One-step generation of continuous-variable quadripartite cluster states in a circuit QED system

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We propose a dissipative scheme for one-step generation of continuous-variable quadripartite cluster states in a circuit QED setup consisting of four superconducting coplanar waveguide resonators and a gap-tunable superconducting flux qubit. With external driving fields to adjust the desired qubit-resonator and resonator-resonator interactions, we show that continuous-variable quadripartite cluster states of the four resonators can be generated with the assistance of energy relaxation of the qubit. By comparison with the previous proposals, the distinct advantage of our scheme is that only one step of quantum operation is needed to realize the quantum state engineering. This makes our scheme simpler and more feasible in experiment. Our result may have useful application for implementing quantum computation in solid-state circuit QED systems.

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I. INTRODUCTION

Quantum entanglement plays a central role in the field of quantum optics and quantum information [1]. It is an indispensable ingredient for implementing various quantum information protocols, such as quantum teleportation and quantum dense coding [2,3]. Among various kinds of entangled states, cluster states are a type of highly entangled state of multiple qubits and have more sophisticated quantum correlations [4]. For example, different from other types of multiparticle entangled states such as the W state or Greenberger-Horne-Zeilinger (GHZ) state, cluster states are more robust against projective measurement; that is, the quantum entanglement cannot be eliminated by the measurement on single qubits. Cluster states are especially useful in the context of one-way quantum computation [5–7]. Moreover, it has been demonstrated that any quantum circuits and quantum gates can be implemented on a suitable cluster state [8–11]. Experimentally, the generation of cluster states has been achieved with photonic qubits [12–14] and cold atoms trapped in optical lattices [15].

In parallel, continuous-variable (CV) cluster states [16], like the amplitude and phase quadratures of an electromagnetic field, have also attracted much attention for implementing one-way quantum computation and constructing quantum networks [17–19]. Due to high efficiency in manipulation and detection of CV quadrature components, the experimental generation of CV cluster states has also been widely investigated [20–24]. By employing nondegenerate optical parametric amplifiers and linear optics, CV cluster states can be created by linear optical transformation of squeezed light with certain phase relations [25,26]. However, it is hard to implement scalable quantum computing with optical CV cluster states due to the inherent limitation of large-scale integration for a linear optics system. Schemes have also been proposed to create CV cluster states with the collective excitations of separated atomic ensembles located inside a ring cavity [27–29], as well as the microwave fields of superconducting resonators [30]. In addition, the generation of CV cluster states has been considered in an optomechanical system with the motional states of mechanical resonators [31]. However, the procedures in those schemes are relatively complex and require multiple steps to realize the target states. Therefore, it is crucial to produce a cluster state in just one step on a scalable circuit.

In this paper, we propose a one-step scheme for generation of four-mode CV cluster states in a circuit QED system. The solid-state superconducting circuit has many merits such as controllability, tunability, and scaling on chip with nanofabrication techniques [32,33]. It has become a leading candidate for present-day realization of quantum information processing [34–45]. The model under consideration consists of four superconducting coplanar waveguide resonators and a gap-tunable qubit. With the appropriate choice of frequencies and phases of the external driving fields, we can tailor an effective Hamiltonian; that is, the qubit is coupled to one of the combined modes of the resonators via the squeezing-type interaction, while the other three combined modes couples with each other via the beam-splitter-type interaction. The key idea of our work is to utilize dissipation of the qubit to directly cool one of the combined modes into a squeezed state, while the other modes are indirectly swapped into the squeezed state via the intramode beam-splitter-type interactions. Thus, all the combined modes are simultaneously driven into the same single-mode squeezed vacuum state at steady state, which is identical to steering the four resonators into a stationary CV cluster state. The distinct advantage of our scheme is that the CV cluster states can be achieved with only one step of quantum operation. It is simpler and more feasible in the experimental implementation. The present work may have useful application for performing quantum computation in a solid-state circuit QED system.

II. MODEL

In Fig. 1, a possible setup for the implementation of the present proposal is shown. Four coplanar waveguide resonators are connected one by one via superconducting quantum interference devices (SQUIDs) and form a closed resonator chain. On the other hand, resonators 1 and 2 are

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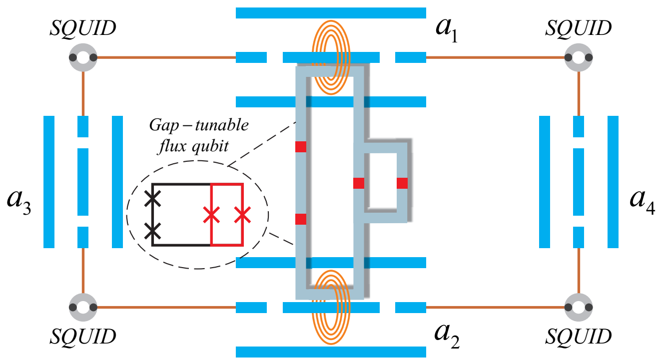

FIG. 1. A possible setup for four-resonator circuit QED with a gap-tunable flux qubit. Four superconducting coplanar waveguide resonators are linearly coupled with each other via SQUIDs, and a gap-tunable superconducting flux qubit is coupled to resonator modes a_1 and a_2 via the mutual inductance. The qubit has a low-inductance dc SQUID loop which is used to realize energy gap tunability, denoted by a red circle.

