Thermal Casimir-Polder forces on a V-type three-level atom

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We study the thermal Casimir-Polder (CP) forces on a V-type three-level atom. The competition between the thermal effect and the quantum interference of the two transition dipoles on the force is investigated. To shed light onto the role of the quantum interference, we analyze two kinds of initial states of the atom, i.e., the superradiant state and the subradiant state. Considering the atom being in the thermal reservoir, the resonant CP force arising from the real photon emission dominates in the evolution of the CP force. Under the zero-temperature condition, the quantum interference can effectively modify the amplitude and the evolution of the force, leading to a long-time force or even the cancellation of the force. Our results reveal that in the finite-temperature case, the thermal photons can enhance the amplitude of all force elements, but have no influence on the net resonant CP force in the steady state, which means that the second law of thermodynamics still works. For the ideal degenerate V-type atom with parallel dipoles under the initial subradiant state, the robust destructive quantum interference overrides the thermal fluctuations, leading to the trapping of the atom in the subradiant state and the disappearance of the CP force. However, in terms of a realistic Zeeman atom, the thermal photons play a significant role during the evolution of the CP force. The thermal fluctuations can enhance the amplitude of the initial CP force by increasing the temperature, and weaken the influence of the quantum interference on the evolution of the CP force from the initial superradiant (subradiant) state to the steady state.

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I. INTRODUCTION

Casimir-Polder (CP) forces, firstly predicted by Casimir and Polder in 1948 [1], are long-range electromagnetic interactions between neutral polarizable particles and a macroscopic object. It is a purely quantum mechanical effect which originates from the zero-point fluctuations of the electromagnetic field and dipole moments [1–5]. As an observable quantum effect, the CP force attracts much fundamental attention. With the development in trapping and manipulating cold atoms or polar molecules near surfaces [6,7], the CP force also becomes an important subject in the applied research. Recently, it was predicted that Rydberg atoms would be effectively excited by the CP interactions combined with optomechanics [8].

Although the CP force is tiny, a number of precise experiments had demonstrated such a quantum phenomenon [9–13]. Aside from the zero-point fluctuations, thermal fluctuations also contribute to the CP force. As the experiments were typically performed at room temperature, the impact of the thermal photons on CP forces has attracted particular interest. In 2007, the temperature-dependent CP force was firstly observed out of thermal equilibrium [14]. Meanwhile, theoretical studies on thermal CP forces have covered various configurations such as spheres [15], cylinders [15–18], planar [19,20], cylindrical [19,21], graphene [22], and carbon nanotubes [23] in order to explore the entangled position and temperature dependences of thermal CP forces. For instance, Ellingsen *et al.* demonstrated that the thermal CP potentials of a molecule near a metal surface can be entirely independent of temperature even if the thermal photon number is large [24].

The studies of CP forces mentioned above are mainly focused on the ground-state atoms in which only the virtual photon fluctuations appear. The CP forces on the excited atoms also inspired a great number of investigations [25–32] owing to the significant amplification compared with those on the ground-state atoms. It is generally recognized that the CP force on an excited atom originates mainly from the real photon emission, exhibiting an oscillatory spatial behavior. In other words, the CP force on an excited atom is dominated by the electromagnetic mode at the atomic transition frequency. Thus it is feasible to control the force by tailoring the electromagnetic property of the material at the atomic frequency. For instance, the metamaterials have been used to trap atoms and produce a long-time force [33].

Though there were some studies concerning the CP force on the excited two-level atom system or multilevel systems [20,30,31,33], the quantum interference effect was rarely in discussion due to the premise that these atom systems had the ladder-type level structures. As a significant fundamental quantum effect, the quantum interference in the atom among different transition channels has aroused great attention, leading to many fascinating phenomena such as coherence trapping of population, lasing without inversion, electromagnetic induced transparency, and gain without inversion [34]. Recently, the quantum interference effect on the CP force on a three-level V-type atom was analyzed [35], which revealed that the quantum interference plays an important role on the CP force. However, the CP force was investigated under the condition of zero temperature, where thermal photons are out of consideration. Thus it is of great

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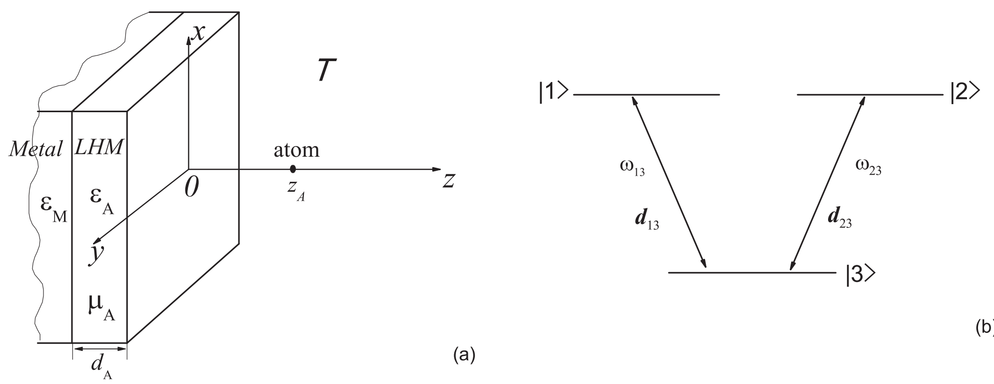

FIG. 1. (a) Scheme of an atom near the structure made of a LHM slab mounted on a metal substrate. (b) The level scheme of the V-type three-level atom.

interest for us to analyze both the thermal and the quantum interference effects in one system to discover how the CP force behaves.

In this paper, we analyze the amplitude and the evolution of the CP force acting on a V-type three-level atom in a thermal reservoir with temperature T. Two kinds of V-type atoms are investigated and compared. The first is an ideal degenerate V-type atom with two parallel transition dipoles, while the other is a Zeeman V-type atom whose two transition dipoles are left rotation and right rotation, respectively. We discuss the evolution of the CP force starting from two kinds of initial states of the atom, i.e., the subradiant and the superradiant state, in order to study the influence of quantum interference. The atom is considered to locate near a structure made of the left-handed metamaterials (LHMs) and metal, similar to the previous work [33,35]. This paper is organized as follows: in Sec. II we introduce the model and give the explicit derivation of the thermal CP force on a V-type atom. In Sec. III, we analyze in detail the thermal CP force on a V-type atom with two kinds of initial states. The effect of the thermal fluctuations as well as quantum interference on the CP force is especially discussed. In Sec. IV, we summarize the results.

