Generic method for lossless generation of arbitrarily shaped photons

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We put forward a generic method that enables lossless generation of pure single photons with arbitrary shape over any degree of freedom or several degrees of freedom simultaneously. The method exploits pairs of entangled photons. One of the photons is the subject for lossy shaping manipulations followed by a specially designed mode-equalizing measurement. A successful measurement outcome heralds the losslessly shaped second photon. The method has three crucial ingredients that define the quantum state of the shaped photon: the initial bipartite state of the photons, modulation of the first photon, and its mode-equalizing detection. We provide a specific recipe with a combination of these ingredients for achieving any desired pure state of the shaped photon.

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I. INTRODUCTION

Complete control over the properties of light up to the level of single photons is an invaluable tool for quantum information science and fundamental studies of light-matter interaction. The crucial prerequisite is the ability to create a spatiotemporal distribution of single-photon electromagnetic field with the desired characteristics, i.e., to shape a photon by design. Despite the ever-growing demand for tuneable single-photon sources, there is a lack of practical, efficient, and scalable methods for photon shaping.

The straightforward approach to generate shaped photons is to manipulate the properties of a quantum emitter (atom, ion, quantum dot, molecule, etc.). The emission of quantum systems is naturally quantized, though its characteristics may differ from the desired ones. To obtain photons with a given shape, one could manipulate properties of the emitter and to some extent tailor the properties of its radiation. The development of such methods is promising, but truly deterministic and flexible single-photon sources based on this approach are quite challenging to implement in practice [1–5].

An alternative approach to single-photon shaping is to manipulate the shape after the photon has already been produced [6,7]. This approach usually requires a less complicated setup and offers higher flexibility. At the same time, a common problem of any direct manipulation are losses, either of technical or of fundamental origin. After the lossy manipulation, the shaped photon appears only probabilistically, which fundamentally limits the scalability of this approach. Even a small loss drastically reduces the probability to generate several shaped photons since the multiphoton shaping rate decreases exponentially with the number of photons. This obstacle hinders the implementation of practically useful multiphoton proposals, e.g., for quantum computation or quantum networks.

In this work we propose a method for shaping single photons with respect to any degree of freedom. The method is lossless, practical, and overcomes the above-mentioned limitations. As the indispensable resource the method exploits pairs of entangled photons that could (but need not) be produced in nonlinear optical processes such as parametric down-conversion. We propose to perform all the *lossy* shaping manipulations on only one of the photons of each pair, while its entangled counterpart is merely the subject for heralding. Such an indirect shaping approach is based on the following two effects. The first effect is conditional state preparation: the detection of one of the entangled photons in a pure state heralds the second photon in a pure state. The heralded state depends on the detection procedure and can be "remotely prepared" [8] by choosing the detection basis and postselecting detection outcomes. The second effect is "ghost" interference with entangled photons [9] that can be interpreted as the conditional preparation of a pure single photon [10,11] with a transverse quantum state defined by the mask in the heralding arm. In this work we bring these effects together to solve a practically important task: produce a single photon with an arbitrary spatiotemporal shape in a pure quantum state in a conditionally lossless way.

The method generalizes existing experiments on nonlocal effects with pairs of entangled photons and remote manipulations with heralded photons [11–15]. In contrast to the previous work, we do not consider specific sources of entangled photons, such as parametric down-conversion, and avoid *ad hoc* solutions for achieving pure heralded photons, but explicitly identify all crucial ingredients that enable the heralded generation of pure shaped photons in the most general case. We describe a photon shaping procedure that involves entanglement, modulation, and detection in terms of arbitrary photonic modes. Furthermore, our consideration also shows that the originally proposed spatial "ghost" effects [9,16] can be performed in an arbitrary degree of freedom as well as in several degrees of freedom simultaneously, having in hand a proper source of entangled photons and performing the heralding photon detection in a specified way.

