

Dispersive radiation and regime switching of oscillating bound solitons in twin-core fibers near zero-dispersion wavelength

Ivan Oreshnikov,^{1,*} Rodislav Driben,² and Alexey Yulin¹

¹*ITMO University, Kronverskiy pr. 49, St. Petersburg, 197101 Russian Federation*

²*Department of Physics and CeOPP, University of Paderborn, Warburger St. 100, D-33098 Paderborn, Germany*

(Received 12 May 2017; published 7 July 2017)

We study resonant radiation generated by bound solitons in a twin-core fiber near a zero-dispersion wavelength, in the presence of higher order dispersion terms. We propose a theoretical description of a dispersive wave generation mechanism and derive resonance conditions. The presence of a third order dispersion term leads to generation of polychromatic dispersive radiation and transition from the regime of center-of-mass oscillations to the regime of amplitude oscillations. Such a transition is not reproduced in the case of symmetric fourth order dispersion.

DOI: [10.1103/PhysRevA.96.013809](https://doi.org/10.1103/PhysRevA.96.013809)

I. INTRODUCTION

Radiative effects of solitary waves propagating in nonlinear waveguides with high order dispersion have been attracting attention for a long time [1]. This has a twofold motivation. On one hand this problem is interesting from the point of view of fundamentals of nonlinear waves dynamics. On the other hand this research has practical importance. Optical supercontinuum generation is one of the most prominent examples of practical application of solitary waves [2].

In an optics context one of the most interesting phenomena is Cherenkov emission of dispersive wave by solitons propagating in nonlinear fibers with high order dispersion. This effect was reported in 1986 [3] and described in detail in Ref. [4]. A similar effect of transitional radiation of solitons moving in periodical systems was considered in Ref. [5].

Interest in the interaction between the dispersive waves and optical solitons was revived when it was found that it plays a crucial role in optical supercontinuum generation [2]. In particular it was found that Cherenkov radiation affects the parameters of solitary waves and can even compensate for Raman self-frequency shift [6,7]. The combined effect of the soliton frequency shift and the resonant radiation modifies the spectrum of the output radiation strongly.

The interaction of solitons with the dispersive waves, either those emitted by the solitons themselves, or those left from the initial pulse in the form of residual radiation, leads to further enrichment of the spectrum of the output signal. The theory of this interaction was developed in Refs. [8,9] and verified experimentally in Refs. [10–12]. The role of the four-wave mixing of the solitons and dispersive waves in supercontinuum generation was revealed in a number of works [13–18].

Resonant radiation of oscillating solitary waves started with the analysis of the radiation of solitons propagating in active fibers with periodically varying linear gain [19]. Recently an analogous effect was reported for conservative systems with periodically varying parameters [20] and for oscillating solitons propagating in spatially uniform single- [21] and multi-mode [22,23] fibers and in filament light bullets [24].

The aim of this paper is to consider resonant radiation of the bound state of solitons propagating in coupled fibers with

high order dispersion. We show that the oscillations of the solitons result in the generation of polychromatic radiation with a discrete spectrum, and we discuss how this radiation affects the dynamics of the bound state of the solitons.

A system of nonlinear Schrödinger equations is a widely accepted model describing propagation of light in linearly coupled optical fibers with higher order dispersion

$$\begin{aligned} i\partial_z u_n + \hat{D}(i\partial_t)u_n + |u_n|^2 u_n + \kappa u_m \\ = 0, \quad n, m = 1, 2 \quad \text{and } n \neq m, \end{aligned} \quad (1)$$

where κ is a coupling parameter and $\hat{D}(i\partial_t)$ is a dispersion operator defined by

$$\hat{D}(i\partial_t) = \sum_{n=2}^{\infty} \frac{\beta_n}{n!} (i\partial_t)^n.$$

Equations (1) have a rich plethora of linear and nonlinear solutions. In spite of the fact that the equations have been actively studied for many decades, new interesting solutions have been found even in the linear limit; see, for example, Ref. [25] where optical Airy breathers were reported.

For the purposes of this paper it is important that twin-core fibers support a family of symmetric and antisymmetric solitons. However, as the stability analysis shows, both of those families are unstable after a certain threshold in η/κ is reached (soliton amplitude to coupling constant ratio) and evolve into so-called asymmetric A and B states [26–28].

Extensive research into adiabatic quasiparticle theory provides more insights into dynamics of solitons interaction. Launching the soliton into a single core of the fiber can, after a power threshold is reached, lead to a periodic switching [29,30]. The symmetric solution with equal amplitude solitons of the same phase can be considered as an equilibrium point [31]. Small perturbation of the solution either due to one of the solitons being delayed with respect to another or uneven soliton amplitudes can lead to soliton parameter oscillations near that equilibrium. For example, a couple of delayed solitons can form a bound state with the solitons oscillating near a common center of mass [31,32]. High order dispersion and Raman effects alter the stability of the symmetric and antisymmetric solutions [33,34]; however, stable oscillating soliton states can be found in the presence of high order dispersion as well.

In the present paper we provide a theoretical description of resonant radiation generated by an oscillating pair of bound

*oreshnikov.ivan@gmail.com

solitons. We also report a transition in the oscillation regime of coupled solitons from center-of-mass oscillations to periodic energy exchange between the channel, which is apparent only in the setting of an asymmetric dispersive characteristic of the medium.