II. MODEL AND FORMULAS

We consider a V-type atom located in the vicinity of a slab with thickness d_A made of LHM with permittivity ε_A and permeability μ_A, shown in Fig. 1(a). The LHM slab is mounted on a metal substrate with permittivity ε_M. The V-type atom located at position $\boldsymbol{r}_A = (0,\ 0,\ z_A)$ has two nearly degenerate upper states $|1\rangle$ and $|2\rangle$, and one ground state $|3\rangle$, as illustrated in Fig. 1(b). The frequencies and the dipole moments of two transition channels are ω_{i3} and d_{i3} $(i = 1,2)$, respectively. Note that the dipole transition between $|1\rangle$ and $|2\rangle$ is forbidden.

The Hamiltonian of the whole system is [36]

$$\hat{H} = \sum_{\lambda=e,m} \int d^3\boldsymbol{r} \int_0^\infty d\omega\hbar\omega \hat{\boldsymbol{f}}^\dagger_\lambda(\boldsymbol{r},\omega)\cdot\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega) + \hbar\omega_{13}|1\rangle\langle 1| + \hbar\omega_{23}|2\rangle\langle 2| - \hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{E}}(\boldsymbol{r}_A). \tag{1}$$

Here $\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega)$ and $\hat{\boldsymbol{f}}^\dagger_\lambda(\boldsymbol{r},\omega)$ are the generalized bosonic structure-assisted operators satisfying the commutation relationship of $[\hat{\boldsymbol{f}}^\dagger_\lambda(\boldsymbol{r},\omega),\hat{\boldsymbol{f}}_{\lambda'}(\boldsymbol{r}',\omega')] = \delta_{\lambda\lambda'}\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(\omega-\omega')$. The atomic electric-dipole operator is defined as $\hat{\boldsymbol{d}} = \boldsymbol{d}_{13}\hat{A}_{13} + \boldsymbol{d}_{31}\hat{A}_{31} + \boldsymbol{d}_{23}\hat{A}_{23} + \boldsymbol{d}_{32}\hat{A}_{32}$. Here, $\boldsymbol{d}_{mn} = \langle m|\hat{\boldsymbol{d}}|n\rangle$ is the transition dipole moment and $\hat{A}_{mn} = |m\rangle\langle n|$ is the atomic flip operator [36]. The time-dependent atomic density matrix $\hat{\sigma} = \sum_{m,n}\sigma_{mn}|m\rangle\langle n|$ is closely related to $\hat{A}_{mn}$ by

$$\langle\hat{A}_{mn}(t)\rangle = \sigma_{nm}(t), \quad m,n = 1,2,3. \tag{2}$$

The electric field operator $\hat{\boldsymbol{E}}(\boldsymbol{r})$ can be expressed in terms of the fundamental bosonic operators $\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega)$ and the classical Green tensor $\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}',\omega)$ (see Eq. (8) in Ref [33].) as

$$\hat{\boldsymbol{E}}(\boldsymbol{r}) = \int d^3\boldsymbol{r}' \int_0^\infty d\omega \overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}',\omega) \cdot\left[\omega\sqrt{\frac{\hbar\varepsilon_0}{\pi}\mathrm{Im}\varepsilon(\boldsymbol{r}',\omega)}\hat{\boldsymbol{f}}_e(\boldsymbol{r}',\omega) + \nabla\times\sqrt{-\frac{\hbar}{\pi\mu_0}\mathrm{Im}\frac{1}{\mu(\boldsymbol{r}',\omega)}}\hat{\boldsymbol{f}}_m(\boldsymbol{r}',\omega)\right] + \mathrm{H.c.} \tag{3}$$

Throughout this paper, the atom is initially prepared in the superposition state:

$$|\psi_A(0)\rangle = c_1|1\rangle + c_2|2\rangle. \tag{4}$$

The density matrix of the thermal field at temperature T is given by [34,36]

$$\hat{\rho}_T = \frac{\exp[-\hat{H}_F/(k_BT)]}{\mathrm{Tr}\{\exp[-\hat{H}_F/(k_BT)]\}}. \tag{5}$$

Here k_B is the Boltzmann constant. $\hat{H}_F$ is the Hamiltonian of the electromagnetic field; i.e., $\hat{H}_F = \sum_{\lambda=e,m}\int d^3\boldsymbol{r}\int_0^\infty d\omega\hbar\omega\hat{\boldsymbol{f}}^\dagger_\lambda(\boldsymbol{r},\omega)\cdot\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega)$. The nonvanishing thermal averages of the electric field are given by [36]

$$\langle\hat{\boldsymbol{E}}^\dagger(\boldsymbol{r},\omega)\hat{\boldsymbol{E}}(\boldsymbol{r}',\omega')\rangle_T = \frac{\hbar\mu_0}{\pi}n\omega^2\mathrm{Im}\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}',\omega)\delta(\omega-\omega'). \tag{6}$$

Here n is the average number of thermal photons in accordance with the Bose-Einstein statistics:

$$n = \frac{1}{e^{\hbar\omega/(k_BT)} - 1}. \tag{7}$$

In the long-wavelength approximation, the CP force is the expectation value of the operator of electromagnetic force on the atom [27]:

$$\boldsymbol{F}(\boldsymbol{r}_A,t) = \left\langle \nabla[\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{E}}(\boldsymbol{r})] + \frac{\mathrm{d}}{\mathrm{d}t}[\hat{\boldsymbol{d}}\times\hat{\boldsymbol{B}}(\boldsymbol{r})]\right\rangle_{\boldsymbol{r}=\boldsymbol{r}_A}. \quad (8)$$

The first term refers to the dipole force and the second term refers to the Lorentz force. It should be noticed that the Lorentz force can be rigorously canceled under the condition of $\omega_{13} = \omega_{23} = \omega_0$ [27,35]. We do make this approximation, i.e., $\omega_{13} \approx \omega_{23} = \omega_0$, in this paper.

After tedious manipulations, we obtain the CP force on a V-type three-level atom in a thermal reservoir with temperature T [27]:

$$\boldsymbol{F}(\boldsymbol{r}_A,t) = \sum_{m,n}\sigma_{nm}(t)\boldsymbol{F}_{mn}(\boldsymbol{r}_A), \quad m,n = 1,2,3. \quad (9)$$

It consists of the time-dependent density-matrix elements $\sigma_{nm}(t)$ and time-independent force amplitude terms $\boldsymbol{F}_{mn}(\boldsymbol{r}_A)$. The density-matrix elements $\sigma_{nm}(t)$ describe the internal atomic dynamics, and are governed by the master equations as [34] follows:

$$\frac{\partial}{\partial t}\sigma_{11}(t) = -(n+1)\gamma_1\sigma_{11}(t) - (n+1)\frac{\kappa}{2}[\sigma_{12}(t)+\sigma_{21}(t)] + n\gamma_1\sigma_{33}(t), \quad (10a)$$

$$\frac{\partial}{\partial t}\sigma_{22}(t) = -(n+1)\gamma_2\sigma_{22}(t) - (n+1)\frac{\kappa}{2}[\sigma_{21}(t)+\sigma_{12}(t)] + n\gamma_2\sigma_{33}(t), \quad (10b)$$

$$\frac{\partial}{\partial t}\sigma_{12}(t) = -\frac{1}{2}(n+1)(\gamma_1+\gamma_2)\sigma_{12}(t) - \frac{1}{2}\kappa(n+1) \times[\sigma_{22}(t)+\sigma_{11}(t)] + n\kappa\sigma_{33}(t). \quad (10c)$$

