Generation of the superposition of mesoscopic states of a nanomechanical resonator by a single two-level system

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We propose measurement-based conditional generation of the superposition of mesoscopic states of a nanomechanical resonator. We consider a two-level quantum mechanical system (qubit) coupled with a nanomechanical resonator through phonon exchange. An interaction, which produces shifts in the state of the nanomechanical resonator depending on the state of the qubit, is realized by driving the qubit through two resonant fields or a single field with controlled phase. The measurement of the state of the qubit produces superposition states of the nanomechanical resonator. We show that the quantum interference between the generated states in the superposition may lead to an arbitrary large displacement in the resonator. We also discuss decoherence of the generated states using the Wigner function.

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I. INTRODUCTION

Nanomechanical resonators (NRs) have a wide range of applications, which includes ultrasensitive mass detection [1], force detection [2,3], imaging, and lithography [4]. Now, it is possible to cool the fundamental mode of a NR to its ground state [5,6] and create single-phonon excitation in a controlled manner [6]. Clearly, it is a milestone in demonstrating quantum mechanical behavior of a mesoscopic system consisting of billions of atoms, which can lead to applications such as quantum limited measurements [7], generating nonclassical mechanical states [8], and the realization of hybrid quantum mechanical systems for quantum information processing [9–14]. Apart from having a high-quality factor, NR can be coupled with various quantum mechanical systems through magnetic, electrical, or optical interactions. Such developments have opened up important possibilities to encode quantum information into a mechanical state and mediate the interaction between different types of quantum systems. Mainly, two types of hybrid systems have been realized by coupling NR to either a cavity where the electromagnetic field directly couples through radiation pressure [9,10] or by coupling with a qubit where states of the qubit can be manipulated by external fields [11–15]. In the case of a cavity-coupled NR, various phenomena such as optomechanically induced transparency [16], single-photon blockade [17], and generation of micro-macro entangled states [18] have been realized.

Due to its versatile nature, a NR has been efficiently coupled to various types of qubits such as superconducting qubit [11,12], quantum dot [15], cold atoms [19], and nitrogen vacancy (NV) center [13,14] which provide a mechanical analog of cavity quantum electrodynamics. Such systems are of vital importance, particularly for controlling and generating nonclassical states of a NR. There have been proposals for cooling [20] or lasing [21] the NR by coupling with a two-level quantum system. A quantum dot (QD) [15] or a NV center [14] embedded in a nanowire naturally couples through the phonon mode by strain-mediated interaction. The effects of such coupling have been witnessed in florescence spectra from a quantum dot or NV center driven by an external field. A QD or NV center embedded in a nanowire also provides very high collection efficiency for emitted photons, which has been utilized in the realization of high-efficiency single-photon sources [22]. However, a major difficulty for the quantum manipulation of mechanical states using a coupled two-level system is that it requires the coupling strength to be at least of the order of the decoherence rate of the two-level system [15]. For a QD and superconducting qubit, where coupling strength could be as large as 500 kHz, the decoherence rate is of the order of GHz. Similarly, for NV centers where the decoherence rate could be as small as a few kHz, the coupling strength has been of the order of 10 Hz. In this paper, we present a scheme for generating the superposition of mesoscopic states of NR by externally deriving a coupled two-level system and measuring the state of the system. Our proposal could be feasible for the NR having frequency equal to the decoherence rate of the qubit, which could be achieved, for example, by using a GHz-frequency resonator [6] coupled with a QD or superconducting qubit and by using a 10-kHz-frequency resonator [13] coupled with a NV center.

Our paper is organized as follows. In Sec. II, we present our model for the realization of effective interaction. Section III presents a method for generating the superposition of mesoscopic states of a NR. In Sec. IV, we discuss the effects of decoherence. Finally, we present possibilities for the experimental realization in the current scenario and conclusions in Sec. V.