II. DESCRIPTION OF THE METHOD

The main idea is illustrated in Fig. 1 and can be outlined as follows. To produce a shaped single photon, which we shall

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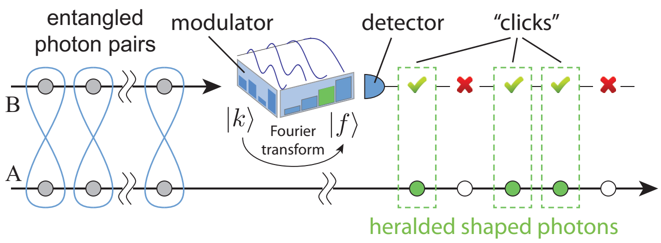

FIG. 1. Schematic of the heralded single-photon shaping method. A source of light generates pairs of entangled photons A and B. The photons B probabilistically pass through a modulator which forms a required single-photon shape in a basis $\{k\}$. A single photon detector is placed after the modulator and detects photons in the basis $\{f\}$ which is Fourier conjugated to the modulation basis $\{k\}$. Clicks of this detector deterministically herald the shaped photons A (marked green), while the "no-click" cases are discarded (marked white).

call photon A, we begin with a pair of photons A and B that are entangled in the degree of freedom that we want to shape. These entangled photons can be addressed individually. Next, we modulate the photon B according to the desired shape in a modulation basis and send it to a single-photon detector. The detector measures the photon in the basis which is Fourier-conjugated to the modulation basis. For example, if the desired shape has a certain temporal profile, we perform modulation in the time domain and detection in the frequency domain [15]; to produce a single photon with the desired transverse spatial shape, we use a spatial mask in the near field and single-mode detection in the far field and so on. Successful detection of the photon B ("click") heralds the shaped photon A, and only such cases are selected. If the photon B is not detected, we discard the photon A. Due to the initial entanglement between photons A and B, modulation of the photon B and selection of "clicks" indirectly affect the shape of the photon A. In turn, single-mode detection of the photon B in the Fourier-conjugated basis ensures purity of the shaped heralded photon A.

Now we present the formal description of our method. The task is to produce a given spatiotemporal distribution of electromagnetic field that contains just one photon, i.e., to produce a shaped photon. Our method can be applied to shaping the photons with respect to various parameters, such as the distribution of amplitude, polarization, phase, spectrum, orbital angular momentum, and so on. At the moment we consider only one parameter and count all the others as fixed. The desired distribution of the single-photon light field can be described in some basis given by a set of N modes [17] enumerated by index $k = 0,1,\ldots,N-1$. The single-photon state of the mode k is defined by applying the creation operator $\hat{a}_k^\dagger$ to the vacuum state and denoted by $|k\rangle = \hat{a}_k^\dagger |0\rangle$. The desired shaped photon can be formally written as a pure superposition state

$$|\phi\rangle = \sum_k \nu_k |k\rangle , \tag{1}$$

i.e., the shape $\{\nu_k\}$ of a photon is given by a probability distribution $|\nu_k|^2$ and relative phases $\arg(\nu_k)$ over the modes $\{k\}$.

As the first essential component of our method, we use a pair of photons (A and B), which are entangled in the degree of freedom that we want to shape. Consider the maximally entangled state

$$|\Phi\rangle_{AB} = \frac{1}{\sqrt{N}} \sum_k |k\rangle_A |k\rangle_B , \tag{2}$$

that provides uniform distribution of both entangled photons over the modes $\{k\}$.

Next, we apply a modulator to the photon B such that the light field after the modulator is distributed according to the desired shape $\{\nu_k\}$:

$$|k\rangle_B \to \nu_k |k\rangle_B . \tag{3}$$

As a result, the joint state $|\Phi\rangle_{AB} \propto \sum_k |k\rangle_A |k\rangle_B$ is transformed to $|\Phi\rangle'_{AB} \propto \sum_k \nu_k |k\rangle_A |k\rangle_B$. In the simplest case, this operation can be realized by a passive filter that attenuates modes $\{k\}$ according to the distribution $\{\nu_k\}$.

