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Shortcuts to adiabaticity in the presence of a continuum: Applications to itinerant quantum state transfer

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We present a method for accelerating adiabatic protocols for systems involving a coupling to a continuum, one that cancels both nonadiabatic errors as well as errors due to dissipation. We focus on applications to a generic quantum state transfer problem, where the goal is to transfer a state between a single level or mode, and a propagating temporal mode in a waveguide or transmission line. Our approach enables perfect adiabatic transfer protocols in this setup, despite a finite protocol speed and a finite waveguide coupling. Our approach even works in highly constrained settings, where there is only a single time-dependent control field available.

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Introduction. Adiabatic quantum evolution provides an efficient and robust way to implement a variety of important quantum operations including state transfer [1–7], state preparation [8–11], and even quantum logic gates [12–14]. While such protocols are robust against timing errors, they are necessarily slow, making them vulnerable to dissipation or fluctuations. There is thus considerable interest in finding ways to accelerate adiabatic protocols, such that fast evolution is possible without significant nonadiabatic errors [15–19]. These techniques are generally referred to as "shortcuts to adiabaticity" (STA), and involve modifying control fields to suppress the net effect of nonadiabatic errors [20–25]. Recent experiments have successfully implemented versions of these strategies [26–30].

A key drawback of the transitionless driving strategy and its higher-order variants [20–25] is that they require the exact diagonalization of a time-dependent Hamiltonian, making them unwieldy for systems with many degrees of freedom. They are thus unsuitable for an important class of quantum state transfer problems, where the goal is to transfer an initial state in a localized system having discrete energy levels to a propagating state in a continuum such as a waveguide or a transmission line (see, e.g., Refs. [31–33]).

In this Rapid Communication, we present a general method for applying STA to the above class of problems. The method is based on first deriving an effective non-Hermitian Hamiltonian, and then constructing dressed states and modified control sequences that suppress *both* nonadiabatic errors (due to finite protocol speed) and "dissipative" errors (due to the coupling to the continuum). We apply our technique to two ubiquitous quantum state transfer problems based on stimulated Raman adiabatic passage (STIRAP) [2]. Such protocols have been discussed in systems ranging from atomic cavity QED setups [31,32] to optomechanics [34]. Remarkably, we show that our method works even in the highly constrained protocol introduced by Duan *et al.* [32], where there is only a single time-dependent control field in the Hamiltonian.

Our work represents a substantial advance over previous work using STA to accelerate adiabatic state transfer [20–25], as these works did not include a coupling to a continuum. It also differs significantly from studies exploring STIRAP-style state transfer to a continuum [35–38], as these did not consider any kind of STA correction. References [39–41] applied STA techniques to phenomenological non-Hermitian Hamiltonians, but in a very different context from the work presented here. In particular, Ref. [39] requires both the diagonalization of a non-Hermitian Hamiltonian, and the use of non-Hermitian control fields. In contrast, the approach that we develop in this Rapid Communication leads to simple modifications of the original pulse sequence, without requiring additional non-Hermitian controls which would be very challenging to experimentally implement.

System. While our approach can be applied to a wide variety of adiabatic protocols [see, e.g., Figs. 1(b) and 1(c) and Ref. [43]], for concreteness we focus on the generic state transfer problem depicted in Fig. 1(a), where three discrete levels A, B, and C are coupled in a Λ-system configuration, with the C state additionally coupled to a waveguide. The goal is to convert an initial state $|A\rangle$ to a propagating mode in the waveguide. The system has two time-dependent couplings $G_1(t), G_2(t)$, and is described by the Hamiltonian $\hat{H} = \hat{H}_0(t) + \hat{H}_{\text{int}} + \hat{H}_{\text{WG}}$, with ($\hbar = 1$):

$$
\begin{aligned}
\hat{H}_0(t) &= G_1(t)|A\rangle\langle B| + G_2(t)|C\rangle\langle B| + \text{H.c.},\\
\hat{H}_{\text{int}} &= \sqrt{\frac{\kappa}{2\pi}} \int_{-\omega_{\max}/2}^{\omega_{\max}/2} d\omega [|C\rangle\langle D_\omega| + |D_\omega\rangle\langle C|],\\
\hat{H}_{\text{WG}} &= \int_{-\omega_{\max}/2}^{\omega_{\max}/2} d\omega\ \omega\, |D_\omega\rangle\langle D_\omega|.
\end{aligned} \tag{1}
$$

