Level-mixing effect induced by blackbody radiation and its influence on the cosmological hydrogen recombination problem

T. Zalialiutdinov,[1] D. Solovyev,[1] L. Labzowsky,[1,2] and G. Plunien[3]

[1]*Department of Physics, Saint Petersburg State University, Petrodvorets, Ulianovskaya 1, 198504 Saint Petersburg, Russia*

[2]*Petersburg Nuclear Physics Institute, 188300 Gatchina, Saint Petersburg, Russia*

[3]*Institut für Theoretische Physik, Technische Universität Dresden, Mommsenstraße 13, D-10162 Dresden, Germany*

(Received 21 April 2017; published 18 July 2017)

Two different effects of the blackbody-radiation-induced atomic line broadening are compared. The first one (stimulated Raman scattering) has been discussed by many authors; the second one (quadratic level mixing) was discussed in our previous work. It is shown that the mixing effect gives the most significant contribution to the line broadening and it is indicated how to distinguish these two effects in laboratory experiments. The influence of the level mixing on the recombination history of primordial plasma is also discussed.

DOI: 10.1103/PhysRevA.96.012512

The influence of external fields on atomic characteristics is still one of the interesting subjects for investigations in modern atomic physics. In particular a question about the blackbody radiation (BBR) influence on atoms is widely discussed. First the BBR-induced effects were observed experimentally and then the theoretical description was given in [1,2] within the framework of the quantum-mechanical (QM) approach. In particular, it was shown that the blackbody radiation induces the ac Stark shift of energy levels and an additional line broadening in atoms. Theoretical calculations of the dynamic Stark shifts and depopulation rates of Rydberg energy levels caused by the BBR and the corresponding experimental measurements were widely discussed in the literature [3–8]. The most important consequence of these investigations corresponds to the improvement of atomic clocks and the development of optical standards of frequency measurements [9].

Finally, in [10] the effect of level mixing induced by the blackbody radiation was described theoretically within the rigorous quantum electrodynamic (QED) theory. The mixing effect for the states of opposite parity arising in the presence of an external electric field leads to significant changes of the decay rates (see, for example, [11,12]). We should note that all effects in the presence of the BBR are similar to the phenomena which take place in an external electric field. Similar to the Stark (static or dynamic) effect in the presence of "ordinary" external electric field the energy shift of atomic levels induced by the BBR can be estimated with the use of the rms value of the field strength of thermal radiation (in a.u.):

$$\langle E^2(\omega)\rangle = \frac{8\alpha^3}{\pi}\omega^3 n_\beta(\omega) = \frac{8\alpha^3}{\pi}\frac{\omega^3}{e^{\beta\omega}-1}, \tag{1}$$

where $\langle E^2(\omega)\rangle$ is the rms electric-field strength and ω is the radiation frequency. The Planck distribution function is presented by $n_\beta(\omega)$ with $\beta = 1/k_B T$, k_B is the Boltzmann constant, T is the temperature in Kelvin, and α is the fine-structure constant. Then the integral rms value of the electric field is

$$\langle E^2\rangle = \frac{1}{2}\int_0^\infty \langle E^2(\omega)\rangle d\omega = \frac{4\pi^3}{15}\alpha^3(k_B T)^4 = (8.319\ \mathrm{V/cm})^2[T(\mathrm{K})/300]^4. \tag{2}$$

In conjunction with the expression (2) the level-mixing effect induced by the thermal radiation can be introduced. The level-mixing effect in an external electric field was considered in connection with the Lamb shift measurements in hydrogen and hydrogen-like ions [13,14] and the corresponding theoretical analysis of the electric-field influence on atomic levels can be found in [15,16]. An accurate description of the level-mixing effect in the hydrogen atom was given in [17]. In particular, the authors of [17] have shown that the mixing of $2s$ and $2p$ states in the hydrogen atom can mimic the parity nonconservation phenomenon.

As a result of the level-mixing effect in the presence of an external electric field [15,17] or in the presence of the BBR [10] the essential modification of the $2s$ state decay in the hydrogen atom arises. This is due to the appearance of the one-photon electric dipole decay channel which was forbidden by the selection rules in the absence of an external field. As a consequence the $2s$ state level in the hydrogen atom does not remain a metastable one in the presence of an external field.

We should note that in what concerns level mixing we include only $2p_{1/2}$ and $2s_{1/2}$ mixing and neglect $2p_{3/2}$ and $2s_{1/2}$ mixing. The reason is that the fine-structure interval $E_{2p_{3/2}} - E_{2s_{1/2}}$ is much larger than the Lamb shift $E_{2s_{1/2}} - E_{2p_{1/2}}$ and consequently the mixing effect for the $2p_{3/2}$ state is much smaller.

