Asymmetric noise sensitivity of pulse trains in an excitable microlaser with delayed optical feedback

Soizic Terrien,* Bernd Krauskopf, and Neil G. R. Broderick
Dodd-Walls Centre for Photonic and Quantum Technologies, University of Auckland, 38 Princes Street, Auckland 1010, New Zealand

Louis Andréoli, Foued Selmi, Rémy Braive, Grégoire Beaudoin, Isabelle Sagnes, and Sylvain Barbay
Centre de Nanosciences et de Nanotechnologies, C2N-UMR9001, CNRS, Université Paris-Sud, Université Paris-Saclay, Site de Marcoussis, Route de Nozay, 91460 Marcoussis, France
(Received 2 May 2017; published 27 October 2017)

A semiconductor micropillar laser with delayed optical feedback is considered. In the excitable regime, we show that a single optical perturbation can trigger a train of pulses that is sustained for a finite duration. The distribution of the pulse train duration exhibits an exponential behavior characteristic of a noise-induced process driven by uncorrelated white noise present in the system. The comparison of experimental observations with theoretical and numerical analysis of a minimal model yields excellent agreement. Importantly, the random switch-off process takes place between two attractors of different nature: an equilibrium and a periodic orbit. Our analysis shows that there is a small time window during which the pulsations are very sensitive to noise, and this explains the observed strong bias toward switch-off. These results raise the possibility of all optical control of the pulse train duration that may have an impact for practical applications in photonics and may also apply to the dynamics of other noise-driven excitable systems with delayed feedback.

DOI: 10.1103/PhysRevA.96.043863

I. INTRODUCTION

Pulsing regimes arise in many different contexts, from biological systems such as neurons to optical devices. They are key elements in many applications, for example, in photonics and optical communications where they can provide clocks for optical processing of information. Excitable systems are of particular interest in this respect. They exhibit an all-or-none response to external perturbations: above a minimal perturbation amplitude (the excitable threshold), a characteristic, pulse-shaped response is elicited. We consider here the effect of delayed self-feedback on an excitable system: a first excitable pulse can trigger a new pulse when it comes back after a fixed delay. In this simple picture, one obtains a controllable pulsing system, producing pulses at a fixed repetition rate.

Optical pulse trains can result from temporal self-localization of light in dissipative optical systems, generally involving a cavity and a nonlinearity, and are thus also called trains of temporal dissipative solitons. These can appear spontaneously from noise or be triggered by a coherent perturbation. They have also been invoked to explain the formation of soliton molecules [1] or the merging of solitons [2] in mode-locked lasers. Optical temporal dissipative solitons have been found in a coherently injected fiber cavity [3], in a driven nonlinear microresonator [4], and in a face-to-face VCSEL configuration [5]. Self-localization of light associated with other degrees of freedom has also been reported in optical cavity systems in Ref. [6] with polarization, and in Ref. [7] where the optical phase was considered and excitable pulses consist of 2π phase shifts in the emitted light. An interesting property of temporal dissipative solitons lies in the possibility to control them [8,9].

In this paper we investigate an excitable micropillar laser with saturable absorber and delayed optical feedback. This system is conceptually simpler and more compact than other systems where excitable pulse regeneration has been found because it consists of only a microcavity and a feedback mirror. We show that it sustains temporal dissipative solitons in the form of trains of optical pulses, triggered by an external perturbation. In particular, several pulse trains can be triggered and sustained simultaneously in the external cavity; see the inset of Fig. 1. A rate equation model for a single-mode laser with saturable absorber subject to incoherent optical delayed feedback [10] emerges as a natural minimal model for this laser system and shows very good agreement with the experimental observations. We determine the important role of noise for the observed stochastic switching dynamics and how it results in the finite duration of pulse trains. We also unveil the physical reason why this random process is strongly asymmetric and biased toward switch-off. Our findings suggest a way to optically control the pulse train duration, which shows that an excitable laser with feedback constitutes a simple system with interesting prospects for applications in photonics.