coupled by a gap-tunable flux qubit. The qubit functions as a quantum switch to mediate switchable coupling between resonators 1 and 2. As usual, a superconducting flux qubit is composed of three Josephson junctions forming a loop, in which two junctions have the same large critical currents but the third junction has a smaller one. Note that the energy gap of a flux qubit at the degenerate point strongly depends on the critical current of the third junction. To realize a gap-tunable flux qubit, the third junction is replaced by two parallel junctions forming a low-inductance dc SQUID loop, as shown in the dashed circle in Fig. 1, and the energy gap can be tuned by applying an external magnetic field penetrating the SQUID loop. If the applied magnetic field is periodic and has multiple frequencies, one can create a σ_z driving such as the one described by Eq. (4). In fact, the *in situ* tunability of the minimum-energy gap of a superconducting flux qubit has been experimentally demonstrated [46]. In current experiments, there are several schemes for the realization of the coupling between superconducting qubits and coplanar waveguide resonators. For example, charge or flux qubits can be fabricated on the ground plane of a coplanar waveguide resonator, and thus, either the electrical coupling [36,47,48] or magnetic coupling [49–51] can be realized. The scheme using a single qubit to couple two coplanar waveguide resonators may be implemented via capacitors at the two ends of the resonators [40,52]. Here, we propose a scheme in which the larger loop of the flux qubit, which contains the three Josephson junctions, is transversely fabricated on the surface of coplanar waveguide resonators 1 and 2 [53]. In this way, the Jaynes-Cummings-type interaction between the qubit and the resonators, described by the Hamiltonian (3), may be realized via the mutual inductance. Here, we assume that the couplings of the qubit with the two resonators are identical and thus have the same strength. It is noted that the coupling of a flux qubit with two microstrip resonators via the mutual inductance has been proposed and investigated in detail [54]. For the setup shown in Fig. 1, we have the following Hamiltonians.

The free Hamiltonian of the whole system is described by ($\hbar = 1$)

$$H_0 = \frac{\delta}{2}\sigma_z + \sum_{j=1}^{4}\omega_j a_j^\dagger a_j, \tag{1}$$

where $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ is the population-inversion operator of the qubit and $a_j^\dagger, a_j$ are the creation and annihilation operators of bosonic resonator modes. The static qubit energy gap between the ground state $|g\rangle$ and the excited state $|e\rangle$ is δ, and the characteristic frequencies of the four resonators are denoted by ω_j.

We consider a time-dependent resonator-resonator interaction, described by

$$H_{RR} = \sum_{m=1}^{2}\sum_{n=3}^{4}\alpha_{mn}(t)(a_m^\dagger + a_m)(a_n^\dagger + a_n), \tag{2}$$

where $\alpha_{mn}(t) = g_{mn}\cos(\omega_{mn}t + \varphi_{mn})$ denotes the time-dependent coupling strength between resonator modes a_m and a_n and ω_{mn} and φ_{mn} are the modulation frequencies and phases, respectively. Experimentally, this Hamiltonian can be implemented by means of a SQUID connecting two resonators [55]. The SQUID is composed of a superconducting loop interrupted by two identical Josephson junctions [56,57]. By means of fluxoid quantization of the loop, the SQUID behaves as a single Josephson junction with a tunable inductance $L(\Phi_{\rm ext}) = (\frac{\Phi_0}{2\pi})^2 \frac{1}{2E_J\cos(\pi\Phi_{\rm ext}/\Phi_0)}$, in which $\Phi_0 = \frac{h}{2e}$ is the flux quanta, $\Phi_{\rm ext}$ is the external flux threading the SQUID loop, and E_J is the Josephson energy [58,59]. The variation of the inductance of the SQUID can change the electrical boundary condition of the resonators and their interaction. When applying a fast-oscillating magnetic field with flux $\Phi(t)$ threading the SQUID, one can realize a time-dependent interaction between the two resonators [55].

The interaction between qubit and resonator modes a_1 and a_2 is given by

$$H_{QR} = (\sigma_+ + \sigma_-)\sum_{i=1,2} g_i(a_i^\dagger + a_i), \tag{3}$$

where g_i is the coupling strength between the qubit and the ith resonator and $\sigma_+,\sigma_- = |e\rangle\langle g|,|g\rangle\langle e|$ are the spin-flip operators of the qubit. In addition, we apply two pairs of the σ_z driving to the qubit, which is described by the time-dependent Hamiltonian [60,61]

$$H_{\rm drive}(t) = -\sum_{l=1}^{2}\sum_{k=1}^{2}\xi_{lk}\omega_{dlk}\cos(\omega_{dlk}t + \phi_{lk})\sigma_z, \tag{4}$$

where ω_{dlk} is the driving frequency, ξ_{lk} is the ratio between the driving amplitude and frequency, and ϕ_{lk} is the phase of the driving ($k,l = 1,2$). This Hamiltonian can induce sideband transitions between the qubit and resonators, which is used to modify the qubit-resonator interactions. This quantum manipulation has been demonstrated in experiment with various types of qubits, such as flux [46,62], charge [63], and transmon [64] qubits.

The time evolution of the whole system is governed by the master equation

$$\frac{d\rho}{dt} = -i[H,\rho] + \Gamma D[\sigma_-]\rho, \tag{5}$$

where $H = H_0 + H_{RR} + H_{QR} + H_{\text{drive}}(t)$, ρ is the density-matrix operator, $D[\sigma_-]\rho = 2\sigma_-\rho\sigma_+ - \rho\sigma_+\sigma_- - \sigma_+\sigma_-\rho$ is the standard Lindblad operator, and Γ is the energy relaxation rate of the qubit. Here, we have assumed that the resonators possess high-quality factors, i.e., with a negligible decay rate κ_j. On the other hand, here, as discussed in the following, we utilize the energy decay of the qubit from the excited state to the ground state to bring the resonator modes to the CV cluster states. Thus, the dephasing process of the qubit is ignored since it cannot change the unique steady state of the system.

III. GENERATION OF CONTINUOUS-VARIABLE QUADRIPARTITE CLUSTER STATES

In this section, we discuss in detail how to generate quadripartite CV cluster states of the four resonators by one step via the dissipation of the qubit. For four entities, there are three kinds of cluster states: linear cluster state, square cluster state, and T-shaped cluster state, as shown in Fig. 2. For a continuous-variable quadripartite cluster state, the variances of the entities obey the condition [24]

$$V\left(P_j - \sum_{i\in N_j} X_i\right) \to 0, \quad j = 1,2,3,4, \tag{6}$$

in the limit of infinite squeezing. In Eq. (6), the Hermitian quadrature amplitude and phase operators X_j and P_j are defined based on the relation $a_j = \frac{1}{\sqrt{2}}(X_j - iP_j)$, which is the photon annihilation operator of jth mode, and $j \in N_i$ means that mode j is among the nearest neighbors of mode i.