The states in the continuum are defined as $|D_\omega\rangle = \hat{c}^\dagger(\omega)|\text{vac}\rangle$, where $\hat{c}(\omega)$ is the photon annihilation operator of a mode at frequency ω in the waveguide, obeying the commutation relation $[\hat{c}(\omega), \hat{c}^\dagger(\omega')] = \delta(\omega - \omega')$, and $|\text{vac}\rangle$ is the vacuum of the whole system. We consider a waveguide with a finite bandwidth $\omega_{\max}$, and also that the amplitude of the interaction between the state $|C\rangle$ and the waveguide states $|D_\omega\rangle$ is frequency independent [$\kappa(\omega) = \kappa, \forall\, |\omega| \leqslant \omega_{\max}/2$]. As is standard, we will consider the Markovian regime throughout this work, where the waveguide bandwidth is much larger than any other frequency scale in the problem. As such, we can take $\omega_{\max} \to \infty$.

The above model corresponds to the basic setup described in Refs. [31,32]; we will consider both the case where $G_1(t)$ and $G_2(t)$ are independently tunable, and the more constrained situation where *only* $G_1(t)$ is tunable. Note that our results will also immediately apply to the model where the discrete levels A, B, C are replaced by bosonic modes, as is the situation in optomechanical state transfer problems [5–7]. In this case, our

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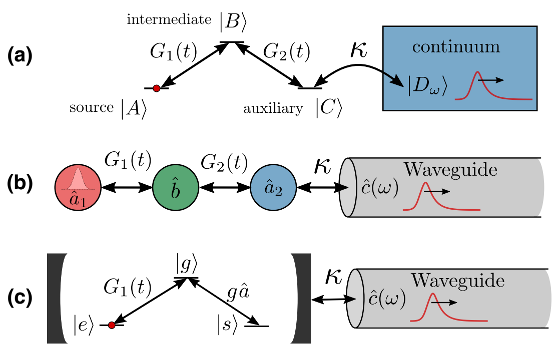

FIG. 1. (a) Three-level Λ system with time-dependent couplings $G_1(t), G_2(t)$. The level $|C\rangle$ is coupled (rate κ) to a waveguide. The goal is to perform a STIRAP-style state transfer to adiabatically transfer state $|A\rangle$ to a propagating mode in the waveguide. (b) System of three bosonic modes coupled in a Λ configuration, as can be realized in optomechanics [42]. $\hat{a}_1$ and $\hat{a}_2$ are photonic modes, $\hat{b}$ is a mechanical mode, and $G_j(t)$ represent many-photon optomechanical couplings. In the single-excitation subspace, this system is completely equivalent to (a). The correspondence also holds for a general initial state due to the linearity of the dynamics [43]. (c) Realization of (a) using the setup introduced in Refs. [31,32], where a three-level system is placed in a cavity (the cavity mode annihilation operator is denoted by $\hat{a}$), which is in turn coupled to a waveguide. Here, there is only a single time-dependent control field [$G_2(t) = g$ is time independent].

protocol can be used to transfer an arbitrary A-mode state to the state of a propagating wave packet in the continuum [43].

The starting point for our accelerated adiabatic protocols is the basic STIRAP approach for moving population from A to C in the case where $\kappa = 0$ [2]. This protocol uses the fact that $\hat{H}_0(t)$ has two instantaneous eigenstates $|\pm(t)\rangle$ of energy $\pm G_0(t)$ (the bright states [43]) and a zero-energy instantaneous eigenstate (the dark state),

$$|\mathrm{dk}(t)\rangle = \cos\theta(t)|A\rangle - \sin\theta(t)|C\rangle, \tag{2}$$

where we have parametrized the control fields as $G_1(t) = G_0(t)\sin\theta(t)$ and $G_2(t) = G_0(t)\cos\theta(t)$. The "dark state" has zero overlap with $|B\rangle$. Standard STIRAP [2] works by evolving $\theta(t)$ continuously from 0 to $\pi/2$, such that $|\mathrm{dk}(t)\rangle$ changes continuously from being $|A\rangle$ at the initial time, to being $|C\rangle$ at the final time. If this is done slowly enough compared to the gap $G_0(t)$ separating $|\mathrm{dk}(t)\rangle$ from the "bright" adiabatic eigenstates $|\pm(t)\rangle$, the system will remain in $|\mathrm{dk}(t)\rangle$ at all times, thus effecting the desired transfer.