II. DESCRIPTION OF DEVICE AND EXPERIMENT

Figure 1 represents a sketch of the experiment. A micropillar laser with intracavity saturable absorber is optically pumped by a laser diode emitting at $\sim$800 nm; see Ref. [11]. The micropillar laser includes two gain and one saturable absorber quantum wells [12,13] and emits light at a wavelength close to 980 nm. It is temperature stabilized by a Peltier element to better than 0.1 °K around −12.1 °C. The pump is sent through a dichroic mirror DM that reflects light at the laser emission wavelength. Part of the light is sent back to the micropillar because of a high-reflectivity feedback mirror FB after being focused by a lens of 5 cm focal length. A 70/30 beamsplitter extracts part of the emitted light to a fast avalanche photodetector (>5 GHz bandwidth). The resulting signal is analyzed by a high-bandwidth oscilloscope (13 GHz).

*Corresponding author: s.terrien@auckland.ac.nz

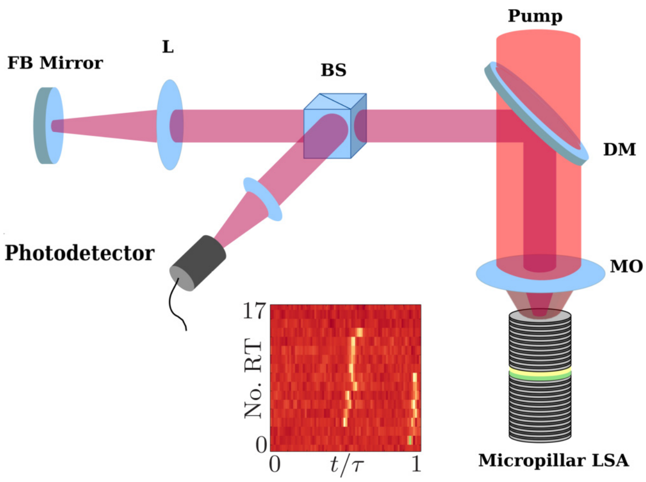

FIG. 1. Sketch of the micropillar semiconductor laser with intracavity saturable absorber, optical pumping, and delayed optical feedback loop; BS: beam splitter, L: lens with $f = 5\,\text{cm}$, FB mirror: high-reflectivity feedback mirror, BS: 70/30 beam splitter, DM: dichroic mirror. The inset shows two coexisting pulse trains in the pseudospace of time and number of round trips.

When the pump of the solitary micropillar laser is increased from below to above the laser threshold to a power density value of about $7.6\,\text{kW/cm}^2$, there is a transition from practically zero emission to emission of a train of Q-switched pulses with average power of some tens of microwatts. This behavior indicates that the system is excitable below threshold [14,15]. Excitability in lasers with saturable absorbers has been experimentally investigated in CO_2 lasers [16], solid-state lasers [17,18], semiconductor lasers with integrated saturable absorber (planar or micropillar structures) [11,19], and graphene fiber lasers [20]. The details of the response properties of the micropillar laser depend on the precise perturbation sent to the system; for either a coherent or incoherent optical pulse, the excitable pulse duration has been measured to range below 200 ps. For a coherent perturbation (at the cavity resonance wavelength), the amplitude of the excitable pulse only depends on the optical pump and not on the perturbation strength, as long as it is greater than the excitable threshold. The response is delayed in a nonlinear fashion with respect to the input perturbation since the system passes near a stationary point [21,22].

We now consider the micropillar laser with an additional delayed optical feedback. An obvious effect of the feedback is to reduce the laser threshold; however, the excitable regime remains. The feedback strength is fixed such that the lasing threshold A_{th} is reduced by 14.6% with respect to that of the solitary laser, and the delay τ is 4.88 ns.