The one-photon decay rate of the mixed $\overline{2s}$ state [11,17] can be expressed as

$$W^{(1\gamma)}_{\overline{2s}\,1s}(\mathbf{k}) = W^{(1\gamma)}_{2s\,1s}(\mathbf{k})\left[1 + ea_0\mathbf{E}\,\mathbf{n}_\mathbf{k}\frac{|\eta|^2\Gamma_{2p}}{w} + e^2a_0^2\frac{|\eta|^2\mathbf{E}^2}{w^2}\right], \tag{3}$$

where $\mathbf{E}$ represents the electric field, $\mathbf{n}_\mathbf{k}$ is the unit vector corresponding to the wave vector $\mathbf{k}$ of the photon, $w = \sqrt{W^{\mathrm{M1}}_{2s\,1s}/W^{\mathrm{E1}}_{2p\,1s}}$, and the electron charge e and the Bohr radius a_0 are written explicitly for clarity. Γ_{2p} is the $2p$ level width, $\eta = (\Delta E^L_{2p2s} - \frac{i}{2}\Gamma_{2p})^{-1}$, ΔE^L_{2p2s} represents the Lamb shift between $2s_{1/2}$ and $2p_{1/2}$ levels, and the one-photon transition probabilities $W^{\mathrm{M1}}_{2s\,1s}$ and $W^{\mathrm{E1}}_{2p\,1s}$ correspond to the emission of the magnetic dipole and electric dipole photons, respectively. Integration over photon emission direction $\mathbf{n}_\mathbf{k}$ and frequency

of the emitted photon $\omega = |\mathbf{k}|$ yields the expression [11,12]

$$W^{(1\gamma)}_{\overline{2s}\,1s} = W^{\mathrm{M1}}_{2s\,1s} + \frac{e^2 a_0^2 \mathrm{E}_0^2}{\left(\Delta E^L_{2p2s}\right)^2 + \frac{1}{4}\Gamma^2_{2p}} W^{\mathrm{E1}}_{2p1s}, \tag{4}$$

where E_0 is the field amplitude. This expression shows that the additional one-photon electric dipole emission channel is allowed for the hydrogenlike atom in the metastable $2s$ state in the presence of an external electric field. The term linear in the field in Eq. (3) vanishes after the integration over photon emission directions. In contrast, the term quadratic in the field does not depend on the photon emission or field directions. This contribution represents the quadratic mixing effect.

Since the decay rate of the E1 transition, $W^{\mathrm{E1}}_{2p1s} = 6.265\times 10^8\,\mathrm{s}^{-1}$, exceeds strongly the one-photon magnetic decay channel, $W^{\mathrm{M1}}_{2s1s} = 2.496\times 10^{-6}\,\mathrm{s}^{-1}$, the second term in Eq. (4) may become the dominant decay channel of the mixed $\overline{2s}$ state with increasing strength of the external electric field. The contribution of the second term in Eq. (4) at the field strength 475 V/cm (easily achievable in laboratory experiments) becomes equal to the decay rate of the $2p$ level in the hydrogen atom (the case of complete mixing) and is much larger than the main two-photon E1E1 decay rate of the $2s$ state in the absence of the electric field, $W^{\mathrm{E1E1}}_{2s1s} = 8.229\,\mathrm{s}^{-1}$. In turn, the same scenario can be considered for the mixing of the $\overline{2p}$ state in the hydrogen atom. In this case there is no essential difference in the decay rate in the external field because of the small additional contribution of the transition rates of the $2s$ level. Note that this effect arises in the presence of a static electric field. According to the description above the rms value $\langle E^2\rangle$ of the electric field caused by the BBR can be estimated by Eq. (2). Thus, the effect of level mixing should arise in the presence of thermal radiation as it was demonstrated in [10].

However, the thermal radiation cannot be described completely as a static electric field. The significant dynamical character of BBR modifies the form of the transition rate.

The full description of the dynamical effects as well as mixing effects was given in [10]. The QED expression for the BBR broadening is

$$\Gamma^{\mathrm{BBR-QED}}_a = \frac{2e^2}{3\pi}\sum_n |\langle a|\mathbf{r}|n\rangle|^2 \int_0^\infty d\omega n_\beta(\omega)\omega^3 \times\left[\frac{\Gamma_{na}}{(\tilde{\omega}_{na}+\omega)^2+\frac{1}{4}\Gamma^2_{na}} + \frac{\Gamma_{na}}{(\tilde{\omega}_{na}-\omega)^2+\frac{1}{4}\Gamma^2_{na}}\right], \tag{5}$$

where a and n denote the set of quantum numbers of the corresponding atomic state, $\tilde{\omega}_{na} \equiv E_n - E_a + \Delta E^L_{na}$, ΔE^L_{na} is the corresponding Lamb shift, and $\Gamma_{na} \equiv \Gamma_n + \Gamma_a$. Expression (5) is the width of the resonant emission line profile in the presence of BBR.

