Temporal shaping of single photons enabled by entanglement

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We present a method to produce pure single photons with an arbitrary designed temporal shape in a heralded way. As an indispensable resource, the method uses pairs of time-energy entangled photons. One photon of a pair undergoes temporal amplitude-phase modulation according to the desired shape. Subsequent frequency-resolved detection of the modulated photon heralds its entangled counterpart in a pure quantum state. The temporal shape of the heralded photon is indirectly affected by the modulation in the heralding arm. We derive conditions for which the shape of the heralded photon is given by the modulation function. The method can be implemented with various sources of time-energy entangled photons. In particular, using entangled photons from parametric down-conversion the method provides a simple means to generate pure shaped photons with an unprecedented broad range of temporal durations, from tenths of femtoseconds to microseconds. This shaping of single photons will push forward the implementation of scalable multidimensional quantum information protocols, efficient photon-matter coupling, and quantum control at the level of single quanta.

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I. INTRODUCTION

Single photons are an indispensable tool in quantum information, quantum communication, and quantum metrology [1]. Depending on a specific application, photons must be tuned in wavelength, bandwidth, polarization, and other degrees of freedom. It has been realized in the last decade that full control over the spatiotemporal shape of a single photon pulse [2,3] is essential for a number of applications. For example, single-photon pulses, having identical Lorentzian power spectral density, can exhibit opposite temporal shapes—exponential decaying or rising—depending on spectral distribution of the phase. As a result, photons with an exponential decaying shape can excite a two-level atom in free space only with 54% efficiency, while exponentially rising photons with 100% efficiency [4–6]. There are other situations where the temporal photon shape is important: symmetric shape is optimal for cavity QED quantum communication [7], and a Gaussian shape is optimal for experiments relying on single-photon interference [8]. Furthermore, developing sources of shaped photons will push forward photonic implementation of high-dimensional quantum information protocols, e.g., quantum key distribution [9–14].

Present-day methods for producing temporally shaped photons can be summarized as follows. Methods of the first group are typically based on the control of quantum emitters (single atom [15,16], ion [17], atomic ensemble [18], quantum dot [19]). Development of these methods is promising for the deterministic production of shaped photons; however, the methods often require sophisticated and costly setups to manipulate single quantum emitters. In addition, the wavelengths of the generated photons are limited to resonant transitions of quantum emitters. Methods of the second group are based either on direct spectral filtering [20] or on amplitude-phase modulation [21,22] of single photons. These methods require less elaborate setups than methods of the first group and show higher flexibility in wavelengths and durations of shaped photons. However, losses inherent to the filtering and modulation of a single photon pulse lead to the probabilistic generation of shaped photons, which prevents efficient production of multiple shaped photons and hinders realization of scalable photonic networks. Methods of the third group use nonlinear optical processes [2,23] or electro-optic phase modulation [24] to implement unitary conversion of a carrying frequency, bandwidth, or spectrotemporal shape of single-photon pulses, while preserving the pulse coherence properties. Different fundamental and technical issues still limit overall conversion efficiency of the methods and prevent their deterministic operation.

It has been shown in [25,26] that shaped single photons can be heralded from photon pairs generated in the process of spontaneous parametric down-conversion with temporally modulated pump pulses. In the work [27], nonlocal shaping effects in the time or spectral profiles of an entangled photon pair emerging from a pulsed parametric down-converter was observed by spectrally or temporally filtering one of the twin beams. In the work [28], the authors herald single photons with a rising exponential temporal envelope using photon pairs from a nondegenerate four-wave mixing in a cold atomic ensemble. In a recent work [29], we propose a generic method that allows us to produce shaped photons in various degrees of freedom in a heralded way and can be implemented with various sources of entangled photon pairs. More specifically, we propose to shape photons indirectly by manipulating and detecting their entangled counterparts that act as heralds. The method is based on the conditional state preparation that also lies at the heart of related effects such as ghost interference [30], nonlocal dispersion cancellation [31], remote state preparation [32], and nonlocal modulation of entangled photons [33]. Heralded

