\Delta < 0$). The $\kappa(1-e^{-2r})$ term is due to the dressed atom-photon interaction, and the Π term comes from the atomic spontaneous emission. Since we have equal variances

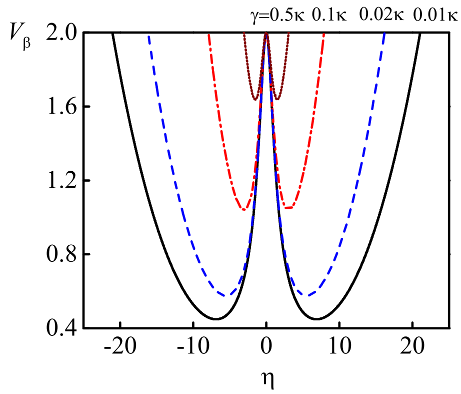

FIG. 4. Spin variance sum V_β versus the nonlinear parameter η for the symmetrical case. The parameters are chosen as $\gamma = 0.5\kappa$ (dotted), 0.1κ (dot-dashed), 0.02κ (dashed), 0.01κ (solid), $g^2N = 10\kappa^2$, and $\phi = 0$.

$\delta^2 X_\beta = \delta^2 P_\beta$, the conditions for entanglement are the same as for the squeezing.

We note that the parameters in Eq. (28) are strongly dependent on the nonlinear parameter $\eta = |\Omega|/\Delta$ through the following explicit expressions:

$$C = \frac{16g^2N}{\kappa\gamma}\frac{1+\eta^2}{(2+\eta^2)(2+3\eta^2)}, \tag{29}$$

$$e^{-2r} = \frac{1}{\sqrt{1+\eta^2}}, \tag{30}$$

$$\Pi = -\frac{1}{2} + \frac{\sqrt{1+\eta^2}}{4} + \frac{1}{4\sqrt{1+\eta^2}}. \tag{31}$$

The cooperativity parameter C falls monotonously with the nonlinear parameter $|\eta|$. The larger the parameter η, the slower the decrease in C. In the limiting cases, we have $C = 4g^2N/(\kappa\gamma)$ for $|\eta| \ll 1$, and $C \to 0$ for $|\eta| \gg 1$. In the weak nonlinearity region ($|\eta| \ll 1, \Pi \ll 1$) and when $C \gg 1 + \kappa/\Gamma$ and $\gamma \ll \kappa$, Eq. (28) reduces to

$$\delta^2 X_\beta = \delta^2 P_\beta \doteq e^{-2r} = \frac{1}{\sqrt{1+\eta^2}} \lesssim 1, \tag{32}$$

which shows that the squeezing degree is negligibly weak even if it is existent. In the other limit (the strong nonlinearity, $\eta \to \infty, C \to 0, \Pi \to \infty$), we have

$$\delta^2 X_\beta = \delta^2 P_\beta > 1, \tag{33}$$

which indicates no squeezing. It is seen from the two limiting cases that only weak squeezing happens for the weak nonlinearity, and no squeezing exists for the too strong nonlinearity.

(ii) *Remarkable enhancement of squeezing via moderate nonlinearity*. The spin variance sum V_β is plotted in Fig. 4 versus the nonlinear parameter η for the symmetrical case. We take the parameters as $\gamma = 0.5\kappa$ (dotted), 0.1κ (dot-dashed), 0.02κ (dashed), 0.01κ (solid), $g^2N = 10\kappa^2$, and $\phi = 0$. The variance displays two dips below the standard quantum level. The variance rises close to 2 (the standard quantum noise level) for small η and rises above 2 for too large η. Good squeezing and entanglement appear for a comparably large range of moderate values of η. The minimal variances $\delta^2 X_\beta = \delta^2 P_\beta$ (half of V_β) approach 0.2, which corresponds to the best squeezing of close to 80% for the X_β and P_β quadratures. Physically, it is the combination of the squeezing parameter with the engineered reservoir dissipation rates that optimizes the squeezing. For the moderate nonlinearity, the correlation is sensitive to the atomic decay rates. The smaller the atomic decay rates, the smaller the minimal two-mode variance. As the atomic decay rates decrease, the two dips become wide and the bottoms for the best squeezing shift towards the outside. Beyond the two dips, the overly saturation η makes the decreased dissipation rate A not dominate over the vacuum dissipation rates, and so the squeezing disappears.