For the dynamical effects in Eq. (5) the frequency-dependent energy denominators are responsible. The mixing effect is incorporated when the summation over n in Eq. (5) extends over the states with the space parity opposite to the parity of the state a. In this case the main contribution comes from the state n close to a. In the hydrogen atom such states ($2s_{1/2}$ and $2p_{1/2}$, for example) are degenerate and $\omega_{an} = \Delta E^L_{an}$ is the Lamb shift. For such levels as $2s$ in hydrogen, which will be of our interest below, the mixing effect becomes dominant.

To clarify the physical situation we have to compare the result Eq. (5) with the well-known effect of the level broadening by the multiple photon scattering [Raman scattering (RS) in the general case] on atomic levels: $i+\gamma \to a \to f+\gamma'$, where i and f denote the initial and final states, respectively, a is the excited intermediate state, and γ represents the emitted or absorbed photon. In case of BBR this effect was described in [2] and has application in cosmological recombination [18]. Below we will show that (1) the mixing broadening cannot be reduced to the RS but is an independent effect; (2) the mixing effect dominates over RS in line broadening; (3) the mixing effect leads to the emission lines that occur at frequencies Lamb shifted from frequencies corresponding to the emission lines broadened by the RS process; and (4) this frequency difference in principle is possible to observe in laboratory experiments that may give access to the study of primordial plasma.

For this purpose below we consider the simple RS process. The multiple photon scattering will be taken into account by introducing the Einstein coefficients, or the number of photons. In case of the BBR the number of photons is defined by the Planck distribution function.

The S-matrix element of the RS process can be written in the form [19–21]

$$\hat{S}^{(2)}_{fi} = (-ie)^2 \int dx_1 dx_2 \overline{\psi}_f(x_1)\gamma_{\mu_1} A^{*(\mathbf{k}_2,\mathbf{e}_2)}_{\mu_1}(x_1) \times S(x_1x_2)\gamma_{\mu_2} A^{(\mathbf{k}_1,\mathbf{e}_1)}_{\mu_2}(x_2)\psi_i(x_2), \tag{6}$$

where $\psi_i(x)$ and Dirac conjugated $\overline{\psi}_f(x)$ represent the wave functions of the initial and final states, respectively, and γ_μ are the Dirac matrices with $\mu = 0,1,2,3$. The photon wave function (electromagnetic field potential) is described by

$$A^{(\mathbf{k},\mathbf{e})}_\mu(x) = \sqrt{\frac{2\pi}{\omega}} e^{(\lambda)}_\mu e^{ik_\mu x_\mu} = A^{(\mathbf{k},\mathbf{e})}_\mu(\mathbf{r})\, e^{-i\omega t}, \tag{7}$$

where $k \equiv (\mathbf{k},\omega)$ is the photon momentum 4-vector, $\mathbf{k}$ is the photon wave vector, $\omega = |\mathbf{k}|$ is the photon frequency, and $e^{(\lambda)}_\mu$ are the components of the photon polarization 4-vector. $A^{(\mathbf{k},\mathbf{e})}_\mu$ and $A^{*(\mathbf{k},\mathbf{e})}_\mu$ in Eq. (6) correspond to the absorbed and emitted photon, respectively. In the Furry picture the eigenmode decomposition of Feynman electron propagator $S(x_1,x_2)$ reads [22]

$$S(x_1x_2) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega\, e^{i\Omega_1(t_1-t_2)} \sum_n \frac{\psi_n(\mathbf{r}_1)\bar{\psi}_n(\mathbf{r}_2)}{E_n(1-i0)+\omega}, \tag{8}$$

where the summation over n in Eq. (8) extends over the entire Dirac spectrum.

The differential absolute probability of emission process resulting from the RS cross section in case of resonant scattering is [22,23]

$$dw_{af}(\omega) = \frac{1}{2\pi}\frac{dW_{af}(\omega)}{(E_a - E_f - \omega)^2 + \frac{1}{4}\Gamma^2_a}, \tag{9}$$

where dW_{af} is the differential partial transition rate $a \to f$.