For κ nonzero, we could again imagine a STIRAP-like protocol, where the dark state changes adiabatically from being localized in A to C. As C is now coupled to the waveguide, in the ideal case the excitation will be transferred to a propagating waveguide excitation. This dark-state approach for stationary to itinerant state transfer has been discussed in numerous works [31,32,34].

Accelerated STIRAP with dissipation. While the above approach has many advantages, any finite speed will lead to nonadiabatic errors which disrupt this transfer. Reference [25] presented a dressed-state approach for mitigating this problem in the case where $\kappa = 0$. In our case, the coupling to the waveguide will create additional errors. We show here how these can also be mitigated by using a dressed-state approach.

We start by writing the solution to the Schrödinger equation (in the original laboratory frame) in the form

$$|\psi(t)\rangle = u_A(t)|A\rangle + u_B(t)|B\rangle + u_C(t)|C\rangle + \int_{-\infty}^{+\infty} d\omega\, u_{\mathrm{WG}}(\omega,t)|D_\omega\rangle. \tag{3}$$

One can next solve the linear equations of motion for the waveguide mode amplitudes $u_{\mathrm{WG}}(\omega,t)$ [43], and use these to simplify the equations for the remaining amplitudes. By assuming that there are no excitations in the waveguide at the initial time t_i, one finds that the equations of motion of the remaining amplitudes correspond to a Schrödinger equation for the effective non-Hermitian Hamiltonian $\hat{H}_1(t) = \hat{H}_0(t) - i(\kappa/2)|C\rangle\langle C|$. This Hamiltonian possesses a special structure: Its Hermitian part [$\hat{H}_0(t)$] possesses a set of adiabatic eigenstates (the dark and bright states), whose existence will allow us to construct and correct a useful state transfer protocol. While this structure is of course not generic to an arbitrary problem with a continuum, it is sufficiently general to apply to many situations of interest.

We next transform to the adiabatic frame [via a time-dependent unitary $\hat{U}_{\mathrm{ad}}(t) = \sum_{k=\pm,\mathrm{dk}} |k(t)\rangle\langle k|$], in which the adiabatic eigenstates of $\hat{H}_0(t)$ have no explicit time dependence. This involves diagonalizing the three-dimensional Hamiltonian $\hat{H}_0(t)$ (and not the full, infinite-dimensional Hamiltonian $\hat{H}$). In this frame, our effective non-Hermitian Hamiltonian takes the form

$$\begin{aligned}\hat{H}_{1,\mathrm{ad}}(t) &= \hat{U}_{\mathrm{ad}}^\dagger(t)\hat{H}_1(t)\hat{U}_{\mathrm{ad}}(t) - i\hat{U}_{\mathrm{ad}}^\dagger(t)\frac{d}{dt}\hat{U}_{\mathrm{ad}}(t)\\ &= G_0(t)(|+\rangle\langle +| - |-\rangle\langle -|) - i\frac{\kappa}{2}\sin^2\theta(t)|\mathrm{dk}\rangle\langle \mathrm{dk}|\\ &\quad - i\frac{\kappa}{2}\cos^2\theta(t)\frac{|+\rangle + |-\rangle}{\sqrt{2}}\,\frac{\langle +| + \langle -|}{\sqrt{2}}\\ &\quad - i\left(\dot{\theta}(t) + \frac{\kappa}{4}\sin[2\theta(t)]\right)\frac{|+\rangle + |-\rangle}{\sqrt{2}}\langle \mathrm{dk}|\\ &\quad - i\left(-\dot{\theta}(t) + \frac{\kappa}{4}\sin[2\theta(t)]\right)|\mathrm{dk}\rangle\frac{\langle +| + \langle -|}{\sqrt{2}}.\end{aligned} \tag{4}$$

The diagonal terms in the second line describe the desired evolution: There is no mixing of the adiabatic eigenstates, and the decay of the dark state corresponds to the desired emission into the waveguide. The remaining off-diagonal terms describe imperfections. In particular, both the dissipation ($\kappa \neq 0$) and the finite protocol speed ($\dot{\theta} \neq 0$) cause a deleterious mixing of adiabatic eigenstates. This implies that, while the deleterious nonadiabatic effects can be arbitrarily reduced by slowing down the protocol, the deleterious effect of the dissipation will still be a problem in this regime.

