

**Correlations of photoemissions in a multiatomic ensemble driven by a cat-state field**

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A system of two-level noninteracting atoms driven by superposition of two Glauber coherent photonic states (a cat state) is studied. The field state is continuously restored by a source explicitly incorporated into the model. Due to its nature, the cat state changes phase by  $\pi$  upon stimulated excitation of any atom, a peculiar kind of coherent quantum feedback. That results in correlations between photonic and atomic subsystems. In the limit of a strong field, the ansatz for the system's density matrix is proposed and an approximate analytical solution to the master equation is obtained in the case of a large number of atoms and slow spontaneous emission. Based on this solution, the steady-state second-order correlation function of atomic photoemissions is evaluated and investigated. The results demonstrate a remarkable difference from the case of the classical field (i.e., the field in a Glauber coherent state).

DOI: [10.1103/PhysRevA.96.063805](https://doi.org/10.1103/PhysRevA.96.063805)**I. INTRODUCTION**

For many years the creation and investigation of exotic quantum states of light has been one of the topical goals for researchers in quantum optics. Examples of such states are squeezed states [1–6], Fock number states, and various superpositions of Glauber coherent states [7–10] as well as their entangled variants [11–16]. Such states have proved to be convenient for various test of foundations of quantum theory and they may eventually be useful for such fields as quantum computation and quantum metrology [17,18]. Of particular interest are superpositions of two Glauber coherent states, sometimes called Schrödinger cat states. They can, for example, serve as qubits for a quantum computer [19,20]. Since to-date experimental techniques are close to producing such states with many photons, it is worth exploring their properties for future research.

The behavior of resonant atoms in highly nonclassical light is still far from being studied more or less completely. However, this matter is intriguing, instructive, and naturally in demand by promising quantum technologies. In [21] the resonance fluorescence of a single two-level atom in a cat-state field was considered. The steady-state regime of fluorescence was guaranteed by explicitly incorporating the source of the cat-state field into the model. The source not only replenishes the field energy but tends to recreate its superpositional coherent nature.

Surprisingly, as it might seem, atoms interacting with cat-state fields puts us into the realm of quantum feedback and control, another rapidly developing branch of quantum optics [22,23] with many applications in studying quantum optical systems. Any control scheme involves usually an artificial modification of the system of interest. For measurement-based quantum feedback, one has to set up an apparatus performing measurements on the system and triggering the feedback loop based on the measurement outcomes. The system thus can be called self-organized in the sense that it demonstrates a certain amount of self-control. In the present article we study

a simple example of such a system: an atomic ensemble interacting with a cat-state field of a special type, with induced photon absorptions causing phase switchings of the field components. The feedback loop may be constructed so as to modify the system's Hamiltonian or its quantum state. In any case that is dissipative feedback since the information about its activation enters the environment. In a series of works [24], a measurement-based feedback loop implemented  $\pi$ -phase switching of the radiation field upon emissions, i.e., spontaneous deexcitations, in a one- or two-atom system. Feedback of quite the opposite, nondissipative (coherent), type is the focus of the present work. This is rather a peculiar kind of feedback due to it being an inherent property of the system and emerging naturally during its evolution. As a consequence of the cat-state nature of the pumping radiation, the same  $\pi$  switching of light is induced by stimulated excitation of any atom in the ensemble. Any individual atom is so able to change the driving field for all other atoms in the ensemble, quite an unusual peculiarity if compared with ordinary nonlinear spectroscopy. This motivates studying multiatomic systems. The aim of the present work is to consider a multiatomic ensemble in its resonance fluorescence in the same cat state. It is the statistics of photoemissions which is studied, the resonance fluorescence property complementary to its spectrum considered in [21].

The paper is organized as follows. In Sec. II we give a theoretical model that allows us to study a steady-state regime of the interaction of atoms with the nonclassical field. In Sec. III we derive the master equation for the atomic part of the system. In Sec. IV we solve the master equation approximately, calculate the second-order correlation function of atomic photoemissions, and compare with the known functions that emerge in the case of classical field. In Sec. V we discuss our results and summarize.