II. MODEL FOR REALIZATION OF EFFECTIVE HAMILTONIAN

We consider a qubit coupled with NR, where the qubit is driven by two external resonant fields, and the fields have phase difference of $\pi/2$. Further, the intensity of one field is much greater than the other. The schematic diagram is shown in Fig. 1. The Hamiltonian of the system in the rotating frame is given by

$$H = \hbar\omega a^{\dagger}a + \hbar\Omega_1(\sigma^+ + \sigma^-) + i\hbar\Omega_2(\sigma^+ - \sigma^-) + \hbar g|e\rangle\langle e|(a + a^{\dagger}), \tag{1}$$

where ω, a ($a^{\dagger}$), Ω_1 (Ω_2), g, σ^-, and σ^+ are, respectively, the frequency of the NR mode, annihilation (creation) operator for the phonon field, coupling constants for the qubit with first (second) external field, coupling constant for the qubit with NR mode, and lowering and raising operators for the qubit.

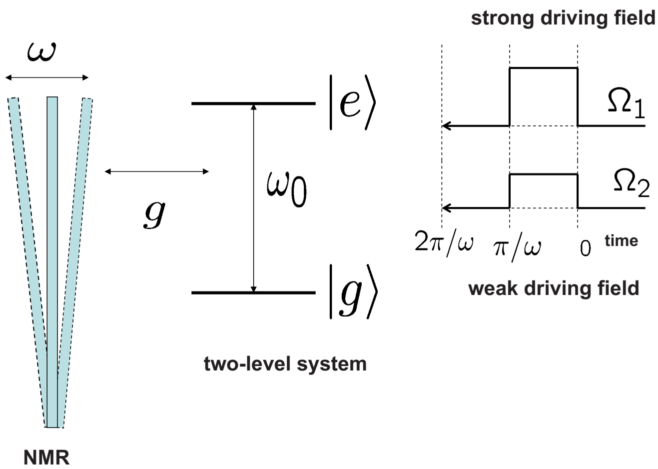

FIG. 1. Schematic for a nanomechanical resonator coupled with a two-level quantum mechanical system (qubit). The qubit is driven by two external fields, where one field is strong and another is weak.

We notice that a similar Hamiltonian was considered earlier by Freedhoff and Ficek [23] in the context of a modification in Mollow's sidebands and the results have been recently verified by He *et al.* [24]. We also notice that the Hamiltonian (1) can also be realized using a single driving field with a controlled phase. For example, if the coupling of the single driving field is $\Omega e^{i\phi}$, then $\Omega_1 = \Omega\cos\phi$ and $\Omega_2 = \Omega\sin\phi$. By fixing the value of ϕ properly, one can have $\Omega_1 \gg \Omega_2$. Further, a singly driven qubit has been utilized in a proposal [25] for generating the coherent state of a NR. In order to get a clear picture of the interaction of the Hamiltonian (1), we perform a time-independent unitary transformation [20,26] $\tilde{H} = e^s H e^{-s}$, with $s = \eta|e\rangle\langle e|(a^\dagger - a)$, where $\eta = g/\omega$, and the above Hamiltonian takes the form

$$\begin{aligned}\tilde{H} &= \hbar\omega a^\dagger a + \hbar\Omega_1[\sigma^+ e^{\eta(a^\dagger - a)} + \sigma^- e^{-\eta(a^\dagger - a)}] \\ &\quad + i\hbar\Omega_2[\sigma^+ e^{\eta(a^\dagger - a)} - \sigma^- e^{-\eta(a^\dagger - a)}]. \end{aligned} \tag{2}$$

Under the above unitary transformation, the state of the system transforms as $|\tilde{\psi}\rangle = e^s|\psi\rangle$, which produces constant shift η in the phonon mode when the qubit is in excited $|e\rangle$. However, the state remains unaffected if the qubit is initially prepared in ground state $|g\rangle$ and detected in $|g\rangle$. For the QD and superconducting qubits ($g \sim 100$ kHz) coupled with NR having $\omega \sim 1$ GHz and for NV centers ($g \sim 1$ kHz) coupled with NR having $\omega \sim 1$ MHz, we have $\eta \sim 10^{-3}$. Therefore, we consider the terms up to second order in η. We also consider that the qubit is strongly driven by one of the fields, say, $\Omega_1 \gg g,\omega,\Omega_2,\Gamma$, where Γ is the decoherence rate for the qubit. We further rewrite the above Hamiltonian in the interaction picture in which the interaction with the stronger field has been diagonalized. In this picture, the state of the system $|\tilde{\psi}\rangle$ and the Hamiltonian $\tilde{H}$ are transformed to

$$|\bar{\psi}\rangle = e^{iht}|\tilde{\psi}\rangle,\ \bar{H} = e^{iht}\tilde{H}e^{-iht};\ h = \Omega_1(\sigma^+ + \sigma^-). \tag{3}$$