To design improved pulses that overcome these limitations, we extend the dressed-state approach introduced in Ref. [25]. One first constructs a "dressed" dark state $|\widetilde{\mathrm{dk}}(t)\rangle \equiv \hat{V}(t)|\mathrm{dk}\rangle$ that coincides with the original dressed state at the initial and final protocol time i.e., [$\hat{V}(t_i) = \hat{V}(t_f) = \mathbb{1}$]. $\hat{V}(t)$ here is the unitary operator which defines the dressing (in the adiabatic frame). Second, one modifies the control pulses $G_1(t), G_2(t)$ such that the dynamics never causes transitions between the dressed dark state and the other two dressed adiabatic eigenstates $|\widetilde{\pm}(t)\rangle \equiv \hat{V}(t)|\pm\rangle$. We describe this modification of the control pulses by an added control Hamiltonian $\hat{H}_{\rm cor}(t)$, such that the original Hamiltonian is modified as $\hat{H}_0(t) \to \hat{H}_0(t) + \hat{H}_{\rm cor}(t)$ (in the laboratory frame).

Formally, the above "no transitions" requirement is best formulated by writing the effective non-Hermitian Hamiltonian $\hat{H}_{1,\rm ad}(t)$ in the frame where the dressed states $\hat{V}(t)|k\rangle$ ($k = +,-,$dk) have no explicit time dependence. This transformed Hamiltonian is given by

$$\hat{H}_{1,\rm dsb}(t) = \hat{V}^\dagger(t)[\hat{H}_{1,\rm ad}(t) + \hat{U}^\dagger_{\rm ad}(t)\hat{H}_{\rm cor}(t)\hat{U}_{\rm ad}(t)]\hat{V}(t) - i\hat{V}^\dagger(t)\frac{d}{dt}\hat{V}(t). \tag{5}$$

The requirement that the dynamics does not cause transitions out of the dressed dark state then becomes

$$\langle\widetilde{+}|\hat{H}_{1,\rm dsb}(t)|\widetilde{\rm dk}\rangle = \langle\widetilde{-}|\hat{H}_{1,\rm dsb}(t)|\widetilde{\rm dk}\rangle = 0. \tag{6}$$

Note that as $\hat{H}_{1,\rm dsb}(t)$ is non-Hermitian, fulfilling the above condition does not also imply $\langle\widetilde{\rm dk}|\hat{H}_{1,\rm dsb}(t)|\widetilde{\pm}\rangle = 0$. This is not a concern, as our initial condition (i.e., we start in the dark state) means that only the matrix elements in Eq. (6) are of relevance.

In order to implement the above strategy, we take a dressing operator

$$\hat{V}(t) = \exp\left[i\mu(t)\left(\frac{|+\rangle - |-\rangle}{\sqrt{2}}\langle{\rm dk}| + {\rm H.c.}\right)\right]. \tag{7}$$

Here, $\mu(t)$ parametrizes the strength of the dressing at time t. The fact that the dressing must turn off at the initial and final times implies that $\mu(t)$ must tend to zero at the start and end of the protocol.

We also parametrize the added correction Hamiltonian via two functions $g_x(t)$ and $g_z(t)$,

$$\hat{H}_{\rm cor}(t) = \hat{U}_{\rm ad}(t)\left[g_x(t)\left(\frac{|+\rangle - |-\rangle}{\sqrt{2}}\langle{\rm dk}| + {\rm H.c.}\right) + g_z(t)(|+\rangle\langle+| - |-\rangle\langle-|)\right]\hat{U}^\dagger_{\rm ad}(t), \tag{8}$$

which leads modifications of the pulses $G_1(t)$ and $G_2(t)$,

$$G_{1,\rm cor}(t) = G_1(t) - g_x(t)\cos\theta(t) + g_z(t)\sin\theta(t),$$
$$G_{2,\rm cor}(t) = G_2(t) + g_x(t)\sin\theta(t) + g_z(t)\cos\theta(t). \tag{9}$$