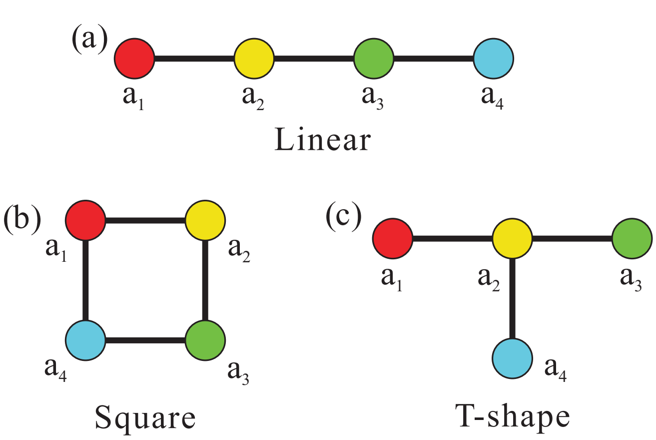

FIG. 2. Schematic of CV quadripartite cluster states: (a) linear cluster state, (b) square cluster state, and (c) T-shaped cluster state.

For a linear cluster state, we readily have the following quadrature combinations according to Eq. (6):

$$\begin{aligned}
P_{a_1} - X_{a_2} &= A_1^\dagger + A_1,\\
P_{a_2} - X_{a_1} - X_{a_3} &= A_2^\dagger + A_2,\\
P_{a_3} - X_{a_2} - X_{a_4} &= A_3^\dagger + A_3,\\
P_{a_4} - X_{a_3} &= A_4^\dagger + A_4,
\end{aligned} \tag{7}$$

where the linearly combined modes A_j are given by

$$\begin{pmatrix} A_1\\ A_2\\ A_3\\ A_4 \end{pmatrix} = \begin{pmatrix} -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0\\ -\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ 0 & -\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ 0 & 0 & -\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} a_1\\ a_2\\ a_3\\ a_4 \end{pmatrix}. \tag{8}$$

Note that the four combined modes A_1, A_2, A_3, A_4 are not orthogonal with each other and cannot be operated independently. To orthogonalize the combined modes (8), we make the unitary transformation

$$\begin{pmatrix} \bar{A}_1\\ \bar{A}_2\\ \bar{A}_3\\ \bar{A}_4 \end{pmatrix} = \begin{pmatrix} -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0\\ -\frac{1}{\sqrt{3}} & -\frac{i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0\\ \frac{i}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & -\frac{2i}{\sqrt{10}} & -\frac{2}{\sqrt{10}}\\ \frac{1}{\sqrt{15}} & \frac{i}{\sqrt{15}} & -\frac{2}{\sqrt{15}} & -\frac{3i}{\sqrt{15}} \end{pmatrix} \begin{pmatrix} a_1\\ a_2\\ a_3\\ a_4 \end{pmatrix}. \tag{9}$$

In terms of these new orthogonal combined modes, the variances of the quadrature amplitude combinations (7) can be written as

$$\begin{aligned}
V_1 &= V(P_{a_1} - X_{a_2}) = V(\bar{A}_1 + \bar{A}_1^\dagger),\\
V_2 &= V(P_{a_2} - X_{a_1} - X_{a_3}) = \frac{3}{2}V(\bar{A}_2 + \bar{A}_2^\dagger),\\
V_3 &= V(P_{a_3} - X_{a_2} - X_{a_4})\\
&= \frac{1}{4}V(\bar{A}_1 + \bar{A}_1^\dagger) + \frac{5}{4}V(\bar{A}_3 + \bar{A}_3^\dagger),\\
V_4 &= V(P_{a_4} - X_{a_3})\\
&= \frac{1}{6}V(\bar{A}_2 + \bar{A}_2^\dagger) + \frac{5}{6}V(\bar{A}_4 + \bar{A}_4^\dagger).
\end{aligned} \tag{10}$$

For a square cluster state, we consider the quadrature combinations

$$\begin{aligned}
P_{a_1} - X_{a_2} - X_{a_4} &= -\frac{1}{\sqrt{2}}[i(a_1 - a_1^\dagger) + (a_2 + a_2^\dagger)\\
&\quad + (a_4 + a_4^\dagger)],\\
P_{a_2} - X_{a_1} - X_{a_3} &= -\frac{1}{\sqrt{2}}[i(a_2 - a_2^\dagger) + (a_1 + a_1^\dagger)\\
&\quad + (a_3 + a_3^\dagger)],\\
P_{a_3} - X_{a_2} - X_{a_4} &= -\frac{1}{\sqrt{2}}[i(a_3 - a_3^\dagger) + (a_2 + a_2^\dagger)\\
&\quad + (a_4 + a_4^\dagger)],\\
P_{a_4} - X_{a_1} - X_{a_3} &= -\frac{1}{\sqrt{2}}[i(a_4 - a_4^\dagger) + (a_1 + a_1^\dagger)\\
&\quad + (a_3 + a_3^\dagger)].
\end{aligned} \tag{11}$$

