

Quantum phase transitions of light in a dissipative Dicke-Bose-Hubbard modelRen-Cun Wu,¹ Lei Tan,^{1,*} Wen-Xuan Zhang,¹ and Wu-Ming Liu²¹*Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, China*²*Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*

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The impact that the environment has on the quantum phase transition of light in the Dicke-Bose-Hubbard model is investigated. Based on the quasibosonic approach, mean-field theory, and perturbation theory, the formulation of the Hamiltonian, the eigenenergies, and the superfluid order parameter are obtained analytically. Compared with the ideal cases, the order parameter of the system evolves with time as the photons naturally decay in their environment. When the system starts with the superfluid state, the dissipation makes the photons more likely to localize, and a greater hopping energy of photons is required to restore the long-range phase coherence of the localized state of the system. Furthermore, the Mott lobes depend crucially on the numbers of atoms and photons (which disappear) of each site, and the system tends to be classical with the number of atoms increasing; however, the atomic number is far lower than that expected under ideal circumstances. As there is an inevitable interaction between the coupled-cavity array and its surrounding environment in the actual experiments, the system is intrinsically dissipative. The results obtained here provide a more realistic image for characterizing the dissipative nature of quantum phase transitions in lossy platforms, which will offer valuable insight into quantum simulation of a dissipative system and which are helpful in guiding experimentalists in open quantum systems.

DOI: [10.1103/PhysRevA.96.033633](https://doi.org/10.1103/PhysRevA.96.033633)**I. INTRODUCTION**

Quantum simulation has become a research frontier and an indispensable tool in quantum information science [1–3]; its remarkable development in experimental realization has led to significant advances in the field of quantum optics and atomic physics [4–9]. Among the recent developments, the system of coupled-cavity arrays embedded with cold atoms has been intensively investigated as a platform to realize and simulate quantum many-body phenomena because of its extremely high tunability, individual addressability, and flexibility in its geometric design [10–14]. A wide range of condensed-matter systems have been theoretically investigated and many proposals for probing them have been proposed including the quantum phase transition [15,16], spin glasses [17,18], photonic crystals [19], the emergence of gauge fields [20], the quantum Hall effects [21], the Pfaffian-like topological state [22], and the supersolid [23,24].

The simplest physical model of light-matter coupling in a coupled-cavity-array system is the Jaynes-Cummings-Hubbard model, which presents an array of optical cavities that each contain a single two-level atom (TLA) in the photon-blockade regime [13,14]. A modified Jaynes-Cummings-Hubbard model based on an array of cavities where each cavity contains an embedded three-level atom has been proposed recently; this model circumvents the drawbacks of the excited-state spontaneous emission and provides a tunable extension of two-polariton bound states of the standard Jaynes-Cummings-Hubbard model [25,26]. As the number of atoms in each cavity increases, the collective effects due to interactions among atoms give rise to intriguing many-body phenomena. In quantum optics the Dicke model is a paradigm of collective behavior [27] that describes the interaction of ensembles of TLAs that are collectively coupled to the single

mode of radiation of a cavity [28]. Numerous investigations of interesting physical effects and their experimental realization [29], such as the superradiation phase [30–32], the superradiant Mott insulator [33], and the dynamical phase transition [34], are discussed. As a first step, a Dicke-Bose-Hubbard (DBH) model that includes more than one identical coupled cavity and N identical TLAs in each cavity has been conducted to study the quantum phase transitions of light without considering the counterrotating terms [35]. The transfer of excitations under a large range of operative conditions is also demonstrated and explored by tuning the control parameters in the DBH model [28]. Both the emergence of a polaritonic glassy phase [36] and the quantum phase transitions from the superfluid to the Bose-glass and the Mott-insulator states [37] have also been studied. Most recently, the localization-delocalization quantum phase transition of photons of the DBH model including counterrotating terms has been presented [38]. The model shows that under the influence of the counterrotating terms, the Mott lobes are fully suppressed.

As is well known, a realistic quantum optical system can rarely be isolated from its surroundings completely, particularly in an experiment; rather it is usually coupled to its external environment with an infinite number of degrees of freedom. To date, an investigation of the quantum phase transition of photons in a dissipative DBH model is still lacking. To treat the interplay between the coupled-cavity array and its environment in a more general setting, we developed a quasibosonic approach to describe the quantum phase transition and photon transport in an open quantum optical system [39,40]. Without the requirement of considering the finite environment's degrees of freedom, the quasibosonic method is a great concept that has a computational advantage. In the present paper, we use the quasibosonic approach to obtain an effective Hamiltonian of the dissipative DBH model. The coordinates of a bath can be eliminated and the system can be considered an ensemble of quasibosons in less time

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than its decay rate. Next, the eigenenergies and the superfluid order parameter of the system are also derived analytically for two TLAs on resonance, and we numerically demonstrate the phase diagram of an arbitrary number of TLAs. We hope that the theoretical analysis presented here will be an essential reference for future experiments to explore the quantum effects for multiatom systems.

The paper is organized as follows. In Sec. II, the dissipative DBH model is introduced based on the quasibosonic approach. Section III is devoted to deriving the eigenvalues and eigenstates for two atoms in each cavity. The analytical solutions of the superfluid order parameter for dressed states are given, and the properties of the quantum phase transition are discussed in Sec. IV. The extension to an arbitrary number of TLAs is also given in this section. Section V gives the conclusion.