The qubit spin operators $\sigma^\pm$ transform as

$$e^{iht}\sigma^\pm e^{-iht} = \sigma^\pm\cos^2\Omega_1 t + \sigma^\mp\sin^2\Omega_1 t \mp i\sigma_z\sin 2\Omega_1 t. \tag{4}$$

We consider that the qubit is driven strongly such that Ω_1 is very large, and therefore we can neglect the highly oscillating terms in Eq. (4), i.e., $\sin 2\Omega_1 t \approx 0$. A similar approximation was extensively used earlier in various contexts [27]. Under this approximation, the effective Hamiltonian becomes

$$\begin{aligned}\bar{H}_{\rm eff} &= \hbar\omega a^\dagger a + i\hbar\Omega_2\eta\sigma_x(a^\dagger - a) \\ &\quad + \frac{\hbar\Omega_1\eta^2}{2}\sigma_x(a^\dagger - a)^2,\end{aligned} \tag{5}$$

where $\sigma_x = \sigma^+ + \sigma^-$. We note that $\bar{H}_{\rm eff}$ commutes with h, thus changing to the previous interaction picture, where the effective Hamiltonian becomes

$$\begin{aligned}H_{\rm eff} &= \hbar\omega a^\dagger a + \hbar\Omega_1'\sigma_x \\ &\quad + i\hbar\Omega_2\eta\sigma_x(a^\dagger - a) - \hbar\Omega_1\eta^2\sigma_x a^\dagger a,\end{aligned} \tag{6}$$

where $\Omega_1' = \Omega_1(1 - \eta^2/2)$. Here we have neglected two phonon transitions which produce a small squeezing in the coherent state of NR when Ω_1 is switched on and the squeezing is undone when Ω_1 is switched off. We relegate the details to the Appendix. The Hamiltonian $H_{\rm eff}$ can be diagonalized in the basis of eigenstates of σ_x as follows:

$$H_{\rm eff} \approx h_+|+\rangle\langle+| + h_-|-\rangle\langle-|, \tag{7}$$

$$h_\pm = \pm\hbar\Omega_1' + \hbar(\omega \pm \Omega_1\eta^2)A_\pm^\dagger A_\pm, \tag{8}$$

where $A_\pm = a \pm i\Omega_2\eta/\omega$. In the expression of $h_\pm$, the first term gives the change in phase and the second term gives the change in phase as well as the displacement in the NR state during evolution of the qubit-NR coupled system. Clearly, the magnitude of displacement in the NR state depends on Ω_2; further, the displacement will be negative or positive depending on the qubit states $|\pm\rangle$. When $\Omega_2 = 0$, the Hamiltonian changes to the optically driven qubit-NR system [25], where the effect of qubit-NR coupling leads to the modifications in qubit energy states as well as a shift in the frequency of the NR. Next, we exploit this interaction for generating the superposition of mesoscopic states of a NR.

III. GENERATING SUPERPOSITION OF MESOSCOPIC STATES

A. Superposition of multiple coherent states and quantum random walk

We initially consider that the qubit is in ground state $|g\rangle$ and the NR is in coherent state $|\alpha_0\rangle$, in which the initial state of the system can be written as

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)|\alpha_0\rangle. \tag{9}$$

In the presence of field Ω_1, when Ω_2 is switched on, the phonon annihilation operator a transforms to $A_\pm$, depending on the qubit state $|\pm\rangle$, i.e., $a \rightarrow D(\mp i\Omega_2\eta/\omega)aD(\pm i\Omega_2\eta/\omega)$. As a result, the state of the NR changes as $|\alpha\rangle \rightarrow D(\mp i\Omega_2\eta/\omega)|\alpha\rangle$, where D is the displacement operator. Similarly, when Ω_2 is switched off, the phonon annihilation operator transforms as $a \rightarrow D(\pm i\Omega_2\eta/\omega)aD(\mp i\Omega_2\eta/\omega)$ and the state of the NR transforms as $|\alpha'\rangle \rightarrow D(\pm i\Omega_2\eta/\omega)|\alpha'\rangle$. Therefore, corresponding to qubit states $|\pm\rangle$, a small positive or negative displacement in the NR state is produced when Ω_2 is switched on or off. In order to get a constructive effect during one on-off cycle of Ω_2, we work on a strategy in which the time between switching on and off of Ω_2 should be such that the phonon field has changed its phase by π. Therefore, we consider that Ω_1 and Ω_2 are switched on for half of the time period of NR, i.e., for $t = 0$ to $t = \pi/\omega$, and remain off for the other half. The state of the system at $t = \pi/\omega$ is given by