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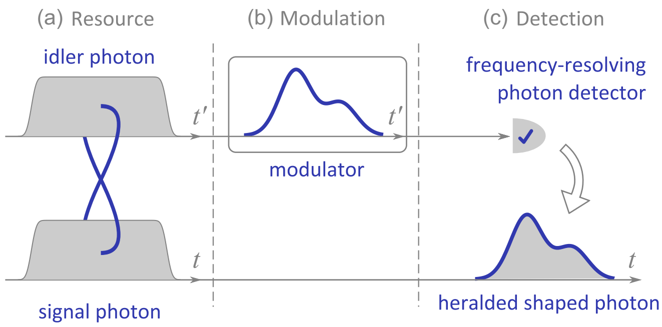

FIG. 1. Schematic depiction of the temporal shaping method: (a) To produce shaped single photons we start with a pair of time-energy entangled photons, conventionally called signal and idler. (b) The idler photon passes through a temporal amplitude-phase modulator with the transmission function which reproduces the desired temporal shape. (c) After the modulator, the idler photon is detected by a frequency-resolving single-photon detector. Provided the successful detection of the idler photon ("click" event), the conditional heralded shape of the signal photon (shown by the blue solid line) follows the modulator function [see Eqs. (2) and (4)].

generation of shaped photons is one of the prerequisites for scalable production of multiples of the shaped photons.

Here we apply the general shaping method of [29] to the time domain and give a detailed description of steps required to produce temporally shaped photons. We present the required shaping ingredients and their relevant parameters. Furthermore, we estimate figures of merit such as heralding probability and state purity of the produced shaped photons as well as achievable temporal durations of the photons. We illustrate the application of the proposed method considering heralded generation of a single photon with the exponentially rising temporal shape with realistic parameters. The method can be implemented with various sources of entangled photon pairs based on spontaneous nonlinear conversion in $\chi^{(2)}$ and $\chi^{(3)}$ materials [34], cascaded nonlinear processes in atomic ensembles [35], or semiconductors [36].

II. METHOD

The principal scheme of the method is presented in Fig. 1. As a resource, the method uses pairs of time-energy entangled photons [37] that are conventionally called signal and idler photons. The photons exhibit both time-time and frequency-frequency Einstein–Podolsky–Rosen-type correlations. To produce a shaped signal photon, we perform an amplitude-phase temporal modulation of an idler photon according to the desired shape and subsequent frequency-resolving detection of the photon. The frequency-resolving detection measures the frequency of idler photons in contrast to more conventional time-resolving detection that measures the photon arrival time. We show further that under these conditions and due to the initial entanglement, successful detection of the idler photon heralds a pure signal photon with the temporal shape affected by the modulation in the idler arm.

We stress that in the described shaping method a potentially lossy shaper (temporal modulator) is brought into the idler arm in contrast to schemes where the modulation is performed directly on signal photons (see, for example, [21,22]). Then the detector "click" heralds a shaped signal photon with certainty in the absence of dark counts of the detector and losses in the signal arm. The shaping method can be complemented with a scheme that rejects nonheralded photons (see, for example, [38]). Multiplexing the method can result in scalable production of multiple shaped photons.

We start the formal description of the method by defining the main resource, namely, signal and idler photons in an entangled state. We assume that the entangled state is described by a joint probability amplitude $\Psi(t,t')$ which has a simple physical meaning—its modulus squared gives a joint probability density to detect a signal photon at a time instant t and idler photon at a time instant t' (here and below arguments with prime refer to the idler photon and ones without prime to the signal photon, unless otherwise stated). The photons are called entangled when the function $\Psi(t,t')$ cannot be factorized with respect to t and t'.