II. THE DISSIPATIVE DICKE-BOSE-HUBBARD MODEL

The system considered is depicted in Fig. 1. The Hamiltonian of the DBH model considering the coupling to its environment is given by (with $\hbar = 1$) [35]

$$\begin{aligned}
 H &= \sum_i H_i^{DM} - \kappa \sum_{ij} a_i^\dagger a_j - \mu \sum_i N_i + H_R, \\
 H_i^{DM} &= \omega_a J_i^+ J_i^- + \omega_c a_i^\dagger a_i + \beta (a_i J_i^+ + a_i^\dagger J_i^-), \\
 H_R &= \sum_i \sum_{\omega_k} \omega_k r_k^\dagger r_k + H_{CR} + H_{AR}, \\
 H_{CR} &= \sum_i \sum_{\omega_k} [\eta_c(\omega_k) r_k^\dagger a + \text{H.c.}], \\
 H_{AR} &= \sum_i \sum_{\omega_k} [\eta_a(\omega_k) r_k^\dagger J^- + \text{H.c.}], \quad (2.1)
 \end{aligned}$$

where the indices i and j are individual cavity ranges over all sites, and a_i^\dagger and a_i are the photon creation and annihilation operators, respectively. $J_i^\pm = \sum_j \delta_j^\pm$ are the atomic collective raising and lowering angular momentum operators, and the total number of excitations is $N_i = a_i^\dagger a_i + J_i^+ J_i^-$. The transition energy of the TLA is ω_a , and ω_c is the frequency of the cavity field. All the atoms couple to cavities with the same coupling β [41]. We assume that the hopping energy of photons $\kappa_{ij} = \kappa$ between sites i and j and the chemical potential in the grand-canonical ensemble $\mu_i = \mu$ are the same for all cavities. The coupling Hamiltonian of the system with the environment and the Hamiltonian of the environment are described as H_R . ω_k is a bath model, and r_k^\dagger (r_k) is the creation (annihilation) operator of the environment in the k th model. H_{CR} is the interaction of the cavity with the environment. The interaction of the atoms with the environment is denoted as H_{AR} .

Considering the influence of the environment, the decoherence of every cavity and the two-level atom would result in the incoherent or dissipative propagation of the incident photon; thus nonequilibrium dynamics for the open quantum many-body system will arise. In general, simulations of nonequilibrium many-body effects for a finite freedom of the system can be performed using the master equation and the mean-field decoupling approximation [42–45]. However it is a formidable task to solve a fairly large parameter space

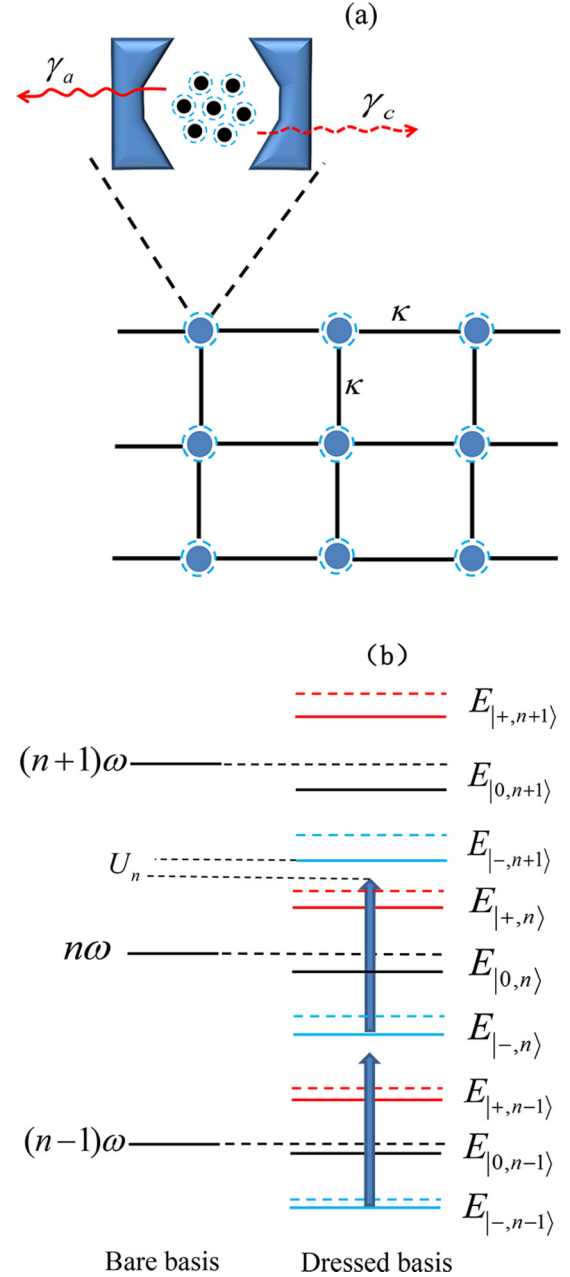


FIG. 1. (a) A schematic of a two-dimensional coupled-cavity-array setup. Each cavity contains N identical two-level atoms (dark gray globes) which couple resonantly to the cavity field (light gray globes). The dotted blue lines represent the interaction with the environment. The dotted red wavy arrow (γ_c) indicates the cavity decay, and the solid red wavy arrow (γ_a) is the decay of atoms. (b) Energy level diagram of the cavity-atom system on the n th site. The system is on resonance, i.e., $\tilde{\omega}_c = \tilde{\omega}_a = \tilde{\omega}$. The on-site repulsion U_n can be defined as $U_n = E_{|-,n+1\rangle} - E_{|-,n\rangle} - \omega$, which impedes the absorption for the next photon.

because of the infinite freedom of the environment. To address this problem, our group proposed a quasibosonic approach to eliminate the infinite freedom of the environment, in which the operators of the environment can be treated as a c-number and then the dissipative system can be solved easily [39,40]. One can obtain an effective Hamiltonian for the system based

on the quasibosonic approach:

$$H = \sum_i H_i^{DM} - \kappa \sum_{ij} \tilde{a}_i^\dagger \tilde{a}_j - \mu \sum_i \tilde{N}_i, \\ H_i^{DM} = \tilde{\omega}_a \tilde{J}_i^+ \tilde{J}_i^- + \tilde{\omega}_c \tilde{a}_i^\dagger \tilde{a}_i + \beta (\tilde{a}_i \tilde{J}_i^+ + \tilde{a}_i^\dagger \tilde{J}_i^-), \quad (2.2)$$

where $\tilde{\omega}_a = \omega_a - i\gamma_a$, $\tilde{\omega}_c = \omega_c - i\gamma_c$. γ_a and γ_c are decay rates of atoms and cavities, respectively. \tilde{a}_i^\dagger (\tilde{a}_i) is a quasiboson creation (annihilation) operator. \tilde{J}_i^+ (\tilde{J}_i^-) is the dressed atomic raising (lowering) angular momentum operator. The dissipation becomes an inherent property for the DBH model considered here.