Now we can relate the two dips of the variance V_β in Fig. 4 to those two peaks of the cross correlation R in Fig. 3. Essentially, the variance dips are the competing consequence of the nonideal dissipation rate A and the squeezing parameter r. Good squeezing is represented by a large parameter r but it is achievable only when the dissipation rate A is dominant over the environmental vacuum-induced dissipation rate γ. However, as shown in Fig. 3, the squeezing parameter r and the dissipation rate A have opposite dependence on the nonlinearity η. For this reason, in a sophisticated way as shown in Eq. (28), the competing effects of the not so ideal dissipation rate A and the squeezing parameter r on the variances depend on the nonlinear parameter η, the cooperation parameter C, and the ratio of atomic to cavity decay γ/κ. This determines that the variances display two characteristic features as follows. First, the variance dips become deeper, wider, and further separated from each other as the atomic decay rate falls. With increasing cooperativity parameter and/or decreasing atomic decay rate, the engineered dissipation is more dominant over the vacuum dissipation and so the best achievable squeezing is enhanced. At the same time, there exists a wider and wider range of the nonlinear parameter η in which the engineered dissipation dominates over the spontaneous dissipation. Second, the variance dips are cut off at two particular points of the borderlines. This is because the engineered reservoir dissipation rate A is no longer enough to overcome the vacuum dissipation at the cutoff points, though the squeezing parameter is large. Once it is the case, squeezing is no longer existent even if the squeezing parameter r is large. Beyond the two dips, the too strong nonlinearity makes the engineered reservoir dissipation rate weaker, and thus squeezing is impossible.

(iii) *Phase dependence*. Beyond the symmetric case, it becomes relatively complicated to calculate the variance. We resort to the spectral method [48]. By expressing the variables in a column vector $O(t) = (\alpha_1,\alpha_2,\beta_1,\beta_2,\alpha_1^*,\alpha_2^*,\beta_1^*,\beta_2^*)^T$ and by defining $\delta O(t) = O(t) - \langle O\rangle$, we write the set of Heisenberg-Langevin equations (27) in a compact form

$$\frac{d}{dt}\delta O(t) = -B\delta O(t) + F(t), \tag{34}$$

where B is the drift matrix and $F(t)$ is the column vector for noises. The noise correlations are expressed as $\langle F(t)F^T(t')\rangle = D\delta(t-t')$. Using the Fourier transformation $\delta O(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\delta O(t)e^{i\omega t}\,dt$, we derive the correlation spectrum as

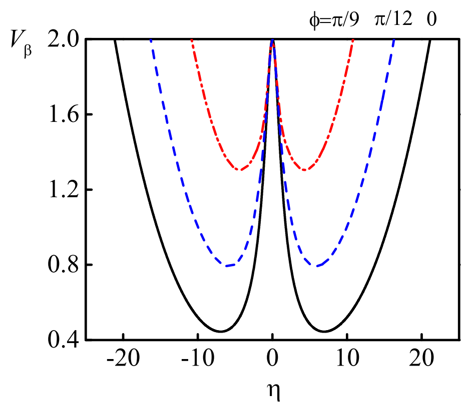

FIG. 5. Spin variance sum V_β versus the nonlinear parameter η for different phases, $\phi = 0$ (solid), $\pi/12$ (dashed), $\pi/9$ (dot-dashed). The other parameters are chosen to be the same for the solid line in Fig. 4.

$S(\omega) = \lim_{t\to\infty} \int_{-\infty}^{+\infty} e^{-i\omega\tau} \langle \delta O(t+\tau)\delta O^T(t)\rangle \, d\tau$, where

$$S(\omega) = (B + i\omega I)^{-1} D (B^T - i\omega I)^{-1}, \tag{35}$$

with I being a unit matrix. The spectrum exists if the steady-state solutions are stable. The stability can be verified by calculating the eigenvalues of the matrix B, which is performed by using a computer. When the real parts of all eigenvalues are positive, then the system is stable. The variance sum V_β in Eq. (26) is obtained as

$$V_\beta = 2 + \frac{1}{2\pi} \int V_\beta(\omega)\, d\omega, \tag{36}$$

where 2 is the standard quantum noise level [43,44] and the second term is due to the fluctuation spectrum under the selected operator ordering

$$V_\beta(\omega) = S_{37}(\omega) + S_{48}(\omega) - S_{34}(\omega) - S_{78}(\omega) + \text{c.c.}, \tag{37}$$