**II. MODEL FOR THE FIELD SOURCE**

We consider an ensemble of two-level atoms with a dipole transition interacting with an external quantized electromagnetic field. The field is supposed to be prepared in the specific superposition of Glauber coherent states ( $G$  states) called

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Yurke-Stoler (YS) states  $|\alpha\rangle_{\text{YS}}$  [25]. This choice is motivated by YS states being dual to ordinary Glauber coherent states  $|\alpha\rangle_G$ . Just in parallel with the definition of  $G$  states as eigenstates of the ordinary annihilation operator  $\hat{a}_G|\alpha\rangle_G = \alpha|\alpha\rangle_G$ , YS states are eigenstates of the gauge-deformed annihilation operator

$$\begin{aligned}\hat{a}_{\text{YS}} &= e^{i\pi\hat{n}}\hat{a}_G, \\ \hat{a}_{\text{YS}}^\dagger &= \hat{a}_G^\dagger e^{-i\pi\hat{n}}, \\ \hat{a}_{\text{YS}}|\alpha\rangle_{\text{YS}} &= \alpha|\alpha\rangle_{\text{YS}}.\end{aligned}\quad (1)$$

It is easy to check that they obey the ordinary bosonic commutation relations  $[\hat{a}_{\text{YS}}, \hat{a}_{\text{YS}}^\dagger] = \hat{1}$ . The number of photons is invariant,  $\hat{n} = \hat{a}_G^\dagger \hat{a}_G = \hat{a}_{\text{YS}}^\dagger \hat{a}_{\text{YS}}$ , which guarantees the mentioned duality. This symmetry allows one to express YS and Glauber states through each other:

$$\begin{aligned}|\alpha\rangle_{\text{YS}} &= \frac{1}{\sqrt{2}}(|i\alpha\rangle_G + i|-i\alpha\rangle_G), \\ |\alpha\rangle_G &= -\frac{i}{\sqrt{2}}(|i\alpha\rangle_{\text{YS}} + i|-i\alpha\rangle_{\text{YS}}).\end{aligned}\quad (2)$$

The property crucial to our consideration is how the Glauber annihilation operator acts on the YS state:

$$\hat{a}_G|\alpha\rangle_{\text{YS}} = \alpha|-i\alpha\rangle_{\text{YS}}. \quad (3)$$

From this equation one can see how irreversible photon loss affects the YS state. Limited controllability of the number of photons which have left the cavity mode prepared initially in the YS state results in a rapid decoherence of the last one. With a photon loss rate  $\nu$ , the coherence of the initial YS state  $|\alpha\rangle_{\text{YS}}$  will be lost in a time  $t \simeq |\alpha_0|^2 \nu^{-1}$  and it will decay into a statistical mixture of Glauber coherent states

$$\begin{aligned}|\alpha_0\rangle_{\text{YS}}\langle\alpha_0| &\rightarrow \frac{1}{2}|i\alpha_t\rangle_G\langle i\alpha_t| + \frac{1}{2}|-i\alpha_t\rangle_G\langle -i\alpha_t|, \\ \alpha_t &= \alpha_0 \exp(-\nu t - i\omega_0 t),\end{aligned}\quad (4)$$

where  $\omega_0$  is the mode frequency. This property is also central to the present work. If one intends to study how the cat-state field interacts with the system of elementary emitters in a steady-state regime, one should somehow include in the model the source mechanism that continuously recreates the state of the field. While various strategies exist for the preparation and protection of cat states against the decoherence [26–29], for the purpose of the work we choose perhaps the simplest mechanism utilizing the aforementioned symmetry between YS states and  $G$  states. It is well known that a quantized mode driven by an external source, an oscillating classical dipole, of strength  $\mu$  (without loss of generality, we consider it real in what follows) and subjected to irreversible photon loss with rate  $\nu$  has the  $G$  state  $|-i\mu/\nu\rangle_G$  as its steady state [30]. For simplicity, the frequency of the source is assumed to be equal to that of the mode. Analogically, a simple change of every  $G$  operator with the YS operator will give a steady YS state

$$\begin{aligned}\partial_t \hat{\rho}_{\text{ph}} &= \Lambda_{\text{ph}}[\hat{\rho}_{\text{ph}}] = -i\mu[\hat{a}_{\text{YS}} + \hat{a}_{\text{YS}}^\dagger, \hat{\rho}_{\text{ph}}] \\ &+ 2\nu\hat{a}_{\text{YS}}\hat{\rho}_{\text{ph}}\hat{a}_{\text{YS}}^\dagger - \nu\{\hat{a}_{\text{YS}}^\dagger\hat{a}_{\text{YS}}, \hat{\rho}_{\text{ph}}\}, \\ \Lambda_{\text{ph}}[|\alpha_{\text{st}}\rangle_{\text{YS}}\langle\alpha_{\text{st}}|] &= 0, \\ \alpha_{\text{st}} &= -i\mu/\nu.\end{aligned}\quad (5)$$