A superfluid order parameter ψ , with the mean-field assumption $\psi \equiv \langle \tilde{a}_i \rangle$, is usually introduced to gain insight into the role of dissipation in the quantum phase transition. For $\psi \neq 0$, the system is in the superfluid phase. When $\psi = 0$, the system is in the Mott-insulator phase. In the present case, the expected value of \tilde{a}_i is in general complex with the formation $\langle \tilde{a}_i \rangle = \psi - i\psi_\gamma$. ψ_γ is a solvable small quantity as a function of decay rates of the system, and vanishes in the limit of ideal cases. Using the decoupling approximation $\tilde{a}_i^\dagger \tilde{a}_j = \langle \tilde{a}_i^\dagger \rangle \tilde{a}_j + \langle \tilde{a}_j \rangle \tilde{a}_i^\dagger - \langle \tilde{a}_i^\dagger \rangle \langle \tilde{a}_j \rangle$, the mean-field Hamiltonian of

Eq. (2.2) can be written as

$$H^{MF} = \sum_i H_i^{MF}, \\ H_i^{MF} = H_i^{DM} - \kappa \psi (\tilde{a}_i^\dagger + \tilde{a}_i) + \kappa |\psi|^2 - \mu \sum_i \tilde{N}_i. \quad (2.3)$$

This mean-field Hamiltonian is assumed to be the same for every site.

III. EIGENVALUES AND EIGENSTATES OF THE DISSIPATIVE DICKE-BOSE-HUBBARD MODEL

In the following, the case of two TLAs in each cavity is investigated as an example to provide a detailed illustration. The extension for an arbitrary number of two-level atoms is given in Sec. IV, which can be easily calculated by using the same approach. Followed by the proposal by Lei *et al.*, the bare states of the system are $|0, e^{\otimes 2}\rangle |n-2\rangle$, $|g, e\rangle |n-1\rangle$, and $|g^{\otimes 2}, 0\rangle |n\rangle$ with photon number n running from 0, 1, 2, 3 to ∞ [35]. For the two TLAs system, the case in which two atoms are in the excited state can be denoted as $|0, e^{\otimes 2}\rangle$, only one atom in the excited state is denoted by $|g, e\rangle$, and $|g^{\otimes 2}, 0\rangle$ is for the case in which the two atoms are in the ground state. Here a total of $3n$ bare state bases form a group for the whole Hilbert space. Based on these states, the matrix elements for H_n^{MF} can be obtained:

$$H_n^{MF} = \begin{vmatrix} 2\tilde{\omega}_a + (n-2)\tilde{\omega}_c - n\mu & \sqrt{2(n-1)\beta} & 0 \\ \sqrt{2(n-1)\beta} & 2\tilde{\omega}_a + (n-1)\tilde{\omega}_c - (n+1)\mu & \sqrt{2n\beta} \\ 0 & \sqrt{2n\beta} & n\tilde{\omega}_c - n\mu \end{vmatrix} + \kappa |\psi|^2 \quad (3.1)$$

with $\tilde{\omega}_c = \tilde{\omega}_a = \tilde{\omega}$ ($\tilde{\omega} = \omega - i\gamma$), $\gamma = \gamma_a + \gamma_c$. The eigenvalues can be obtained by diagonalizing the matrix in Eq. (3.1), and the corresponding eigenstates can be found:

$$E_{|0,n\rangle}^{(0)} = n\tilde{\omega}, \quad (3.2)$$

$$E_{|\pm,n\rangle}^{(0)} = \frac{(2n+1)\tilde{\omega} \pm \beta R(n, \frac{\tilde{\omega}}{\beta})}{2}, \quad (3.3)$$

$$|0,n\rangle = \frac{-\sqrt{n-1}|0, e^{\otimes 2}\rangle |n-2\rangle + \sqrt{n}|g^{\otimes 2}, 0\rangle |n\rangle}{\sqrt{2n-1}}, \quad (3.4)$$

$$|\pm,n\rangle = \frac{\sqrt{n}|0, e^{\otimes 2}\rangle |n-2\rangle + \frac{1}{2\sqrt{2}} [\frac{\tilde{\omega}}{\beta} \pm R(n, \frac{\tilde{\omega}}{\beta})] |g, e\rangle |n-1\rangle + \sqrt{n-1}|g^{\otimes 2}, 0\rangle |n\rangle}{\sqrt{2n-1 + \{ \frac{1}{2\sqrt{2}} [\frac{\tilde{\omega}}{\beta} \pm R(n, \frac{\tilde{\omega}}{\beta})] \}^2}}. \quad (3.5)$$

Here $R(n, \frac{\tilde{\omega}}{\beta}) = \sqrt{8(2n-1) + (\frac{\tilde{\omega}}{\beta})^2}$ is the effective Rabi frequency. The energy levels split into three branches corresponding to the upper branch $E_{|+,n\rangle}^{(0)}$, center branch $E_{|0,n\rangle}^{(0)}$, and the lower branch $E_{|-,n\rangle}^{(0)}$, as shown in Fig. 1(b).