II. RESONANCE CONDITION

We are interested in the evolution of two coupled solitons with their parameters periodically changing, either in the regime of amplitude oscillations, when the solitons in the neighboring channels periodically exchange energy, or in the regime of center-of-mass oscillations, when both solitons sway around common center of mass. Both these configurations can arise from an initial condition

$$u_{1,2}(z=0, t) = \eta_{1,2} \operatorname{sech}[\eta_{1,2}(t \pm \tau_0)],$$

where $\eta_{1,2}$ are the amplitudes of the solitons and $2\tau_0$ is an initial soliton delay.

An exact analytical solution to Eq. (1) is not feasible. However, in the limit of weak coupling $\kappa \ll 1$ and in the absence of higher-order dispersion terms, an approximate quasiparticle solution can be constructed by use of a common *ansatz* of a soliton with slowly changing amplitude $\eta_n(z)$, position $\tau_n(z)$, phase $\phi_n(z)$, and frequency $\omega(z)$,

$$U_n(z, t) = A_n(z, t) \exp(iqz),$$

$$A_n(z, t) = \eta_n \operatorname{sech}[\eta_n(t - \tau_n)] \exp(-i\omega_n t + i\phi_n),$$

where $q = \kappa + \eta^2/2$ is a wave number corresponding to a symmetric soliton solution. A concrete expressions for the slowly changing soliton parameters can be found either by means of perturbation theory [32], using integrals of motion [31], or by a variational approach [33]. As an example, let us consider symmetric center-of-mass oscillations, with both amplitudes being equal and constant $\eta_1 = \eta_2 = \eta$, and soliton positions $\tau_n(z)$ being periodic functions of z with period Z_τ . The solitons are oscillating out-of-phase, $\tau_n(z + Z_\tau/2) = -\tau_n(z)$, which also means that

$$A_1(z, t) = A_2(z + Z_\tau/2, t). \quad (2)$$

To account for dispersive radiation we seek for the solution in the form of $u_n = U_n + \tilde{u}_n$, where \tilde{u}_n is a small perturbation on top of an approximate quasiparticle solution U_n . Substituting this into Eq. (1) and linearizing with respect to small \tilde{u}_n we arrive at the equation

$$\begin{aligned} i\partial_z \tilde{u}_n + \hat{D}(i\partial_t) \tilde{u}_n + 2|U_n|^2 \tilde{u}_n + U_n^2 \tilde{u}_n^* + \kappa \tilde{u}_m \\ = R[U_n, U_m] - [\hat{D}(i\partial_t) - 1/2 \partial_{tt}^2] U_n, \end{aligned} \quad (3)$$

where $R[U_n, U_m]$ is a residue term left from substituting quasiparticle solution U_n in the original equation Eq. (1) in the absence of higher order dispersion

$$R[U_n, U_m] = i\partial_z U_n + 1/2 \partial_{tt}^2 U_n + |U_n|^2 U_n + \kappa U_m.$$

Far from the solitons Eq. (3) simplifies to $i\partial_z \tilde{u}_n + \hat{D}(i\partial_t) \tilde{u}_n + \kappa u_m = 0$ and admits to a plane-wave solution $u_n = a_n \exp[ik(\omega)z - i\omega t]$ with parameters

$$k_\pm(\omega) = \hat{D}(\omega) \pm \kappa, \quad [a_1, a_2]_\pm = [1, \pm 1]. \quad (4)$$

This defines the asymptotics of the radiating states of Eq. (3). The most important feature of the solution is that the upper “+” branch of the dispersive curve is symmetric, while the lower “-” branch is antisymmetric [35].

Because of the periodicity of quasiparticle solutions $U_{1,2}(z, t)$ and property (2) we can represent it as a sum of spatial harmonics in the form of the Fourier series

$$\begin{bmatrix} U_1(z, t) \\ U_2(z, t) \end{bmatrix} = \sum_n \begin{bmatrix} +1 \\ (-1)^n \end{bmatrix} C_n(t) \exp[i(q + nk_0)z],$$

where $k_0 = 2\pi/Z_\tau$ and $C_n(t)$ is defined by an integral $C_n(t) = (1/Z_\tau) \int_{-Z_\tau/2}^{+Z_\tau/2} A_n(z, t) \exp(-ink_0 z) dz$. Here we can note an important property in $[U_1, U_2]$, namely, that even spatial harmonics are symmetric and odd harmonics are antisymmetric. Similar representations can be written for both terms in the right-hand side of Eq. (3), $R[U_n, U_m]$ and $(\hat{D} - 1/2 \partial_{tt}^2)U_n$.