$$\psi(\pi/\omega) = \frac{1}{\sqrt{2}}[e^{-i\phi}e^{-il_1\mathrm{Re}(\alpha_0)}|(\alpha_0 - il_1)e^{-i(\omega-\Omega_1\eta^2)\frac{\pi}{\omega}}\rangle|+\rangle - e^{i\phi}e^{il_1\mathrm{Re}(\alpha_0)}|(\alpha_0 + il_1)e^{-i(\omega+\Omega_1\eta^2)\frac{\pi}{\omega}}\rangle|-\rangle] \quad (10)$$

$$= \frac{1}{\sqrt{2}}[e^{-i\phi}e^{-il_1\mathrm{Re}(\alpha_0)}|(-\alpha_0 + il_1)e^{il_2\pi}\rangle|+\rangle - e^{i\phi}e^{il_1\mathrm{Re}(\alpha_0)}|(-\alpha_0 - il_1)e^{-il_2\pi}\rangle|-\rangle], \quad (11)$$

where Re represents the real part, $\phi = \Omega_1'\pi/\omega$, $l_1 = \Omega_2\eta/\omega$, and $l_2 = \Omega_1\eta^2/\omega$. For time $t = \pi/\omega$ to $t = 2\pi/\omega$, fields Ω_1 and Ω_2 are switched off, and therefore the state of the system is given by

$$\psi(2\pi/\omega) = \frac{1}{\sqrt{2}}[e^{-i\phi}e^{-il_1\mathrm{Re}[\alpha_0+(\alpha_0-il_1)e^{il_2\pi}]} \times |(\alpha_0 - il_1)e^{il_2\pi} - il_1\rangle|+\rangle - e^{i\phi}e^{il_1\mathrm{Re}[\alpha_0+(\alpha_0+il_1)e^{-il_2\pi}]} \times |(\alpha_0 + il_1)e^{-il_2\pi} + il_1\rangle|-\rangle]. \quad (12)$$

If we measure the qubit in the ground state $|g\rangle$ or in the excited state $|e\rangle$, the state of the NR is projected into a coherent superposition of the mesoscopic state $|(\alpha_0 - il_1)e^{il_2\pi} - il_1\rangle$ and $|(\alpha_0 + il_1)e^{-il_2\pi} + il_1\rangle$. Clearly, for small values of l_1 and l_2, these states overlap considerably, and therefore the effects of quantum interference become significant. If we assume that the qubit is measured in ground state $|g\rangle$ after switching off the fields, the projected state of the NR is given by

$$\psi(2\pi/\omega) = [e^{-i\phi}e^{-il_1\mathrm{Re}[\alpha_0+(\alpha_0-il_1)e^{il_2\pi}]}|(\alpha_0 - il_1)e^{il_2\pi} - il_1\rangle + e^{i\phi}e^{il_1\mathrm{Re}[\alpha_0+(\alpha_0+il_1)e^{-il_2\pi}]}|(\alpha_0 + il_1)e^{-il_2\pi} + il_1\rangle]. \quad (13)$$

After sending one pair of pulses, consisting of one square pulse of each field Ω_1 and Ω_2, which are on for half of the time period of the NR and off for the other half simultaneously, the phonon field is displaced anticlockwise or clockwise along a circle in a random fashion. If we pass n such pulse pairs and every time detect the qubit in its ground state, the state of the NR will be equivalent to an n-step random walk along a circle [28]. The state of the NR can be expressed as

$$|\psi_{ph}(n)\rangle = C[e^{-i\phi}\hat{O}(-l_1,-l_2) + e^{i\phi}\hat{O}(l_1,l_2)]^n|\alpha_0\rangle, \quad (14)$$