A temporal amplitude-phase modulator in the idler arm has a time-dependent transmission of the optical field and is described by a complex transmission coefficient $\mathcal{A}(t)$, such that $|\mathcal{A}(t)| \leqslant 1$. We assume that the transmission is nonzero only within a finite time window $t_{\rm m}$. If the idler photon has passed the modulator then the joint amplitude of the photon pair transforms as

$$\Psi(t,t') \to \mathcal{A}(t')\Psi(t,t'). \quad (1)$$

After the modulator, we perform a frequency-resolving measurement of the idler photon. Detection of the idler photon with a frequency ω' (counted with respect to a photon's central frequency) heralds a signal photon in a conditional quantum state, described by the following probability amplitude (for details, see [39]):

$$\psi(t|\omega') \propto \int dt' \mathcal{A}(t')\Psi(t,t')e^{i\omega' t'}. \quad (2)$$

The obtained probability amplitude has the following meaning: its modulus squared defines the probability density distribution to detect the signal photon at a time instant t given the idler photon measurement outcome ω'. We refer to this amplitude $\psi(t|\omega')$ as a conditional or heralded temporal shape of the signal photon. The proportionality symbol denotes that the amplitude is not normalized, which we use out of convenience.

III. TEMPORAL SHAPE OF THE HERALDED PHOTON

The generic expression for the shape of the signal photon $\psi(t|\omega')$ in Eq. (2) depends on the initial joint state $\Psi(t,t')$, the applied modulation $\mathcal{A}(t)$, and the obtained measurement outcome ω' in the idler arm. The joint control of these parameters tailors the photon shape.

Now we analyze the shape of the heralded photon $\psi(t|\omega')$ depending on the joint state $\Psi(t,t')$ of photon pairs. We characterize the joint state with two time scales: unconditional $t_{\rm u}$ and conditional $t_{\rm c}$ temporal widths of photon distributions (see Fig. 2) that have the following meaning: unconditional individual measurements of either signal or idler photons with fast time-resolving single-photon "click" detectors show a statistical uncertainty with a characteristic time $t_{\rm u}$. Measurements of the signal photon, conditioned on the measurement of the

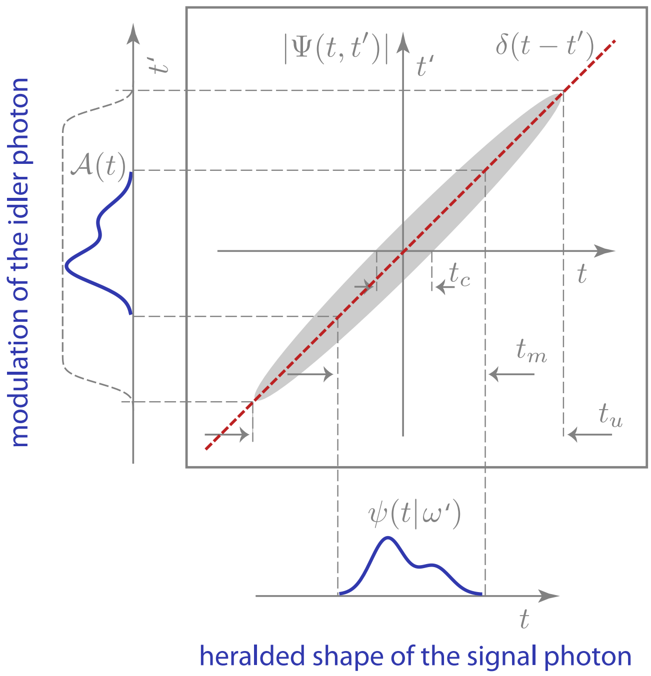

FIG. 2. Temporal modulation of the idler photon with the function $\mathcal{A}(t)$ affects the temporal shape of the heralded signal photon due to the time-energy entanglement between the signal and idler photons, and frequency-resolving detection of the idler photon. The joint probability amplitude $\Psi(t,t')'$ is visualized by the ellipse. The condition $t_c \ll t_m < t_u$ corresponds to the case of perfect correlations, i.e., the joint amplitude is approximated as $\Psi(t,t') \simeq \delta(t-t')$ (depicted by the dashed red line).

idler photon at a particular time instant (or vice versa), indicate a statistical spread t_c which is smaller than t_u due to the temporal correlations between the signal and idler photons. The stronger the temporal correlations between the photons, the higher the ratio t_u/t_c (see, for example, [40]). Particularly, for photon pairs generated in spontaneous parametric down-conversion, t_c is the second-order cross-correlation time, while the time t_u depends on the regime of photon pair generation. In the continuous-wave pump regime, t_u is defined by the modulation period of the idler photons. In the pulsed pump regime it is defined by the duration of pump pulses that drive the parametric process unless the modulation is faster than the pump pulses.