with S_{kl} being the elements of $S(\omega)$ at the kth row and at the l column. Plotted in Fig. 5 is the spin variance sum V_β versus the nonlinear parameter η for different phases $\phi = 0$ (solid), $\pi/12$ (dashed), $\pi/9$ (dot-dashed). The other parameters used in the calculation are chosen to be the same as for the solid line in Fig. 4. In order to show the equivalence between the spectrum integration method and the direct analytic calculation method, we also plot in Fig. 5 the solid line for exactly the same parameters as for the solid line in Fig. 4. It is clearly shown that these two curves are exactly the same. Therefore, both methods are valid for the calculation of the variances. It is clearly seen from Fig. 5 that the two wings for the variance sum shift up and remarkably shrink as the phase increases. This indicates that the deviation of the phase from zero always weakens the squeezing. Further increase in the phase leads to disappearance of the squeezing. For not too large deviation from $\phi = 0$, however, two wide dips still exist, which indicate a wide range of the moderate nonlinearity for good squeezing.

(iv) *Effects of asymmetrical detunings*. So far we have assumed the symmetric nonlinearities. Now we pay attention to the asymmetric detunings. In Fig. 6 we plot the spin variance sum V_β versus the nonlinear parameter $\eta_1 = |\Omega_1|/\Delta_1$ for asymmetrical case ($\eta_2 \neq \eta_1$). This also requires $\Omega_1 \neq \Omega_2$ to satisfy the conditions in Eq. (14). The parameter $\zeta = -\Delta_2/\Delta_1$ is chosen as $\zeta = -1$ (solid), -0.7 (dashed), and -0.4 (dot-dashed). The other parameters used for the calculation are the same as for the solid line in Fig. 4. Note that squeezing is not existent at $\eta_1 = 0$. It is seen from Fig. 6 that the best achievable squeezing becomes weak, and the wings for squeezing also become narrow as the ratio Δ_2/Δ_1 increases. Combining Fig. 5 with Fig. 6 we see that the deviation of the parameters from the symmetrical case generally spoils the squeezing and narrows the nonlinearity regime for squeezing.

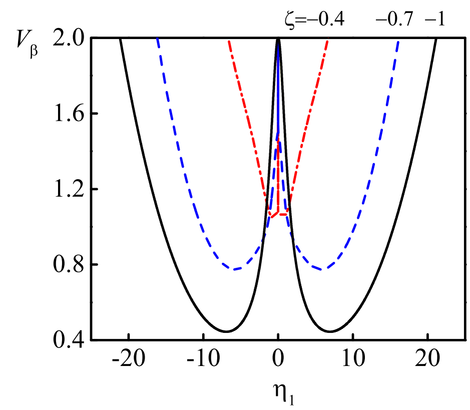

FIG. 6. Spin variance sum V_β versus the nonlinear parameter $\eta_1 = |\Omega_1|/\Delta_1$ for asymmetrical nonlinearities ($\eta_2 \neq \eta_1$). The parameter $\zeta = -\Delta_2/\Delta_1$ is set as $\zeta = -1$ (solid), -0.7 (dashed), and -0.4 (dot-dashed). The other parameters are chosen to be the same as for the solid line in Fig. 4.

So far we have shown the double-dip structure for the moderate nonlinearity-induced enhancement of two-mode spin squeezing. Within the double dips, the nonlinearity compromises the squeezing parameter and the engineered reservoir dissipation rates and thus optimizes the spin correlations. Beyond the two dips, there no longer are the squeezing and entanglement due to the fact that the too strong nonlinearity overly reduces the engineered reservoir dissipation rates.

V. TWO-MODE FIELD SQUEEZING

As a comparison, we consider the dependence of the field correlations on the nonlinearity when we exchange the atomic ensembles as an engineered reservoir for the cavity fields. Similarly, for the symmetric case, we can also introduce the Bogoliubov modes for the cavity fields [43]

$$b_1 = a_1 \cosh r - a_2^\dagger \sinh r,$$
$$b_2 = a_2 \cosh r - a_1^\dagger \sinh r, \tag{38}$$

and rewrite the interaction Hamiltonian (16) for $\Delta > 0$ as

$$H_I = \sum_{l=1}^{2} \hbar G_l (b_l \tilde{\sigma}_l^+ + \tilde{\sigma}_l b_l^\dagger). \tag{39}$$