Introducing the orthogonal combined modes

$$\begin{pmatrix}\bar{A}_1\\ \bar{A}_2\\ \bar{A}_3\\ \bar{A}_4\end{pmatrix} = \begin{pmatrix}\frac{i}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}} & \frac{i}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{2i}{\sqrt{15}} & \frac{1}{\sqrt{15}} & \frac{3i}{\sqrt{15}} & \frac{1}{\sqrt{15}}\\ \frac{1}{\sqrt{15}} & -\frac{2i}{\sqrt{15}} & \frac{1}{\sqrt{15}} & \frac{3i}{\sqrt{15}}\end{pmatrix}\begin{pmatrix}a_1\\ a_2\\ a_3\\ a_4\end{pmatrix} \qquad (12)$$

and their Hermitian conjugates, one can write the variances of the quadrature combinations (11) as

$$\begin{aligned}V_1 &= V(P_{a_1} - X_{a_2} - X_{a_4}) = -\frac{\sqrt{6}}{2}V(\bar{A}_1 + \bar{A}_1^\dagger),\\ V_2 &= V(P_{a_2} - X_{a_1} - X_{a_3}) = -\frac{\sqrt{6}}{2}V(\bar{A}_2 + \bar{A}_2^\dagger),\\ V_3 &= V(P_{a_3} - X_{a_2} - X_{a_4})\\ &= -\frac{\sqrt{6}}{3}V(\bar{A}_1 + \bar{A}_1^\dagger) - \frac{\sqrt{30}}{6}V(\bar{A}_3 + \bar{A}_3^\dagger),\\ V_4 &= V(P_{a_4} - X_{a_1} - X_{a_3})\\ &= -\frac{\sqrt{6}}{3}V(\bar{A}_2 + \bar{A}_2^\dagger) - \frac{\sqrt{30}}{6}V(\bar{A}_4 + \bar{A}_4^\dagger). \qquad (13)\end{aligned}$$

For a T-shaped cluster state, we consider the quadrature amplitude combinations

$$\begin{aligned}P_{a_2} - X_{a_1} - X_{a_3} - X_{a_4} &= -\frac{1}{\sqrt{2}}[i(a_2 - a_2^\dagger) + (a_1 + a_1^\dagger)\\ &\quad + (a_3 + a_3^\dagger) + (a_4 + a_4^\dagger)],\\ P_{a_1} - X_{a_2} &= -\frac{1}{\sqrt{2}}[i(a_1 - a_1^\dagger) + (a_2 + a_2^\dagger)],\\ P_{a_3} - X_{a_2} &= -\frac{1}{\sqrt{2}}[i(a_3 - a_3^\dagger) + (a_2 + a_2^\dagger)],\\ P_{a_4} - X_{a_2} &= -\frac{1}{\sqrt{2}}[i(a_4 - a_4^\dagger) + (a_2 + a_2^\dagger)].\end{aligned} \qquad (14)$$

Introducing the four orthogonal combined modes $\bar{A}_j$ by the unitary transformation

$$\begin{pmatrix}\bar{A}_1\\ \bar{A}_2\\ \bar{A}_3\\ \bar{A}_4\end{pmatrix} = \begin{pmatrix}\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0\\ \frac{1}{2} & \frac{i}{2} & \frac{1}{2} & \frac{1}{2}\\ -\frac{i}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{2i}{\sqrt{6}} & 0\\ -\frac{i}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{i}{2\sqrt{3}} & \frac{3i}{2\sqrt{3}}\end{pmatrix}\begin{pmatrix}a_1\\ a_2\\ a_3\\ a_4\end{pmatrix} \qquad (15)$$

and their Hermitian conjugates, one can write the variances of the quadrature combinations (14) as

$$\begin{aligned}V_1 &= V(P_{a_1} - X_{a_2}) = -V(\bar{A}_1 + \bar{A}_1^\dagger),\\ V_2 &= V(P_{a_2} - X_{a_1} - X_{a_3} - X_{a_4}) = -\sqrt{2}V(\bar{A}_2 + \bar{A}_2^\dagger),\\ V_3 &= V(P_{a_3} - X_{a_2})\\ &= -\frac{1}{2}V(\bar{A}_1 + \bar{A}_1^\dagger) - \frac{\sqrt{3}}{2}V(\bar{A}_3 + \bar{A}_3^\dagger),\\ V_4 &= V(P_{a_4} - X_{a_2}) = -\frac{1}{2}V(\bar{A}_1 + \bar{A}_1^\dagger)\\ &\quad -\frac{1}{2\sqrt{3}}V(\bar{A}_3 + \bar{A}_3^\dagger) - \frac{2}{\sqrt{6}}V(\bar{A}_4 + \bar{A}_4^\dagger). \qquad (16)\end{aligned}$$

Obviously, the variances of these three types of CV cluster states are determined by the combined quadratures $\bar{A}_j + \bar{A}_j^\dagger$. If we can prepare the four orthogonal modes $\bar{A}_1,\bar{A}_2,\bar{A}_3,\bar{A}_4$ in amplitude squeezed vacuum states with a large squeezing degree, the variances V_j will satisfy the conditions $V_j \to 0$ in Eq. (6). This is equivalent to preparing the four resonator modes a_1,a_2,a_3,a_4 in the CV cluster states. Therefore, the main task of our work is to steer all the combined modes $\bar{A}_j$ into single-mode squeezed vacuum states. In the following we proceed to engineer an effective Hamiltonian which can simultaneously cool down the four modes $\bar{A}_1,\bar{A}_2,\bar{A}_3,\bar{A}_4$ to single-mode squeezed states via the dissipation of the qubit. Since the strategies for the generation of the linear, square, and T-shaped cluster states are similar, as an example, here, we discuss only the case of preparing a linear cluster state.

In order to work out the effective Hamiltonian for the generation of a linear cluster state, we perform a rotating transformation $U_1 = e^{-iH't}$, with $H' = \frac{\delta}{2}\sigma_z + \sum_{j=1}^4 \omega_{0j} a_j^\dagger a_j$, to Eq. (5), where we have set $\omega_{01} - \omega_1 = 2\Omega$, $\omega_2 - \omega_{02} = 2\Omega$, $\omega_{03} - \omega_3 = 2\Omega$, $\omega_4 - \omega_{04} = 2\Omega$. Then, the resulting Hamiltonian is

$$H = H_0' + H_{QR}' + H_{RR}' + H_{\mathrm{drive}}(t), \qquad (17)$$

where

$$H_0' = 2\Omega(-a_1^\dagger a_1 + a_2^\dagger a_2 - a_3^\dagger a_3 + a_4^\dagger a_4), \qquad (18)$$

$$H_{QR}' = \sigma_+ e^{i\delta t}\sum_{j=1,2} g_j(a_j^\dagger e^{-i\omega_{0j}t} + a_j e^{i\omega_{0j}t}) + \text{H.c.}, \qquad (19)$$

$$\begin{aligned}H_{RR}' = \sum_{m=1}^{2}\sum_{n=3}^{4} & g_{mn}\cos(\omega_{mn}t + \varphi_{mn})\\ &\times(a_m^\dagger e^{-i\omega_{0m}t} + a_m e^{i\omega_{0m}t})(a_n^\dagger e^{-i\omega_{0n}t} + a_n e^{i\omega_{0n}t}).\end{aligned} \qquad (20)$$