After the modulator, we place a single-photon detector to measure the photon B in the Fourier conjugated basis $\{f\}$:

$$|f\rangle_B = \frac{1}{\sqrt{N}} \sum_k e^{i\frac{2\pi}{N}kf} |k\rangle_B , \quad f = 0,1,\ldots,N-1. \tag{4}$$

A "click" of the detector unambiguously tells us in which Fourier-conjugated mode f the photon B is detected. Thus we can conclude that the heralded state of the photon A is

$$|\phi\rangle'_A \propto \langle f|_B \Phi\rangle'_{AB} \propto \sum_k \nu_k e^{-i\frac{2\pi}{N}kf} |k\rangle_A . \tag{5}$$

In the case $f = 0$ this state has the required single-photon shape (1), which heralds successfully completed shaping operation. This constitutes one of the main results of this work.

Now, we clarify the role of all the components of our method (namely entanglement, mode-selective modulation, Fourier-conjugated detection) in detail.

First of all, to get the photon A in a pure state (1), photons A and B must be entangled. In principle, any maximally entangled state would solve the task. The only difference between different maximally entangled states is a certain type of symmetry between the modulator applied to the photon B and the shape of the heralded photon A. For example, the use of generalized Bell states [18] instead of (2) leads to the reshuffled order of coefficients $\{\nu_k\}$ due to the modified correlation symmetry between entangled photons [19].

It is crucial to understand that classically correlated photons, e.g., in the separable state $\rho_{AB} \propto \sum_k |k\rangle\langle k|_A \otimes |k\rangle\langle k|_B$, do not solve the task. Indeed, applying the same procedure [mode-selective modulation (3) and detection in the Fourier conjugated basis (4)] to the photon B, we get the heralded photon A in the mixed state

$$\rho_A = \sum_k |\nu_k|^2 |k\rangle\langle k|_A . \tag{6}$$

Such photon has the same probability distribution $|\nu_k|^2$ over the modes $\{k\}$ as the the desired state (1), but possesses no coherence between them.

The second cornerstone of our method is the way of detecting the photon B. In the above derivation, we employed single-photon detection in the Fourier-conjugated modes. To understand the reason for it, consider detection of the photon B without Fourier transformation, just in the same basis $\{k\}$ as the modulator is applied. Instead of (5), we would obtain the state $|\phi\rangle''_A \propto \langle k|_B\Phi\rangle_{AB} \propto |k\rangle_A$, which merely reflects the original correlations between entangled photons in the basis $\{k\}$ but carries no imprint of the modulator on the photon A. In contrary, we want to detect the photon B in such a way that all modulated modes $\{k\}$ are taken equiprobably. The detection of the photon B in the basis $\{f\}$, which is complementary, or unbiased, to the modulation basis $\{k\}$, ensures that the modulated modes are taken with equal probabilities, or "equalized" after the measurement. Due to entanglement between the photons A and B and postselection of "clicks," this equalization also acts on the photon A and makes it fully coherent in the basis $\{k\}$.

Important to note that the modulated modes must be equalized (i.e., taken with equal probabilities) in a coherent way, which makes them indistinguishable. The use of a mode-insensitive detector that collects all the photon field B without mode discrimination (the so-called bucket detector) does not result in the shaped pure state (1). Indeed, if the coherence between the modulated modes is ignored, there is no difference whether we have A and B photons in the entangled or separable joint state. Mathematically this is described as the trace of the joint state over the photon B, i.e., $\rho_A = Tr_B \rho_{AB}$, which leads to the mixed state of the photon A (6).