where C is the normalization constant and operator $\hat{O}(l_1,l_2)|\alpha\rangle = D(il_1)e^{-il_2\pi a^\dagger a}D(il_1)$. We note that operators $\hat{O}(l_1,l_2)$ and $\hat{O}(-l_1,-l_2)$ commute each other: $[\hat{O}(l_1,l_2),\hat{O}(-l_1,-l_2)] = 0$ for real l_1,l_2 and $\hat{O}(l_1,l_2)\hat{O}(-l_1,-l_2)|\alpha\rangle = |\alpha\rangle$. Therefore, the state of the NR can be written as

$$|\psi_{ph}(n)\rangle = C\sum_{m=0}^{n}\binom{n}{m}[e^{-im\phi}\hat{O}^m(-l_1,-l_2) \times e^{i(n-m)\phi}\hat{O}^{n-m}(l_1,l_2)]|\alpha_0\rangle,$$

$$= C\sum_{m=0}^{n}\binom{n}{m}e^{i(n-2m)\phi}\hat{O}^{n-2m}(l_1,l_2)|\alpha_0\rangle,l \quad (15)$$

$$= C\sum_{m=0}^{N}\binom{n}{m}e^{i(n-2m)\phi}e^{-i\theta_{n-2m}}|\alpha_{n-2m}\rangle, \quad (16)$$

where we have recursive expressions $\theta_j = \theta_{j-1} + l_1\mathrm{Re}(\alpha_{j-1} + \alpha_j)$ and $\alpha_j = (\alpha_{j-1} + il_1)e^{-il_2\pi} + il_1$. Now, expressing this result in coordinate representation, we get the wave function of the NR $\psi_{ph}(n,x) = \langle x|\psi_{ph}(n)\rangle$,

$$\psi_{ph}(n,x) = C\sum_{m=0}^{n}\binom{n}{m}e^{i(n-2m)\phi}e^{-i\theta_{n-2m}}\psi_{\alpha_{n-2m}}(x), \quad (17)$$

where $\psi_{\alpha_j}(x)=\pi^{-1/4}\exp\{-[x - \sqrt{2}\mathrm{Re}(\alpha_j)]^2/2 + i\sqrt{2}\mathrm{Im}(\alpha_j)x - i\mathrm{Re}(\alpha_n)\mathrm{Im}(\alpha_n j)\}$; $\mathrm{Im}(\alpha_j)$ is the imaginary part of α_j. In Fig. 2, we plot the probability distribution for the displacement of NR $|\psi_{ph}(n,x)|^2$, using initial state $|\alpha_0\rangle = |0\rangle$ and typical values of ϕ, l_1, l_2 for different values of n. The displacement of the NR depends on l_1, l_2, ϕ and the number of pulse pairs n. For $l_1 = 0.1$ and $l_2 = 0.01$, we calculate values of $\alpha_1 = 0.00314107591 + 0.199950656i$, $\alpha_5 = 0.0783720116 + 0.995810825i$, and $\alpha_{10} = 0.311558267 + 1.96710148i$, with $\alpha_{-n} = \alpha_n^*$. The maximum expected value of displacement x for a typical value of n is given by $\langle x\rangle_n = \sqrt{2}\mathrm{Re}(\alpha_n)$. From Eq. (17), it is clear that the final state of the NR is the coherent superposition of $n + 1$ coherent states; further, their relative phases depend on ϕ and θ_s. For small values of l_1 and l_2, these coherent states overlap considerably, leading to dominating quantum interference effects. The unexpected displacement in the state of NR is due to the constructive interference between the coherent states generated after passing n pulse pairs. Here we must emphasize

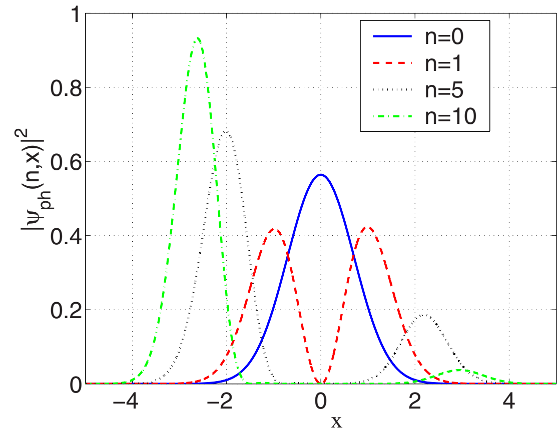

FIG. 2. Probability distribution $|\psi_{ph}(n,x)|^2$ for the amplitude of the nanomechanical resonator after passing different numbers of pulse pairs. The parameters are $l_1 = 0.1$, $l_2 = 0.01$, $\phi = 9\pi/2$, and initial state as $|0\rangle$.