III. FINITE PULSE TRAINS

In the experiment, a pulse is triggered incoherently by a 80 ps duration optical pulse at a wavelength close to 800 nm [22]. This response pulse, at the cavity resonance wavelength, is reinjected after the delay τ and may trigger a new pulse. Note that the first pulse in a train has a larger amplitude because it is triggered incoherently. Figure 2 illustrates the experimentally observed pulsing dynamics. Figure 2(a) shows a typical temporal trace: a train of regularly spaced pulses, with significant variation of the amplitude, stops after approximately $40\times\tau$. Figure 2(b) shows the same time trace folded at the feedback delay time τ. In this pseudospace map, the vertical axis represents the number of round trips in the external cavity. The straight line of light intensity, with positive slope, highlights a regular pulse repetition rate. It is slightly larger than τ, due to the response time of the excitable laser to a perturbation [21,22]. Figures 2(c1)–2(c3) present, for three different values of the pump A, log-scale histograms of the number of pulses in each of 20 000 recorded pulse trains. They clearly unveil an exponential behavior, whose slope of the logarithmic histograms varies considerably with A. Figure 2(d) shows that the natural logarithm of the associated decay rate ν (related to the reciprocal of the slope) increases linearly with A. Importantly, once the pulse train has stopped, it never starts again from noise: an external perturbation is required to trigger a first pulse. We conclude that the laser is bistable, and that the pulsing solution has a small basin of attraction compared to the off-state.

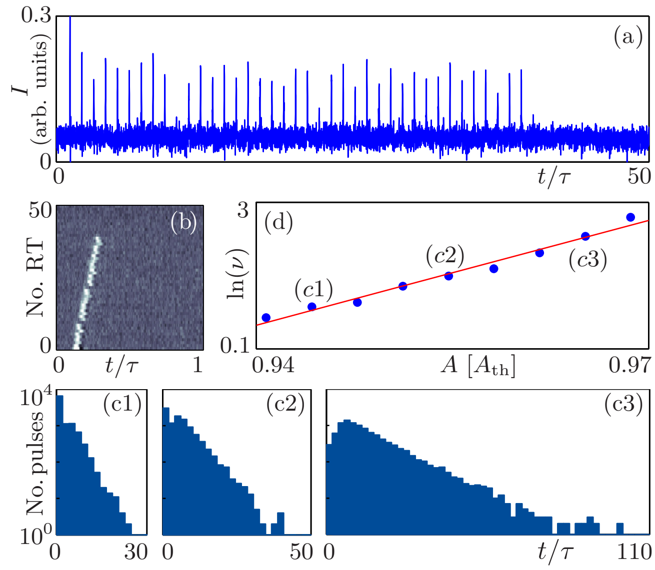

FIG. 2. Experimental measurements: (a) Typical pulse train, with finite duration. (b) Pseudospace mapping of the time trace (see text). (c) Log-scale histograms of the number of pulses in 20 000 recorded pulse trains, for pump currents of 94.4%, 95.4%, and 96.5% of the laser threshold A_{th}. (d) Logarithm of the corresponding decay rate ν (scaled to the feedback delay τ) versus scaled pump A.

These observations strongly suggest that the laser switch-off is a noise-induced process, following Kramers' law [23,24]. Different sources of noise manifest themselves experimentally: quantum noise because of spontaneous emission processes, and classical noise due to parameter fluctuations. While the system operates with low powers, pump noise is expected to be dominant. We have checked that polarization fluctuations inherent to circularly symmetric lasers are not responsible for the observed switch-off phenomenon by testing laser samples where the polarization direction is fixed at the fabrication stage. Moreover, we never observe a trend of decreasing pulse amplitudes during an individual pulse train, but rather an abrupt switch-off event. Therefore, the experimental measurements cannot be explained in terms of transient pulse trains. The increasing duration of pulse trains with A can be understood by recalling that the excitability threshold decreases when A is increased [21]: the higher A, the easier it is for the reinjected pulse to regenerate the pulse, and the less pronounced is the noise sensitivity.

IV. YAMADA MODEL WITH FEEDBACK

To test these ideas, we investigate the Yamada model with incoherent feedback [10,25], and the addition of both spontaneous emission and pump noise. This model for Q-switched pulsing lasers is written, in a dimensionless form, as a system of three differential equations for the gain G, absorption Q, and intensity I:

$$\dot{G} = \gamma_G(A - G - GI) + \epsilon(t),$$
$$\dot{Q} = \gamma_Q(B - Q - aQI),$$
$$\dot{I} = (G - Q - 1)I + \kappa I(t-\tau) + R_{\rm sp}(G+\eta)^2 + F(t), \quad (1)$$