With these definitions in hand, we can now constrain the dressing and modified control pulses so that they fulfill Eq. (6), the condition which prevents transitions out of the dressed dark state (either by nonadiabatic errors, or by dissipation). One finds [43]

$$g_x(t) = -\dot{\mu}(t) + \frac{\kappa}{4}\sin^2[\theta(t)]\sin[2\mu(t)], \tag{10}$$

$$g_z(t) = \frac{1}{\tan\mu(t)}\left(\dot{\theta}(t) + \frac{\kappa}{4}\sin[2\theta(t)]\right) - G_0(t). \tag{11}$$

We thus have an infinite number of corrected protocols that can yield a perfect fidelity despite nonzero κ and $\dot{\theta}$: For any possible dressing function $\mu(t)$ that starts and ends at zero, one simply needs to use modified control pulses that satisfy Eqs. (10) and (11).

Accelerated protocol in the presence of dissipation. In the case where both $G_1(t)$ and $G_2(t)$ are controllable, one can find a simple correction by choosing the dressing strength $\mu(t)$ so that the control-correction $g_z(t) = 0$. Using Eq. (11), we obtain easily

$$\mu(t) = \arctan\left[\frac{\dot{\theta}(t) + (\kappa/4)\sin[2\theta(t)]}{G_0(t)}\right]. \tag{12}$$

Recall that for STIRAP, $\theta(t)$ varies from 0 to $\pi/2$ during the protocol; hence, the above $\mu(t)$ is guaranteed to vanish at the start and end of the protocol (as required) as long as the original uncorrected protocol is sufficiently smooth. With $\mu(t)$ determined, the needed modification of the control pulses is given immediately by Eqs. (10) and (9).

For $\kappa = 0$, the dressed states defined by this choice of $\mu(t)$ correspond to the instantaneous eigenstates of the adiabatic Hamiltonian $\hat{H}_{1,\rm ad}$ (the so-called superadiabatic states). The corresponding corrected pulse sequence then coincides with that described in Refs. [25,29] and is termed superadiabatic transitionless driving (SATD). With nonzero κ, we see that *both* the choices of dressed states and control fields are modified [via the second term in Eq. (12)]. This modification ensures that irrespective of the size of κ, we can still have a perfect state transfer from $|A\rangle$ to a propagating temporal mode in the waveguide. We term this correction scheme "SATD+κ."

With the correction implemented, the dynamics is easy to describe. One prepares the system in $|A\rangle$ at the initial time t_i, which coincides with the dressed dark state, $|A\rangle = |\widetilde{\rm dk}(t_i)\rangle$. At $t > t_i$, the correction ensures that the system only has amplitude to be in the dressed dark state $|\widetilde{\rm dk}(t)\rangle$ or in the waveguide; the remaining dressed states $|\widetilde{\pm}(t)\rangle$ are never occupied. Defining $\tilde{u}_{\rm dk}(t) = \langle\widetilde{\rm dk}(t)|\psi(t)\rangle$, one obtains

$$|\tilde{u}_{\rm dk}(t)|^2 = \exp\left[-\int_{t_i}^t dt'\,\kappa_{\rm eff}(t')\right], \tag{13}$$

where $\kappa_{\rm eff}(t') = \frac{\kappa}{2}\sin^2[\theta(t')]\cos^2[\mu(t')]$. The physics is thus that the dressed dark state simply leaks directly into the waveguide at an effective instantaneous rate $\kappa_{\rm eff}(t)$. The fidelity of the state transfer operation at time t, $F(t)$, can then be defined as the probability of having the initial excitation in the waveguide, i.e.,

$$F(t) = \int d\omega |u_{\rm WG}(\omega,t)|^2 = \kappa\int_{t_i}^t d\tau |u_C(\tau)|^2, \tag{14}$$

where in the last equality we made use of the expression of $u_{\rm WG}(\omega,t)$ in the Markovian limit [43]. A full transfer to the waveguide will thus necessarily require a total protocol time $t_{\rm tot} > 1/\kappa$. There is, however, no additional constraint on the size of the adiabatic gap $G_0(t)$ relative either to protocol time *or* the size of the dissipation.