that the choice of ϕ is also critical for the final displacement in the NR state. For ϕ equal to odd multiples of $\pi/2$, we get maximum displacement, and for even multiples of $\pi/2$, the displacement is negligible. For ϕ equal to odd multiples of $\pi/4$, only one peak in the probability distribution appears which has displacement between the maximum and minimum values. For $n = 1$, we get two equal peaks symmetrically placed on the positive and negatives sides around $x = 0$. When the value of n increases, the peak on the negative side starts dominating; in fact, for $n \geqslant 10$, the peak along the positive side becomes negligible. The final state of the NR is equivalent to the quantum random walk defined by Aharnov *et al.* [29,30] along a circle.

B. Superposition of two coherent states

Next, we discuss how one can generate a superposition state of two coherent states which are well separated in phase space. We consider that the strong driving field Ω_1 is on for a few cycles of mechanical oscillations and the weak driving field is modified in a similar fashion as discussed above. We consider the initial state of the system as $|g\rangle|\beta_0\rangle$ with $\beta_0 = 0$. If after n cycles of mechanical oscillations, when n pulses of Ω_2 have been passed, Ω_1 is switched off and we measure the state of the qubit in ground state $|g\rangle$, then the projected state of the NR is given by

$$|\psi_{\rm cat}(n)\rangle = K(e^{-i\phi'}e^{i\theta'_{-n}}|\beta_{-n}\rangle - e^{i\phi'}e^{i\theta'_n}|\beta_n\rangle), \qquad (18)$$

where K is the normalization constant and $\phi' = \Omega'_1 2n\pi/\omega$, $\beta_j = (\beta_{j-1} + il_1)e^{-2il_2\pi} + il_1e^{-il_2\pi}$, $\theta'_j = \theta'_{j-1} + l_1\text{Re}[\beta_{j-1} + (\beta_{j-1} + il_1)e^{-2il_2\pi}]$, with $\theta'_0 = 0$. Clearly, the above method can be used to generate the superposition of two mesoscopic states similar to Schrodinger cat states.

IV. DECOHERENCE

In Sec. III, we have shown that the unexpected displacement in the NR state is produced due to quantum interference. Therefore, it is important to maintain coherence in the system during the generation of the superposition of mesoscopic states of the NR. For a nanomechanical resonator having high-quality factor, we can neglect the effects of decoherence in the generated superposition state due to phonon-mode damping [31]. In the following, we consider the decoherence of the generated states due to spontaneous decay of the qubit. During the generation of states (16) and (18), the qubit remains in dressed states $|+\rangle$ and $|-\rangle$. Using the density matrix of qubit ρ_q, we evaluate the density matrix elements at time t, where the diagonal elements remain constant, $\langle +|\rho_q(t)|+\rangle = \langle -|\rho_q(t)|-\rangle = 1/2$, and the off-diagonal elements decay as $\langle \pm|\rho_q(t)|\mp\rangle = \langle \pm|\rho_q(0)|\mp\rangle \exp(-3\Gamma t/4)$ [32]. We include the effect of qubit decoherence in the generated state (16) as follows. We calculate density matrix $\rho_{ph}(n)$ for the generated NR state after n pulse pairs are passed using the recursion relation,

$$\begin{aligned}\rho_{ph}(n) = C[&\hat{O}(l_1,l_2)\rho_{ph}(n-1)\hat{O}(l_1,l_2) + \hat{O}(-l_1,-l_2)\\ &\times\rho_{ph}(n-1)\hat{O}(-l_1,-l_2) + e^{2i\phi}e^{-\xi}\hat{O}(l_1,l_2)\rho_{ph}\\ &\times(n-1)\hat{O}(-l_1,-l_2) + e^{-2i\phi}e^{-\xi}\hat{O}\\ &\times(-l_1,-l_2)\rho_{ph}(n-1)\hat{O}(l_1,l_2)],\end{aligned} \qquad (19)$$