Note that  $\alpha_{\text{st}}$  is purely imaginary in this definition. This approach was introduced in [21], where we direct the interested reader for details. Since we are focused on the atom-field interaction in the steady-state regime, the distinctive properties of the cat-state field source are not of particular importance as long as it gives the desired cat state in a stationary regime, and the introduced simple mathematical model is by assumption sufficient for our needs. In the remainder of the paper we will assume that  $|\alpha_{\text{st}}\rangle_{\text{YS}}$  is the initial state of the field.

### III. MASTER EQUATION FOR A SYSTEM OF PHOTONS AND ATOMS

Let us now insert the ensemble of  $N$  two-level atoms into the field. The  $i$ th atom has its ground state  $|g^{(i)}\rangle$  and excited state  $|e^{(i)}\rangle$ . We will neglect their interaction with each other and assume that every atom experiences the same field as the others, i.e., there is no need to introduce an atomic coordinate into the field operators. This is true, for example, for closely spaced atoms (a dense ensemble) or when the atoms are arranged in a plane orthogonal to the field's wave vector. We thus have to introduce the Hamiltonian terms into master equations that describe the internal dynamics of the atoms and their (dipole) interaction with the field:

$$\partial_t \hat{\rho}_{\text{tot}} = -i[\hat{H}_{\text{at}} + \hat{H}_{\text{int}}, \hat{\rho}_{\text{tot}}] + \Lambda_{\text{ph}}[\hat{\rho}_{\text{tot}}], \quad (6)$$

$$\hat{H}_{\text{at}} = \Delta \hat{S}_0, \quad \hat{H}_{\text{int}} = \lambda(\hat{a}_G \hat{S}_+ + \hat{a}_G^\dagger \hat{S}_-), \quad (7)$$

where  $\Delta$  is the field detuning from the atomic resonance frequency,  $\lambda$  is the Rabi frequency, and  $\hat{S}_{\pm,0}$  are collective atomic pseudospin operators

$$\begin{aligned}\hat{S}_0 &= \sum_{i=1}^N \hat{S}_0^{(i)} = \frac{1}{2} \sum_{i=1}^N (|e^{(i)}\rangle\langle e^{(i)}| - |g^{(i)}\rangle\langle g^{(i)}|), \\ \hat{S}_+ &= \sum_{i=1}^N \hat{S}_+^{(i)} = \sum_{i=1}^N |e^{(i)}\rangle\langle g^{(i)}|, \\ \hat{S}_- &= \hat{S}_+^\dagger.\end{aligned}\quad (8)$$

We will also assume that the spontaneous decay process is slow enough compared to the evolution due to the atom-field interaction that it can be neglected on the typical time scale of the atom-field interaction characterized by the Rabi frequency  $\lambda$  (assumed to be real without loss of generality). In this case one can talk about self-organized feedback: The Hamiltonian (7) suggests that the phase of the coherent state's parameter is switched by  $\pi$  upon every absorption of the field photon [see (3)]. This is fundamentally different from the artificially constructed feedback studied in our previous papers, where the feedback was engineered to switch the phase of the external (classical) field upon every *spontaneous* photodetection.