where time has been rescaled to the cavity photon lifetime. The pump A is the main control parameter in the experiment. The parameters describing the laser's material are chosen to represent the experimental conditions; they are $B = 2$ for the nonsaturable absorption, $a = 10$ for the saturation parameter, and $\gamma_G = 5\times10^{-4}$ and $\gamma_Q = 10^{-3}$ for the carrier recombination rates in the gain and absorption media, respectively [11,19]. The delayed term in the intensity equation describes the incoherent feedback, with delay τ and strength κ. The spontaneous emission noise term is proportional to the factor $R_{\rm sp}$, and $F(t) = (G+\eta)\sqrt{2R_{\rm sp}I}\xi(t)$ is the associated Langevin force, with white Gaussian noise $\xi(t)$ and the carrier density at transparency $\eta = 1$ [21]. The effect of spontaneous emission noise on the carrier density is not taken into account since pump noise, described by white Gaussian noise $\epsilon(t)$ with standard deviation n_G, is dominant. Because pulse reinjection by the external mirror occurs with a random polarization, and in a state where the electric field is practically zero, there are no phase effects and the feedback can be modeled as being incoherent. This was supported by showing theoretically that system (1) and a Lang-Kobayashi-type model show effectively the same dynamics in the excitable regime [10].

Equations (1) form a system of delay-differential equations (DDEs), that have an infinite-dimensional phase-space [26]. We used a dedicated solver [27] for its numerical integration, and the toolbox DDE-Biftool [28,29] for continuation of equilibria, periodic orbits, and their bifurcations in the parameters space.

A. Bifurcation analysis

Since experimental parameters values cannot be estimated easily, the choice of parameters is motivated by the agreement of the model dynamics with the experimental observations. A bifurcation analysis has shown that a wealth of dynamics can be found as A, κ, and τ are varied [10]. Figure 3 represents the phase portrait for realistic values $\kappa = 0.04$ and $\tau = 1000$ (corresponding to a feedback loop of approximately one meter, with τ much larger than the pulse duration), and for a range of A for which the solitary laser is in the excitable regime. We observe, as in the experiment, the coexistence of one stable equilibrium with zero intensity and a single stable pulsing periodic solution, with period close to τ. There are also three unstable periodic solutions and two unstable equilibria.

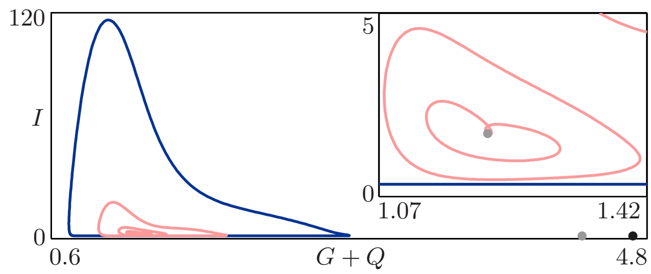

FIG. 3. Phase portrait of (1) in the $(G+Q,I)$-plane, for A=2.7, $R_{\rm sp} = 0$, and $n_G = 0$. Dots are equilibria, and closed curves are periodic orbits. Stable and unstable periodic orbits are represented in blue (dark gray) and pink (light gray), respectively; stable and unstable equilibria are represented in black and gray, respectively.

B. Influence of noise

In the presence of noise, system (1) becomes a system of stochastic DDEs and needs to be investigated by simulation. We fix the noise parameters at $n_G = 0.025$ and $R_{\rm sp} = 10^{-5}$ [30]. A first pulse is triggered by an incoherent perturbation $\Delta G = 3$ on the gain variable. A typical simulated temporal trace is represented in Fig. 4(a), showing a pulse train of finite duration. It corresponds to the stable periodic orbit in Fig. 3, but subject to the effect of noise. As for the experiment, the representation of the same data in the pseudospace map [Fig. 4(b)] shows a slanted straight line. The histograms in