Mode-selective modulation (3) is the third key component of our method, which essentially defines the shape of the photon A (1). To increase the overall heralding efficiency, i.e., to increase the detection rate of the photons B, the modulator can be equivalently replaced by the use of a nonmaximally entangled state that already has the desired distribution ν_k over the modes $\{k\}$. Indeed, the action of the modulator (3) can be treated as a replacement of the joint maximally entangled state $|\Phi\rangle_{AB}$ by a nonmaximally entangled

$$\sum_k |k\rangle_A\,|k\rangle_B \rightarrow \sum_k \nu_k\,|k\rangle_A\,|k\rangle_B \tag{7}$$

without the need for modulation afterwards. It makes no essential difference whether we use a maximally entangled state with a modulator or a nonmaximally entangled state without the modulator. Though the latter option might have technical advantages in experimental realizations of our method. For example, if entangled photons A and B are produced in optical parametric processes, then their joint-entangled state can be tuned by controlling the pump that drives the process [16,20,21]. The use of pump modulation provides higher heralding rate since one can compensate the pump modulation losses by the corresponding increase of the pump intensity.

It is worth mentioning that the above-described combination of mode-selective modulation and mode-equalizing detection can be realized in different ways. First, mode-equalizing detection via a measurement in the Fourier-conjugated basis is not a unique solution. In a broad sense, we can rely on Heisenberg's uncertainty principle which states that the more precisely we detect an observable, the less knowledgeable is the complementary one. Thus we can use any unbiased basis for this purpose or employ generalized measurements that do not form orthogonal bases. Second, mode-selective modulation and mode-equalizing detection are not necessarily separate steps, but can be realized jointly. For example, within the context of temporal shaping [15], one can use an unbalanced interferometer in such a way, that two consecutive temporal modes do overlap after the interferometer (mode equalization), while controllable losses in one arm of the interferometer define mode amplitudes (mode modulation) [13]. The latter approach allows for temporal shaping of single photons, albeit the number of photon modes is limited by 2 (number of interferometer arms).

III. SEVERAL DEGREES OF FREEDOM

The proposed shaping method is very versatile and can be used to shape single photons in any degree of freedom, provided the required resources are experimentally available. Moreover, the method allows for shaping photons in several degrees of freedom simultaneously. Consider two degrees of freedom, represented by two sets of modes $\{k\}$ and $\{l\}$, in which we want to produce a single photon with a desired state

$$|\phi\rangle_A = \sum_{k,l} \nu_{kl}\,|k,l\rangle_A\,, \tag{8}$$

analogous to the state (1). To realize such a state, we can use exactly the same method as above, after proper modifications of the main components.

First, the joint state $|\Phi\rangle_{AB}$ must be entangled in both degrees of freedom. There are different types of multivariable entanglement, and for our purpose we use the following:

$$|\Phi\rangle_{AB} \propto \sum_{k,l} |k,l\rangle_A\,|k,l\rangle_B\,. \tag{9}$$

This type of state, called hyperentangled, can be experimentally generated for various degrees of freedom [22,23]. Next, the modified modulator should consist of two consecutively applied modulators, each of them shapes the photon in different degrees of freedom. Finally, the detection must be performed in such a way that a measurement outcome may correspond to each modulated mode with equal probability.

For illustration purposes only, let k be the transverse spatial binary coordinate ("left or right slit"), l is polarization ("H/V"), and the desired single-photon state corresponds to the coherent superposition of two orthogonally polarized slits:

$$|\phi\rangle_A = \frac{1}{\sqrt{2}}(|\text{left},V\rangle_A + |\text{right},H\rangle_A). \tag{10}$$

The required joint state $|\Phi\rangle_{AB}$ must be hyperentangled in spatial and polarization degrees of freedom. The combined modulator consists of two consecutively applied modulators: one of them shapes the spatial amplitude profile (a double slit mask), and the other one shapes the polarization profile (horizontally and vertically oriented polarizers behind the right and left slits, correspondingly), as illustrated in Fig. 2(a). The joint state after the modulator transforms to $|\Phi\rangle_{AB} \propto |\text{left},V\rangle_A\,|\text{left},V\rangle_B + |\text{right},H\rangle_A\,|\text{right},H\rangle_B$.