In the next step, we perform another unitary transformation $U_2 = Te^{-i\int H_{\mathrm{drive}}(t')dt'}$, where T is the time-order operator. Note that the Hamiltonian $H_{\mathrm{drive}}(t)$ contains only the operator σ_z and commutes at a different time. Thus, directly working out the integral and sum, we have $U_2 = \exp[i\sum_{l=1}^2\sum_{k=1}^2 \xi_{lk}\sin(\omega_{dlk}t + \phi_{lk})\sigma_z]$. Under the unitary transformation U_2, the operators $\sigma_\pm$ acquire the total phases factor $\exp[\pm i\sum_{l=1}^2\sum_{k=1}^2 \xi_{lk}\sin(\omega_{dlk}t + \phi_{lk})]$. If the driving parameters ξ_{lk} are chosen to be small, we make a Taylor expansion of the phase factor with respect to ξ_{lk} and keep all the terms up to the first order of ξ_{lk}. Then, the Hamiltonian H_{QR}' can be written after the unitary transformation U_2 in the form

$$\begin{aligned}H_{QR}' = \sigma_+ e^{i\delta t} & \sum_{j=1,2} g_j(a_j^\dagger e^{-i\omega_{0j}t} + a_j e^{i\omega_{0j}t})\\ &\times\Big[1 - \sum_{l=1}^{2}\sum_{k=1}^{2}\xi_{lk}(e^{i(\omega_{dlk}t+\phi_{lk})} - e^{-i(\omega_{dlk}t+\phi_{lk})})\Big]\\ &+\text{H.c.}\end{aligned} \qquad (21)$$

In order to obtain the desired qubit-resonator interaction, we choose external driving frequencies and phases to satisfy the following relations:

$$\begin{aligned} \omega_{d11} &= \delta - \omega_{01} = \delta - \omega_1 - 2\Omega, \\ \omega_{d12} &= \delta + \omega_{01} = \delta + \omega_1 + 2\Omega, \\ \omega_{d21} &= \delta - \omega_{02} = \delta - \omega_2 + 2\Omega, \\ \omega_{d22} &= \delta + \omega_{02} = \delta + \omega_2 - 2\Omega, \end{aligned} \tag{22}$$

and

$$\phi_{11} = -\frac{\pi}{2}, \phi_{12} = \frac{\pi}{2}, \phi_{21} = \phi_{22} = \pi. \tag{23}$$

We retain the resonant terms while discarding those fast-oscillating terms in Eq. (21) under the rotating-wave approximation condition $\{\omega_j, \delta, \omega_{d1l}, \omega_{d2l}\} \gg g, \Omega$. Then, the Hamiltonian H'_{QR} can be approximated to

$$H''_{QR} = \sigma_+[ig_1(\xi_1 a_1^\dagger - \xi_2 a_1) - g_2(\xi_1 a_2^\dagger + \xi_2 a_2)] + \text{H.c.} \tag{24}$$

Here, we have set $\xi_{11} = \xi_{21} = \xi_1$ and $\xi_{12} = \xi_{22} = \xi_2$.

To further simplify the resonator-resonator interaction, a suitable choice of the driving frequencies ω_{mn} allows us to select the resonant interaction terms in Hamiltonian H'_{RR}. By choosing the frequencies to satisfy the relations

$$\begin{aligned} \omega_{13} &= \omega_{01} - \omega_{03} = \omega_1 - \omega_3, \\ \omega_{23} &= \omega_{02} - \omega_{03} = \omega_2 - \omega_3 - 4\Omega, \\ \omega_{14} &= \omega_{01} - \omega_{04} = \omega_1 - \omega_4 + 4\Omega, \\ \omega_{24} &= \omega_{02} - \omega_{04} = \omega_2 - \omega_4 \end{aligned} \tag{25}$$

and the phases

$$\varphi_{13} = \pi, \quad \varphi_{23} = -\frac{\pi}{2}, \quad \varphi_{14} = \frac{\pi}{2}, \quad \varphi_{24} = 0 \tag{26}$$

and performing the rotating-wave approximation under the condition $\{\omega_{cmn}, \omega_{0m}, \omega_{0n}\} \gg g_{mn}$ and neglecting the fast-oscillating terms in H'_{RR}, we have

$$H''_{RR} = -\frac{\Omega}{2} a_1^\dagger a_3 - i\frac{3\Omega}{2} a_2^\dagger a_3 + i\frac{\Omega}{2} a_1^\dagger a_4 + \frac{\Omega}{2} a_2^\dagger a_4 + \text{H.c.}, \tag{27}$$

where $g_{13} = g_{14} = g_{24} = \Omega$ and $g_{23} = 3\Omega$. Finally, we achieve the effective Hamiltonian of the whole system,

$$\begin{aligned} H_{\text{eff}} &= H'_0 + H''_{QR} + H''_{RR} \\ &= 2\Omega(-a_1^\dagger a_1 + a_2^\dagger a_2 - a_3^\dagger a_3 + a_4^\dagger a_4) \\ &\quad + \Big\{ -\frac{\Omega}{2} a_1^\dagger a_3 - i\frac{3\Omega}{2} a_2^\dagger a_3 + i\frac{\Omega}{2} a_1^\dagger a_4 + \frac{\Omega}{2} a_2^\dagger a_4 \\ &\quad + \sigma_+[ig_1(\xi_1 a_1^\dagger - \xi_2 a_1) - g_2(\xi_1 a_2^\dagger + \xi_2 a_2)] + \text{H.c.} \Big\} \end{aligned} \tag{28}$$