The master equation (6) is quite complicated as it describes the evolution of a combined (infinite-dimensional) system of field and atoms. We will follow the approach from [21] and assume that the field subsystem is fast compared to that of atoms and contains many photons, i.e., the coherent state parameter of the steady state is big enough:  $\mu \gg \nu$ . At the same time, this is the most interesting case to study, as the YS state with a large number of photons (as is well known, the average photon number in  $|\alpha_{\text{st}}\rangle$  is  $|\alpha_{\text{st}}|^2$ ) demonstrates strong

nonclassical features, involving the superposition of almost orthogonal coherent states:  $\langle \alpha_{\text{st}} | -\alpha_{\text{st}} \rangle \ll 1$  (this inequality is valid for both  $G$  and  $YS$  states). The most general form of the solution to the master equation in this case is as follows:

$$\hat{\rho}_{\text{tot}} = \hat{\varrho}^{(+)} \otimes |\alpha_{\text{st}}\rangle_{\text{YS}} \langle \alpha_{\text{st}}| + \hat{\varrho}^{(-)} \otimes |-\alpha_{\text{st}}\rangle_{\text{YS}} \langle -\alpha_{\text{st}}| + \hat{R} \otimes |\alpha_{\text{st}}\rangle_{\text{YS}} \langle -\alpha_{\text{st}}| + \hat{R}^{\dagger} \otimes |-\alpha_{\text{st}}\rangle_{\text{YS}} \langle \alpha_{\text{st}}|. \quad (9)$$

In [21] we have seen that interaction with but one atom is enough to destroy the coherence of the field state and make it an incoherent mixture of  $|\alpha_{\text{st}}\rangle_{\text{YS}}$  and  $|-\alpha_{\text{st}}\rangle_{\text{YS}}$ . However, keeping the last two terms in (9) (nondiagonal with respect to field variables) is important. While they vanish when tracing over field variables and hence do not contribute to the atomic state (since we have assumed the practical orthogonality of  $|\alpha_{\text{st}}\rangle_{\text{YS}}$  and  $|-\alpha_{\text{st}}\rangle_{\text{YS}}$ ), they represent the correlation that builds up between the atom and the field during evolution. As we will see below, the time evolution of  $\hat{\varrho}^{(\pm)}$  and  $\hat{R}$  is interrelated, meaning that the said terms are indispensable for a correct description of all the physical phenomena in the system.

When substituting this ansatz into a master equation and tracing over field variables, it appears that the result looks particularly simple in terms of new operators

$$\begin{aligned} \hat{\varrho} &= \hat{\varrho}^{(+)} + \hat{\varrho}^{(-)}, \\ \hat{r} &= \hat{R} - \hat{R}^{\dagger}. \end{aligned} \quad (10)$$

The operators (10) obey the following system of equations:

$$\begin{aligned} \partial_t \hat{\varrho} &= -i\Delta[\hat{S}_0, \hat{\varrho}] + i\lambda\alpha_{\text{st}}[\hat{r}, \hat{S}_+ + \hat{S}_-], \\ \partial_t \hat{r} &= -i\Delta[\hat{S}_0, \hat{r}] - \Gamma\hat{r} - i\lambda\alpha_{\text{st}}[\hat{\varrho}, \hat{S}_+ + \hat{S}_-], \end{aligned} \quad (11)$$

where  $\Gamma = 4\mu^2/\nu$ , and in the case of a strong field it dominates the time dependence of  $\hat{r}$ . It is then well justified to neglect the derivative of  $\hat{r}$  in the second equation, assuming that  $\hat{r}$  reaches its steady value very rapidly compared to the time scale of the evolution of  $\hat{\varrho}$ . Note that for  $\Delta = 0$  the evolution of  $\hat{\varrho}$  is merely a result of random infinitesimal rotations around the first axis in the three-dimensional space of a collective atomic pseudospin. For nonzero detunings this simple evolution interwinds with precession around the third axis. As a result, the steady-state solution to (11) is a maximally mixed  $(N+1) \times (N+1)$  density matrix  $\hat{\varrho}^{\text{st}} = \hat{1}/(N+1)$ .