Here, $\gamma_1(\gamma_2)$ is the spontaneous decay rate from the upper level $|1\rangle(|2\rangle)$ to the ground level $|3\rangle$, and κ is the collective damping rate due to the quantum interference. They have expressions as follows:

$$\gamma_1 = 2\mu_0\omega_0^2\boldsymbol{d}_{13}\cdot\mathrm{Im}\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0)\cdot\boldsymbol{d}_{31}/\hbar, \quad (11a)$$

$$\gamma_2 = 2\mu_0\omega_0^2\boldsymbol{d}_{23}\cdot\mathrm{Im}\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0)\cdot\boldsymbol{d}_{32}/\hbar, \quad (11b)$$

$$\kappa = 2\mu_0\omega_0^2\boldsymbol{d}_{13}\cdot\mathrm{Im}\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0)\cdot\boldsymbol{d}_{32}/\hbar. \quad (11c)$$

Equation (10) reveals that the thermal photon number n plays a significant role in the evolution of the V-type atom. The time-independent force amplitudes $\boldsymbol{F}_{mn}(\boldsymbol{r}_A)$ in Eq. (9) are given by

$$\boldsymbol{F}_{11}(\boldsymbol{r}_A) = -\frac{\mu_0 k_B T}{\pi}\sum_{j=0}^{\infty}{}'\xi_j^2\nabla\mathrm{tr}\{[\overleftrightarrow{\boldsymbol{\alpha}}_{11}(i\xi_j)+\overleftrightarrow{\boldsymbol{\alpha}}_{11}(-i\xi_j)]\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,i\xi_j)\}_{\boldsymbol{r}=\boldsymbol{r}_A} + \left\{(n+1)\mu_0\omega_0^2\nabla[\boldsymbol{d}_{13}\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)\cdot\boldsymbol{d}_{31}] + \mathrm{c.c.}\right\}_{\boldsymbol{r}=\boldsymbol{r}_A}, \quad (12a)$$

$$\boldsymbol{F}_{22}(\boldsymbol{r}_A) = -\frac{\mu_0 k_B T}{\pi}\sum_{j=0}^{\infty}{}'\xi_j^2\nabla\mathrm{tr}\{[\overleftrightarrow{\boldsymbol{\alpha}}_{22}(i\xi_j)+\overleftrightarrow{\boldsymbol{\alpha}}_{22}(-i\xi_j)]\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,i\xi_j)\}_{\boldsymbol{r}=\boldsymbol{r}_A} + \left\{(n+1)\mu_0\omega_0^2\nabla[\boldsymbol{d}_{23}\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)\cdot\boldsymbol{d}_{32}] + \mathrm{c.c.}\right\}_{\boldsymbol{r}=\boldsymbol{r}_A}, \quad (12b)$$

$$\boldsymbol{F}_{12}(\boldsymbol{r}_A) = \boldsymbol{F}_{21}(\boldsymbol{r}_A) = -\frac{\mu_0 k_B T}{\pi}\sum_{j=0}^{\infty}{}'\xi_j^2\nabla\mathrm{tr}\{[\overleftrightarrow{\boldsymbol{\alpha}}_{12}(i\xi_j)+\overleftrightarrow{\boldsymbol{\alpha}}_{12}(-i\xi_j)]\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,i\xi_j)\}_{\boldsymbol{r}=\boldsymbol{r}_A} + \left\{(n+1)\mu_0\omega_0^2\nabla[\boldsymbol{d}_{23}\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)\cdot\boldsymbol{d}_{31}] + \mathrm{c.c.}\right\}_{\boldsymbol{r}=\boldsymbol{r}_A}, \quad (12c)$$

$$\boldsymbol{F}_{33}(\boldsymbol{r}_A) = -\frac{\mu_0 k_B T}{\pi}\sum_{j=0}^{\infty}{}'\xi_j^2\nabla\mathrm{tr}\{[\overleftrightarrow{\boldsymbol{\alpha}}_{33}(i\xi_j)+\overleftrightarrow{\boldsymbol{\alpha}}_{33}(-i\xi_j)]\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,i\xi_j)\}_{\boldsymbol{r}=\boldsymbol{r}_A} - \left\{n\mu_0\omega_0^2\nabla[\boldsymbol{d}_{13}\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)\cdot\boldsymbol{d}_{31}] + n\mu_0\omega_0^2\nabla[\boldsymbol{d}_{23}\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)\cdot\boldsymbol{d}_{32}] + \mathrm{c.c.}\right\}_{\boldsymbol{r}=\boldsymbol{r}_A}. \quad (12d)$$

Here, $\xi_j = j2\pi k_B T/\hbar$ denotes the Matsubara frequencies [27,36] and "c.c." means the complex conjugate. $\overleftrightarrow{\boldsymbol{G}}^{(1)}$ is the scattering part of the Green tensor (see Eq. (9) in Ref. [33]) and $\overleftrightarrow{\boldsymbol{\alpha}}$ is the atomic polarizability tensor [36] given by

$$\overleftrightarrow{\boldsymbol{\alpha}}_{mn} = \frac{1}{\hbar}\sum_k\left(\frac{\boldsymbol{d}_{nk}\boldsymbol{d}_{km}}{\omega_{km}-\omega} + \frac{\boldsymbol{d}_{km}\boldsymbol{d}_{nk}}{\omega_{kn}+\omega}\right), \quad m,n,k = 1,2,3. \quad (13)$$

We neglect the shift and width of the atomic transition frequency in Eqs. (12) and (13) due to the weak coupling between the atom and the field. It should be noticed that σ_{13} and σ_{23} as well as $\boldsymbol{F}_{13}(\boldsymbol{r}_A)$ and $\boldsymbol{F}_{23}(\boldsymbol{r}_A)$ are neglected here owing to the inhibition of the transition between $|1\rangle$ and $|2\rangle$. As a consequence, there are five density-matrix elements [i.e., σ_{11}, σ_{22}, σ_{12}, σ_{21}, and σ_{33}] as well as five time-independent force amplitudes [i.e., $\boldsymbol{F}_{11}(\boldsymbol{r}_A)$, $\boldsymbol{F}_{22}(\boldsymbol{r}_A)$, $\boldsymbol{F}_{12}(\boldsymbol{r}_A)$, $\boldsymbol{F}_{21}(\boldsymbol{r}_A)$, and $\boldsymbol{F}_{33}(\boldsymbol{r}_A)$] contributing to the CP force.