Dispersive radiation is generated due to a resonance between one of the source terms in the right-hand side and a radiating solution of Eq. (3), which happens when a wave number of a source term matches a wave number of a radiating solution (see Ref. [4] for details). Due to the opposite symmetries of upper and lower branches of the dispersive curves, as well as even and odd harmonics of the source, an additional restriction applies: the upper branch is excited by even harmonics, and odd harmonics excite the lower branch. Following these statements we write the resonance conditions:

$$\hat{D}(\omega) + \kappa = 2n k_0 + q, \quad (5a)$$

$$\hat{D}(\omega) - \kappa = (2n + 1) k_0 + q. \quad (5b)$$

In the case of a moving source $U_{1,2}$ we can employ Galilean invariance of Eq. (1) and change to the solitons’ reference frame. The resonance conditions in this case change to

$$\hat{D}(\omega) + \kappa = 2n k_0 + q + v\omega, \quad (6a)$$

$$\hat{D}(\omega) - \kappa = (2n + 1) k_0 + q + v\omega, \quad (6b)$$

where v is the central velocity of the coupled solitons. This approach to resonance conditions is analogous to one used earlier for higher-order solitons [21] and solitons in dispersion oscillating fibers [20], but the situation is complicated by double dispersion curve and alternating symmetries in soliton harmonics and dispersive solutions.

The reasoning above can be repeated for pure amplitude oscillations involving two solitons periodically exchanging energies between the channels without any temporal delay: a state that evolves from the initial condition with $\eta_1 \neq \eta_2$ and $\tau_0 = 0$. An intermediate regime, featuring both oscillation in the soliton amplitudes η_n and central position τ_n , is not so tractable. We can suppose that in the intermediate regime the symmetry-based resonance exclusion would not hold, and all resonances would contribute to the dispersive radiation.

III. SECOND ORDER DISPERSION

We start our numeric analysis with a simple case of second order dispersion with $\hat{D}(i\partial_t) = 1/2 \partial_{tt}^2$. To make the radiation more prominent at shorter simulation distances we choose

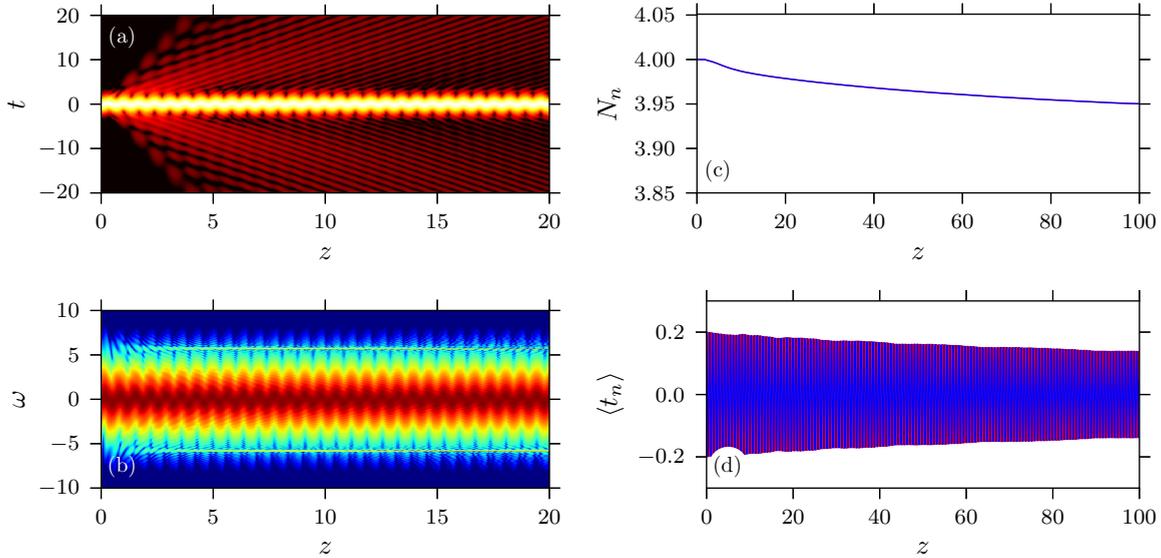


FIG. 1. Second order dispersion case. (a) Time domain and (b) frequency domain plots of the field intensity in the first channel of the coupler $|u_1(z,t)|^2$ in logarithmic scale. The field in the second channel $|u_2(z,t)|^2$ is symmetric with respect to $t = 0$. (c, d) Number of photons N_n and central soliton positions $\langle t_n \rangle$ for both channels.

$\eta = 2$. It is known from the bifurcation analysis [26] that for $q \gtrsim \sqrt{1.2\kappa}$ the symmetric solution (in our case $2\tau_0 = 0$) is unstable and if launched evolves toward a stable asymmetric A-type state. In order to avoid that we set the coupling $\kappa = 4$.

Launching the solitons with an initial relative delay $2\tau = 0.4$ we observe propagation of a bound state with the solitons oscillating near the common center of mass [for a time domain plot of the first channel see Fig. 1(a)]. We also see that the pair of solitons immediately begin to generate resonance radiation (both time and frequency domain plots in Figs. 1(a) and 1(b). Figures 1(c) and 1(d) display the first two moments of intensity distribution functions $|u_n(z,t)|^2$, namely,

$$N_n(z) = \int_{-T/2}^{+T/2} |u_n(z,t)|^2 dt, \quad (7a)$$

$$\langle t_n \rangle(z) = \frac{1}{N_n} \int_{-T/2}^{+T/2} t |u_n(z,t)|^2 dt \quad (7b)$$

as functions of distance z , where the integrals are taken over a finite time window of $T = 20$. The integral N_n is a characteristic of total energy attributed to a pulse, sometimes also called a *number of photons*, and $\langle t_n \rangle$ is the central position of a pulse. In Figs. 2(c) and 2(d) we can notice that due to shedding of dispersive radiation both solitons lose energy N_n . However, an additional decay in the amplitude of position $\langle t_n \rangle$ oscillations is present. This indicates that the internal oscillation mode loses energy due to radiation damping.