To herald the shaped photon in a pure state, we have to detect its entangled counterpart without any possibility to distinguish which slit and polarizer it may have passed through, i.e., to equalize the modes after the modulators. To solve this task, one can perform measurement in the basis which is Fourier conjugated with respect to each degree of freedom, i.e., place

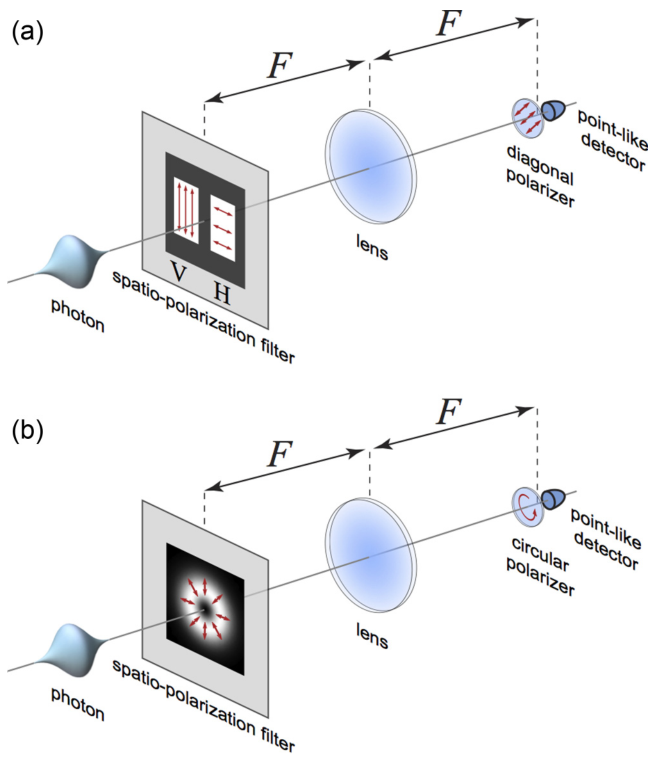

FIG. 2. Explicit schemes for producing shaped photons in quantum state (a) (10) and (b) (11). The transverse amplitude-polarization profile is defined by combining an intensity filter and polarizers: (a) double-slit mask + H/V polarizers; (b) doughnut-like gradient + radial polarizer. Detection of the photon B is realized by a point-like single-photon detector placed in the back focal plane of a lens ("F-F" configuration) with a polarizer [(a) diagonal and (b) circular] in front of the detector.

a point-like detector in the far field after the double slit mask (or use a lens in "F-F" configuration, which realizes spatial Fourier transform) and a diagonally oriented polarizer in front of the detector (polarization Fourier transform). A "click" of the detector in this configuration carriers no information about H/V polarization of the photon B (projection to the diagonal polarization $|D\rangle \propto |H\rangle + |V\rangle$), and no information about which slit the photon B has passed though (the projection to the center of the focal plane $|\text{center}\rangle \propto |\text{left}\rangle + |\text{right}\rangle$). As a result, both spatial and polarization modes are equalized, and one obtains the desired pure state (10).

In the generic multidimensional case, the structure of the required equalizing measurement appears to be highly nontrivial, and we leave it for future study. Here we only note that, using our approach, it is possible to produce photons in *any* quantum state. This can be immediately exploited in continuous-alphabet high-dimensional quantum communication [24] or generation of photons in the radially polarized doughnut mode; essential for the efficient free-space excitation of a single atom [25]. It has been shown that, with the direct spatiopolarization filtering, the doughnut mode can be generated with a power efficiency of 70% [26]. By applying our method, one can, in principle, achieve 100% efficiency, which we show below.