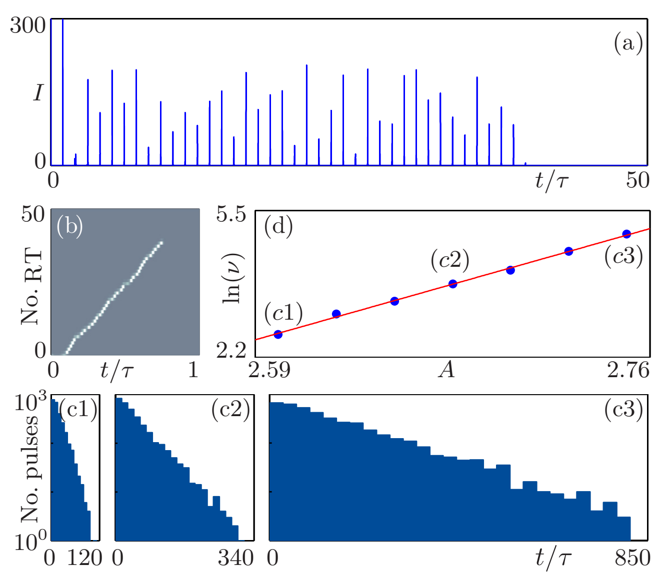

FIG. 4. Simulation of system (1) with noise: (a) typical time trace for $A = 2.7$, $n_G = 0.025$, $\kappa = 0.04$, and $\tau = 1000$ and (b) its representation in the pseudospace map. (c) Log-scale histograms of the number of pulses in 5000 simulated pulse trains, for three values of $A = 2.6$, $A = 2.675$ and $A = 2.75$. (d) Logarithm of ν versus A.

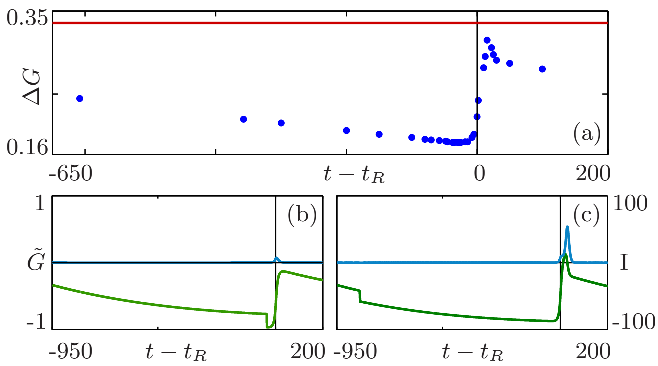

FIG. 5. Evaluation of noise bias: (a) Minimal gain perturbations $\Delta G_{\rm off}$ (blue dots) and $\Delta G_{\rm on}$ (red line) required to terminate a pulse train and trigger a first pulse, respectively, versus $t - t_R$, with t_R the reinjection time of the previous pulse. (b, c) Net gain $\widetilde{G}$ (green lower line) and intensity I (blue upper line) versus $t - t_R$, for a perturbation $\Delta G = -0.2$ at $t - t_R = -35$ and at $t - t_R = -850$, respectively.

Figs. 4(c1)–4(c3) highlight the exponential behavior of the distribution of the number of pulses in 5000 pulse trains. In Fig. 4(d), the natural logarithm of the corresponding decay rate ν increases linearly with A. Overall, Fig. 4 shows excellent qualitative agreement with the experimental observations in Fig. 2.

We checked that in the model the noise-induced escape phenomenon does not depend on the way the first pulse is triggered: both the value of ν and its dependence on A are very similar for both an initial perturbation ΔG on the gain or ΔI on the intensity. This demonstrates that the system forgets its initial conditions. Hence, the switch-off is a nondeterministic process, only driven by noise. Moreover, as expected in a noise-induced escape process following Kramers' law, an increase of the pump noise level n_G results in shorter pulse trains. We stress that the noise-induced escape process takes place between two different kinds of attractors: an equilibrium and a periodic orbit. We thus expect a peculiar behavior of the escape process with respect to the position on the orbit, as we will evidence below. Interestingly, the escape is triggered only by pump noise: even an unrealistically high level of spontaneous emission noise is not, alone, sufficient to terminate a pulse train. Once the laser is off, it is highly unlikely to start pulsing from noise, and an external perturbation is required to trigger a first pulse.