In the output spectrum the dispersive radiation manifests itself as two sharp peaks near $\omega \approx \pm 6$ [see Fig. 2(a) for the spectral density of both channels at $z = 50$]. To compare the simulated spectrum with the predictions of resonance conditions (5) we take the field $u_1(z,t)$ in the simulation domain and perform a two-dimensional Fourier transform, moving into the ω - k plane. The intensity plot of the resulting spectrum [Fig. 2(b)] clearly shows two branches of the dispersive curve (parabolas starting at $\pm\kappa$) as well as a set of

solitons' spatiotemporal harmonics (horizontal line). The line passing through $k = 6$ is the fundamental $n = 0$ harmonics of the soliton ($k = q$). The horizontal line below it is the $n = -1$ harmonics of the soliton. It does intersect with the upper branch of the dispersive curve near $\omega \approx \pm 4$. However, due to the different symmetries of the solution these resonances do not contribute to the radiation. The horizontal line corresponding to $n = -2$ harmonics (faint, passing through $k = -13$) is in resonance with both the upper and the lower branches of the dispersive curve. Due to the difference in the symmetry, the resonance with the lower branch does not lead to generation of dispersive waves. The only contributing resonance is between the $n = -2$ harmonics of the soliton and the upper branch of the dispersive curve, which is indicated by the cross-hair pattern centered at the intersection points.

To confirm our reasoning about the soliton's harmonic and dispersive curves' parity we additionally look at the case of pure amplitude oscillations with $\eta_1 = 2.1$, $\eta_2 = 1.9$, and no delay between the solitons $2\tau_0 = 0$ [see Fig. 2(c)] and then the case of a mixed regime with $\eta_1 = 2.1$, $\eta_2 = 1.9$, and $2\tau_0 = 0.4$ [see Fig. 2(d)]. In the regime of pure amplitude oscillations the same parity selection rule holds, as is evident from no resonance between the $n = -1$ harmonic of the soliton and the upper branch of the dispersive curve. In the case of a mixed oscillation regime ($\eta_1 = 2.1$, $\eta_2 = 1.9$, and $2\tau_0 = 0.4$) no harmonic is either purely symmetric or antisymmetric, rather all of them are asymmetric and thus every intersection of the soliton's harmonics with the dispersive curves contributes to the dispersive radiation.

IV. THIRD AND FOURTH ORDER DISPERSION

We proceed with the case of third order dispersion using the operator $\hat{D} = -1/2 \partial_t^2 - i\beta_3/6 \partial_{ttt}^3$ with $\beta_3 = 0.2$. We probe three different relative delays $2\tau_0 = 0.0, 0.1, \text{ and } 0.4$. Figure 3 demonstrates field intensity in time domain for all the relative