Consider the physical situation described by the inequality

$$t_c \ll t_m < t_u, \tag{3}$$

with t_m being a characteristic modulation time. We approximate the joint state as maximally entangled $\Psi(t,t') \propto \delta(t-t')$ in the integral (2), hence the temporal shape of the heralded signal photon is

$$\psi(t|\omega') \propto \mathcal{A}(t)\, e^{i\omega' t}. \tag{4}$$

This expression represents the main result of the paper. Namely, the temporal shape of the heralded single-photon pulse is defined by the modulation function in the heralding arm. The complex exponent shows that the carrier frequency of the heralded photon depends on the outcome of the frequency-resolving detector in the heralding arm. The detection of the idler photon with the frequency ω' (which is counted with respect to the carrier frequency) heralds a signal photon with the carrier frequency offset $-\omega'$. Thus, postselection of outcomes of the frequency-resolving detector is important to produce shaped photons with the desired carrier frequency, for example, to produce photons which are suitable to interface specific atomic transitions [41].

We stress that to herald pure shaped signal photons the following conditions are crucial: initial entanglement between signal and idler photons, modulation of the idler photon, and a frequency-resolving measurement of the idler photon. We consider these conditions in more detail. First, if photon pairs possess only classical correlations and are described by a mixed quantum state, then implementation of our method results in signal photons heralded in a mixed state [29,42]. Second, in the extreme case of disentangled and uncorrelated photon pairs, the detection of the idler photon after the modulator does not affect the postselection temporal shape of the signal photon and its conditional state does not change. Third, consider the time-resolved measurement instead of the frequency-resolved one. Then detection of the idler photon after the modulator at time instant t' heralds the signal photon in the conditional state not affected by the modulation. These conditions can be seen in Eq. (1), where the left- and right-hand sides are the same functions of t.

IV. FREQUENCY-RESOLVING MEASUREMENT

Now we analyze an implementation of the frequency-resolving measurement of the idler photon required in the proposed method. The measurement can be realized with a spectral filter followed by a single-photon time-resolving "click" detector. The filter can be implemented with an optical cavity, atomic ensemble, or another frequency-selective element that transmits idler photons around a particular frequency that we denote ω'. We analyze this part of the shaping method in more detail using a specific model of the filter. We describe the filter operation in the time domain. The filter delays the idler photon by the time τ with the probability proportional to the modulus squared of the impulse response of the filter, which we denote as $\mathcal{F}(\tau)e^{-i\omega'\tau}$. Due to the probabilistic delay, the filter "removes information" at which time instant the photon has passed the modulator. A click of the detector at a time instant t' heralds the signal photon with the conditional amplitude given by the convolution of the filter response and joint amplitude of photons before the filter [Eq. (1)]:

$$\psi(t|t',\omega') \propto \int d\tau \mathcal{F}(t'-\tau)e^{-i\omega'(t'-\tau)}\mathcal{A}(\tau)\Psi(t,\tau). \tag{5}$$

Let us consider the case of perfectly correlated photons satisfying condition (3). Then the shape of the heralded photon is given by the expression $\psi(t|t',\omega') \propto \mathcal{A}(t)\mathcal{F}(t'-t)e^{-i\omega'(t'-t)}$. The shape is defined by the modulation function $\mathcal{A}(t)$ multiplied with the impulse response of the filter $\mathcal{F}(t'-t)e^{-i\omega'(t'-t)}$. Consider an experimental situation such that the characteristic response time of the filter, defined by the inverse filter bandwidth ω_f^{-1}, is longer than the modulation time, i.e.,

$$t_m \ll \omega_f^{-1}, \tag{6}$$

and we can approximate $\mathcal{F}(t'-t)$ by a constant. Then the shape of the heralded photon is determined by the modulation function $\psi(t|t',\omega') \propto \mathcal{A}(t)e^{i\omega' t}$, provided postselection on the idler photons detected after the end of the modulation. It is the same result as Eq. (4) obtained for a particular model of frequency-resolving measurement. We conclude that the heralded photon is synchronized with the modulation and its shape does not depend on a specific heralding instant t'; all idler clicks after the end of the modulation are suitable as heralds. Furthermore, due to condition (6), most of the heralding clicks happen after the modulation.