While such simple considerations are enough to get a steady-state solution, obtaining the actual time dependence of the density matrix is not so straightforward. The discussed atomic system is totally equivalent to the angular momentum  $j = N/2$  and  $\hat{S}_+, \hat{S}_-, \hat{S}_0$  operate in subspaces specified by  $j$ . The technique of irreducible tensor operators [31,32] is convenient when treating such systems. Instead of solving the equations in the  $jm$  basis, one can decompose any bounded operator  $\hat{O}$  over the set of operators  $\{\hat{T}_{\kappa q}; \kappa = 0, 1, \dots, 2j; q = -\kappa, \dots, \kappa\}$ , which are the components of the irreducible tensor operator  $\hat{\mathbf{T}}_{\kappa}$ :

$$\hat{O} = \sum_{\kappa=0}^{2j} \sum_{q=-\kappa}^{\kappa} O_{\kappa q} \hat{T}_{\kappa q}, \quad O_{\kappa q} = \text{Tr}(\hat{T}_{\kappa q}^{\dagger} \hat{O}), \quad (12)$$

where by definition

$$\hat{T}_{\kappa q} = \sum_{m, m'=-j}^j (-1)^{j-m'} C_{m, -m', q}^{j, j, \kappa} |jm\rangle \langle jm'|, \quad (13)$$

with  $C_{m, m', q}^{j, j', \kappa}$  the Clebsch-Gordan coefficients. The operators  $\hat{T}_{\kappa q}$  satisfy certain commutation relations with the angular momentum operators. They take the most compact form when using spherical components of angular momentum [31]:

$$[\hat{S}_m, \hat{T}_{\kappa q}] = \sqrt{\kappa(\kappa+1)} C_{q, m, q+m}^{\kappa, 1, \kappa} \hat{T}_{\kappa q+m}, \quad m = 0, \pm 1, \quad (14)$$

where  $\hat{S}_{\pm 1} = \mp \hat{S}_{\pm}/\sqrt{2}$  and  $\hat{S}_0$  is the same as in (8).

Applying these to (11) (with  $\partial_t \hat{r} = 0$ , as discussed above), it is straightforward to get the expression for  $r_{\kappa q}$ ,

$$r_{\kappa q} = \frac{i\lambda\alpha_{\text{st}}\sqrt{2\kappa(\kappa+1)}}{\Gamma + i\Delta q} (C_{q, -1, q-1}^{\kappa, 1, \kappa} \varrho_{\kappa q-1} - C_{q, 1, q+1}^{\kappa, 1, \kappa} \varrho_{\kappa q+1}), \quad (15)$$

and substitution into first equation of (11) yields

$$\partial_t \varrho_{\kappa q} = -i\Delta q \varrho_{\kappa q} + (C_{q+1, -1, q}^{\kappa, 1, \kappa} F_{\kappa q+1} - C_{q-1, 1, q}^{\kappa, 1, \kappa} F_{\kappa q-1}), \quad (16)$$

$$F_{\kappa q} = \frac{2\lambda^2\alpha_{\text{st}}^2\kappa(\kappa+1)}{\Gamma + i\Delta q} (C_{q+1, -1, q}^{\kappa, 1, \kappa} \varrho_{\kappa q+1} - C_{q-1, 1, q}^{\kappa, 1, \kappa} \varrho_{\kappa q-1}). \quad (17)$$

As one can see, it splits into independent subsystems for each individual  $\kappa$ . These systems can be solved numerically; however, for a large number of atoms this approach becomes impractical.

#### IV. APPROXIMATE SOLUTION AND CORRELATIONS OF ATOMIC PHOTOEMISSIONS

In [21] we dealt with the spectrum of resonance fluorescence from a single atom interacting with a field in the  $YS$  state. In the present paper we study another important characteristic quantity, second-order correlation of photoemissions from the atomic ensemble

$$G(t) = \text{Tr}[\hat{S}_+ \hat{S}_+(t) \hat{S}_-(t) \hat{S}_- \hat{\varrho}^{\text{st}}]. \quad (18)$$

Terms with  $\hat{r}$  do not contribute due to the aforementioned orthogonality of  $YS$  states with opposite parameters in the strong-field limit.