It is also interesting to ask about the temporal mode shape $f(t)$ of the state produced in the waveguide. This is defined via the amplitude $u_{\rm WG}(\omega,t)$ [cf. Eq. (3)] at the end of the protocol, and is completely determined by the time-dependent amplitude associated with $|C\rangle$, $u_C(t)$,

$$f(t) = \lim_{T\to\infty}\int_{-\infty}^{+\infty}\frac{d\omega}{\sqrt{2\pi}}e^{-i\omega(t-T)}u_{\rm WG}(\omega,T) = -i\sqrt{\kappa}u_C(t). \tag{15}$$

Finally, the perfect fidelity possible with the corrected protocol does come with a price: The use of a dressed dark state means that at intermediate times, the level $|B\rangle$ will have a nonzero occupancy given by

$$|u_B(t)|^2 = \sin^2[\mu(t)] \exp\left[-\int_{t_i}^{t} dt' \, \kappa_{\text{eff}}(t')\right]. \qquad (16)$$

As the dressing strength $\mu(t)$ is proportional to $\dot{\theta}(t)$ [see Eq. (12)], the faster the protocol speed, the greater is the population of $|B\rangle$ at intermediate times.

To demonstrate the utility of SATD+κ, we use it to correct the optimal STIRAP pulses discussed by Vitanov *et al.* in Ref. [44]. They are defined by $G_0(t) = G_0$ and $\theta(t) = \pi/[2(1+e^{-\nu t})]$, and only turn on and off asymptotically as $t \to \pm\infty$. To mimic a realistic experiment, we truncate the pulses to a finite time interval $-t_i = t_f \simeq 7.4/\nu$, which ensures $G_1(t_i) = G_2(t_f) = 10^{-3}G_0$. Figure 2(a) shows the asymptotic behavior of fidelity $\lim_{t\to\infty} F(t)$ for this protocol versus the protocol speed ν, with comparisons against both our SATD+κ correction, and the $\kappa = 0$ correction. The SATD+κ correction yields a several orders of magnitude improvement. Note that the only reason it fails to be perfect is due to constraining the pulses to a finite time interval. Moreover, even when we include incoherent decay on the intermediate level $|B\rangle$ [by adding a non-Hermitian term $-i(\Gamma/2)|B\rangle\langle B|$ to $\hat{H}_1(t)$] at a rate $\Gamma = 10^{-3}G_0$ (diamonds), the SATD+κ correction still yields a several orders of magnitude improvement. Figures 2(b) and 2(c) show the form of the corrected pulse sequences, while the inset of Fig. 2(a) shows the final outgoing temporal mode when using the SATD+κ correction.

Accelerated STIRAP using a single control field. Adiabatic state transfer to a waveguide is also possible in systems where $G_2(t) = g$ is a fixed constant, and only $G_1(t)$ is controllable [e.g., Fig. 1(b)] [31,32]. The SATD+κ approach for correcting errors is no longer viable, as it requires both $G_1(t)$ and $G_2(t)$ to be time dependent. Nonetheless, by using an alternate form of dressing, we can still obtain a perfectly corrected protocol in this more constrained setting.

When $G_2 = g$ is constant, the uncorrected adiabatic transfer protocol here involves slowly ramping $G_1(t)$ up from zero until it is $\gg g$ at a time $t \sim t_{\text{mid}}$, so that the adiabatic dark state is just $|C\rangle$. One then waits for a time $\sim t_0 > 1/\kappa$ for the state to decay to the waveguide, and then ramps $G_1(t)$ back down to zero [31,32]. A simple pulse shape that accomplishes this is cf. Fig. 3(b)