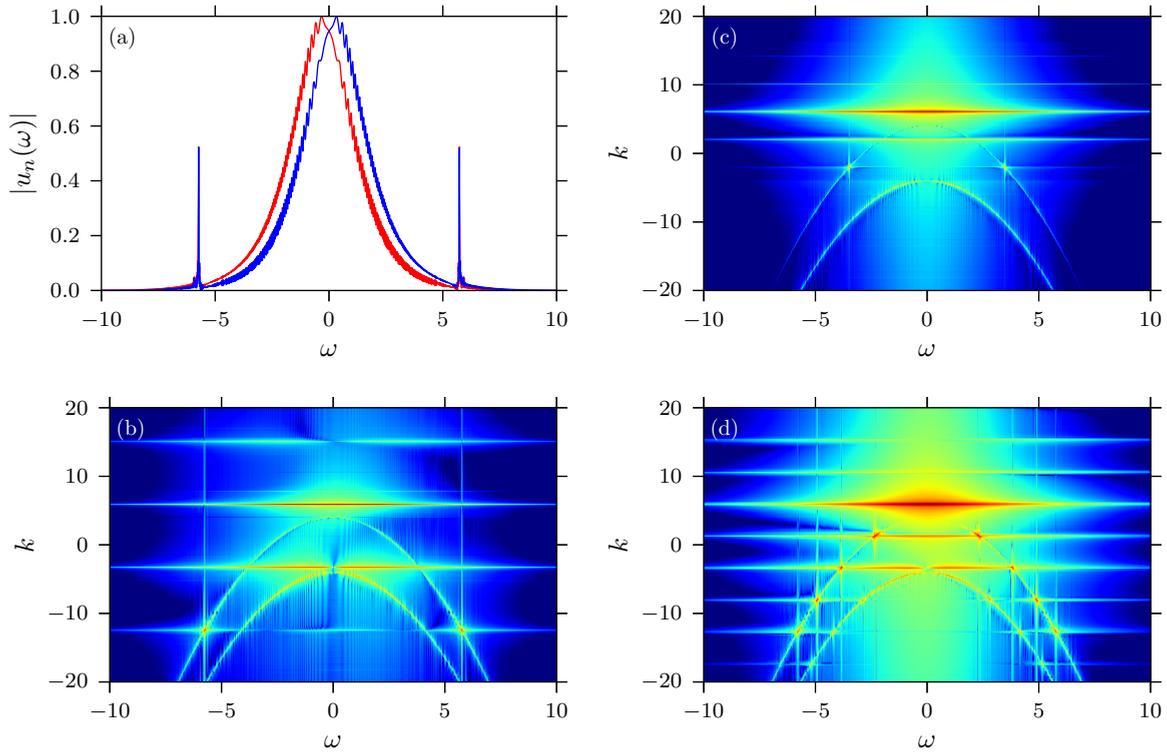


FIG. 2. Second order dispersion case. (a) Output spectrum for a coupled state with initial delay of $2\tau_0 = 0.4$ taken at $z = 50$. (b) ω - k diagram of the field in the first channel $u_1(z,t)$ for $\eta_1 = \eta_2 = 2$; (c) the same diagram for the case of pure amplitude oscillations $\eta_1 = 2.1$ and $\eta_2 = 1.9$, $2\tau_0 = 0$; (d) mixed regime at $\eta_1 = 2.1$, $\eta_2 = 1.9$, and $2\tau_0 = 0.4$.

delays $2\tau_0$ probed. The case of no relative delay $2\tau_0 = 0$ behaves exactly as a single soliton of a scalar equation in the presence of third order dispersion perturbation, i.e., the soliton generates monochromatic resonant radiation at the frequency predicted by a resonance condition as is shown in Figures 4(a) and 4(d). Introduction of a small relative delay $2\tau_0 = 0.1$ leads to a splitting of a single spectral line into a tight frequency pair [Fig. 4(b)]. This additional spectral line is due to contribution of the resonance between the $n = -1$ harmonic of a soliton pair and the lower dispersive curve [compare with Fig. 4(d)]. However, a larger delay (in our simulations $2\tau_0 \geq 0.2$) reveals a more complicated dynamics. As an example, let us look at the output spectrum in Fig. 4(c) for the extreme case of $2\tau_0 = 0.4$. Aside from the pronounced spectral lines near the spectrum spectrum of the solitons we can see that the vicinity of Cherenkov resonance is filled with a number of

separated frequency pairs [inset in Fig. 4(c)]. The ω - k diagram in Fig. 4(f) indicates, that in addition to a set of harmonics corresponding to the center of mass oscillations of the soliton pair, the wave number spectrum contains a series of additional lines between the original harmonics. In addition to that, the resonance exclusion based on the symmetries does not work anymore, and every intersection between a dispersive curve and a soliton harmonic contributes to the dispersive radiation.

To track the origin of the additional spatial harmonics in the solitons' spectrum in Fig. 4(d) we look at the moments N_n and $\langle \tau_n \rangle$ in Fig. 5. Insets demonstrate input and output parts of the simulation. As it is evident from the plots, the center-of-mass oscillations (with period $Z_\tau \approx 0.7$) decay rapidly [Fig. 5(b)]. The system, however, does not evolve towards a steady state, but instead develops amplitude oscillations with period $Z_\eta \approx 1.4$ [Fig. 5(a)]. There are known approximations for

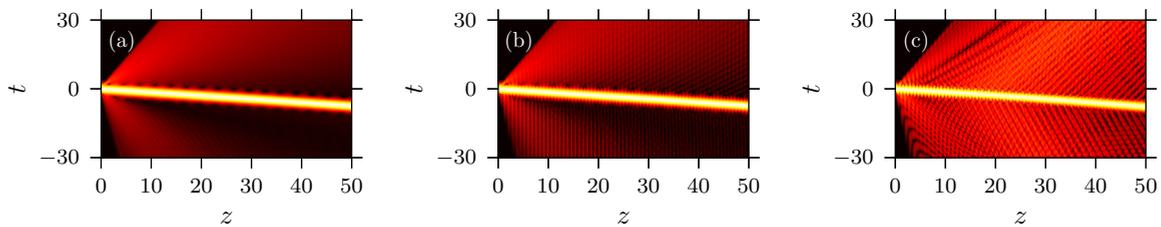


FIG. 3. Third order dispersion case. Time domain plots of intensity distribution in the first channel $|u_1|$ for the oscillating soliton states (with $\eta_1 = 2.1$ and $\eta_2 = 1.9$) at different initial values of relative delay $2\tau_0$. (a) The symmetric case $2\tau_0 = 0.0$, (b) the case of $2\tau_0 = 0.1$, and (c) the case of $2\tau_0 = 0.4$.