V. HERALDING PROBABILITY AND PHOTON PURITY

The probability to herald shaped signal photons in the proposed method can be estimated as

$$R \approx \omega_f t_c \frac{t_m}{t_u}. \tag{7}$$

The expression has the following meaning. The ratio t_m/t_u gives a probability that idler photon with unconditional temporal spread t_u passes the temporal modulator within the time window t_m. Such modulation shaping losses also appear in the direct shaping of single photons (see, for example, [43]). Furthermore, $\omega_f t_c$ is a probability to detect an idler photon of the bandwidth t_c^{-1} within the pass band ω_f of the spectral filter.

There is a tradeoff between requirements (3) and (6) of the shaping method and the condition (7) of high heralding probability. The probability can be increased if the modulation of idler photons is replaced with a pulsed photon pair generation. For example, one can employ parametric down-conversion with the pulsed pump with the shaped envelope $\mathcal{A}(t)$ [25,26,44–46]. Then frequency-resolved detection of the idler photon heralds the signal photon with the shape (4) defined by the pump envelope. The corresponding heralding probability is $R \approx \omega_f t_c$ which is higher than the probability (7). We notice that this approach enables us to not only increase the heralding probability of shaped photons, but also reach femtosecond duration of the heralded shaped photons by using shaped femtosecond pump pulses; this range is not easily achievable with the direct modulation of idler photons using acousto- or electro-optical modulators.

One may naturally expect that experimental imperfections can affect the parameters of the shaped signal photons. Let us consider an example. Expression (4) shows that carrier frequency of the heralded photon depends on an outcome ω' of the frequency-resolving measurement. Statistical uncertainty of the measurement, which we denote as ω_d, results in the corresponding uncertainty of the carrier frequency of heralded photons. The effect is negligible when the uncertainty is smaller than bandwidth of the heralded photon, i.e., $t_m\omega_d \ll 1$. The effect can be quantitatively characterized calculating the purity of the quantum state of the heralded photon depending on the uncertainty ω_d. Let us model the statistical distribution of measurement outcomes ω' with the function $\gamma(\omega'-\omega_0')$, given the frequency of the impinging photon is ω_0'. Then a detector click at the frequency ω_0' heralds the signal photon, whose state is described by the following density matrix:

$$\rho(t,t'|\omega_0') = \int d\omega'\, \gamma(\omega'-\omega_0')\psi(t|\omega')\psi^*(t'|\omega'). \tag{8}$$

Here both arguments t and t' refer to the signal photon and $\psi(t|\omega')$ is given by expression (2). We calculate the purity $\mu \equiv \mathrm{Tr}\,\rho^2$ of this state as a function of four experimentally relevant parameters t_c, t_u, t_m, and ω_d using the following Gaussian approximations:

$$\Psi(t,t') = e^{-(t-t')^2/2t_c^2}\, e^{-(t+t')^2/2t_u^2}\sqrt{2/\pi t_c t_u}, \tag{9}$$

$$\mathcal{A}(t) = e^{-t^2/2t_m^2}, \tag{10}$$

$$\gamma(\omega) = e^{-\omega^2/\omega_d^2}/\sqrt{\pi}\,\omega_d. \tag{11}$$

The resulting general expression for the purity [47] can be applied for various experimental conditions. In a particular case of condition (3) of highly correlated photon pairs, such that $t_u \to \infty$ and $t_c \to 0$, we get the following estimation:

$$\mu \approx \frac{1}{\sqrt{1+t_m^2\omega_d^2}}. \tag{12}$$

The state of the heralded photon is approximately pure when the uncertainty of the frequency-resolved measurement is smaller than the modulation bandwidth, i.e., $t_m\omega_d \ll 1$.