$$G_1(t) = \frac{G_{\max}}{2}(\tanh[\nu t] - \tanh[\nu(t-t_0)]). \qquad (17)$$

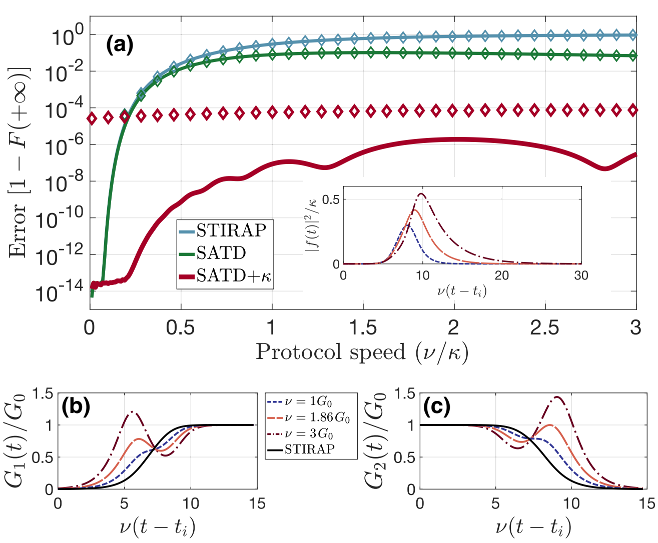

FIG. 2. SATD+κ correction for STIRAP-style state transfer to a waveguide based on the optimal STIRAP pulses described in the text. We take κ to be equal to the adiabatic gap G_0, a regime in which dissipative errors are large. (a) Asymptotic fidelity $\lim_{t\to\infty} F(t)$ as a function of protocol speed ν for the uncorrected STIRAP protocol (light blue top line), the SATD protocol (green middle line), and the SATD+κ protocol (thick red bottom line). The incoherent decay rate of the middle $|B\rangle$ level is either $\Gamma = 0$ (solid curves) or $\Gamma = 10^{-3}\kappa$ (diamonds whose positions correspond to those of the solid lines). For $\Gamma = 0$, the fidelity error of the SATD+κ protocol is only limited by our truncation of the pulses: The initial and final time have been chosen such that $G_1(t_i) = G_2(t_f) = 10^{-3}G_0$. Inset: Shape of the emitted temporal mode in the waveguide when using the corrected pulse sequence [same labeling as in (b)]. (b), (c) Time dependence of uncorrected and corrected pulse amplitudes $G_1(t)/G_0$ and $G_2(t)/G_0$ during the protocol.

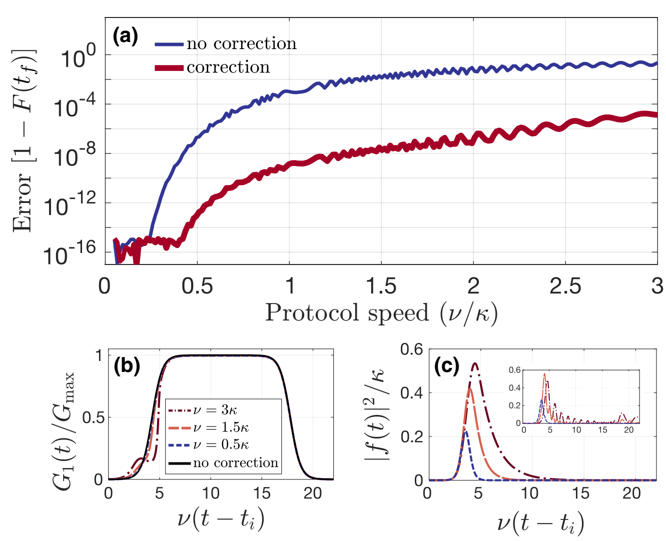

FIG. 3. STA correction for constrained STIRAP-style state transfer, where $G_2(t) = g$ at all times. Corrections are based on the adiabatic control pulse given in Eq. (17); we set $g = 6\kappa$, and $G_{\max} = 30\kappa$. (a) Fidelity error at the end of the protocol as a function of protocol speed ν for the uncorrected and corrected protocols. The correction yields $\sim$6 orders of magnitude improvement for a wide range of protocol speeds. The fidelity error of the corrected protocol is only limited by our truncation of the control pulses [initial and final time have been chosen such that $G_1(t_i) = G_1(t_f) = 10^{-3}g$], and the finite amplitude $G_{\max}$ of the pulse at intermediate time. (b) Evolution of the control field $G_1(t)$ during the protocol, without (solid line) and with (dashed and dashed-dotted lines) correction; the legend indicates the value of protocol speed ν. (c) Shape of the emitted temporal mode in the waveguide when using the corrected pulse (the same quantity has been represented in the inset for the noncorrected pulse); the curves are for different values of ν [same labeling as in (b)].