IV. THE QUANTUM PHASE TRANSITION

In this section, we use the perturbation theory to obtain the superfluid order parameter and study the quantum phase transition by changing the controlling parameters. We have

assumed that cavities are coupled weakly to each other; thus, the interaction term between cavities can be considered as a perturbation term when the two-level atom system is coupled strongly to the cavity field. The effective Hamiltonian Eq. (2.3) thus reads

$$H_i^{MF} = H_i^{DM} + H_i', \\ H_i' = -\kappa \psi (\tilde{a}_i^\dagger + \tilde{a}_i) + \chi |\psi|^2 - \mu \tilde{N}_i, \quad (4.1)$$

which is valid on each site; we therefore drop the subscript i in the following. Considering the analogy of the transition

from the Mott-insulator to superfluid state between the Jaynes-Cummings model and the Bose-Hubbard model and the fact that the analytical results obtained by the second- and fourth-order perturbations are in good agreement with the exact-diagonalization numerical calculation [46], we derive the analytical solution of the system in terms of the second-order perturbation for simplicity. Equation (3.2) and Fig. 1(b) show that a center energy level $E_{|0,n\rangle}$ is required to perform the translation; thus, the on-site repulsion U_n based on the center branch state $|0,n\rangle$ is independent of the atom-cavity coupling β , which is different from the one defined by the state $|\pm,n\rangle$. To study the quantum phase transition in detail, the superfluid order parameter must be calculated separately for different cases.

Preparing in the center branch of the dressed state. According to the definition of the superfluid order parameter $\psi = \langle \Phi_n(t) | \tilde{a}_i | \Phi_n(t) \rangle$, $|\Phi_n(t)\rangle$ can be obtained based on the second-order perturbation theory. We first obtain the second-order corrections of energy eigenvalues $E_{|0,n\rangle}^{(2)}$ and (normalized) eigenstates $\tilde{\phi}_{|0,n\rangle}^{(2)}$ with respect to the dressed basis Eq. (3.4):

$$E_{|0,n\rangle}^{(2)} = \frac{(n-1)(2n-2)^2\kappa^2\psi^2}{(2n-1)(2n-3)(\epsilon-i\gamma)} + \frac{4n^3\kappa^2\psi^2}{(2n-1)(2n+1)(-\epsilon+i\gamma)}, \quad (4.2)$$

$$\tilde{\phi}_{|0,n\rangle}^{(2)} = \frac{\sqrt{n-1}(2n-2)(-\kappa\psi)}{\sqrt{(2n-1)(2n-3)}(\epsilon-i\gamma)}|n-1\rangle + \frac{2n\sqrt{n}(-\kappa\psi)}{\sqrt{(2n-1)(2n+1)}(\epsilon-i\gamma)}|n+1\rangle, \quad (4.3)$$

where $\epsilon = \omega - \mu$. Therefore, the eigenvalue of the dissipative system based on the second-order perturbation theory is

$$E_{|0,n\rangle} \equiv E_s + iE_\gamma \quad (4.4)$$

with

$$E_s = n\epsilon + \kappa|\psi|^2 + \frac{(-8n^3 + 12n^2 + 4n - 4)\kappa^2\psi^2\epsilon}{(2n-1)(2n-3)(2n+1)(\epsilon^2 + \gamma^2)},$$

$$E_\gamma = n\gamma + \frac{(-8n^3 + 12n^2 + 4n - 4)i\kappa^2\psi^2\gamma}{(2n-1)(2n-3)(2n+1)(\epsilon^2 + \gamma^2)}.$$

When the system is in the Mott-insulator state, $\psi = 0$, we have $E_\gamma = n\gamma$. When $\psi \neq 0$, one can take $E_\gamma \approx n\gamma$ because we assume that the coupling strength κ between cavities is weak. Up to second order, the expression for the (normalized) eigenstates is

$$\phi_{|0,n\rangle} = \frac{1}{\sqrt{\tilde{N}}}\tilde{\phi}_{|0,n\rangle},$$

$$\tilde{\phi}_{|0,n\rangle} = \frac{\sqrt{n-1}(2n-2)(-\kappa\psi)}{\sqrt{(2n-1)(2n-3)}(\epsilon-i\gamma)}|n-1\rangle + |n\rangle + \frac{2n\sqrt{n}(-\kappa\psi)}{\sqrt{(2n-1)(2n+1)}(\epsilon-i\gamma)}|n+1\rangle,$$

$$\tilde{N} = 1 + \frac{(n-1)(2n-2)^2\kappa^2\psi^2}{(2n-1)(2n-3)(\epsilon^2 + \gamma^2)} + \frac{4n^3\kappa^2\psi^2}{(2n-1)(2n+1)(\epsilon^2 + \gamma^2)}; \quad (4.5)$$

\tilde{N} is the normalized constant. For the open system considered here, the superfluid order parameter ψ is time-dependent. According to Eq. (4.5), the (normalized) eigenstate is a function of time; however, its time derivative can be ignored because of the second-order correction. Thus, the approximate time-dependent wave function of the system can be written as

$$\Phi_n(t) = f(t)\phi_{|0,n\rangle}.$$

Using the Schrödinger equation, one can find

$$\Phi_n(t) = \phi_{|0,n\rangle}e^{-iE_{|0,n\rangle}t}. \quad (4.6)$$

Therefore, the superfluid order parameter ψ for the state $|0,n\rangle$ can be obtained:

$$\psi_1 = e^{-n\gamma t} \sqrt{\frac{(8n^3 - 12n^2 - 4n + 4)\epsilon}{(16n^4 - 32n^3 + 12n^2 + 4n - 4)\kappa} - \frac{(2n-1)(2n+1)(2n-3)(\epsilon^2 + \gamma^2)}{(16n^4 - 32n^3 + 12n^2 + 4n - 4)\kappa^2 e^{-2n\gamma t}}}. \quad (4.7)$$

Equation (4.7) shows that ψ_1 is a function of the parameters κ , γ , t , and μ (in the present case, μ is a constant). The superfluid order parameter evolves and decays with time with a decay rate proportional to the number of photons n .