Equations (12a)–(12d) show that all force amplitudes can be decomposed of the dispersion (off-resonant) part and the resonant part. The dispersion part from virtual photons is characterized by the summation of Matsubara frequencies

$\sum' \xi_j^2(\cdots)$. The prime in the Matsubara sum indicates the half weight for the term $j = 0$. On the other hand, the resonant part is due to the real photon emission or absorption.

From Eqs. (10a)–(10c) it is clear that the quantum interference (κ) as well as thermal photons (n) can alter the evolution of the CP force by changing the time-dependent parts $\sigma_{mn}(t)$. Furthermore, Eqs. (12a)–(12d) demonstrate that the time-independent amplitudes $\boldsymbol{F}_{mn}(\boldsymbol{r}_A)$ are also changed by the above two effects. For instance, Eq. (12c) reveals that $\boldsymbol{F}_{12}(\boldsymbol{r}_A)$ and $\boldsymbol{F}_{21}(\boldsymbol{r}_A)$ arise from the quantum interference effect due to the cross coupling between $\boldsymbol{d}_{13}$ and $\boldsymbol{d}_{23}$ [35]. Simultaneously, the thermal photons enhance all the resonant-force elements, such as changing the prefactor of $\boldsymbol{F}_{ij}(\boldsymbol{r}_A)\,(i,j = 1,2)$ from 1 in the zero-temperature case to $(n + 1)$ in the finite-temperature case. On the other hand, thermal fluctuations change the dispersion part of the CP force from integral in the zero-temperature case to the Matsubara summation in the finite-temperature case. Additionally, $\boldsymbol{F}_{33}(\boldsymbol{r}_A)$, i.e., Eq. (12d), exhibits the resonant force arising from thermal fluctuation photons, which is different from the zero-temperature case [35] where only the dispersion force appears. As a consequence, the CP forces on the V-type atom in the finite temperature would exhibit different behavior from those in the zero-temperature condition.

III. ANALYSIS

We now analyze in detail the effect of thermal fluctuations on the CP force. To get a complete picture of the role of thermal photons, we study two versions of the V-type systems: an ideal atom with two parallel dipoles and a Zeeman atom with two perpendicular dipoles. The aforementioned two kinds of superposition states, i.e., the subradiant state and the superradiant state, are assumed as the initial states to obtain the maximum quantum interference. Thus the competition between thermal photons and the quantum interference on the evolution of the CP force can be discussed.

A. Ideal atom with two parallel dipoles, $\boldsymbol{d}_{13} \parallel \boldsymbol{d}_{32} = \boldsymbol{d}_0$

In terms of the atom with two parallel transition dipoles, $\boldsymbol{d}_{13}$ and $\boldsymbol{d}_{32}$ are equal to each other; i.e., $\boldsymbol{d}_{13} \parallel \boldsymbol{d}_{32} = \boldsymbol{d}_0$. Such an atom system was extensively studied in the previous works concerning the quantum interference [37,38]. Whether the environment is involved or not, the corresponding decay rates and the collective damping of the two channels are the same; i.e., $\gamma_1 = \gamma_2 = \kappa = \gamma$. By virtue of $\boldsymbol{d}_{13} = \boldsymbol{d}_{32} = \boldsymbol{d}_0$, the time-independent force amplitudes are given by

$$\begin{aligned}\boldsymbol{F}_{11}(\boldsymbol{r}_A) &= \boldsymbol{F}_{22}(\boldsymbol{r}_A) = \boldsymbol{F}_{12}(\boldsymbol{r}_A) = \boldsymbol{F}_{21}(\boldsymbol{r}_A)\\ &= -\frac{\mu_0 k_B T}{\pi}\sum_{j=0}^{\infty}{}'\xi_j^2\nabla\,\mathrm{tr}\{[\overleftrightarrow{\boldsymbol{\alpha}}_{11}(i\xi_j)+\overleftrightarrow{\boldsymbol{\alpha}}_{11}(-i\xi_j)]\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,i\xi_j)\}_{\boldsymbol{r}=\boldsymbol{r}_A}\\ &\quad+2(n+1)\mu_0\omega_0^2\,\mathrm{Re}\nabla\{\boldsymbol{d}_0\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)\cdot\boldsymbol{d}_0\}_{\boldsymbol{r}=\boldsymbol{r}_A}, \qquad (14a)\end{aligned}$$

$$\begin{aligned}\boldsymbol{F}_{33}(\boldsymbol{r}_A) &= -\frac{\mu_0 k_B T}{\pi}\sum_{j=0}^{\infty}{}'\xi_j^2\nabla\mathrm{tr}\{[\overleftrightarrow{\boldsymbol{\alpha}}_{33}(i\xi_j)+\overleftrightarrow{\boldsymbol{\alpha}}_{33}(-i\xi_j)]\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,i\xi_j)\}_{\boldsymbol{r}=\boldsymbol{r}_A}\\ &\quad-4n\mu_0\omega_0^2\,\mathrm{Re}\nabla\{\boldsymbol{d}_0\cdot\overleftrightarrow{\boldsymbol{G}}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)\cdot\boldsymbol{d}_0\}_{\boldsymbol{r}=\boldsymbol{r}_A}. \qquad (14b)\end{aligned}$$

The resonant parts as well as the dispersion parts of the forces are the same in terms of $\boldsymbol{F}_{11}(\boldsymbol{r}_A)$, $\boldsymbol{F}_{22}(\boldsymbol{r}_A)$, $\boldsymbol{F}_{12}(\boldsymbol{r}_A)$, and $\boldsymbol{F}_{21}(\boldsymbol{r}_A)$, revealing that thermal photons make the same contributions to these four elements. Especially, Eq. (14b) demonstrates that the resonant part of $\boldsymbol{F}_{33}(\boldsymbol{r}_A)$ completely originates from thermal photons, characterized by the mean thermal photon number n.

In the previous work [35], this model was discussed in the condition of $T = 0\,\mathrm{K}$. It was found that when the atom is prepared initially in the subradiant state $|\psi(t=0)\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$, it is immune from the CP force by virtue of the destructive quantum interference. In contrast, for the initial superradiant state $|\psi(t=0)\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$, due to the constructive quantum interference, the amplitude of the CP force is double enhanced and decays twice as fast than in the case of a two-level atom.