The case for $\Delta < 0$ is treated similarly. In Eq. (39) we note that annihilation (creation) of the new modes $b_{1,2}$ is always accompanied with the excitation (deexcitation) of the atoms, respectively. The atomic ensembles constitute the engineered dissipation reservoir for the Bogoliubov field modes. For the symmetric case as above and under the adiabatic condition ($\gamma \gg \kappa$), we derive the master equation for the reduced density operator $\rho_c = \mathrm{Tr}_{\mathrm{atoms}}\rho$ of the cavity fields for $\Delta > 0$ as [44]

$$\dot{\rho}_c = \frac{B_1}{2}\sum_{l=1}^{2}(2b_l\rho_c b_l^\dagger - b_l^\dagger b_l\rho_c - \rho_c b_l^\dagger b_l) + \frac{B_2}{2}\sum_{l=1}^{2}(2b_l^\dagger\rho_c b_l - b_l b_l^\dagger\rho_c - \rho_c b_l b_l^\dagger), \tag{40}$$

where $B_{1,2} = 2g^2N_1^{\mp}|\cos(2\theta)|/\Gamma$ are the absorption and gain coefficients [44] of Bogoliubov field modes b_l by the dressed atoms, respectively. The difference from the spin case is the presence of the gain terms. The net rate for the engineered dissipation is $B_1 - B_2$. The cross correlation between the two cavity fields is obtained as

$$R' = -\left(\frac{B_1 - B_2}{\kappa}\right)\sinh(2r), \tag{41}$$

which is the same as the cross spin correlation except a minus sign

$$R' = -R. \tag{42}$$

This means that the field correlation displays the similar dependence on the nonlinearity except for the exchange of the relative values of κ and γ. Taking into account the differences in the plus and minus signs of the spin and field Bogoliubov operators (18) and (38) and of the cross correlations (42), we will have two-mode squeezing and entanglement for the same collective quantities. It is the case indeed, as shown in what follows.

We use the quadrature operators (24) for the fields with $o = a_{1,2}$. Then we express the two-mode quadrature operators as $\delta X_\alpha = \delta x_{a_1} - \delta x_{a_2}$ and $\delta P_\alpha = \delta p_{a_1} + \delta p_{a_2}$. If the variance of any quadrature is less than unity, $\delta^2 X_\alpha < 1$ and/or $\delta^2 P_\alpha < 1$, the two-mode squeezing occurs for the two cavity fields [43,44]. The cavity fields are entangled with each other if [53]

$$V_\alpha = \delta^2 X_\alpha + \delta^2 P_\alpha < 2. \tag{43}$$

The steady-state variances for the collective cavity fields are calculated from the set of Heisenberg-Langevin equations (27) for the symmetric case in an analytic form

$$\delta^2 X_\alpha = \delta^2 P_\alpha = 1 - \frac{\Gamma(1 - e^{-2r}) - 2\Pi\Gamma e^{-2r}}{(\kappa + \Gamma)(1 + C^{-1})}. \tag{44}$$

After exchanging the atomic ensembles as the engineered reservoir, we can find the similar dependence of the field correlations on the nonlinear parameter η. We plot in Fig. 7 the field variance sum V_α versus the nonlinear parameter η for the symmetrical situation. The parameters we used are $\kappa = \gamma$ (dot-dashed), 0.2γ (dashed), 0.05γ (solid), $g^2N = 10\gamma^2$, and $\phi = 0$. It is seen from Fig. 7 that the variance sum is smaller than the standard quantum noise level, $V_\alpha < 2$, so long as $\eta \neq 0$. In the weak nonlinearity limit, $\eta \to 0$, the variance rises towards 2 and the squeezing is weak. Within a very large range of strong nonlinearity, $0 < \eta < \infty$, the squeezing is considerably strong. As the cavity decay rate κ decreases, the two wings shift down. Like the case for the spin atomic ensembles, the best squeezing always appears for moderate nonlinearity for given parameters. However, there are two remarkably different features from the spin case. First, the minimal variances $\delta^2 X_\alpha = \delta^2 P_\alpha$ (half of V_α) approach 0.5, which corresponds to squeezing of close to 50% for the X_α and P_α quadratures. Second, squeezing is existent for the entire regime of the nonlinearity except at $\eta = 0$, though the squeezing is weak for strong nonlinearity. This is because the cavity fields just reduce to their vacuum state even though the engineered reservoir dissipation rates are not enough to overcome the environment vacuum dissipation. Compared with the spin case, the best field squeezing is reduced, while the parameter range for squeezing is greatly widened.