Preparing in the negative branch of the dressed state. Assume that each site is prepared in the negative branch of the dressed state $|- , n\rangle$. We can find the second-order deviations using a similar procedure, although the calculations become quite tedious when using our current formulation. The superfluid order parameter ψ_2 can be obtained by solving the following equation:

$$\psi_2 = \text{Re} \left\{ \frac{e^{-2\gamma nt}}{\tilde{N}'} \left[\frac{2[2\sqrt{n(n-1)(n-2)} + \frac{\sqrt{n-1}}{8}(\frac{\omega+i\gamma}{\beta} - R_{n-1}^\dagger)(\frac{\omega-i\gamma}{\beta} - R_n)]^2(-\kappa\psi_2)}{[2\epsilon + 2i\gamma - \beta(R_n^\dagger - R_{n-1}^\dagger)][2n-1 + \frac{1}{8}(\frac{\omega-i\gamma}{\beta} - R_{n-1})^2][2n-3 + \frac{1}{8}(\frac{\omega+i\gamma}{\beta} - R_{n-1}^\dagger)^2]} \right. \right. \\ \left. \left. + \frac{2[2\sqrt{n(n-1)(n+1)} + \frac{\sqrt{n}}{8}(\frac{\omega+i\gamma}{\beta} - R_n^\dagger)(\frac{\omega-i\gamma}{\beta} - R_{n+1})]^2(-\kappa\psi_2)}{[-2\epsilon + 2i\gamma - \beta(R_n - R_{n+1})][2n-1 + \frac{1}{8}(\frac{\omega+i\gamma}{\beta} - R_n^\dagger)^2][2n+1 + \frac{1}{8}(\frac{\omega-i\gamma}{\beta} - R_{n+1})^2]} \right] \right\}; \quad (4.8)$$

\tilde{N}' (see the Appendix) is the normalized constant. In what follows, we use Eqs. (4.7) and (4.8) to numerically investigate the features of the quantum phase transition arising from the competition between the on-site repulsion U_n and the hopping rate under the influences of the environment.

Analyses. These are illustrated in Fig. 2. First, we start with a superfluid phase. The time evolution of the superfluid order parameter ψ for different hopping rates κ and decay rates γ is shown. Comparing Figs. 2(a) and 2(b) with 2(c) and 2(d), a clear quantum phase transition is found for different initial states. The ideal cases are also given in Fig. 2 for comparison, which shows that the system is still in the coherent state that was prepared initially. The evolution of the dissipative system clearly reflects the expected decay of the coherence, which is the most obvious characteristic different from ideal cases. For a small t , although ψ decreases slightly, the system remains in a superfluid state. At a sufficiently large t , the effects of the environment become large, and the coherence of the system is initially destroyed in a pronounced manner and is then gradually reduced. Thus, ψ decays rapidly and the system undergoes a phase transition into a Mott-insulating phase. With the increase of the photon number n , the long-range order parameter will decrease rapidly, as shown in Figs. 3(b) and 3(d), because the decay time is proportional to n . The critical point t_c is a function of the control parameters and can be found by setting $\psi = 0$ in Eqs. (4.7) and (4.8), which yields $t_c = \frac{1}{2n\gamma} \ln \frac{(4n^2-4n-4)\kappa}{(2n+1)(2n-3)\epsilon}$. It follows that, for certain cavity decay rates κ , one may change the other control parameters according to t_c to enable the dissipative system to maintain coherence for a relatively long time. Obviously, when the external environment is considered, the decoherence of every resonator and the TLAs would result in the decay of the superfluid order parameter. In the experiment, dynamical decoupling [47] and feedback control [48] have been proposed to hamper the decay of the cavity field and TLAs and thus improve the coherence time.

In contrast, as shown in Fig. 3, we seek to determine how a Mott-insulator state in the beginning can restore the coherence by changing the intercavity hopping rate κ for a dissipative system. For a small κ , there are not enough excitations for hopping between cavities. By raising the hopping rate to a certain value $\kappa_c = \frac{(8n^3-12n^2-2n+3)\epsilon e^{2n\gamma t}}{(8n^3-12n^2-4n+4)}$, the system will restore its long-range coherence and a phase transition from the Mott-insulator to the superfluid phase appears. According to Eqs. (4.7) and (4.8), the photon hopping rate is also found to decrease because of the effect of the environment; thus the long-range coherence can only occur when the increase of the photon hopping rate is faster than its decay. Figures 3(a) and 3(c) also demonstrate that the influence of environment accumulates over time. With an increase in time, a large hopping rate is required to restore the coherence. Because the system and the environment have been recognized as a whole system in the effective Hamiltonian, the dissipation is the inherent nature of the system. Therefore, for $t = 0$, the system is also dissipative, and the hopping rate required for the phase transition to occur is higher than the rate expected in the ideal case. In addition, increasing the number of photons to n , the dissipation of the system is also enhanced