VI. ACHIEVABLE DURATIONS OF PHOTONS

We estimate durations of shaped single-photon pulses that can be produced using the proposed method. Condition (3) shows that the possible durations are limited by the temporal characteristics of the entangled photon pairs. For a given source of entangled photons, the method provides a way to produce shaped photons longer than the correlation time of photon pairs t_c and shorter than their unconditional temporal spread t_u. Furthermore, condition (6) shows that the maximal possible duration of the heralded photon is also limited by the bandwidth of the frequency-resolving measurement. Widely used sources of entangled photons are based on spontaneous parametric down-conversion in a second-order nonlinear medium that can be implemented both in a single-pass [48] and a cavity-assisted [49] configuration. The parametric sources provide entangled photons in various spectral ranges and bandwidths. The unconditional temporal spread of photons t_u is typically defined by the properties of the strong coherent pump light that drives the parametric process. In the continuous regime, it is the coherence time of the pump (μs$-ms$ range). In the pulsed regime, it is the duration of pump pulses (up to tens of fs). The correlation time of parametrically generated photons t_c is defined either by a phase-matching bandwidth in a nonlinear medium (typically, fs–ps range) or a cavity bandwidth in a cavity-assisted configuration (ns range). Therefore, the use of parametric down-conversion as a source of time-energy entangled photons enables generation of shaped photons in a very broad range of temporal durations, from fs to μs.

VII. EXAMPLE: GENERATION OF AN EXPONENTIALLY RISING SINGLE-PHOTON PULSE

Here we illustrate an application of the proposed method for the heralded generation of a single photon with the exponentially rising temporal shape. Such shaped photons are prerequisites for efficient coupling to two-level quantum emitters [4,5,50]. We model time-energy entangled states of photon pairs, which are required to implement the shaping method, with the following joint temporal amplitude:

$$\Psi(t,t') \propto e^{-|t-t'|/2t_c}, \quad \text{where } -t_u \leqslant t,t' \leqslant 0. \tag{13}$$

The expression implies that individual detection of signal and idler photons results in homogeneous distribution of photocounts within the time interval t_u, while joint detection reveals correlation of photons within the characteristic time t_c. Such an entangled state approximates photon pairs from cavity-assisted spontaneous nonlinear conversion in a second- or third-order nonlinear medium [34]. The correlation time t_c is defined by the lifetime of photons in the cavity; the unconditional temporal duration t_u is defined either by the duration of the pump pulses in the pulsed regime or by the repetition rate of the shaping procedure in the continuous pump regime. Low power of the pump ensures that there are predominantly single photon pairs within the time interval t_u.

The idler photon undergoes amplitude modulation with the rising exponential waveform with the rise time t_m that matches the desired temporal shape. An ideal exponentially rising waveform, which is required for the perfect excitation of a two-level quantum emitter, would span an infinite time interval from $-\infty$ to 0. In an experiment, the modulation is performed within the finite time window t_u. In Ref. [4], it is shown theoretically that for pulses of five lifetimes, i.e., $t_u = 5t_m$, 99% excitation probability can be achieved. We approximate the amplitude transmission of the temporal modulator with the function

$$\mathcal{A}(t) = e^{t/2t_m}, \quad \text{where } -t_u \leqslant t \leqslant 0. \tag{14}$$

The transmission of the modulator increases exponentially within the time interval t_u and reaches unity at the end of the interval.

After the modulator the idler photon is sent to a spectral filter. The central frequency of the filter is assumed to be tuned to the central frequency of the photon. Transmission of the filter at the central frequency is equal to unity and the bandwidth of the filter is ω_f. We model the impulse response of the filter with an exponentially decaying function with the response time ω_f^{-1}

$$\mathcal{F}(t) = \frac{\omega_f}{2} e^{-\omega_f t/2}\Theta(t), \tag{15}$$

where $\Theta(t)$ is the Heaviside step function. Such a filter can be implemented with a stabilized high-finesse optical cavity.