The rate ν here sets both the rate of the initial ramp and the time $t_{\rm mid}$, and t_0 sets the delay between the turn-on and turn-off phases. This pulse would give a perfect transfer in the limit $G_{\rm max} \gg \nu,\kappa,g$.

Our goal is to make the above protocol perfect even when nonadiabatic and dissipative effects are important, i.e., when $\nu/G_{\rm max},\kappa/G_{\rm max}$ are finite. We start by insisting that our correction does not modify the amplitude $G_2(t) = g$ [i.e. $G_{2,\rm cor}(t) = g$], which implies [cf. Eq. (9)] $g_x(t)\sin\theta(t) + g_z(t)\cos\theta(t) = 0$. Using this constraint in Eqs. (10) and (11) results in a differential equation for the dressing amplitude $\mu(t)$,

$$\dot{\mu}(t)\sin\theta(t)\sin\mu(t) = \dot{\theta}(t)\cos\theta(t)\cos\mu(t) - g\sin\mu(t) + \frac{\kappa}{2}\sin\theta(t)\cos\mu(t)[1-\sin^2\theta(t)\cos^2\mu(t)]. \tag{18}$$

Finding a pulse that corrects for nonadiabatic and dissipative errors thus requires solving Eq. (18) with the boundary condition $\mu(t_i) = 0$. This, however, is not enough: We also require that the dressing strength $\mu(t)$ vanish in the middle of the protocol (i.e., $t \sim t_{\rm mid}$), so that the dressed dark state is just $|C\rangle$ and can decay fully into the waveguide. *A priori*, there is no guarantee that, in general, the solution of Eq. (18) [with $\mu(t_i) = 0$] fulfills this condition.

Serendipitously, for the uncorrected pulse sequence in Eq. (17), we find via an explicit numerical integration of Eq. (18) that the dressing $\mu(t)$ does indeed almost completely turn off in the middle of the protocol as desired [43]. We use Eqs. (18) and (9) to find the corrected pulse $G_{1,\rm cor}(t)$ on the interval $(-\infty,t_0/2)$ [43]. For $t > t_0/2$, the transfer is effectively complete, and it does not matter how we turn off the pulse i.e., there is no need to correct [$G_1(t)$]. We thus have the pulse turn off exactly the same way as the uncorrected pulse, i.e., $G_{1,\rm cor}(t) = AG_1(t)$ (where the constant A is chosen to ensure continuity).

Figure 3 shows corrected pulses and fidelity improvements resulting from this approach. We use finite initial and final times, chosen so that $G_1(t_i) = G_1(t_f) = 10^{-3}g$, and also pick the delay time $t_0 = -2t_i + 5/\nu$ to scale with $1/\nu$; the result is that the total pulse duration scales inversely with the speed parameter ν. Figure 3(a) demonstrates an impressive six orders of magnitude suppression of the fidelity error in regimes where both adiabatic and dissipative errors contribute equally. Note that for extremely fast pulses $\nu \gg \kappa$, both corrected and uncorrected protocols are limited by there not being enough time for the state to decay to the waveguide. Figure 3(b) demonstrates that the correction to the pulses is extremely simple, corresponding to a simple "wiggle" being added during the turn-on phase.

Finally, our correction also has the benefit of resulting in extremely simple and smooth temporal mode shapes. Figure 3(c) shows the temporal mode shapes resulting from the corrected protocol, while the inset shows the mode shapes obtained in the original, uncorrected protocol. The fast oscillations here (which are absent when one uses the correction) would make subsequent "catch" operations extremely difficult.

Conclusions. We have presented a general strategy for using STA techniques to accelerate adiabatic processes for systems which include an infinite-dimensional continuum. Focusing on the problem of adiabatic state transfer between a discrete system and a waveguide, our technique allows one to both accelerate standard STIRAP-style adiabatic approaches *and* completely counteract dissipative errors generated by the coupling to the continuum. The application of this method on two experimentally relevant situations shows an improvement of the fidelity by several orders of magnitude, even when the intermediate level is subject to damping. In the future, this technique could be generalized to describe more general many-body systems, where part of the system could be modeled as an effective continuum.

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