Here we extend the previous research to the finite-temperature case. When the atom is initially prepared in the subradiant state $|\psi(t=0)\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$, the initial density-matrix elements read

$$\begin{aligned}\sigma_{11}(0) &= \sigma_{22}(0) = -\sigma_{12}(0)\\ &= -\sigma_{21}(0) = 1/2; \quad \text{others are zero.} \qquad (15)\end{aligned}$$

Substituting them into Eqs. (10a)–(10c), these density-matrix elements remain constant because of the destructive interference. It reveals that the destructive interference effect leads to the trapping of the atom in the subradiant state regardless of thermal photons. In this case, the atom is immune from the CP force since

$$\begin{aligned}\boldsymbol{F}(\boldsymbol{r}_A,t) &= \sigma_{11}(t)\boldsymbol{F}_{11}(\boldsymbol{r}_A)+\sigma_{22}(t)\boldsymbol{F}_{22}(\boldsymbol{r}_A)+\sigma_{21}(t)\boldsymbol{F}_{12}(\boldsymbol{r}_A)\\ &\quad+\sigma_{12}(t)\boldsymbol{F}_{21}(\boldsymbol{r}_A)+\sigma_{33}(t)\boldsymbol{F}_{33}(\boldsymbol{r}_A)\\ &= [\sigma_{11}(t)+\sigma_{22}(t)+\sigma_{21}(t)+\sigma_{12}(t)]\boldsymbol{F}_{11}(\boldsymbol{r}_A) = 0. \qquad (16)\end{aligned}$$

It implies that although all the force elements $\boldsymbol{F}_{mn}(\boldsymbol{r}_A)$ are enhanced by thermal fluctuation photons, the net CP force

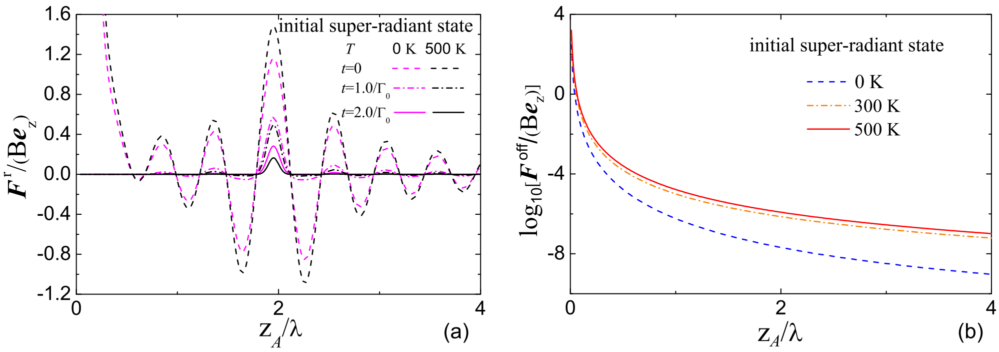

FIG. 2. (a) The resonant Casimir-Polder forces $\boldsymbol{F}^r$ on the ideal V-type atom as a function of the atomic position at different temperatures T and time t. The black and pink (gray) curves refer to temperatures $T = 500\,\mathrm{K}$ and 0 K, respectively. The dash, dash-dot, and solid curves refer to the evolution times $t = 0$, $1/\boldsymbol{\Gamma}_0$, and $2/\boldsymbol{\Gamma}_0$ respectively. (b) The dispersion Casimir-Polder forces $\boldsymbol{F}^{\text{off}}$ on this atom at the initial time as a function of the atomic position. The dashed (blue), dash-dot (orange), and solid (red) curves refer to the temperatures $T = 0\,\mathrm{K}, 300\,\mathrm{K}, 500\,\mathrm{K}$, respectively. The atom is prepared initially in the superradiant state. $\boldsymbol{e}_z$ denotes the unit vector in the z direction. $\boldsymbol{\Gamma}_0$ is the decay rate of a two-level atom with frequency ω_0 in free space.

still disappears due to their superposition. Thermal fluctuation photons thus do not have any impact on the net CP force as a result of the domination of the strong destructive quantum interference.

On the other hand, when the atom is prepared in the superradiant state initially, i.e., $|\psi(t=0)\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$, the corresponding initial density-matrix elements are given by

$$\sigma_{11}(0) = \sigma_{22}(0) = \sigma_{12}(0) = \sigma_{21}(0) = 1/2; \quad \text{others are zero.} \tag{17}$$

With the time evolution, the time-dependent density-matrix elements take the following expressions:

$$\begin{aligned}\sigma_{11}(t) &= \sigma_{22}(t) = \sigma_{12}(t) = \sigma_{21}(t)\\ &= (n+1)/(4n+2)e^{-(4n+2)\gamma t} + n/(4n+2),\\ \sigma_{33}(t) &= -(n+1)/(2n+1)e^{-(4n+2)\gamma t} + (n+1)/(2n+1).\end{aligned} \tag{18}$$

It reveals that the thermal photon number has a significant influence on the evolution of the atom. It can be seen from the exponential part that the prefactor of the decay rate is enhanced from 2 to $(4n+2)$. Thus in the finite-temperature case, thermal photons can accelerate the evolution of the CP force. When the atom evolves into the steady state, the two upper states still have populations arising from thermal fluctuations. By recalling Eq. (9), it might give us a hint that the net resonant CP force may not disappear in the steady state, since the time-dependent parts $\sigma_{ij}(\infty)$ do not vanish anymore. For comparison, it is generally recognized that in the zero-temperature case, the resonant force always vanishes in the steady state.

To analyze this strictly, we write the net CP force at an arbitrary time according to Eq. (9) as

$$\begin{aligned}\boldsymbol{F}(\boldsymbol{r}_A,t) = {}& 4\left[\frac{n+1}{4n+2}e^{-(4n+2)\gamma t} + \frac{n}{4n+2}\right]\boldsymbol{F}_{11}(\boldsymbol{r}_A)\\ &+ \left[-\frac{n+1}{2n+1}e^{-(4n+2)\gamma t} + \frac{n+1}{2n+1}\right]\boldsymbol{F}_{33}(\boldsymbol{r}_A).\end{aligned} \tag{19}$$

To better understand the effect of thermal photons on the CP force, we focus on the resonant force, since the dispersion (off-resonant) force can be neglected compared with the resonant force when the distance between the atom and the surface is larger than the atomic transition wavelength λ. To prove it, we assume that the atomic transition frequency is $\omega_0 = 1.0\times10^{14}$ rad/s, and the orientations of the dipoles are both along the x axis. By recalling Eq. (7), the thermal photon numbers at 300 and 500 K are 0.085 and 0.277, respectively. The parameters of the LHM slab are the same as the previous work [35] where the indices are $\varepsilon_A(\omega_0) = \mu_A(\omega_0) \approx -1.001 + 0.006i$, and the thickness is $d_A = 2\lambda$. Figures 2(a) and 2(b) illustrate the resonant force $\boldsymbol{F}^{\text{r}}$ and the dispersion (off-resonant) force $\boldsymbol{F}^{\text{off}}$ on this ideal V-type atom for different temperatures, respectively. The unit of force is $B = \mu_0|d_0|^2\omega_0^4/4\pi^2c^2$. As the temperature increases from 0 to 500 K, the dispersion force increases by nearly two orders of magnitude. This is in agreement with the previous results that thermal fluctuations can enhance the dispersion CP force [36]. However, the value of the dispersion force exhibits an exponential decrease as the distance between the atom and the slab increases. This makes the dispersion force negligible compared with the resonant force when we focus on the distance comparable to the atomic resonant wavelength λ. For convenience, we only focus on the resonant part of the CP force in the following.