Expression (9) follows from separation of absorption and emission processes, which become independent within the

resonant approximation. The result (9) represents the emission line profile, i.e., the photon emission occurs at the resonant frequency $\omega_{af} = E_a - E_f$. Integrating Eq. (9) over frequency ω we find

$$w_{af} = \frac{W_{af}^{\mathrm{E1}}(\omega_{af})}{\Gamma_a}, \tag{10}$$

where w_{af} is the absolute transition probability $a \to f$ and $W_{af}^{\mathrm{E1}}(\omega_{af})$ denotes the electric dipole one-photon spontaneous emission rate. Expression (10) is given for the case of one-photon emission process that arises as a result of Raman scattering in the resonance approximation. However, the presence of a photon field (in particular BBR) induces additional emission with the same frequency [24]. According to [24] an induced photon emission probability is expressed via the number of photons. In our case this number is defined by the Planck distribution function, n_β:

$$W_{af}^{\mathrm{ind}} = n_\beta(\omega_{af}) W_{af}^{\mathrm{E1}}(\omega_{af}). \tag{11}$$

Thus, the emission probability corresponding to the stimulated RS process is given by

$$W_{af}(T) = [1 + n_\beta(\omega_{af})] W_{af}^{\mathrm{E1}}(\omega_{af}). \tag{12}$$

Then the total BBR-induced level broadening via RS process for an arbitrary level a in the nonrelativistic limit (neglecting all the types of photons except E1) looks like

$$\Gamma_a^{\mathrm{BBR-RS}} = \frac{4e^2}{3} \sum_f |\langle a|\mathbf{r}|f\rangle|^2 \frac{\omega_{af}^3}{e^{\beta\omega_{af}} - 1}, \tag{13}$$

and coincides precisely with the result obtained in [2] within the QM approach. Here we have used an explicit expression for W_{af}^{E1}. Expression (13) is the sum of all the partial transition probabilities including the higher excited states. The one-photon emission occurs at the corresponding resonant frequency ω_{af}.

Thus, we have two effects induced by the BBR: Raman scattering and level mixing. They have similar structure. The former one is described by the expression (13). The expression (5) is more general: it incorporates both effects. The result (13) can be obtained from Eq. (5) if we fully neglect the widths of the states n,a. The limit $\Gamma_{na} \to 0$ in square brackets in Eq. (5) gives the sum of two delta functions $\delta(\tilde{\omega}_{na} + \omega) + \delta(\tilde{\omega}_{na} - \omega)$ and, therefore, the RS result (13) arises immediately.

The QM mixing effect follows from Eq. (4) by substituting the rms value Eq. (2) instead of E_0^2. This result also can be obtained from Eq. (5) but using different approximations than for the RS effect. To do this we have to set $\omega = 0$ in square brackets in Eq. (5). It can be justified if we remember that function $n_\beta(\omega)$ is concentrated at small ω values for not too high temperatures. Moreover Eq. (4) follows from Eq. (5) when we take the contribution Γ_n from the sum $\Gamma_{na} = \Gamma_n + \Gamma_a$ in the numerators in square brackets in Eq. (5). The contribution of Γ_a also appears to be important, but it corresponds not to the mixing but to the general "dynamic" effect.

According to [15,17] the mixing of atomic levels with opposite parity occurs in a static electric field. Such a field cannot induce any electron transitions in atoms. In laboratory experiments an electric field leading to the complete mixing of $2s$, $2p$ states in the hydrogen atom produces the Stark shift, which is much smaller than the Lamb shift. This circumstance allows the measurements of the Lamb shift with a high accuracy [15,16]. The one-photon dipole emission (3) occurs at the resonant frequency $\omega_{\overline{2s}1s} = \omega_{2s1s} + \Delta E_{2s}^{\mathrm{Stark}} \approx \omega_{2s1s}$, which is Lamb shifted from Ly_α transition frequency $\omega_{\overline{2p}1s} \approx \omega_{2p1s}$. With the growth of strength of the electric field the intensity of emitted photons at the frequency ω_{2s1s} increases and reaches the Ly_α value at the field strength corresponding to the complete mixing of $2s$, $2p$ states. The difference of emission frequencies can serve as a tool for distinguishing of mixing and RS processes in laboratory experiments (see Fig. 1).