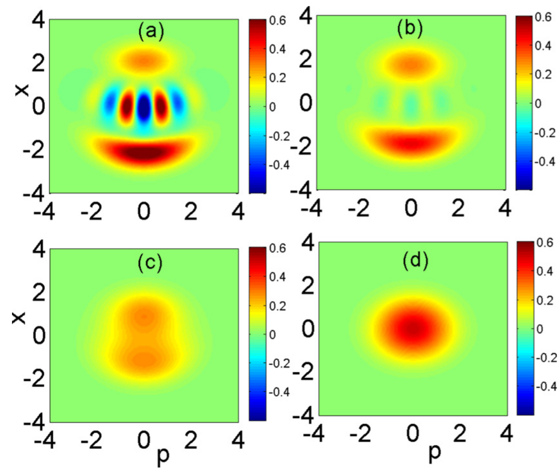

FIG. 3. Wigner function $W(x,p)$ of the generated superposition of multiple mesoscopic states (17) for $n = 5$. (a) $\xi = 0$, (b) $\xi = 0.2$, (c) $\xi = 0.5$, and (d) $\xi = 1$; other parameters are the same as in Fig. 2.

with $\rho_{ph}(0) = |\alpha_0\rangle\langle\alpha_0|$, where $\xi = 3\Gamma T/8$, $T = 2\pi/\omega$ and C is the normalization constant. In Fig. 3, we plot the Wigner function for state (19) for $n = 5$ using different values of ξ. The Wigner function for the density matrix ρ is defined as

$$W(x,p) = \frac{1}{\pi\hbar}\int e^{2ipy/\hbar}\langle x-y|\rho|x+y\rangle dy. \qquad (20)$$

In Fig. 3(a), for $\xi = 0$, the Wigner function shows two peaks at $x \approx \pm 2$: one smaller peak at $x = 2$ and one dominating peak at $x = -2$. The interference fringes are visible between these peaks which are a direct signature of coherence. The Wigner function acquires negative values in the region between the peaks. The negative value of the Wigner function clearly indicates the nonclassical nature of the generated superposition state. Further squeezing in the x quadrature [33] is also visible, which is also clear in Fig. 2. In Figs. 3(a)–3(d), as the value of ξ increases, the interference diminishes progressively. In Fig. 3(d), for $\xi = 1$, the generated superposition state (16) turns into a mixed state completely and the displacement in the NR state becomes approximately zero.

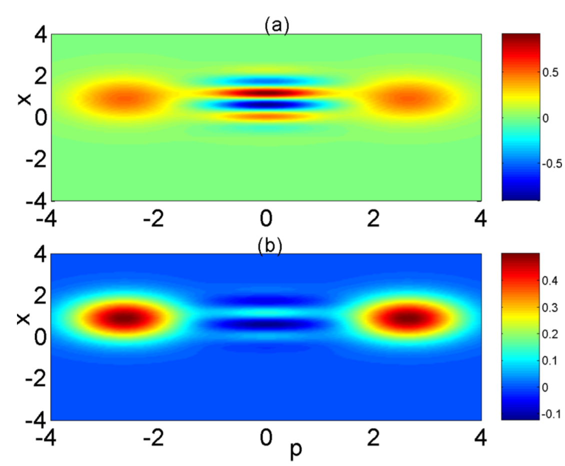

FIG. 4. Wigner function $W(x,p)$ of the generated superposition of two mesoscopic states (18) for $n = 10$. (a) $\Gamma = 0$, (b) $3n\Gamma T/4 = 2$; other parameters are the same as in Fig. 2.

We follow similar calculations for generated state (18). In Fig. 4(a), we plot the Wigner function for state (18) using $n = 10$ for $\Gamma = 0$. In Fig. 4(b), we plot the Wigner function for the same parameters used in Fig. 4(a) except the value of the spontaneous decay rate is chosen such that $3n\Gamma T/4 = 2$. From Figs. 3 and 4, it is clear that the lifetime of the qubit is a crucial factor for generating the superposition of mesoscopic states of the NR.