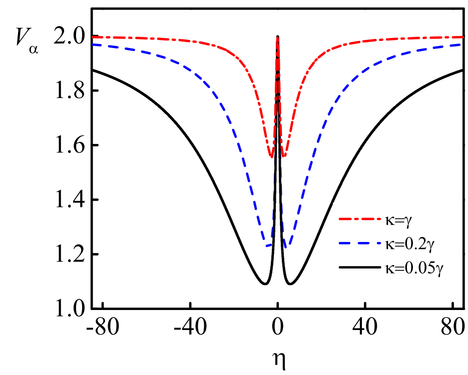

FIG. 7. Field variance sum V_α versus the nonlinear parameter η for the symmetrical case. The used parameters are $\kappa = \gamma$ (dot-dashed), 0.2γ (dashed), 0.05γ (solid), $g^2N = 10\gamma^2$, and $\phi = 0$.

As a possible experimental feasibility, one can employ a cloud of cold alkali atoms prepared in a standard magneto-optic trap, as for squeezing of an atomic ensemble [54–56]. The cesium D_2 transition hyperfine structure $|6S_{1/2}, F = 4\rangle \to |6P_{3/2}, F' = 5\rangle$ (wave length 852 nm) can be used for our working transition. The $\sigma^\pm$ circularly polarized fields of the same frequency are coupled to the σ^+ transition $|F = 4, m_F\rangle \to |F' = 5, m_F + 1\rangle$ and σ^- transition $|F = 4, m_F\rangle \to |F' = 5, m_F - 1\rangle$, respectively. The closest sublevel is 251 MHz below the excited state. The natural line width of the transition is $2\pi \times 2.6$ MHz. Thus the detuning and the Rabi frequency can take the range $2\pi \times 2.6 \ll (\Omega, \Delta) \ll 2\pi \times 251$ MHz. One static magnetic field is used to shift the magnetic sublevels of the different ensembles towards the opposite directions and thus to produce the opposite detunings. The transition $|6S_{1/2}, F = 3\rangle \to |6P_{1/2}, F' = 3,4\rangle$ (wave length 894 nm) is used to initially populate and later keep the atoms in the upper ground state. The spin entanglement of atomic ensembles can be prepared when $\kappa \gg 2\pi \times 2.6$ MHz, while the light entanglement requires $\kappa \ll 2\pi \times 2.6$ MHz. The mediate nonlinearity ($\eta = |\Omega|/\Delta$) is controlled and kept by stabilizing the amplitude and frequency of the dressing field under the conditions $2\pi \times 2.6 \ll \sqrt{|\Omega|^2 + \Delta^2} \ll 2\pi \times 251$ MHz. On the other hand, the variance as a function of η displays two wide dips, each of which means a wide range of moderate nonlinearity for the enhanced squeezing. This indicates that good squeezing can be kept even when the nonlinear parameter η changes within a range of the mediate values.

The present two-level scheme is scalable because the parameters on which quantum correlations are dependent are dimensionless quantities ($C,\eta,\gamma/\kappa,\phi$), as shown above. The interacting atoms and fields can have their frequencies from the microwave to the optical wave, and so the two-level system can be applied to a wide regime of frequencies. Potential candidates include ensembles of electron spins in solid media [57] or coupled to superconducting transmission line cavities [58]. Superconducting circuits as single quantum devices also are considerably advantageous for preparing squeezed and entangled states of microwave fields due to the strong coupling to the fields [59].

Finally, there exist similar enhancement effects of the moderate nonlinearities in multilevel atomic or molecular systems, in which the dissipation is engineered for the two-mode squeezed and entangled states. Essentially, the compatible increase in the squeezing parameter and the engineered reservoir dissipation rates originates from the relative strengths of beam-splitter-like and down-conversion-like atom-field interactions, as shown in Eq. (16). The two kinds of interaction strengths can be neither in too great a disparity nor too close. It is not difficult to imagine that, in many cases, at least in principle, the relative strengths have to be balanced for the compatibly large values of the squeezing parameter and the engineered reservoir dissipation rates.

VI. CONCLUSION

In conclusion, we have presented the nonlinear effects on the quantum correlations in reservoir engineering that is based on the simultaneous interactions of two two-level atomic ensembles with two cavity quantum fields. The squeezing parameter and the dissipative rates, the former of which determines the Bogolibov-modes-like collective interactions and the latter of which determines the engineered dissipation of the Bogoliubov modes, are reversely dependent on the dressing atom-field nonlinearities. It has been shown that their optimization for an enhancement of quantum correlations is achievable for the moderate nonlinearity. Such effects hold for the two-mode squeezing and entanglement of separated spin atomic ensembles or the different optical fields. This represents an advantage of two-level systems with the moderate nonlinearities for quantum information associated with reservoir engineering.

ACKNOWLEDGMENTS

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