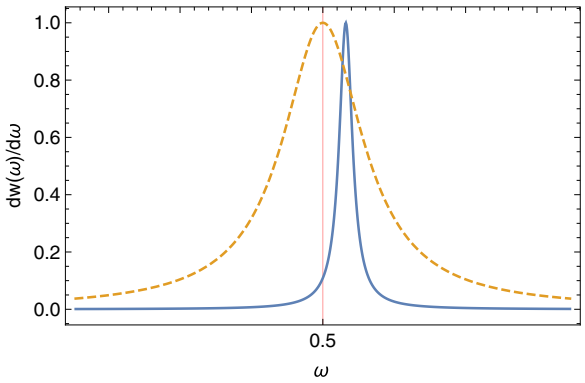

FIG. 1. Emission line profiles Eq. (14) for the transitions $\overline{2p} \to 1s + \gamma(\mathrm{E1})$ (yellow dashed line) and $\overline{2s} \to 1s + \gamma(\mathrm{E1})$ (blue solid line) at the BBR temperature $T = 3000$ K. The magnitudes of emission line profiles and frequency interval are normalized to unity. The Ly_α frequency corresponds to $\omega = 0.5$. The corresponding values of transition rates and level widths are presented in Table I.

To make this physical picture clearer, the RS process should be considered in the case when the excited state a is mixed (we denote it by $\overline{a}$). Formally, it can be obtained by the substitutions $a \to \overline{a}$ and $E_a \to E_{\overline{a}} = E_a + \Delta E_a^{\mathrm{Stark}} \approx E_a$, $\Gamma_{\overline{a}} = \Gamma_a + \Gamma_a^{\mathrm{BBR-QED}}$, $W_{af} \to W_{\overline{a}f}(\omega)$ into Eq. (9). Then in the resonant approximation we find

$$dw_{\overline{a}f}(\omega) = \frac{1}{2\pi} \frac{dW_{\overline{a}f}(\omega_{\overline{a}f})}{(E_{\overline{a}} - E_f - \omega)^2 + \frac{1}{4}\Gamma_{\overline{a}}^2}. \tag{14}$$

In this expression the resonance frequency $\omega_{\overline{a}f} \approx \omega_{af}$. Thus the presence of the static electric field does not change the resonant character of the RS effect.

Therefore, there are two independent processes. The first one is given by Eqs. (12) and (13), when the photon emission occurs at the resonant frequency ω_{af}. The second one can be obtained with QED description. It can be characterized by the one-photon emission from the field-modified level $\overline{2s}$, given by Eq. (14) with the photon emission frequency $\omega_{\overline{a}f}$. In the case of two neighboring $2s$ and $2p$ states (the energies of which are equal in the nonrelativistic limit) the frequencies of these two emission lines differ by the Lamb shift:

$$\begin{aligned} \omega_{2p1s} - \omega_{\overline{2s}1s} &= E_{2p} - E_{2s} + \delta E_{2p}^L - \delta E_{2s}^L - \Delta E^{\mathrm{Stark}} \\ &= \delta E_{2p}^L - \delta E_{2s}^L - \Delta E^{\mathrm{Stark}} \approx \Delta E_{2s2p}^L, \end{aligned} \tag{15}$$

where δE_a^L denotes the Lamb shift of the state a. In total, the situation with $2p$ and $2s$ levels in the BBR field looks as

TABLE I. Numerical values $\Gamma_a^{\mathrm{BBR-QED}}$ for $2p$ and $2s$ states in the hydrogen atom in s^{-1} (the last two columns) for different values of radiation temperature T in Kelvin (first column). The corresponding $\Gamma_{2p}^{\mathrm{BBR-RS}}$ and $\Gamma_{2s}^{\mathrm{BBR-RS}}$ in s^{-1} are listed in second and third columns. The number in parentheses indicates the power of 10.

T (K)	$\Gamma_{2p}^{\mathrm{BBR-RS}}$	$\Gamma_{2s}^{\mathrm{BBR-RS}}$	$\Gamma_{2p}^{\mathrm{BBR-QED}}$	$\Gamma_{2s}^{\mathrm{BBR-QED}}$
3	4.782(−8)	1.434(−7)	0.475	1.42
300	4.743(−6)	1.422(−5)	3.572(3)	1.070(4)
1000	0.033	2.023(−2)	5.265(4)	1.208(5)
2000	1.916(3)	11.783(2)	7.134(8)	1.207(8)
3000	7.583(4)	470.062(2)	2.759(10)	4.651(9)
5000	1.522(6)	967.091(3)	5.198(11)	8.760(10)

follows. Both levels are broadened by RS and mixing effects. The broadening of the $2p$ level modifies the Ly_α spectral emission line but does not change the emission frequency. The broadening of the $2s$ level leads to the arrival of the new spectral line (one-photon E1 transition) with the frequency Lamb shifted from Ly_α. This happens exclusively due to the mixing effect, not by RS. The broadening effects produced by the mixing are much stronger than the broadening effects produced by RS.