Making use of the equivalence between the Schrödinger and Heisenberg pictures [33], we rewrite (18),

$$G(t) = \text{Tr}[\hat{S}_+ \hat{S}_- \hat{\varrho}(t)], \quad \hat{\varrho}(0) = \hat{S}_- \hat{\varrho}^{\text{st}} \hat{S}_+. \quad (19)$$

In the irreducible tensor representation the expression for the correlation function yields

$$\begin{aligned} G(t) &= \sum_{\kappa=0}^{2j} \sum_{q=-\kappa}^{\kappa} \text{Tr}(\hat{S}_+ \hat{S}_- \hat{T}_{\kappa q}) \varrho_{\kappa q}(t) \\ &= 2j(j+1) \sum_{\kappa=0}^{2j} \sum_{m=-j}^j \varrho_{\kappa 0}(t) (-1)^{j-m} \\ &\quad \times (C_{m, -1, m-1}^{j, 1, j})^2 C_{m, -m, 0}^{j, j, \kappa}. \end{aligned} \quad (20)$$

We see that only  $\rho_{\kappa q}(t)$  with  $q = 0$  contribute to the correlation function. To obtain them, we need to solve the whole system (16) and (17), and doing this analytically for an arbitrary number of atoms is troublesome. We propose a simplification of (16) and (17) based on the following considerations. Initially, all  $\varrho_{\kappa q}$  with  $q \neq 0$  are absent. That is not the case for  $t > 0$ . Nevertheless, there are perspectives for a simplified “diagonal” modification of (16) and (17). Note that for a large number of atoms,  $\varrho_{\kappa 0}(t)$  with large  $\kappa$  values contribute to (20). For large  $\kappa$  the Clebsch-Gordan coefficients in (16) and (17) depend smoothly on  $q$ . One may expect that a similar smooth dependence (at least in the vicinity of  $q = 0$ ) is shared by  $\varrho_{\kappa q}(t)$  during an intermediate period of evolution. These observations give us a variety of options to simplify (16) and (17). It is, for example, possible to just assume that  $\rho_{\kappa q} \approx \rho_{\kappa q \pm 2}$ . A smarter way would be to also adjust the diffusion rates appropriately. When considering possible simplifications, our goal is to better simulate the time dependence of the correlation function  $G(t)$ . With this in mind, we have chosen the following approximation to (17):

$$F_{\kappa q-1} = F_{\kappa q+1} \approx 2\lambda^2 \alpha_{\text{st}}^2 \kappa(\kappa+1) \varrho_{\kappa q} \times \left( \frac{C_{q,-1,q-1}^{\kappa,1,\kappa}}{\Gamma + i\Delta(q-1)} - \frac{C_{q,1,q+1}^{\kappa,1,\kappa}}{\Gamma + i\Delta(q+1)} \right). \quad (21)$$

We have shifted the  $q$  index in the terms of  $F_{\kappa q \pm 1}$  that contain  $\rho_{\kappa q \pm 2}$ . It could seem that the more natural approach would be to also change the Clebsch-Gordan coefficients explicitly written in (16) in the same way. However, it was found to give a worse approximation in terms of the function  $G(t)$ . The substitution into (16) yields a set of independent equations on  $\varrho_{\kappa q}$  and we will only be interested in the case  $q = 0$ . Using an explicit expression of Clebsch-Gordan coefficients, we arrive at a rather simple solution

$$\varrho_{\kappa 0}(t) = \varrho_{\kappa 0}(0) \exp(-A_{\kappa} t), \quad A_{\kappa} = \frac{4\Gamma\lambda^2 |\alpha_{\text{st}}|^2 \kappa(\kappa+1)}{\Gamma^2 + \Delta^2}, \quad (22)$$

where  $\varrho_{\kappa 0}(0)$  is evaluated from (19), assuming that  $\hat{\rho}^{\text{st}}$  is a maximally mixed state. Interestingly, while the approximation (21) was initially developed for a high number of atoms, it also works quite well for small numbers too.

It is instructive to compare the calculated correlation functions in a YS field with the same function in the  $G$  field. For the  $G$  field, there is no need to use the ansatz (9). The master equation for the case of the  $G$  field is obtained from (6) by replacing all YS operators with  $G$  operators (including those in  $\Lambda[\hat{\rho}_{\text{tot}}]$ ). If the source of the field is strong, as it is for the YS field, the field with good precision occurs in its steady state  $|\alpha_{\text{st}}\rangle_G$ , with  $\alpha_{\text{st}}$  taken from (5):  $\hat{\rho}_{\text{tot}} = \hat{\rho} \otimes |\alpha_{\text{st}}\rangle_G \langle \alpha_{\text{st}}|$ . Tracing over field variables, we see that the atomic density matrix then obeys the von Neumann equation (neglecting the slow spontaneous emission)

$$\partial_t \hat{\rho} = -i[\lambda \alpha_{\text{st}} (\hat{S}_+ - \hat{S}_-) + \Delta \hat{S}_0, \hat{\rho}]. \quad (23)$$