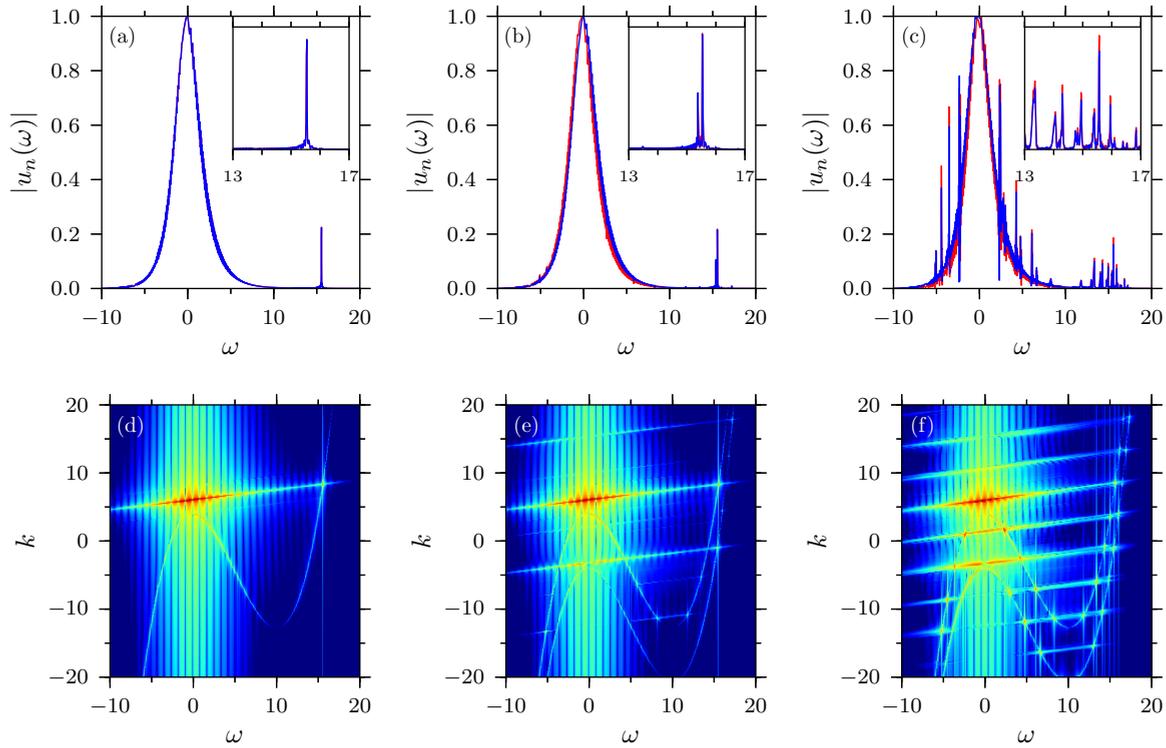


FIG. 4. Third order dispersion case. Output spectra at $z = 0$ and ω - k diagrams for the oscillating soliton state (with $\eta_1 = 2.1$ and $\eta_2 = 1.9$) at different initial values of relative delay $2\tau_0$. (a, d) A symmetric case $2\tau_0 = 0$, (b, e) the case of $2\tau_0 = 0.1$, and (c, f) the case of $2\tau_0 = 0.4$. Insets in (a, b, c) show the vicinity of Cherenkov resonance.

oscillation periods in both regimes provided that amplitudes are small [31,32]. Unfortunately, neither weak coupling nor small amplitude approximations are applicable to our case.

To compare this to the case of symmetric fourth order dispersion we consider the dispersion operator $\hat{D} = -1/2 \partial_{tt}^2 + \beta_4/24 \partial_{tttt}^4$ with $\beta_4 = 0.1$. Launching the solitons with initial relative delay of $2\tau_0 = 0.4$ does not lead to the oscillation regime switching; see Figs. 6(c) and 6(d). Indeed the output spectrum and the ω - k diagram at Fig. 6(b) are similar to the case of second order dispersion. We can propose a preliminary hypothesis that in the presence of an asymmetric dispersion profile center-of-mass oscillations of a soliton pair become unstable, while the amplitude oscilla-

tions are not affected by such a perturbation. The detailed analysis of the regime stability lies outside the scope of the current paper.

V. CONCLUSIONS

In this paper we have considered resonant radiation of oscillating bound states of solitons propagating in the coupled fibers with high order dispersion. The resonant condition is derived analytically, and it is shown that the parity of the soliton bound state defines the parity of the radiated dispersive waves. An excellent agreement between the frequencies of the radiation predicted by the resonance condition and the

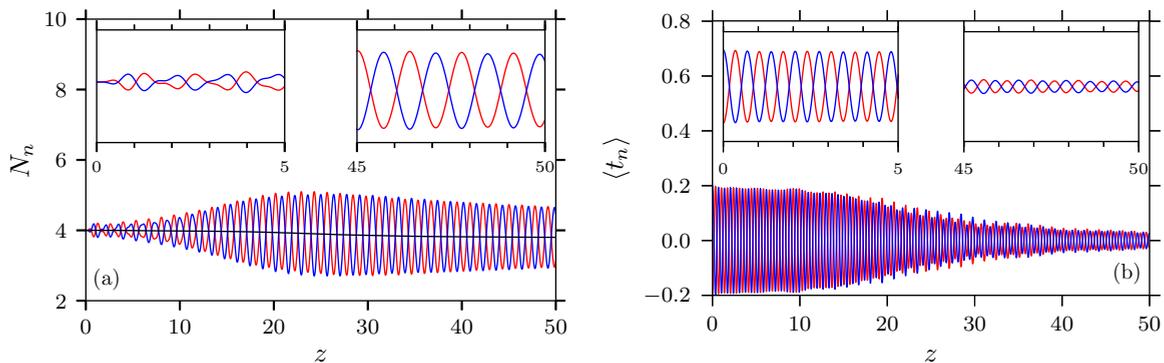


FIG. 5. Third order dispersion case. (a) The number of photons N_n and (b) the soliton position $\langle t_n \rangle$ for the initial conditions of equal amplitude solitons $\eta_1 = \eta_2 = 2$ with significant temporal delay $2\tau_0 = 0.4$ [corresponds to Figs. 4(c) and 4(f)]. Black line is the average of N_1 and N_2 demonstrating the loss of total energy by the soliton pair.