V. CONCLUSIONS

We have presented a scheme to generate the superposition of mesoscopic states of a nanomechanical resonator by coupling with a two-level quantum mechanical system. We have shown that the displacement amplitude of the resonator could be exceptionally large as a result of quantum interference. We also find some squeezing in the position quadrature. We have discussed decoherence of the generated states due to spontaneous decay of a two-level quantum mechanical system. For $\xi = 3\Gamma T/8 = 0.2$, we find small changes in the Wigner function. This condition can be satisfied for resonator frequency $\omega \approx \Gamma$; thus a quantum dot coupled with a nanomechanical resonator of frequency 1 GHz could be considered for possible experimental realization of our results. In NV center qubits, the lifetime of the qubit is very large and $\eta = g/\omega \sim 10^{-3}$ can also be achieved. Therefore, NV centers as a qubit using a resonator of frequency 1 MHz could be another system for potential realization.

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APPENDIX: NEGLECTING TWO-PHONON TRANSITIONS IN EFFECTIVE HAMILTONIAN

The Hamiltonian (5) in the original picture becomes

$$H_{\text{eff}} = \hbar\omega a^\dagger a + i\hbar\Omega_2\eta\sigma_x(a^\dagger - a) + \hbar\Omega_1\sigma_x - \hbar\Omega_1\eta^2\sigma_x(a^\dagger - a)^2. \tag{A1}$$

The term containing Ω_2 commutes with the terms containing Ω_1. In order to understand the effect of the last term in Eq. (A1), we consider the case when $\Omega_2 = 0$. The Heisenberg equations of motion for phonon field operators are

$$\dot{a} = -i(\omega - \Omega_1\eta^2\sigma_x)a - i\Omega_1\eta^2\sigma_x a^\dagger, \tag{A2}$$

$$\dot{a}^\dagger = i(\omega - \Omega_1\eta^2\sigma_x)a^\dagger + i\Omega_1\eta^2\sigma_x a. \tag{A3}$$

For the qubit state, if we choose to use eigenstates of σ_x as the basis, the solution of these equations at any time t gives

$$a(t) = a(0)e^{-i\omega' t} - \frac{i\Omega_1\eta^2\lambda}{2\omega'}\sin(\omega' t)a^\dagger(0), \tag{A4}$$

$$a^\dagger(t) = a^\dagger(0)e^{i\omega' t} + \frac{i\Omega_1\eta^2\lambda}{2\omega'}\sin(\omega' t)a(0), \tag{A5}$$

where $\lambda = \pm 1$ corresponds to the qubit states $|\pm\rangle = (|e\rangle + |g\rangle)/\sqrt{2}$ and we have used the approximation, for $\Omega_1\eta^2 \ll \omega$, $\sqrt{(\omega^2 - 2\omega\Omega_1\eta^2\lambda} = \omega'$, with $\omega' = \omega - \Omega_1\eta^2\lambda$. For the first-order approximation, we notice that $a(t) \approx a(0)e^{-i\omega' t}$, $a^\dagger(t) \approx a^\dagger(0)e^{i\omega' t}$. We substitute first-order values of $a(t)$ and $a^\dagger(t)$ in the above expressions and rearrange the results in the form

$$a(t) + \frac{\Omega_1\eta^2\lambda}{2\omega'}a^\dagger(t) = \left[a(0) + \frac{\Omega_1\eta^2\lambda}{2\omega'}a^\dagger(0)\right]e^{-i\omega' t}, \tag{A6}$$

$$a^\dagger(t) + \frac{\Omega_1\eta^2\sigma_x}{2\omega'}a(t) = \left[a^\dagger(0) + \frac{\Omega_1\eta^2\lambda}{2\omega'}a(0)\right]e^{i\omega' t}. \tag{A7}$$

Clearly, the effect of the last term in Eq. (A1) is equivalent to transforming the NR frequency as $\omega \to \omega'$ and to transforming the operators as $a \to a + \frac{\Omega_1\eta^2\lambda}{2\omega'}a^\dagger$, $a^\dagger \to a^\dagger + \frac{\Omega_1\eta^2\lambda}{2\omega'}a$. For a coherent state of the NR, this transformation in the field operator leads to qubit state-dependent squeezing by a small factor of $\sqrt{1 - \Omega_1\eta^2/\omega}$ in the phonon field quadratures. However, when driving field Ω_1 is "off," the field operators transform back as $a + \frac{\Omega_1\eta^2\lambda}{2\omega'}a^\dagger \to a$, $a^\dagger + \frac{\Omega_1\eta^2\lambda}{2\omega'}a \to a^\dagger$ and the squeezing is undone. Therefore, we neglect the transformation in the field operators and consider the transformation in frequency only in Eq. (6).

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