In terms of $\bar{A}_1, \bar{A}_2, \bar{A}_3, \bar{A}_4$, we have

$$\begin{aligned} H_{\text{eff}} &= \frac{1}{\sqrt{2}}\sigma_+\Big[\frac{1}{\sqrt{2}} g_+(\xi_2 \bar{A}_1 + \xi_1 \bar{A}_1^\dagger) - \frac{i}{\sqrt{3}}\Delta g(\xi_2 \bar{A}_2 - \xi_1 \bar{A}_2^\dagger) \\ &\quad + \frac{1}{\sqrt{10}}\Delta g(\xi_2 \bar{A}_3 + \xi_1 \bar{A}_3^\dagger) + \frac{i}{\sqrt{15}}\Delta g(\xi_2 \bar{A}_4 - \xi_1 \bar{A}_4^\dagger)\Big] \\ &\quad - i\sqrt{6}\Omega \bar{A}_1^\dagger \bar{A}_2 + i\sqrt{\frac{5}{6}}\Omega \bar{A}_2^\dagger \bar{A}_3 - i\frac{5}{\sqrt{6}}\Omega \bar{A}_3^\dagger \bar{A}_4 + \text{H.c.}, \end{aligned} \tag{29}$$

where $g_+ = g_1 + g_2$ and $\Delta g = g_2 - g_1$.

As shown in Eq. (10), the linear cluster state requires that variances of the four combined modes must simultaneously approach zero. Thus, one can realize the linear cluster state if the four combined modes are simultaneously in squeezed states. However, Eq. (29) shows that the combined modes $\bar{A}_{1,3}$ may undergo the squeezing transformation $C_{1,3} = \cosh(r)\bar{A}_{1,3} + \sinh(r)\bar{A}_{1,3}^\dagger$ and $\bar{A}_{2,4}$ may undergo the antisqueezing transformation $C_{2,4} = \cosh(-r)\bar{A}_{2,4} + \sinh(-r)\bar{A}_{2,4}^\dagger$, with $\cosh(r) = \xi_2/\sqrt{\xi_2{}^2 - \xi_1{}^2}$ and $\sinh(r) = \xi_1/\sqrt{\xi_2{}^2 - \xi_1{}^2}$ via the qubit. Thus, to get the four combined modes to be in squeezed states, we first set $g_1 = g_2 = g$. In this ideal case, the effective Hamiltonian is simplified to

$$\begin{aligned} H_{\text{eff}} &= g\sigma_+(\xi_2 \bar{A}_1 + \xi_1 \bar{A}_1^\dagger) - i\sqrt{6}\Omega \bar{A}_1^\dagger \bar{A}_2 \\ &\quad + i\sqrt{\frac{5}{6}}\Omega \bar{A}_2^\dagger \bar{A}_3 - i\frac{5}{\sqrt{6}}\Omega \bar{A}_3^\dagger \bar{A}_4 + \text{H.c.} \end{aligned} \tag{30}$$

The master equation (5) is now written as

$$\frac{d\tilde{\rho}}{dt} = -i[H_{\text{eff}}, \tilde{\rho}] + \Gamma D[\sigma_-]\tilde{\rho}, \tag{31}$$

where $\tilde{\rho} = U_2^\dagger U_1^\dagger \rho U_1 U_2$. It is seen that the first term in Hamiltonian (30) describes a squeezing-type interaction between the qubit and mode $\bar{A}_1$. So the dissipator $\Gamma D[\sigma_-]\tilde{\rho}$ will bring mode $\bar{A}_1$ into the single-mode squeezed vacuum. Due to the intramode interaction described by the rest of the terms of Hamiltonian (30), the squeezing may be transferred from $\bar{A}_1$ to the other modes, $\bar{A}_2, \bar{A}_3, \bar{A}_4$. To make the process clearer and more concrete, we apply the squeezing transformation $\bar{\rho} = S^\dagger \tilde{\rho} S$ to Eq. (31), in which $S = \otimes_{j=1}^4 S(\bar{A}_j)$ and $S(\bar{A}_j) = \exp(-\frac{1}{2} r \bar{A}_j^2 + \frac{1}{2} r \bar{A}_j^{\dagger 2})$ is the standard squeezing operator with the squeezing degree $r = \tanh^{-1}(\xi_1/\xi_2)$. Then, the master equation (31) becomes

$$\frac{d\bar{\rho}}{dt} = -i[\bar{H}_{\text{eff}}, \bar{\rho}] + \Gamma D[\sigma_-]\bar{\rho}, \tag{32}$$

where

$$\begin{aligned} \bar{H}_{\text{eff}} &= g\sqrt{\xi_2^2 - \xi_1^2}\,\sigma^\dagger \bar{A}_1 - i\sqrt{6}\Omega \bar{A}_1^\dagger \bar{A}_2 + i\sqrt{\frac{5}{6}}\Omega \bar{A}_2^\dagger \bar{A}_3 \\ &\quad - i\frac{5}{\sqrt{6}}\Omega \bar{A}_3^\dagger \bar{A}_4 + \text{H.c.} \end{aligned} \tag{33}$$