According to Eq. (19), the resonant force on the atom at the initial time is given by

$$\boldsymbol{F}^{\text{r}}(\boldsymbol{r}_A,t=0) = 2\boldsymbol{F}_{11}^{\text{r}}(\boldsymbol{r}_A) = 2(n+1)\boldsymbol{F}_{\parallel}(\boldsymbol{r}_A). \tag{20}$$

Here $\boldsymbol{F}_{\parallel} = 2\mu_0\omega_0^2 d_0^2 \mathrm{Re}\nabla[G_{xx}^{(1)}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)]_{r=\boldsymbol{r}_A}$ is the resonant part of the CP force on the dipole parallel to the slab. It reveals that the thermal photons enhance the force at the initial time, changing the prefactor from 2 in the zero-temperature case to $2(n+1)$ in the finite-temperature case; cf. Fig. 2(a). Besides, Fig. 2(a) demonstrates that the forces exhibit a faster decrease as the atom evolves at higher temperatures. The reason is that thermal photons accelerate the decay of the upper states, and shorten the life of the CP force. When the atom evolves into the steady state ($t \to \infty$), the density-matrix elements given by Eq. (18) are reduced to $\sigma_{11} = \sigma_{22} = \sigma_{12} = \sigma_{21} = n/$

$(4n+2)$ and $\sigma_{33} = (n+1)/(2n+1)$. Substituting these elements into Eq. (9), we obtain an interesting result— the net resonant force in the steady state vanishes:

$$\boldsymbol{F}^{\mathrm{r}}(\boldsymbol{r}_A, t \to \infty) = 4\sigma_{11}(t)\boldsymbol{F}^{\mathrm{r}}_{11}(\boldsymbol{r}_A) + \sigma_{33}(t)\boldsymbol{F}^{\mathrm{r}}_{33}(\boldsymbol{r}_A) = 0. \quad (21)$$

Contrary to what we expected above, the superposition of all elements of the resonant forces leads to the vanishment of the net force. Strictly, it should be mentioned here that the disappearance of the resonant net CP force is a result based on the statistical averages.

As a conclusion, for the ideal V-type atom with two parallel transition dipoles, when the atom is initially prepared in the subradiant state, the destructive interference will override the effect of thermal fluctuation photons. As a result, the atom is immune from both the dispersion and resonant CP force, i.e., still a free atom. On the other hand, when the atom is initially prepared in the superradiant state, thermal photons will enhance the resonant force at the initial time. With the decay of the atom, the force at higher temperatures decreases faster by virtue of the contribution to the decay constant from thermal photons. However, when the atom evolves to the steady state, the net resonant force disappears even though the atom still has populations in the upper states.

B. Zeeman atom with two perpendicular dipoles, $\boldsymbol{d}_{13} \perp \boldsymbol{d}_{32}$

From the realistic perspective, the V-type atom with two perpendicular dipoles is more relevant to the experiments. Two transition dipoles with nearly the same frequency can be achieved through the Zeeman splitting of a two-level atom under an applied magnetic field. These two dipoles are perpendicular to each other naturally; i.e., one is left-rotating polarized and the other is right-rotating polarized. Such an atom is called a Zeeman atom, and its atomic dipole moments are represented by $\boldsymbol{d}_{13} = d_0\boldsymbol{\varepsilon}_+$ and $\boldsymbol{d}_{23} = d_0\boldsymbol{\varepsilon}_-$. Here, $\boldsymbol{\varepsilon}_+ = (\boldsymbol{e}_z + i\boldsymbol{e}_x)/\sqrt{2}$ and $\boldsymbol{\varepsilon}_- = (\boldsymbol{e}_z - i\boldsymbol{e}_x)/\sqrt{2}$ refer to the right-rotating and the left-rotating unit vectors, respectively.

As two transition dipoles are perpendicular to each other, there is no quantum interference in free space. However, Agarwal mentioned that the anisotropic environment can revive the quantum interference within the Zeeman atom [39]. In terms of the structure shown in Fig. 1(a), the anisotropy is embodied by the difference between the diagonal elements of the Green tensor, $G_{xx}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0)$ and $G_{zz}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0)$. Correspondingly, the decay rates and the collective damping rate are given by

$$\gamma_1 = \gamma_2 = \frac{\mu_0}{\hbar}\omega_0^2 d_0^2[\mathrm{Im}G_{xx}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0) + \mathrm{Im}G_{zz}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0)],$$

$$\kappa = \frac{\mu_0}{\hbar}\omega_0^2 d_0^2[\mathrm{Im}G_{zz}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0) - \mathrm{Im}G_{xx}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega_0)]. \quad (22)$$

In the anisotropic environment, the collective damping rate κ has a nonzero value arising from $G_{zz} \neq G_{xx}$, leading to the appearance of the quantum interference. By recalling Eqs. (12a)–(12d), the corresponding resonant CP forces are given by

$$\begin{aligned}\boldsymbol{F}^{\mathrm{r}}_{11}(\boldsymbol{r}_A) &= \boldsymbol{F}^{\mathrm{r}}_{22}(\boldsymbol{r}_A) = (n+1)\mu_0\omega_0^2 d_0^2 \mathrm{Re}\nabla \\ &\quad \times \big[G^{(1)}_{zz}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0) + G^{(1)}_{xx}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)\big]_{\boldsymbol{r}=\boldsymbol{r}_A} \\ &= (n+1)(\boldsymbol{F}_\perp + \boldsymbol{F}_{||})/2, \quad (23a)\\ \boldsymbol{F}^{\mathrm{r}}_{12}(\boldsymbol{r}_A) &= \boldsymbol{F}^{\mathrm{r}}_{21}(\boldsymbol{r}_A) = (n+1)\mu_0\omega_0^2 d_0^2 \mathrm{Re}\nabla \\ &\quad \times \big[G^{(1)}_{zz}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0) - G^{(1)}_{xx}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)\big]_{\boldsymbol{r}=\boldsymbol{r}_A} \\ &= (n+1)(\boldsymbol{F}_\perp - \boldsymbol{F}_{||})/2, \quad (23b)\\ \boldsymbol{F}^{\mathrm{r}}_{33}(\boldsymbol{r}_A) &= -2n\mu_0\omega_0^2 d_0^2 \mathrm{Re}\nabla\big[G^{(1)}_{zz}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0) \\ &\quad + G^{(1)}_{xx}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)\big]_{\boldsymbol{r}=\boldsymbol{r}_A} \\ &= -n(\boldsymbol{F}_\perp + \boldsymbol{F}_{||}). \quad (23c)\end{aligned}$$