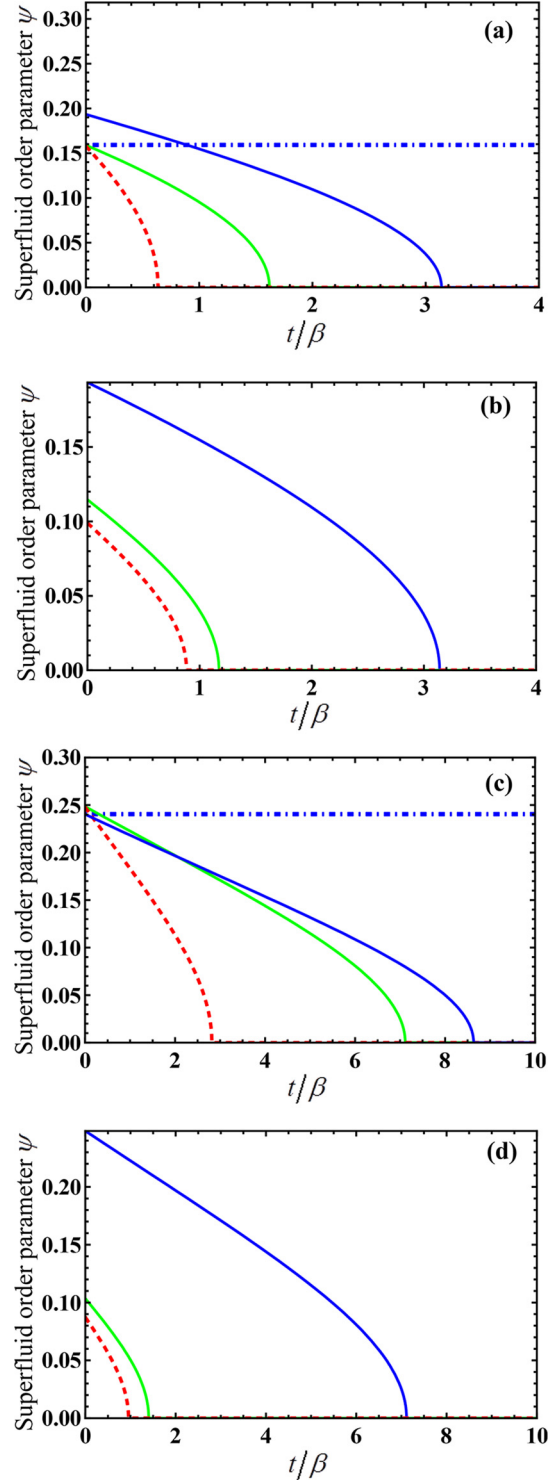


FIG. 2. The decrease of the superfluidity on each site for a initial superfluid state. (a) and (b) are for the state $|0, n\rangle$, but (c) and (d) are for the state $|- , n\rangle$. The resonance frequency $\omega = 10$, $\epsilon = 0.7836$. The superfluid order parameter decays continuously and beyond t_c the system behaves as a Mott-insulator-like state. In (a) and (c), the parameters are $n = 3$, $(\gamma/\beta, \kappa/\beta) = (0, 1)$ (dot-dashed blue line), $(0.02, 1)$ (light gray solid line), $(0.05, 1)$ (dashed red line), and $(0.02, 1.2)$ (dark gray solid line). In (b) and (d) $\gamma/\beta = 0.02$, $\kappa/\beta = 1.2$, with different n : $n = 3$ (dark gray solid line), $n = 9$ (light gray solid line), and $n = 12$ (dashed red line). The long-range order decays rapidly when n increases.

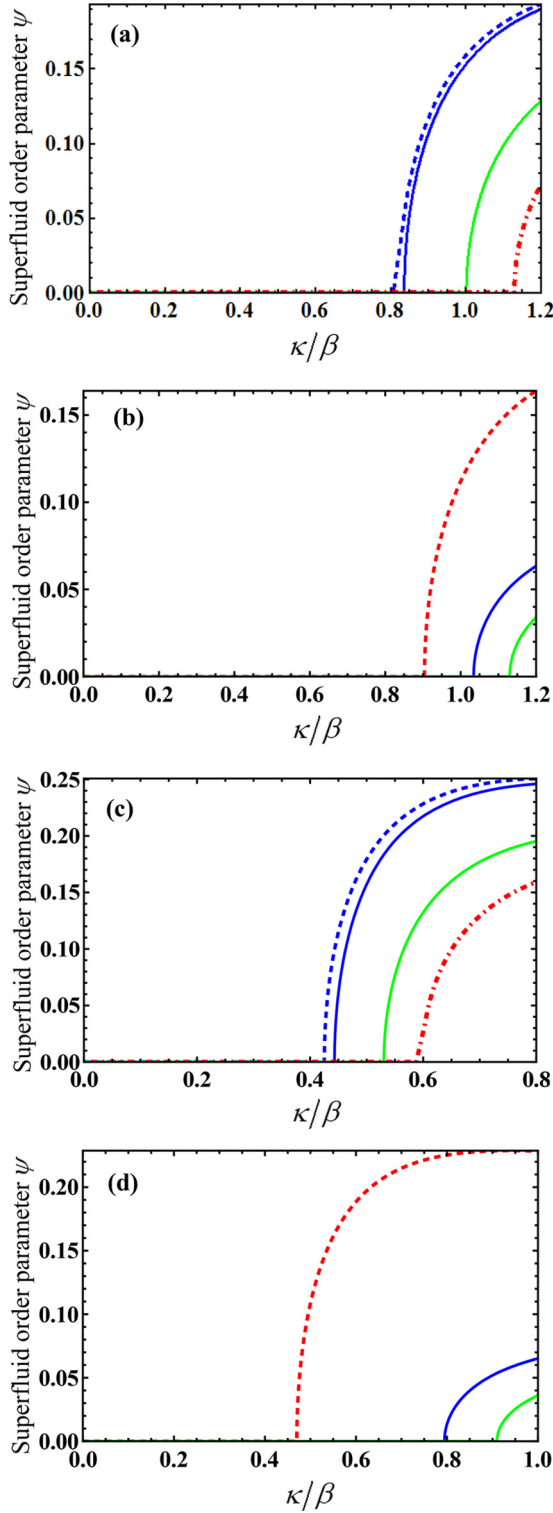


FIG. 3. The restoring of long-range coherence from the Mott-insulator state. (a) and (b) are for the state $|0, n\rangle$, (c) and (d) are for the state $|-, n\rangle$. The system can reach a superfluid phase with continuously increasing intercavity coupled rate κ . In (a) and (c), the parameters are $n = 3$, $(\gamma/\beta, t) = (0, 0)$ (dashed blue line), $(0.05, 0)$ (dark gray solid line), $(0.05, 0.3)$ (light gray solid line), and $(0.05, 0.5)$ (dot-dashed red line). In (b) and (d) $\gamma/\beta = 0.05$, $t = 0.3$, with different n : $n = 3$ (dashed red line), $n = 9$ (dark gray solid line), and $n = 12$ (light gray solid line). With an increase of n , a larger value of the intercavity hopping is needed.