The desired shaped photon has axially invariant doughnut-like amplitude distribution $A(r) \propto r^2 e^{-r^2}$ that depends on the distance r from the axis, and up to normalization can be written as

$$|\phi\rangle_A \propto \int |\vec{r}|^2 e^{-|\vec{r}|^2} |\vec{r}\rangle \, dr. \tag{11}$$

Similarly to the double-slit example, we consecutively apply two filters to photon B: the first one shapes the "doughnut" amplitude profile $A(r) \propto r^2 e^{-r^2}$, and the second one shapes radial polarization $P \propto \vec{r}$ [see Fig. 2(b)]. After the filters, the photon B has to be detected via the mode-equalizing measurement. Such measurement can be realized by a point-like detector placed in the far field and a circular polarizer in front of it. Far-field detection ensures equalization of spatial modes (as discussed in the double-slit example), and the circular polarizer equalizes polarization modes (after detecting the circularly polarized photon, all radially polarized modes have equal probabilities). A click of the detector heralds the photon B in the pure state (11).

It is important to note that our approach offers 100% efficient heralded shaping, compared to the 70% efficiency achieved before [26]. Even a small change in efficiency can drastically influence the scalability of multiphoton shaping: if the probability to generate one photon is 70%, then the probability to generate, for example, 10 shaped photons is equal to $0.7^{10} \simeq 3\%$ and the probability to generate 15 photons is less than 0.5%. Moreover, the wavelength of the shaped photon should match the desired atomic transition. For example, in our case of interest the required wavelength is 252 nm [26], which makes it experimentally challenging to shape the photon directly. Using the indirect shaping approach, entangled photons may have very different wavelengths, say, ultraviolet and visible. It is much easier to operate with the visible photon and indirectly shape the ultraviolet one. Of course, such an approach implies availability of the proper source of entangled photons.

IV. CONCLUSION

In summary, we propose a simple and practical method for heralded lossless shaping of pure single photons. The proposed heralded shaping method has several distinctive features making it highly appealing to a wide range of applications.

Our method is very versatile and can be used to shape single photons with respect to any degree of freedom. Moreover, with the use of hyperentanglement and mode-equalizing measurements, shaping can be performed with respect to several degrees of freedom simultaneously. This feature enables complete control over the spatiotemporal distribution of the single-photon electromagnetic field. Thus we expect our method to be particularly useful for constructing widely tuneable single-photon sources and point to the realistic way towards making efficient photonic interfaces and coherent control of quantum systems at the single-photon level. To experimentally verify the shape of the heralded photon obtained by the proposed method, one can perform informationally complete measurements and mode reconstruction [27–30] or directly observe the enhancement of light-matter interaction efficiency [25,31,32].

The proposed method does not use any direct manipulation with a photon which is subject for shaping, but only with its entangled counterpart. Thus such a shaping procedure is conditionally lossless with respect to the shaped photon. It points the way towards the scalable generation of shaped photons, which is vital for the realization of multiphoton quantum information processing tasks, such as quantum computations, repeaters, networks, memory, and so on. It can be directly integrated with current technologies since all the required components are readily accessible. Experiments on nonlocal effects with pairs of photons entangled in spatial [11] or temporal [12–14] degrees of freedom confirm the experimental feasibility of the proposed method.

The method can be implemented with various sources of entangled photon pairs, providing shaped photons in various degrees of freedom and a wide range of parameters. In addition, if the direct shaping of the photon is technically difficult, then the heralded shaping can be a very attractive practical alternative, which is especially interesting when wavelengths of entangled photons are significantly different (e.g., micron- and angstrom-range [33]).

Finally, we consider the shaping of single photons, but exactly the same ideas can be applied to the larger class of quantum systems, such as multiphoton states, atomic ensembles, phonons, (quasi)particles, and so on. One can definitely say that entanglement finds a new practically useful application, which can turn into a novel powerful tool for quantum control of various physical systems. Thus we anticipate that our results, primarily targeted to single photons, can also be of significant interest for the other research fields as well.

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