Here $\boldsymbol{F}_\perp = 2\mu_0\omega_0^2 d_0^2 \mathrm{Re}\nabla[G^{(1)}_{zz}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)]_{\boldsymbol{r}=\boldsymbol{r}_A}$ is the resonant part of the CP force on the dipole $\boldsymbol{d}_0$ perpendicular to the surface, while $\boldsymbol{F}_{||} = 2\mu_0\omega_0^2 d_0^2 \mathrm{Re}\nabla[G^{(1)}_{xx}(\boldsymbol{r},\boldsymbol{r}_A,\omega_0)]_{\boldsymbol{r}=\boldsymbol{r}_A}$ is the one on the dipole parallel to the surface.

Now we consider the influence of thermal fluctuations on the amplitude as well as the evolution of the CP force. When the Zeeman atom is initially prepared in the subradiant state, the resonant CP force at the initial time is $\boldsymbol{F}^{\mathrm{r}}(\boldsymbol{r}_A, t = 0) = (n+1)\boldsymbol{F}^{\mathrm{r}}_{||}(\boldsymbol{r}_A)$ as found from Eqs. (9) and (23). It is equivalent to the resonant CP force acting on a two-level atom with the x-oriented dipole moment. The reason is that the destructive quantum interference cancels the force component of the z-oriented dipole [37]. On the other hand, in the case of the initial superradiant state, the initial resonant force is

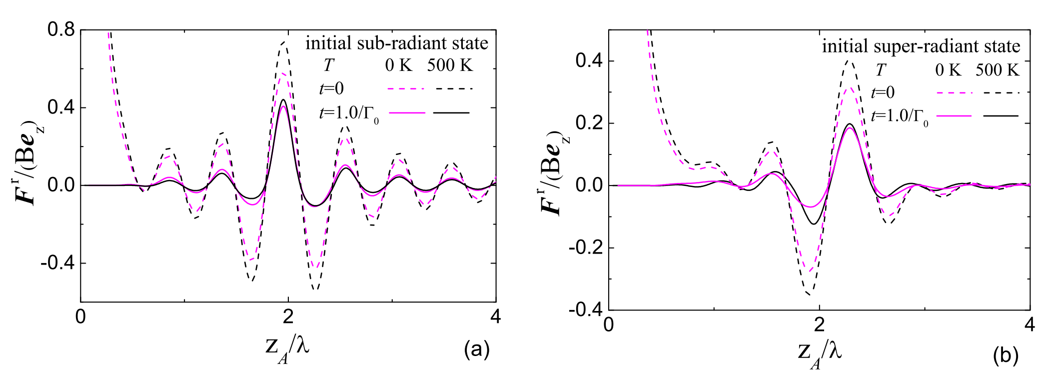

FIG. 3. The resonant Casimir-Polder force as a function of the atomic position at different temperatures and evolution time when the Zeeman atom is prepared initially in (a) the subradiant state and (b) the superradiant state. The black and pink (gray) curves refer to temperatures $T = 500$ K and 0 K, respectively. The solid and dashed curves refer to the evolution times $t = 0$ and $t = 1/\Gamma_0$, respectively.

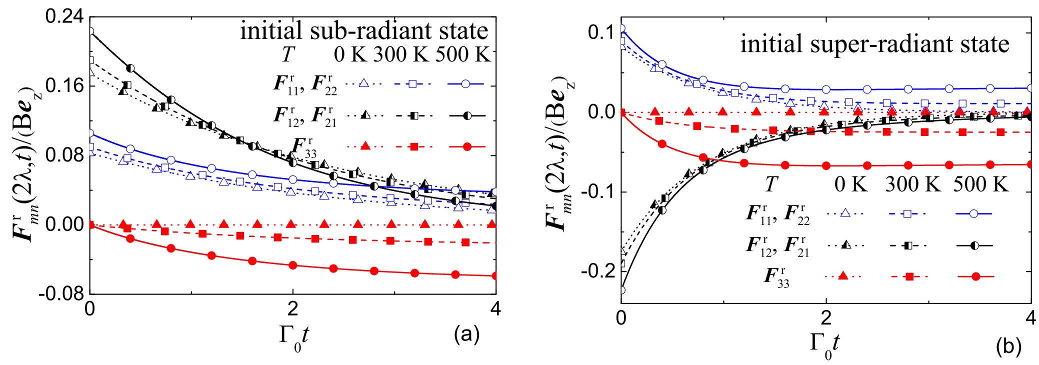

FIG. 4. The evolution of the resonant elements $\boldsymbol{F}^{\mathrm{r}}_{mn}(\boldsymbol{r}_A,t)$ on the Zeeman atom at different temperatures. (a) The Zeeman atom is prepared initially in the subradiant state. (b) The Zeeman atom is prepared initially in the superradiant state. The blue (red) curves with open (full) symbols refer to diagonal terms of $\boldsymbol{F}^{\mathrm{r}}_{11}$ and $\boldsymbol{F}^{\mathrm{r}}_{22}$ ($\boldsymbol{F}^{\mathrm{r}}_{33}$). The black curves with half-open symbols refer to the off-diagonal terms $\boldsymbol{F}^{\mathrm{r}}_{12}$ and $\boldsymbol{F}^{\mathrm{r}}_{21}$. The solid, dashed, and dotted curves refer to temperatures $T = 500\,\mathrm{K}$, $300\,\mathrm{K}$, $0\,\mathrm{K}$, respectively. The atom is placed at the focus point $z_A = 2\lambda$.

$\boldsymbol{F}^{\mathrm{r}}(\boldsymbol{r}_A,t=0) = (n+1)\boldsymbol{F}^{\mathrm{r}}_{\perp}(\boldsymbol{r}_A)$. It resembles the force on the single dipole perpendicular to the surface, arising from the constructive quantum interference that cancels the x-oriented dipole parts. Different from the case of the ideal V-type atom at the subradiant state, the net CP force on the Zeeman V-type atom at the initial time still exists owing to the incomplete quantum interference. Figures 3(a) and 3(b) demonstrate that thermal photons enhance the initial CP forces as the temperature T increases from 0 to 500 K for both of the initial states.