The numerical calculations of $\Gamma_a^{\mathrm{BBR-QED}}$ [10] show that the mixing effect is dominant in comparison to the RS process (see Table I).

In the following the BBR-induced level-mixing effect (5) is discussed in application to the astrophysical investigation of the cosmological recombination epoch of the early universe (in SI units). The corresponding contribution can be evaluated similarly for the level mixing in the helium atom caused by the spin-orbit interaction [25,26]. Within the "three-level" approach [18] only the emission line corresponding to the one-photon decay in RS $2p \to 1s + \gamma(\mathrm{E1})$ together with the two-photon decay of the $2s$ state in hydrogen $2s \to 1s + 2\gamma(\mathrm{E1})$ are taken into account. According to the discussion above the additional electric dipole decay channel $\overline{2s} \to 1s + \gamma(\mathrm{E1})$ should be included into the rate equations.

The latter can be transformed to the differential equation for the ionization fraction $x_e = n_e/n_{\mathrm{H}}$, where n_e is the free-electron number density and n_{H} is the total number density of hydrogen atoms and ions. The time evolution of the density number of free electrons in a homogeneous, isotropic expanding universe can be described by the following differential equation:

$$\frac{dn_e}{dt} = -\sum_{nl}(\alpha_{\mathrm{H},nl} n_e n_p - \beta_{\mathrm{H},nl} n_{nl}) - 3n_e, \tag{16}$$

where n_{nl} is the number density of neutral hydrogen in the state with principal quantum number n and orbital momentum l, $n_p \simeq n_e$ is a number density of protons, $\alpha_{\mathrm{H},nl}$ is the recombination coefficient for the level nl, and $\beta_{\mathrm{H},nl}$ is the corresponding ionization coefficient. The last term in Eq. (16) describes the decreasing of number density n_e due to the cosmological expansion. The redshift z is related to time by the expression $dz/dt = -(1+z)H(z)$, where $H(z)$ is the Hubble factor [18]. The radiation temperature T_R is related to redshift as $T_R = T_0(1+z)$, where $T_0 = 2.725$ K is the present cosmic microwave background (CMB) temperature.

Then Eq. (16) can be rewritten in terms of ionization fraction x_e with the notations $x_p = n_p/n_{\mathrm{H}}$ and $x_{2s} = n_{2s}/n_{\mathrm{H}}$:

$$\frac{dx_e}{dt} = -(\alpha_{\mathrm{H}} x_e x_p - \beta_{\mathrm{H}} x_{2s}) \equiv J_{\mathrm{H}}, \tag{17}$$

where α_{H} and β_{H} are the total coefficients of recombination and ionization, respectively. Assuming that all the uncompensated transitions to the ground state J_{H} proceed via the two-photon decay $2s \to 1s + 2\gamma(\mathrm{E1})$ and escape of Ly_α photons $2p \to 1s + \gamma(\mathrm{E1})$ due to the cosmological expansion [27,28] we arrive at the balance condition

$$J_{\mathrm{H}} = J_{2s}^{\mathrm{E1E1}} + J_{2p}^{\mathrm{E1}}, \tag{18}$$

where J_{2s}^{E1E1} and J_{2p}^{E1} are the corresponding uncompensated transition rates.

Continuing to derivation of the differential equation for the ionization fraction x_e [18] we can introduce the contribution J_{2s}^{E1} for the one-photon transition rate $\overline{2s} \to 1s + \gamma(\mathrm{E1})$:

$$\tilde{J}_{\mathrm{H}} = J_{2s}^{\mathrm{E1E1}} + J_{2p}^{\mathrm{E1}} + J_{2s}^{\mathrm{E1}}. \tag{19}$$

The contribution of J_{2s}^{E1} can be written in the same form as J_{2p}^{E1} [18]:

$$J_{2s}^{\mathrm{E1}} = P_{2s1s} A_{\overline{2s}1s}\left[x_{2s} - \exp\left(-\frac{E_{2s} - E_{1s}}{k_B T}\right) x_{1s}\right]. \tag{20}$$

The two terms in the right-hand side of Eq. (20) represent the difference between forward and backward one-photon transitions $\overline{2s} \leftrightarrow 1s$. The Einstein coefficient $A_{\overline{2s}1s}$ is defined as the partial transition rate in Eq. (5), i.e., $A_{\overline{2s}1s} = \Gamma_{aa_0}^{\mathrm{BBR-QED}}$, where only one term from the sum over n is retained with $n = a_0 = 2p$ and $a = 2s$.