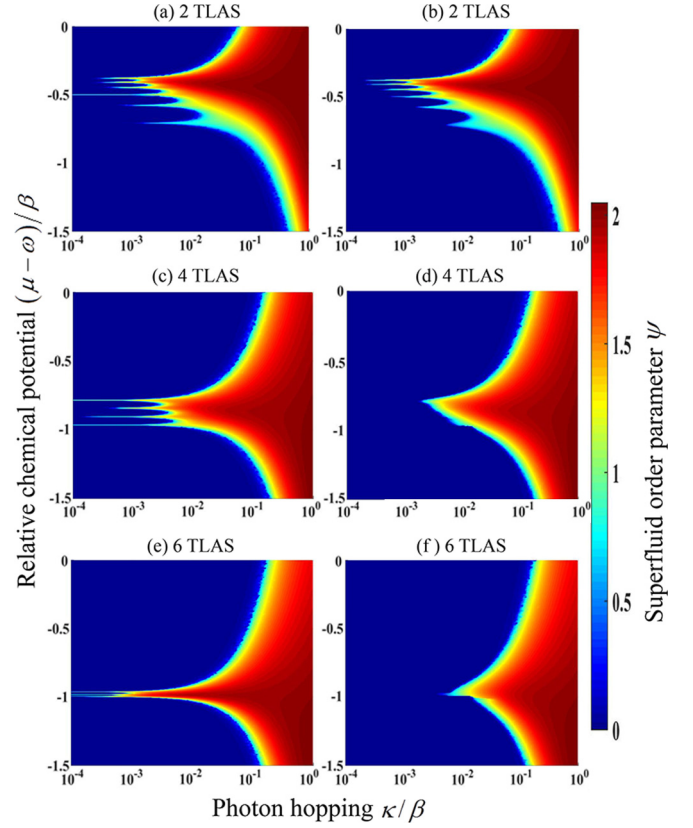


FIG. 4. The superfluid order parameters as a function of the relative chemical potential and the hopping rates for arbitrary number of TLAs. (a), (c), and (e) are for the ideal cases; others are for the dissipative cases. We have chosen $n = 8$ and $\gamma = 0.2$. The change of Mott-insulator and superfluid phase boundary can be seen clearly.

correspondingly; a higher hopping energy is thus required to induce a phase transition, as shown in Figs. 3(b) and 3(d).

Compared with the results in our previous paper [40], the quantum phase transition for the two two-level atoms case has two distinct characteristics. (1) The time for maintaining coherence and the hopping rate for restoring coherence are different. (2) Each site can be prepared both in the negative and the positive branch of the dressed state. Furthermore, the site can also be prepared in the center branch of the dressed state. When each site is prepared in the center branch of the dressed state, the initial superfluidity will not decay for the ideal cases, while the initially prepared Mott insulator in the same dressed state will restore coherence with the increasing of the hopping rate. One may use the effective repulsive potential U_n to understand this difference between the Jaynes-Cummings-Hubbard and Dicke-Bose-Hubbard models. Our numerical results show that the dissipation induces a renormalization on the effective repulsive potential and leads to a larger effective repulsive potential in the Dicke-Bose-Hubbard model than in the Jaynes-Cummings-Hubbard model for the same controlling parameters.

In what follows, we extend the model to an arbitrary number of TLA cases. The dressed-state basis can be written by the general method to diagonalize the effective DBH Hamiltonian (1.1) by numerical computation. The phase diagrams of the dissipative DBH model are plotted in Fig. 4. For comparison,

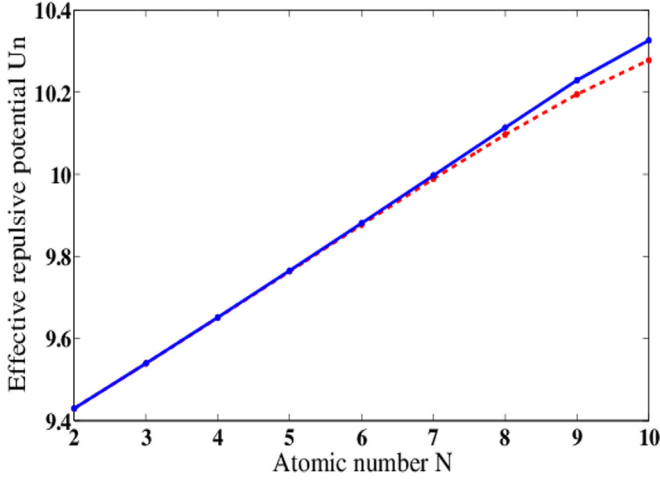


FIG. 5. Effective repulsive potential U_n versus different atomic number N , with parameters $\beta = 1, n = 14, \omega = 10$. The dashed red line is for the ideal case, and the solid blue line is for the dissipative case ($\gamma = 0.2$).

we also show the ideal cases. In the dissipative cases, we choose $t = 0$, which implies that the dissipative system is nearly in equilibrium. As shown in Fig. 4, as interaction with the environment destroys the coherence of the system, the Mott lobes becomes smaller and the area of the coherent phase decreases. Besides, one may find that the Mott-lobe structures depend crucially on the numbers of atoms and photons of each site. More interestingly, with the increase of the photon number n and keeping the number of two-level atoms constant in each cavity, more and more Mott lobes emerge. Next, the realization of the superfluid state requires a large hopping rate to derive the localized photons in each cavity. It can also be found that, in a regime with a small hopping rate κ , fewer TLAs could cause the system to become a localized phase compared with the ideal cases. With an increase in the number of TLAs, the coherent state may disappear rapidly for the dissipative system. The renormalized effective repulsive potential with the increase of atomic numbers for both cases is shown in Fig. 5. It can be found that with an increase of the number of TLAs, the effects of the dissipation will become more obvious, and the effective repulsive potential gets bigger than the one for the ideal case for the same number of atoms. Thus, for a strong dissipation, the pronounced repulsive potential and the enhanced collective decay rate make the superfluid order

parameter gradually decay, which leads the coherent state to disappear rapidly and the system tends to be more localized for the dissipative system. Then, the Mott-insulating region increases accordingly.