C. Bias to switch-off

To investigate the bias toward switch-off, we consider the effect of deterministic gain perturbations on the switching behavior of system (1), without noise ($R_{\rm sp} = n_G = 0$). In Fig. 5(a), dots represent the amplitude $\Delta G_{\rm off}$ of the minimal (negative) gain perturbation required to switch off a pulse train, with respect to the instant when the perturbation is applied along the periodic orbit. Conversely, the red line represents the minimal (positive) gain perturbation $\Delta G_{\rm on}$ that triggers a first pulse when the laser is initially off. This shows that the pulsing laser is particularly sensitive to perturbations in a small time window immediately preceding the pulse reinjection time t_R: for $t - t_R = -15$, $\Delta G_{\rm off}$ is 47% smaller than $\Delta G_{\rm on}$. After the reinjection, for $t - t_R = +15$, $\Delta G_{\rm off}$ remains still 7% smaller than $\Delta G_{\rm on}$. The fact that $\Delta G_{\rm on}$ is always larger than $\Delta G_{\rm off}$ is enough to explain an asymmetry in the switch processes. However, in the small time window immediately preceding the reinjection, a small amount of noise is sufficient to terminate a pulse train, while this level is extremely unlikely to trigger an initial pulse. This explains the observed strong asymmetry and time dependance of the switch process.

Physically, this bias toward switch-off can be understood by considering the effect of perturbations ΔG on the cavity net gain $\widetilde{G} = (G - Q - 1)$. The solitary laser in the excitable regime has been shown to release a pulse if the applied external perturbation brings $\widetilde{G}$ to a positive value [11]. Although the dynamics of the laser with feedback is more complex, the initiation of a first pulse follows the same rule. As Figs. 5(b) and 5(c) show, a perturbation is more likely to inhibit the pulse regeneration when introduced just before the reinjection, than when introduced earlier when the gain G has time to recover. Once the short time window during which reinjection occurs has been missed, $\widetilde{G}$ is well below zero and the noise alone is highly unlikely to bring it above zero. This is of practical interest because the switch-on process can be controlled reliably even in the presence of noise.

The amplitude variations observed in Figs. 2(a) and 4(a) can be explained similarly. In the solitary excitable laser, the response amplitude increases with the energy of the incoherent perturbation [11]. With feedback, what is important is the gain recovery at reinjection time. If $\widetilde{G}$ is increased by noise at that time, even small pulses observed in experiment and simulation can still regenerate the pulse and, hence, not interrupt the pulse train. The amplitude of the reinjected pulse also has an influence on the timing of the next pulse [22], but this effect makes a small contribution to the timing jitter; see Figs. 2(b) and 4(b).

V. CONCLUSIONS

We have shown that an excitable micropillar laser with delayed optical feedback sustains trains of optical temporal dissipative solitons. We demonstrated both experimentally and in simulation that the observed finite duration of pulse trains results from a noise-induced escape from a stable periodic solution to a stable equilibrium, following Kramers' law. Investigating the underlying dynamics of the Yamada model provides a theoretical explanation for the strong bias toward switch-off in the escape process, arising from a pronounced time dependence of the noise sensitivity along the periodic orbit. This work impacts on the fast optical control of pulse train duration and for neuromorphic regenerative memories [5,31]. Since the mechanism for self-pulsations we reported is very general and relies only on excitability and self-feedback, our results might also be of interest to more general pulsing systems beyond the particular device studied.

AKNOWLEDGMENT

The research of S.T. has been funded by a Postdoctoral Fellowship of The Dodd-Walls Centre for Photonic and Quantum Technologies.

[1] P. Grelu and J. Soto-Crespo, in *Dissipative Solitons: From Optics to Biology and Medicine* (Springer, Berlin, 2008), pp. 1–37.

[2] S. Chouli and P. Grelu, Opt. Express **17**, 11776 (2009).

[3] F. Leo, S. Coen, P. Kockaert, S.-P. Gorza, P. Emplit, and M. Haelterman, Nat. Photon. **4**, 471 (2010).

[4] T. Herr, V. Brasch, J. D. Jost, C. Y. Wang, N. M. Kondratiev, M. L. Gorodetsky, and T. J. Kippenberg, Nat. Photon. **8**, 145 (2014).

[5] M. Marconi, J. Javaloyes, S. Balle, and M. Giudici, Phys. Rev. Lett. **112**, 223901 (2014).