To explore the thermal effect on the evolution of the CP force thoroughly, it is of great importance to focus on all elements of the force. From Eq. (9), the element of the force is determined by the amplitude part $\boldsymbol{F}^{r}_{mn}(\boldsymbol{r}_A)$ and the evolution part $\sigma_{mn}(t)$; i.e., $\boldsymbol{F}^{r}_{mn}(\boldsymbol{r}_A,t) = \sigma_{mn}(t)\boldsymbol{F}^{r}_{mn}(\boldsymbol{r}_A)$. Thermal photons strengthen all time-independent amplitudes $\boldsymbol{F}^{r}_{mn}(\boldsymbol{r}_A)$ according to Eqs. (23a)–(23c). In Fig. 4 we plot the time evolutions of all force elements at different temperatures when the atom is located at the focal point nearby the LHM structure; i.e., $\boldsymbol{r}_A = (0,0,2\lambda)$. Note that since the excited and ground states represent two opposite processes of emitting and absorbing photons, the diagonal elements $\boldsymbol{F}^{r}_{11}$ and $\boldsymbol{F}^{r}_{22}$ (blue curves) are positive and repulsive while $\boldsymbol{F}^{r}_{33}$ (red curves) are negative and attractive. In addition, $\boldsymbol{F}^{r}_{11}$ and $\boldsymbol{F}^{r}_{22}$ decrease in absolute value while $\boldsymbol{F}^{r}_{33}$ increases in absolute value. The absolute values of $\boldsymbol{F}^{r}_{21}(\boldsymbol{r}_A,t)$ and $\boldsymbol{F}^{r}_{12}(\boldsymbol{r}_A,t)$ also decrease with time. In contrast to the zero-temperature case, the diagonal elements $\boldsymbol{F}^{r}_{11}$, $\boldsymbol{F}^{r}_{22}$, and $\boldsymbol{F}^{r}_{33}$ do not vanish when the atom decays into the steady state at finite temperatures, i.e., $T = 300\,\mathrm{K}$ and 500 K. This persistence of the diagonal elements arises from thermal photon fluctuations which lead to the weak excitation of the atom in the steady state. On the other hand, the off-diagonal terms $\boldsymbol{F}^{r}_{12}(\boldsymbol{r}_A,t)$ and $\boldsymbol{F}^{r}_{21}(\boldsymbol{r}_A,t)$ arise from the quantum interference, and their initial values are enhanced as the temperature rises from 0 to 500 K, shown in Figs. 4(a) and 4(b). In contrast to the diagonal terms, the off-diagonal terms vanish in the steady state by virtue of the decoherence between the two upper states in the V-type atom. In addition, Fig. 4(a) demonstrates that when the atom is initially prepared in the subradiant state, the off-diagonal terms exhibit faster decreases at higher temperatures due to the acceleration of the decoherence arising from the thermal photons.

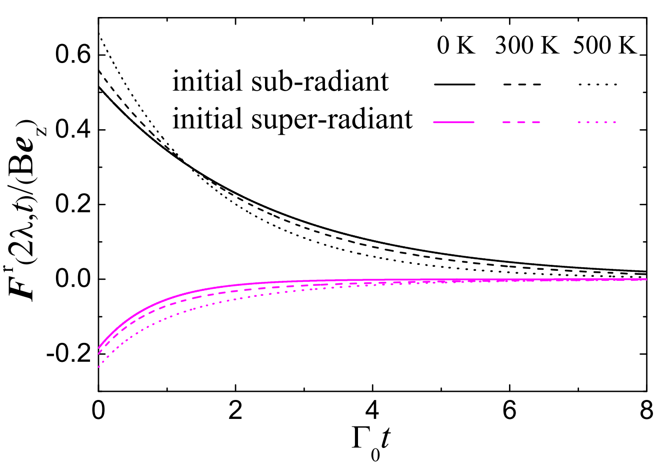

FIG. 5. The evolution of the resonant Casimir-Polder force on the Zeeman atom at different temperatures. The solid, dashed, and dotted curves refer to temperatures $T = 0\,\mathrm{K}, 300\,\mathrm{K}, 500\,\mathrm{K}$, respectively. The black and pink (gray) curves refer to the initial subradiant and the superradiant state, respectively. The Zeeman atom is placed at the focus point $z_A = 2\lambda$.

Now we consider the net resonant force in the steady state. By solving the master equations, we get the density-matrix elements in the steady state as $\sigma_{11}(\infty) = \sigma_{22}(\infty) = n/(3n+1)$, $\sigma_{12}(\infty) = \sigma_{21}(\infty) = 0$, and $\sigma_{33}(\infty) = (n+1)/(3n+1)$. Such a steady state is independent of the initial state. It is interesting to find that the net resonant CP force is $\boldsymbol{F}^{\mathrm{r}}(\boldsymbol{r}_A, t \to \infty) = 0$. As can be seen in Fig. 5, although thermal photons enhance the initial net CP forces by increasing the temperature, the net resonant CP forces all trend to zero in the steady state, regardless of the temperature and the initial state. As mentioned above, the disappearance of the resonant CP force in the steady state is a statistical-average result. From our previous work, in the zero-temperature case, the force on the atom prepared in the superradiant state decreases faster than that in the subradiant state owing to the quantum interference effect [34]. Here, by virtue of the existence of thermal photons, the force in the subradiant state exhibits a faster decrease as the temperature increases due to the faster decay of the off-diagonal elements as analyzed above. On the other hand, for the initial superradiant state, the force decreases more slowly as the temperature increases. It means that the thermal photons play an important role against the quantum interference in the evolution of the CP force on the Zeeman atom.

IV. CONCLUSION

In this paper, the thermal CP force of a V-type three-level atom is studied. We focus on the thermal effect on the amplitude and the evolution of the force on two versions of the atomic system. For the ideal degenerate V-type atom with two parallel dipoles, if the initial state is the subradiant state, there exists no force on the atom regardless of the temperature due to the complete quantum interference. On the other hand, for the initial superradiant state, as the temperature increases, the force is enhanced by thermal photons initially, and then decreases faster. Finally the net resonant force vanishes in the steady state. In terms of the Zeeman atom whose two transition dipoles are left rotation and right rotation, respectively, the CP forces on it exhibit different behavior from those on the ideal atom. For the initial subradiant state, the Zeeman atom is not immune from the CP force during the evolution because of the incomplete quantum interference. In addition, for both sub- and superradiant states, the resonant CP force on the atom at the initial time is enhanced by thermal photon fluctuations. When we look closely at all elements of the resonant CP forces, we find that the diagonal elements of the force still exist in the steady state, because there are tiny populations in the upper states due to thermal fluctuations, which is completely different from the zero-temperature case. However, the net resonant CP force disappears in the steady state regardless of the temperature, which means the second law of thermodynamics still works. Furthermore, it is known that the quantum interference can accelerate or decelerate the atomic evolution. Here we find that the thermal fluctuations always weaken the effect of the quantum interference on the Zeeman atom.

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