Most recently, a promising scheme to implement the two-mode Dicke-lattice model in circuit quantum electrodynamics has been proposed [49]. In general, the circuit quantum electrodynamics (CQED) lattices are inherently open systems; the dissipation leads to the inevitable loss of photons and the atomic decay from CQED. The scheme is motivated by the recent experimental observation of the dissipative phase transition in a one-dimensional chain of 72 microwave cavities with each coupled to a superconducting qubit [50]. In the above analysis, the numerical results are plotted under the condition that all the parameters follow the experimental parameters regime. We hope that the desired quantum phase transition with different parameters predicted here can be observed based on the above scheme.

V. CONCLUSION

Based on the quasibosonic approach, a realistic situation of a DBH model coupled to its environment was considered. The analytical solution of the superfluid order parameter for two TLAs per cavity was derived. The transition behaviors of the superfluid to Mott-insulating phase and the restoring coherence were discussed. The phase diagram for an arbitrary number of TLAs was also investigated. As the number of TLAs increases, Mott lobes may disappear and such a system tends to be classical. Most importantly, the atomic number is far lower than that under ideal circumstances. This work can provide parameters for reference to simulate strongly correlated many-body systems in the actual operation.

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APPENDIX

Assume that each site is prepared in the negative branch of the dressed state $|-, n\rangle$, and it can be obtained by diagonalizing the matrix in Eq. (3.1):

$$|-, n\rangle = \frac{\sqrt{n}|0, e^{\otimes 2}\rangle|n-2\rangle + \frac{1}{2\sqrt{2}}\left[\frac{\hat{\omega}}{\beta} - R\left(n, \frac{\hat{\omega}}{\beta}\right)\right]|g, e\rangle|n-1\rangle + \sqrt{n-1}|g^{\otimes 2}, 0\rangle|n\rangle}{\sqrt{2n-1 + \left\{\frac{1}{2\sqrt{2}}\left[\frac{\hat{\omega}}{\beta} - R\left(n, \frac{\hat{\omega}}{\beta}\right)\right]\right\}^2}}. \quad (\text{A1})$$

Based on the second-order perturbation theory, the expression for the second-order corrections of eigenstates can be obtained:

$$\begin{aligned} \tilde{\phi}_{|-, n\rangle} = & \frac{2\sqrt{n(n-1)(n-2)} + \frac{\sqrt{n-1}}{8}\left(\frac{\omega+i\gamma}{\beta} - R_n^\dagger\right)\left(\frac{\omega-i\gamma}{\beta} - R_{n-1}\right)}{[2\epsilon - 2i\gamma - \beta(R_n - R_{n-1})]\sqrt{\left[2n-1 + \frac{1}{8}\left(\frac{\omega+i\gamma}{\beta} - R_n^\dagger\right)^2\right]\left[2n-3 + \frac{1}{8}\left(\frac{\omega-i\gamma}{\beta} - R_{n-1}\right)^2\right]}}|n-1\rangle \\ & + |n\rangle + \frac{2\sqrt{n(n-1)(n+1)} + \frac{\sqrt{n}}{8}\left(\frac{\omega+i\gamma}{\beta} - R_n^\dagger\right)\left(\frac{\omega-i\gamma}{\beta} - R_{n+1}\right)}{[-2\epsilon + 2i\gamma - \beta(R_n - R_{n+1})]\sqrt{\left[2n-1 + \frac{1}{8}\left(\frac{\omega+i\gamma}{\beta} - R_n^\dagger\right)^2\right]\left[2n+1 + \frac{1}{8}\left(\frac{\omega-i\gamma}{\beta} - R_{n+1}\right)^2\right]}}|n+1\rangle, \quad (\text{A2}) \end{aligned}$$

where $R_n = \sqrt{8(2n-1) + (\frac{\omega}{\beta})^2}$. In order to simplify the above formula, we can assume that the expression follows A and B . The normalized eigenstate is $\phi_{|-,n)} = \frac{1}{\sqrt{N'}} \tilde{\phi}_{|-,n)}$. Thus the expression of the normalized constant $\tilde{N}' = \langle \tilde{\phi}_{|-,n)} | \tilde{\phi}_{|-,n)} \rangle$ can be expressed as follows:

$$A = \frac{2\sqrt{n(n-1)(n-2)} + \frac{\sqrt{n-1}}{8}(\frac{\omega+i\gamma}{\beta} - R_n^\dagger)(\frac{\omega-i\gamma}{\beta} - R_{n-1})}{[2\epsilon - 2i\gamma - \beta(R_n - R_{n-1})]\sqrt{[2n-1 + \frac{1}{8}(\frac{\omega+i\gamma}{\beta} - R_n^\dagger)^2][2n-3 + \frac{1}{8}(\frac{\omega-i\gamma}{\beta} - R_{n-1})^2]}}$$

$$B = \frac{2\sqrt{n(n-1)(n+1)} + \frac{\sqrt{n}}{8}(\frac{\omega+i\gamma}{\beta} - R_n^\dagger)(\frac{\omega-i\gamma}{\beta} - R_{n+1})}{[-2\epsilon + 2i\gamma - \beta(R_n - R_{n+1})]\sqrt{[2n-1 + \frac{1}{8}(\frac{\omega+i\gamma}{\beta} - R_n^\dagger)^2][2n+1 + \frac{1}{8}(\frac{\omega-i\gamma}{\beta} - R_{n+1})^2]}}$$

$$\tilde{N}' = 4\kappa^2\psi^2 AA^\dagger + 4\kappa^2\psi^2 BB^\dagger. \quad (A3)$$

The conjugates of A and B are A^\dagger and B^\dagger , respectively. Using the Schrödinger equation, one can find the time-dependent eigenstates $\Phi_n(t) = \phi_{|-,n)} e^{-iE_{|-,n)}t}$. Therefore, the superfluid order parameter can also be derived by $\psi_2 = \langle \Phi_n(t) | \tilde{a}_i | \Phi_n(t) \rangle$.

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