In this transformed picture, the Hamiltonian $\bar{H}_{\text{eff}}$ describes a set of Jaynes-Cummings-type interactions. It is clear that the qubit will continuously absorb energy from modes $\bar{A}_1, \bar{A}_2, \bar{A}_3, \bar{A}_4$ and then decay back to its ground state with the assistance of energy relaxation. According to the above dissipative repumping process, the whole system will be driven into the unique steady state, i.e., the ground state $\otimes_{j=1}^4 |0_{\bar{A}_j}\rangle \otimes |g\rangle$, in which $\otimes_{j=1}^4 |0_{\bar{A}_j}\rangle$ represents the tensor product vacuum of the combined modes $\bar{A}_1, \bar{A}_2, \bar{A}_3, \bar{A}_4$. Going back to the original representation, we will get the steady state of the system,

$$|\psi_S\rangle = \otimes_{j=1}^4 \exp\Big[-\frac{r}{2}(\bar{A}_j^{\dagger 2} - \bar{A}_j^2)\Big]|0_{\bar{A}_j}\rangle \otimes |g\rangle, \tag{34}$$

with all the combined modes $\bar{A}_j$ stabilized onto the same single-mode squeezed state at the stationary state, and the variances of quadrature combinations (10) will be

$$\begin{aligned} V_1 &= V(\bar{A}_1 + \bar{A}_1^\dagger) = e^{-2r}, \\ V_2 &= \tfrac{3}{2} V(\bar{A}_2 + \bar{A}_2^\dagger) = \tfrac{3}{2} e^{-2r}, \\ V_3 &= \tfrac{1}{4} V(\bar{A}_1 + \bar{A}_1^\dagger) + \tfrac{5}{4} V(\bar{A}_3 + \bar{A}_3^\dagger) = \tfrac{3}{2} e^{-2r}, \\ V_4 &= \tfrac{1}{6} V(\bar{A}_2 + \bar{A}_2^\dagger) + \tfrac{5}{6} V(\bar{A}_4 + \bar{A}_4^\dagger) = e^{-2r}. \end{aligned} \tag{35}$$

When the squeezing degree r is infinite, all four variances V_j tend to zero. This means that the four resonators are prepared in the steady-state linear cluster state. Compared with the previous schemes [27–31], the distinct advantage of our work is that it eliminates the complex procedures; that is, only one step of quantum operation is needed to produce the CV cluster state. Moreover, our scheme is based on a form of reservoir engineering which can stabilize the CV cluster at steady state. Hence, our scheme has significant advantages in practice and is more feasible in experiment.

Now let us discuss the effects of several imperfect factors to the preparation of the cluster state. For this purpose, we numerically solve the master equation (31) with the effective Hamiltonian (28). In the numerical simulation, the system is initially set to be the ground state $|g\rangle \otimes |0,0,0,0\rangle$, and all the parameters are chosen such that the approximations used in deriving the effective Hamiltonian (28) are valid.

We first consider the ideal case without photon leakage from the resonators. In Fig. 3, it is observed that the variances V_1, V_2, V_3, V_4 converge to the constant close to zero. We can further decrease the variances V_j by increasing the ratio ξ_1/ξ_2. The numerical simulation proves that the steady state of the four resonators is a CV linear cluster state.

In Fig. 3 the results with the nonzero resonator decay rate are also shown. The decay rate of the resonators can be written as $\kappa_j = Q_j/\omega_j$, where Q_j is the quality factor. If we set $\kappa_j/2\pi = 0.001\Gamma = 10$ kHz, corresponding to a superconducting resonator with the frequency $\omega_j/2\pi = 10$ GHz and the quality factor $Q_j = 10^6$, which is the most achievable value in current experiments [39], as seen in Fig. 3,

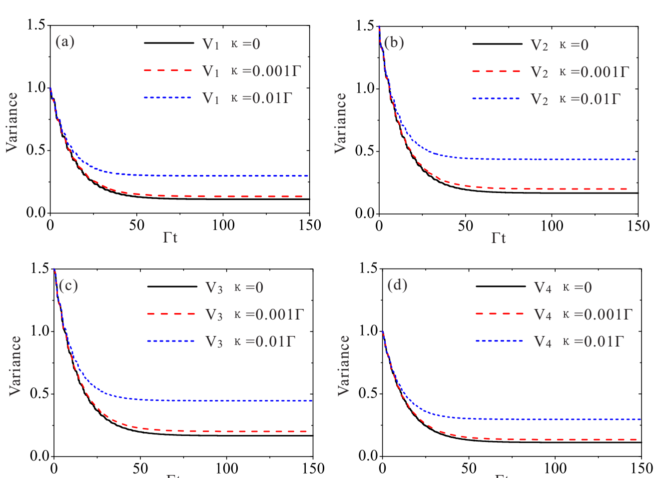

FIG. 3. Time evolution of variances (a) V_1, (b) V_2, (c) V_3, and (d) V_4 for different values of the resonator decay rate. The parameters are chosen as $g/2\pi = 50$ MHz, $\xi_2 = 0.2$, $\xi_1 = 0.16$, and $\Omega/2\pi = 5$ MHz, $\Gamma/2\pi = 10$ MHz, $\Delta g = 0$.

the variances deviate only slightly from those of the ideal case. Actually, the rapid development in superconducting quantum circuits holds great promise for implementing our scheme. A transmission-line resonator with a quality factor $Q > 10^7$ has been fabricated in experiment [65]. Additionally, a superconducting qubit coupled to a high-quality microwave resonator can easily reach the strong-coupling regime; that is, g is approximately hundreds of megahertz [41,48], and the time-dependent coupling strength $\alpha_{mn}(t)$ between the two resonators is several times $\omega_j \times 10^{-3}$ [55]. The present scheme also needs the superconducting qubit with a large decay rate Γ, which can be implemented by coupling it to an auxiliary bath, such as an open transmission line that provides a relaxation channel [61]. Therefore, the parameters in the numerical simulation are reachable in current experiment.

For quantum information and computation on CV quantum states, as usual, the homodyne measurement scheme of quadrature components of fields is employed. In the microwave-domain scheme, a single-mode resonator as the field source is coupled to a transmission line via a leaky mirror, and then linear amplifiers and a microwave quadrature mixer using a local oscillator are used to measure the quadrature amplitudes [66–68]. Thus, in a realistic situation, not only the resonator loss due to coupling with various nonresonance modes and intracavity imperfection but the loss of the leaky mirror must be considered. In Fig. 3, the results with the larger decay rate $\kappa = 0.01\Gamma$ are also shown. By comparing the results with different values of the resonator decay rate, one can see that the present scheme can still be used to prepare the CV cluster states even if the loss of the leaky mirror in the homodyne measurement is included. On the other hand, we may separate the preparation of CV states from the measurement processes. For example, flux or charge qubits used to measure intracavity modes may be synthesized into the resonators. In the preparation step, the qubits are detuned far from the resonators. In the measurement step after the preparation, the qubits are tuned into resonance with the resonators, and the resonator fields are extracted via the qubits to the transmission lines or detectors. In this way, the loss of the measurement to the preparation of the CV cluster states may be ignored.