[6] M. Marconi, J. Javaloyes, S. Barland, S. Balle, and M. Giudici, Nat. Photon. **9**, 450 (2015).

[7] B. Garbin, J. Javaloyes, G. Tissoni, and S. Barland, Nat. Commun. **6**, 5915 (2015).

[8] J. K. Jang, M. Erkintalo, S. Coen, and S. G. Murdoch, Nat. Commun. **6**, 7370 (2015).

[9] J. Javaloyes, P. Camelin, M. Marconi, and M. Giudici, Phys. Rev. Lett. **116**, 133901 (2016).

[10] S. Terrien, B. Krauskopf, and N. G. R. Broderick, SIAM J. Appl. Dyn. Syst. **16**, 771 (2017).

[11] F. Selmi, R. Braive, G. Beaudoin, I. Sagnes, R. Kuszelewicz, and S. Barbay, Phys. Rev. Lett. **112**, 183902 (2014).

[12] T. Elsass, K. Gauthron, G. Beaudoin, I. Sagnes, R. Kuszelewicz, and S. Barbay, Eur. Phys. J. D **59**, 91 (2010).

[13] S. Barbay, Y. Ménesguen, I. Sagnes, and R. Kuszelewicz, Appl. Phys. Lett. **86**, 151119 (2005).

[14] E. Izhikevich, *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting* (MIT Press, Cambridge, MA, 2007).

[15] B. Krauskopf, K. Schneider, J. Sieber, S. Wieczorek, and M. Wolfrum, Opt. Commun. **215**, 367 (2003).

[16] F. Plaza, M. G. Velarde, F. T. Arecchi, S. Boccaletti, M. Ciofini, and R. Meucci, Europhys. Lett. **38**, 85 (1997).

[17] A. M. Yacomotti, M. C. Eguia, J. Aliaga, O. E. Martinez, G. B. Mindlin, and A. Lipsich, Phys. Rev. Lett. **83**, 292 (1999).

[18] M. A. Larotonda, A. Hnilo, J. M. Mendez, and A. M. Yacomotti, Phys. Rev. A **65**, 033812 (2002).

[19] S. Barbay, R. Kuszelewicz, and A. M. Yacomotti, Opt. Lett. **36**, 4476 (2011).

[20] B. J. Shastri, M. A. Nahmias, A. N. Tait, A. W. Rodriguez, B. Wu, and P. R. Prucnal, Sci. Rep. **6**, 19126 (2016).

[21] J. L. A. Dubbeldam, B. Krauskopf, and D. Lenstra, Phys. Rev. E **60**, 6580 (1999).

[22] F. Selmi, R. Braive, G. Beaudoin, I. Sagnes, R. Kuszelewicz, T. Erneux, and S. Barbay, Phys. Rev. E **94**, 042219 (2016).

[23] H. A. Kramers, Physica **7**, 284 (1940).

[24] P. Hanggi, J. Stat. Phys. **42**, 105 (1986).

[25] B. Krauskopf and J. J. Walker, in *Nonlinear Laser Dynamics* (Wiley-VCH, Berlin, 2012), pp. 161–181.

[26] D. Roose and R. Szalai, in *Numerical Continuation Methods for Dynamical Systems* (Springer, New York, 2007), pp. 359–399.

[27] L. F. Shampine and S. Thompson, Appl. Numer. Math. **37**, 441 (2001).

[28] K. Engelborghs, T. Luzyanina, and G. Samaey, DDE-BIFTOOL v. 2.00: A Matlab package for bifurcation analysis of delay differential equations, Technical Report, Department of Computer Science, KU Leuven, Leuven, Belgium (2001).

[29] J. Sieber, K. Engelborghs, T. Luzyanina, G. Samaey, and D. Roose, DDE-BIFTOOL v. 3.1 manual—Bifurcation analysis of delay differential equations, Technical Report, arXiv:1406.7144 (2015).

[30] G. P. Agrawal and N. K. Dutta, *Semiconductor Lasers*, 2nd ed. (Van Nostrand Reinhold, New York, 1993).

[31] B. Romeira, R. Avó, J. M. L. Figueiredo, S. Barland, and J. Javaloyes, Sci. Rep. **6**, 19510 (2016).