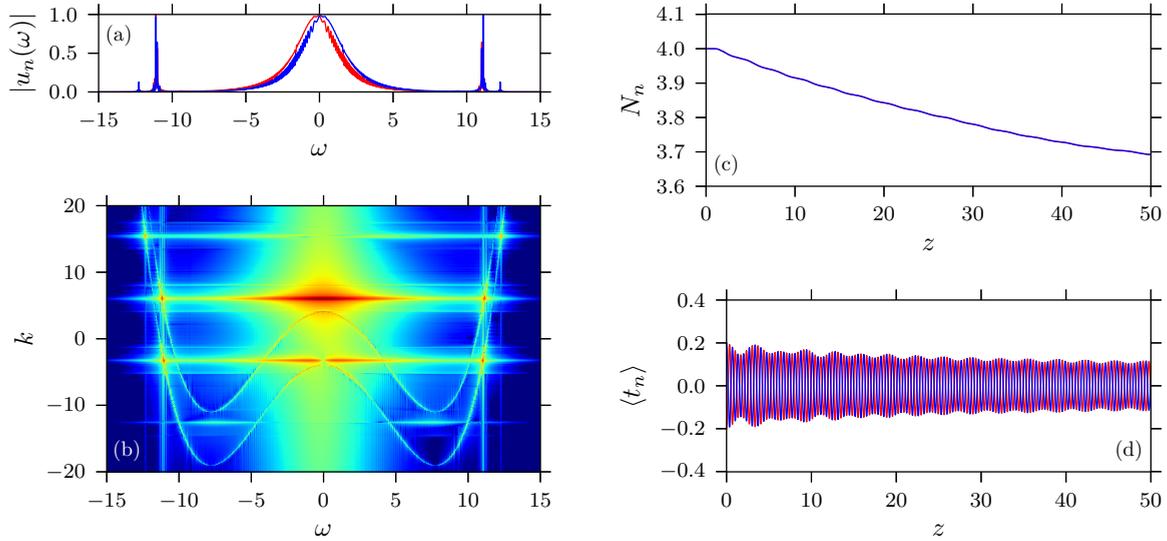


FIG. 6. Fourth order dispersion case. (a) Output spectrum for a coupled state with initial delay of $2\tau_0 = 0.4$ taken at $z = 50$. (b) ω - k diagram of the field in the first channel $u_1(z,t)$.

positions of the spectral lines observed in direct numerical simulations is demonstrated.

It is shown that the resonant radiation can be caused either by the oscillations of the mutual delay between the solitons or by periodic energy exchange between the solitons. The oscillations of the mutual delay and the soliton amplitudes have different periods and, consequently, lead to emission of the resonant radiation of different frequencies. The mutual delay and the amplitude oscillations can occur simultaneously resulting in the formation of rich radiation spectrum. By numerical simulations it was shown that the energy of the oscillations goes into the resonant radiation and finally a nonoscillating bound state of the solitons forms. The energy of the solitons in the bound state decreases if the condition of Cherenkov synchronism is fulfilled.

It was observed and investigated numerically that the recoil from the resonant radiation can result in the drastic change of the dynamics of the bound state of the solitons. In particular in the case of symmetric (in the sense $k \rightarrow -k$) dispersion two identical solitons launched with a small delay in the first and in the second fiber exhibit oscillations of the soliton mutual

delay, but the amplitudes of the solitons are not changing during propagation. As is mentioned above the oscillations slowly decay because of the radiation of the resonant mode.

However, in the case of asymmetric dispersion the oscillation of the soliton mutual delay decays much quicker and gives rise to the quasiperiodical energy transfer between the solitons. In their turn the oscillations of the soliton amplitudes result in the radiation of resonant modes with different frequencies and relatively slow decay with the propagation distance.

The reported results explain the dynamics of the bound states of the solitons in coupled nonlinear fibers with high-order dispersion and are important from the fundamental point of view and can possibly be used for better understanding of the process of supercontinuum generation in complex waveguiding systems, in particular in dual core silica fibers [36].

ACKNOWLEDGMENTS

A.Y. acknowledges support from the Government of the Russian Federation (Grant 074-U01) through the ITMO University Early Career Fellowship.