The Sobolev escape probability P_{2s1s} and optical depth τ_{2s} can be written as [18]

$$P_{2s1s} = \frac{1 - e^{-\tau_{2s}}}{\tau_{2s}}, \tag{21}$$

$$\tau_{2s} = \frac{A_{\overline{2s}1s} n_{1s} c^3}{8\pi H(z) \nu_{2s1s}^3} \frac{g_{2s}}{g_{1s}}. \tag{22}$$

Here g_{2s} and g_{1s} are the statistical weights of the states $2s$ and $1s$, respectively, ν_{2s1s} is the corresponding transition frequency. Insertion of Eqs. (19)–(22) into Eq. (17) gives the differential equation for the variable x_e.

Then the modified equation for the ionization fraction x_e with respect to the redshift z is

$$\frac{d\tilde{x}_e}{dz} = C_{\mathrm{H}} \frac{\left[\alpha_{\mathrm{H}} n_e \tilde{x}_e - \beta_{\mathrm{H}} \exp\left(-\frac{\Delta E_{21}}{k_B T}\right)(1 - \tilde{x}_e)\right]}{H(z)(1+z)}, \tag{23}$$

$$C_{\mathrm{H}} = \frac{\frac{g_{2p}}{g_{1s}} A_{2p1s}^{\mathrm{r}} + \frac{g_{2s}}{g_{1s}} A_{\overline{2s}1s}^{\mathrm{r}} + A_{2s1s}}{\beta_{\mathrm{H}} + \frac{g_{2p}}{g_{1s}} A_{2p1s}^{\mathrm{r}} + \frac{g_{2s}}{g_{1s}} A_{\overline{2s}1s}^{\mathrm{r}} + A_{2s1s}}. \tag{24}$$

Here we have used the short notation for the effective coefficient $A_{2s(2p)1s}^{\mathrm{r}} \equiv P_{2s(2p)1s} A_{\overline{2s}(2p)1s}$. Thus, an additional decay channel of the $\overline{2s}$ level has arisen in C_{H}, that gives the difference from the standard case Eq. (18).

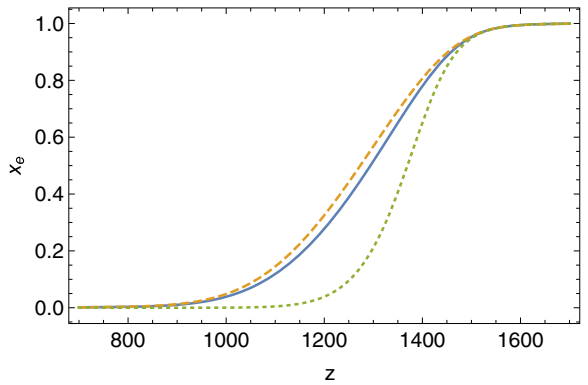

FIG. 2. Ionization fraction x_e as a function of redshift z. The dotted green line corresponds to the LTE case (Saha equation), the dashed yellow line is given by the evaluation of the "ordinary" rate equation, and the solid blue line represents the ionization fraction accounting for BBR-induced level mixing.

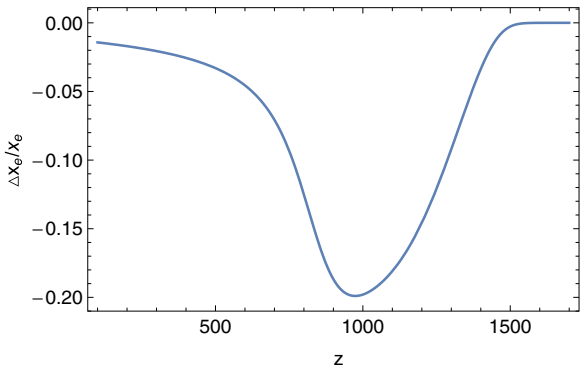

FIG. 3. Relative difference $\Delta x_e/x_e$ as a function of redshift z.

The ionization fraction x_e was evaluated with the use of the MATHEMATICA code. All necessary cosmological parameters are taken from [29]. The corresponding graph is presented in Fig. 2, where the function $x_e(z)$ for the case of local thermodynamic equilibrium, i.e., evaluated with the use of the Saha equation, is depicted as a dotted green line. Evaluation of the "ordinary" rate equation [18] is shown by the dashed yellow line and the solid blue line represents the ionization fraction accounting for BBR-induced level mixing.