In the above discussions, we assume that the two coupling strengths of the qubit with resonators 1 and 2 are equal. In realistic situations, however, it is difficult to keep the couplings the same. As pointed out above, the combined modes $\bar{A}_{1,3}$ may undergo the squeezing transformation. and $\bar{A}_{2,4}$ may undergo the antisqueezing transformation via the decay of the qubit if the two coupling strengths are not equal. On the other hand, there are swapping-type interactions between $\bar{A}_{1,2}$, $\bar{A}_{2,3}$, and $\bar{A}_{3,4}$ in Eq. (29). Note that the swapping interactions are invariable under the squeezing transformation, which means that modes $\bar{A}_{2,4}$ may be brought into the squeezed state by the swapping interaction if modes $\bar{A}_{1,3}$ are in the squeezed states. Therefore, even if the two coupling constants are not equal, the linear cluster state may also be created by properly balancing the antisqueezing interaction and the swapping interaction. In Fig. 4, the variances of the four combined modes for the cases with $\Delta g = 0g, 0.01g, 0.03g$ are compared. It is clearly observed that the coupling-constant difference which may occur in the realistic case has no essential influence on the

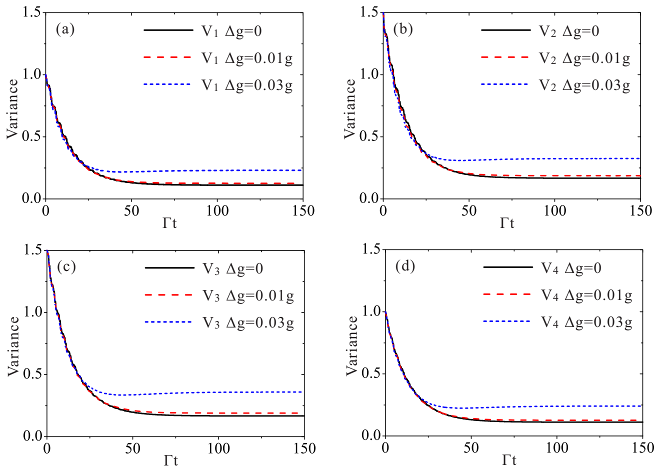

FIG. 4. Time evolution of variances (a) V_1, (b) V_2, (c) V_3, and (d) V_4 for various values of the qubit-resonator coupling-strength difference. The parameters are chosen as $g/2\pi = 50$ MHz, $\xi_2 = 0.2$, $\xi_1 = 0.16$, and $\Omega/2\pi = 5$ MHz, $\Gamma/2\pi = 10$ MHz.

present scheme. In other words, the present scheme is robust against this imperfection.

In the present scheme, the two pairs of driving fields are applied to the qubit. The fluctuation of either the amplitude or phase of the driving fields may affect the present scheme. In general, one should perform the statistical average of the results over the fluctuation distribution of the amplitude or phase of the driving fields. Since only the linear terms of the amplitude ξ_{lk} are kept, the average over the noise distribution of the amplitude leads to the replacement of ξ_{lk} by its mean value. Thus, the fluctuation of the amplitude may have no strong effect on the present scheme. In Hamiltonian (21), however, the phase appears as $e^{i\phi_{lk}}$. In general, $\langle e^{i\phi_{lk}}\rangle \neq e^{i\langle\phi_{lk}\rangle}$. However, to remove the time-dependent factors and extract the required resonant interaction in Hamiltonian (21), the phase conditions (23) have to be satisfied, and the exponential function $e^{i\phi_{lk}}$ is required. For this reason, the phases of the driving fields must be strictly locked to the required values shown in Eq. (23). Therefore, the phase fluctuation of the driving fields may have a strong effect on the present scheme.

In the above investigation, only the amplitude decay of the qubit is considered. When the Lindblad operator for pure dephasing $\frac{\Gamma_\phi}{2}D[\sigma_z]\rho$ does not commute with the full Hamiltonian, the unitary transformation used to diagonalize the Hamiltonian alters the dissipation process and leads to the phenomenon of dressed dephasing [69,70]. Note that the Lindblad operator for pure dephasing $\frac{\Gamma_\phi}{2}D[\sigma_z]\rho$ is invariable under the unitary transformations U_1 and U_2. Thus, if including the dephasing terms in Eq. (5), after the unitary transformations, we obtain an equation the same as Eq. (31) but with the additional term $\frac{\Gamma_\phi}{2}D[\sigma_z]\tilde{\rho}$. Moreover, it can easily be verified that $\frac{\Gamma_\phi}{2}D[\sigma_z]\tilde{\rho} = 0$, where $\tilde{\rho} = |\psi_S\rangle\langle\psi_S|$ and $|\psi_S\rangle = \otimes_{j=1}^{4}\exp[-\frac{r}{2}(\bar{A}_j^{\dagger 2} - \bar{A}_j^2)]|0_{\bar{A}_j}\rangle \otimes |g\rangle$. Thus, the pure dephasing of the qubit will not affect the steady state of the present scheme.

IV. CONCLUSION

In conclusion, we have proposed generating quadripartite CV cluster states in a circuit QED system consisting of four superconducting coplanar waveguide resonators and a superconducting flux qubit. With a suitable choice of the qubit-resonator and resonator-resonator interactions, we showed that the dissipation of the qubit can be utilized to bring the four resonators into CV cluster states at steady state. The distinct advantage of our scheme is that a one-step quantum operation is adequate for achieving the quantum state production, which has a great advantage in experiment.

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