Here we have used the fact that  $\alpha_{\text{st}}$  is purely imaginary. Its solution, as is well known, demonstrates Rabi oscillations (and they can be seen in the correlation function as thin solid lines

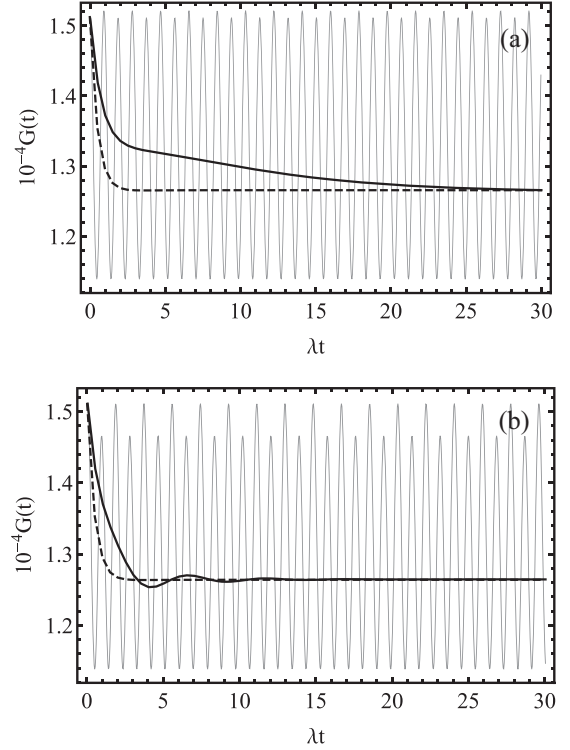


FIG. 1. Correlation function of atomic photoemissions evaluated at (a)  $\Delta = \lambda/10$  and (b)  $\Delta = 2\lambda/3$ . The other parameters are similar for all the curves:  $\mu = 10\lambda$ ,  $\nu = 3\lambda$ , and  $N = 25$ . Correlation functions in the  $G$  field are represented by thin solid lines, thick solid lines are for exact correlation functions in the YS field, and approximate functions in the YS field are shown by dashed lines.

in Fig. 1) and is evidently drastically different from the case of the YS field.

Correlation functions evaluated for different values of the system's parameters are given in Fig. 1 in comparison with the exact correlation function evaluated using the numerical solution of (16) and (17) and correlation functions in the  $G$  field evaluated using the solution of (23). As one can see, the approximation is able to catch the general shape of the curve, although the details related to the  $\Delta$  dependence are not present (the agreement between the exact solution and the approximation is better for greater detunings). However, the asymptotic behavior

$$\lim_{t \rightarrow \infty} G(t) = \left( \frac{2j(j+1)}{3} \right)^2 \quad (24)$$

is the same for the approximate and exact solutions due to eventual extinction of all  $\varrho_{\kappa 0}$  with  $\kappa > 0$  for both types of solution.

## V. CONCLUSION

We have studied correlations of photoemissions in the ensemble of two-level atoms interacting with light in the special superposition of Glauber coherent states. The explicit incorporation of the light source into the model is of principal importance. Despite the source tending to restore the coherence of the cat state, the light mode is inevitably



getting correlated with atoms. These correlations turn the atomic ensemble into an open quantum system due to the openness of the cat-state source. Spontaneous transitions in the atomic system were neglected in the evolution equation and only inserted later “by hand” when considering the correlation function. Nevertheless, correlations of spontaneous emissions revealed by  $G(t)$  are purely of irreversible type. That drastically differs from the case of atoms interacting with the  $G$  field, where the steady state displays Rabi oscillations.

The system demonstrates a self-induced feedback, the phase switchings of the field components upon every induced absorption of a photon. It is the feedback that induces correlations between atomic and photonic subsystems. The case of many atoms and a multiphotonic field is of particular

interest. A somewhat simplistic model appears to be in qualitative agreement with the exact numerical calculation. We believe that investigation of more complex systems with self-organized feedback of the type described in the present paper could potentially reveal further interesting phenomena.

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