We find a significant influence of the BBR-induced mixing effect on the ionization fraction in the cosmological recombination epoch of the early universe. However, the period of recombination is almost the same. Thus, the possible modification of the CMB temperature fluctuations map can be expected in the far tail of multipole expansion. The relative difference between the ionization fraction from Eq. (23) and calculated within the ordinary approach $\Delta x_e/x_e \equiv (\tilde{x}_e - x_e)/x_e$ is presented in Fig. 3. It is shown that the mixing effect is important during the period of cosmological recombination and reaches 20% at $z \approx 1000$. Therefore the contribution of the level-mixing effect should be taken into account in detailed investigation of the cosmological recombination epoch. It is important that the existence of the level-mixing effect can be tested in laboratory experiments as it was discussed above. Hence the laboratory studies may give access to the details of cosmological recombination.

The work of T.Z. was supported by foundation for the advancement of theoretical physics "BASIS". T.Z. also acknowledges the German-Russian Interdisciplinary Science Center. D.S. acknowledges support from Technische Universität Dresden (DAAD Programm Ostpartnerschaften).

[1] T. F. Gallagher and W. E. Cooke, Phys. Rev. Lett. **42**, 835 (1979).
[2] J. W. Farley and W. H. Wing, Phys. Rev. A **23**, 2397 (1981).
[3] S. G. Porsev and A. Derevianko, Phys. Rev. A **74**, 020502(R) (2006).
[4] C. Degenhardt, H. Stoehr, C. Lisdat, G. Wilpers, H. Schnatz, B. Lipphardt, T. Nazarova, P.-E. Pottie, U. Sterr, J. Helmcke, and F. Riehle, Phys. Rev. A **72**, 062111 (2005).
[5] M. S. Safronova, M. G. Kozlov, and C. W. Clark, Phys. Rev. Lett. **107**, 143006 (2011).
[6] W. M. Itano, L. L. Lewis, and D. J. Wineland, Phys. Rev. A **25**, 1233 (1982).
[7] I. L. Glukhov, E. A. Nikitina, and V. D. Ovsiannikov, J. Phys. B **49**, 035003 (2016).
[8] T. Middelmann, C. Lisdat, St. Falke, J. S. R. Vellore Winfred, F. Riehle, and U. Sterr, IEEE Trans. Instrum. Meas. **60**, 2550 (2011).
[9] C. Lisdat *et al.*, Nat. Commun. **7**, 12443 (2016).
[10] D. Solovyev, L. Labzowsky, and G. Plunien, Phys. Rev. A **92**, 022508 (2015).
[11] D. Solovyev, V. Sharipov, L. Labzowsky, and G. Plunien, J. Phys. B **43**, 074005 (2010).
[12] D. Solovyev and E. Solovyeva, Phys. Rev. A **91**, 042506 (2015).
[13] G. W. F. Drake, Bull. Am. Phys. Soc. **22**, 1315 (1977).
[14] H. Gould and R. Marrus, Bull. Am. Phys. Soc. **22**, (1977).
[15] P. J. Mohr, Phys. Rev. Lett. **40**, 854 (1978).
[16] M. Hillery and P. J. Mohr, Phys. Rev. A **21**, 24 (1980).
[17] Ya. I. Azimov, A. A. Anselm, A. N. Moskalev, and R. M. Ryndin, Zh. Eksp. Teor. Fiz. **67**, 17 (1974) [Sov. Phys. JETP **40**, 8 (1975)].
[18] S. Seager, D. D. Sasselov, and D. Scott, Astrophys. J. Suppl. Ser. **128**, 407 (2000).
[19] L. N. Labzowsky, D. A. Solovyev, G. Plunien, and G. Soff, Phys. Rev. Lett. **87**, 143003 (2001).
[20] O. Y. Andreev, L. N. Labzowsky, G. Plunien, and D. A. Solovyev, Phys. Rep. **455**, 135 (2008).
[21] T. Zalialiutdinov, Y. Baukina, D. Solovyev, and L. Labzowsky, J. Phys. B **47**, 115007 (2014).
[22] A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics* (Wiley, New York, 1965).
[23] L. Labzowsky, V. Karasiev, and I. Goidenko, J. Phys. B **27**, L439 (1994).
[24] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics* (Pergamon Press, Oxford, 1982).

[25] V. K. Dubrovich and S. I. Grachev, Astron. Lett. **31**, 359 (2005).
[26] W. Y. Wong and D. Scott, Mon. Not. R. Astron. Soc. **375**, 1441 (2007).
[27] P. J. E. Peebles, Astrophys. J. **153**, 1 (1968).
[28] Ya. B. Zel'dovich, V. G. Kurt, and R. A. Sunyaev, Zh. Eksp. Teor. Fiz. **55**, 278 (1968) [Sov. Phys. JETP **28**, 146 (1969)].
[29] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **594**, A13 (2016).