After the filter, the idler photon is detected with a time-resolving single-photon detector with the resolution which is assumed to be better than the response time of the filter. Detection of the idler photon at time instant t' heralds a signal photon in a pure state with the conditional temporal amplitude that can be calculated from expression (5), where one has to put $\omega' = 0$, since the central frequency of the filter is chosen to be tuned to the central frequency of the idler field. Figure 3 shows temporal distribution of signal photons, i.e., $|\psi(t|t',\omega' = 0)|^2$, heralded on the detection of idler photons at time instants $t' \geqslant 0$ for specifically chosen values of t_u, t_c, t_m, and ω_f. One sees that the distribution reproduces smoothed temporal transmission of the modulator in the heralding arm. The smoothing comes from the finite correlation time of entangled photons. A slightly faster exponential growth comes from finite spectral resolution of the filter. The fidelity of the generated temporal shape can be increased by choosing photon pairs with smaller correlation time and filters with better spectral resolution.

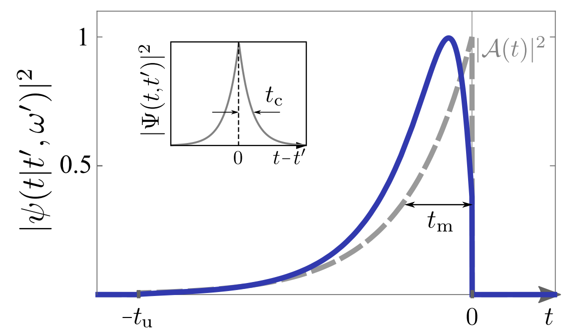

FIG. 3. Temporal distribution of signal photons (blue curve) produced with the proposed shaping method using as a resource time-energy entangled photon pairs [Eq. (13)], temporal modulation of idler photons [Eq. (14)] with exponentially rising waveform (grey dashed curve), frequency-resolving detection of idler photons with an optical cavity with impulse response [Eq. (15)]. The functions are normalized to unity at their maximum. The relation between parameters is the following: $t_u : t_m : t_c = 50 : 10 : 1$ and $\omega_f t_u = 1$. The heralding probability of shaped photons is 0.0056. The inset depicts schematically a cross-correlation function of entangled signal-idler photon pairs with the characteristic correlation time.

For this example, the heralding probability of the shaped photons is calculated as $R = \int_0^\infty |\psi(t|t',0)|^2\, dt' \approx 0.0056$. This value agrees well with the estimation from Eq. (7) which is 0.004. As mentioned above, there is a tradeoff between the fidelity of the shaping and the heralding probability. A higher heralding rate can be achieved generating photon pairs with temporally modulated entanglement, such that the modulation of idler photons is not required. Particularly, for photon pairs generated in a cavity-assisted spontaneous nonlinear conversion, the entanglement modulation can be performed via proper temporal modulation of pump pulses.

VIII. CONCLUSION

In conclusion, we have proposed a method to produce single photons with an arbitrary temporal shape in a pure quantum state. Our method utilizes time-energy entangled photon pairs as an indispensable resource. Shaped photons are heralded upon temporal modulation and successful frequency-resolved measurement of their entangled counterparts. We derive the explicit expression for the temporal shape of heralded photons. The shape is determined by the initial entanglement of photon pairs, modulation function, and measurement outcome in the heralding arm. The joint control of these parameters affects the shape of the heralded photons.

Two features of the method are worth noting. First, shaped photons are produced in a heralded way. This feature is one of prerequisites for scalable production of multiple shaped photons.

Second, our method is ready to implement with present-day photonic technologies, allowing us to produce shaped photons in a broad range of wavelengths and bandwidths. In particular, using parametric down-conversion as a source of entangled photons, the method is a promising means to produce shaped photons in an immense range of temporal durations, from tenths of femtoseconds to microseconds. Experimental results on nonlocal temporal modulation [51] and remote preparation of time-encoded single-photon ebits [52] confirm the experimental feasibility of the proposed shaping method. To experimentally verify the shape of heralded photons the method can be accompanied with one of the tomographic techniques, provided informationally complete measurements are performed on the photon [53–57].

We expect that simplicity and the heralding feature of our method will make generation of temporally shaped photons a common practice in quantum optics and will push forward experiments that require shaped photons, e.g., efficient photon-matter coupling, multidimensional quantum information protocols, and coherent control of quantum systems at the level of single quanta. Besides, the proposed shaping method reveals an application of quantum entanglement.

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