-
- [1] Y. S. Kivshar and B. A. Malomed, *Rev. Mod. Phys.* **61**, 763 (1989).
 - [2] D. V. Skryabin and A. V. Gorbach, *Rev. Mod. Phys.* **82**, 1287 (2010).
 - [3] P. K. A. Wai, C. R. Menyuk, Y. C. Lee, and H. H. Chen, *Opt. Lett.* **11**, 464 (1986).
 - [4] N. Akhmediev and M. Karlsson, *Phys. Rev. A* **51**, 2602 (1995).
 - [5] A. V. Yulin, D. V. Skryabin, and P. S. J. Russell, *Phys. Rev. Lett.* **91**, 260402 (2003).
 - [6] D. Skryabin, F. Luan, J. Knight, and P. S. J. Russell, *Science* **301**, 1705 (2003).
 - [7] F. Biancalana, D. V. Skryabin, and A. V. Yulin, *Phys. Rev. E* **70**, 016615 (2004).
 - [8] A. V. Yulin, D. V. Skryabin, and P. S. J. Russell, *Opt. Lett.* **29**, 2411 (2004).
 - [9] D. V. Skryabin and A. V. Yulin, *Phys. Rev. E* **72**, 016619 (2005).
 - [10] A. Efimov, A. J. Taylor, F. G. Omenetto, A. V. Yulin, N. Y. Joly, F. Biancalana, D. V. Skryabin, J. C. Knight, and P. S. J. Russell, *Opt. Express* **12**, 6498 (2004).
 - [11] A. Efimov, A. V. Yulin, D. V. Skryabin, J. C. Knight, N. Joly, F. G. Omenetto, A. J. Taylor, and P. Russell, *Phys. Rev. Lett.* **95**, 213902 (2005).
 - [12] A. Efimov, A. J. Taylor, A. V. Yulin, D. V. Skryabin, and J. C. Knight, *Opt. Lett.* **31**, 1624 (2006).
 - [13] A. V. Gorbach and D. V. Skryabin, *Nature Photonics* **1**, 653 (2007).

- [14] R. Driben, F. Mitschke, and N. Zhavoronkov, *Opt. Express* **18**, 25993 (2010).
- [15] A. V. Yulin, R. Driben, B. A. Malomed, and D. V. Skryabin, *Opt. Express* **21**, 14481 (2013).
- [16] R. Driben, A. V. Yulin, A. Efimov, and B. A. Malomed, *Opt. Express* **21**, 19091 (2013).
- [17] R. Driben and N. Zhavoronkov, *Opt. Express* **18**, 16733 (2010).
- [18] A. Demircan, S. Amiranashvili, and G. Steinmeyer, *Phys. Rev. Lett.* **106**, 163901 (2011).
- [19] S. W. Y. Kodama, M. Romagnoli and M. Midrio, *Opt. Lett.* **19**, 165 (1994).
- [20] M. Conforti, S. Trillo, A. Mussot, and A. Kudlinski, *Sci. Rep.* **5**, 9433 (2015).
- [21] R. Driben, A. V. Yulin, and A. Efimov, *Opt. Express* **23**, 19112 (2015).
- [22] L. G. Wright, S. Wabnitz, D. N. Christodoulides, and F. W. Wise, *Phys. Rev. Lett.* **115**, 223902 (2015).
- [23] K. Krupa, A. Tonello, A. Barthélémy, V. Couderc, B. M. Shalaby, A. Bendahmane, G. Millot, and S. Wabnitz, *Phys. Rev. Lett.* **116**, 183901 (2016).
- [24] C. Brée, I. Babushkin, U. Morgner, and A. Demircan, *Phys. Rev. Lett.* **118**, 163901 (2017).
- [25] R. Driben, V. V. Konotop, and T. Meier, *Opt. Lett.* **39**, 5523 (2014).
- [26] N. Akhmediev and A. Ankiewicz, *Phys. Rev. Lett.* **70**, 2395 (1993).
- [27] J. M. Soto-Crespo and N. Akhmediev, *Phys. Rev. E* **48**, 4710 (1993).
- [28] N. Akhmediev and J. M. Soto-Crespo, *Phys. Rev. E* **49**, 4519 (1994).
- [29] P. L. Chu, G. D. Peng, and B. A. Malomed, *Opt. Lett.* **18**, 328 (1993).
- [30] P. L. Chu, B. A. Malomed, and G.-D. Peng, *J. Opt. Soc. Am. B* **10**, 1379 (1993).
- [31] Y. S. Kivshar and B. A. Malomed, *Opt. Lett.* **14**, 1365 (1989).
- [32] F. K. Abdullaev, R. Abrarov, and S. Darmanyan, *Opt. Lett.* **14**, 131 (1989).
- [33] N. F. Smyth and A. L. Worthy, *J. Opt. Soc. Am. B* **14**, 2610 (1997).
- [34] B. A. Umarov, F. K. Abdullaev, and M. R. B. Wahiddin, *Opt. Commun.* **162**, 340 (1999).
- [35] C. M. Arabí, F. Bessin, A. Kudlinski, A. Mussot, D. Skryabin, and M. Conforti, *Phys. Rev. A* **94**, 063847 (2016).
- [36] R. Buczynski, D. Pysz, T. Martynkien, D. Lorenc, I. Kujawa, T. Nasilowski, F. Berghmans, H. Thienpont, and R. Stepien, *Laser Phys. Lett.* **6**, 575 (2009).