Transparency under double detuning-induced stimulated Raman adiabatic passage in atoms with hyperfine structure

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We theoretically study the transparency in the generalized two-level system with hyperfine structure by utilizing double detuning-induced stimulated Raman adiabatic passage (double D-STIRAP). The double D-STIRAP is carried out by sequentially applying the three pulses, one near-resonant pump pulse and two far-off resonant Stark pulses before and after the pump pulse. From the study of single-atom response we can roughly learn the transparency conditions, since the full recovery of the system to the initial state is associated with the perfect transparency. After the numerical calculations we find that, for the perfect transparency, the pulse intensities of double D-STIRAP for the generalized two-level systems with hyperfine structure has to be stronger than that for the ideal two-level system. More precisely, we find that the ratio of amplitude to time for the Rabi frequency of the pump pulse and the detuning induced by the Stark pulse have to be close to each other to satisfy the adiabatic conditions. The above conditions, however, are necessary conditions we can learn from the single-atom response, and to ensure that they are indeed sufficient for perfect transparency, we perform the propagation calculations to obtain the temporal profile of the pump pulse at arbitrary propagation depths to find that double D-STIRAP, when applied to the generalized two-level system with hyperfine structure, is indeed robust for perfect transparency.

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I. INTRODUCTION

Nonlinear procedures have been widely investigated when optical fields interact with quantum systems. One of the most interesting phenomena is the transparency in quantum systems due to its potential applications in many areas such as optical fiber communication [1–3] and quantum information [4–6]. Among the mechanisms for transparency, self-induced transparency (SIT) [7] in simple two-level systems, simultons [8], and electromagnetically induced transparency (EIT) [9] in simple three-level systems are well known for researchers and they have been studied for many years. The underlying physics for transparency can be understood in the way that if the quantum system can be fully recovered after interacting with the pulse, then the laser field will also stay at its initial status.

Many quantum control theories are first proposed in simple systems. However, for the experimental realization they have to be applied to much more complicated systems. One of the reasons is that in many cases, the hyperfine structure in atoms or molecules cannot be spectrally resolved and it has to be taken into consideration. This is true especially when pulsed lasers are employed instead of cw lasers. During the interaction, the hyperfine structure is found to play an important role and the quantum control theories need to be treated carefully.

For the transparency mechanisms in simple quantum systems mentioned above, the situation is also the same. Thus in order to clarify the effect of hyperfine structure on the transparency mechanisms, many efforts have been devoted to it [10–22]. The results show that, as expected, the existence of hyperfine structure does influence the transparency mechanisms to some extent. Taking the EIT as an example, Xia *et al.* [14] pointed out that the medium will become opaque if the two levels coupled by the strong field involve hyperfine structure. So, if we want to obtain the transparency again, the control field and the probe field need to be tuned carefully to the respective center of gravity of the two transitions. On the other hand, if the two levels coupled by the weak probe pulse contain hyperfine structure, Kis *et al.* [15] showed that the probe pulse with elliptical polarization will separate into the two modes during propagation, in which one mode survives via EIT while the other mode undergoes damping.

Recently, a new mechanism for transparency in a two-level system was reported [23]. The basic idea of the mechanism can be shown in Fig. 1. It is based on the stimulated Raman adiabatic passage (STIRAP) in a two-level system [24]. The authors of Ref. [24] pointed out that the optical Bloch equation for the resonantly driven two-level system is mathematically similar to the Schrödinger equation for a three-level system. Therefore, using the elements of density matrix for the two-level system where $|1\rangle$ and $|2\rangle$ are the lower and upper levels, respectively, the three parameters in the Bloch equation, which are the population inversion, $w = \rho_{22} - \rho_{11}$, and the real and the imaginary parts of the coherence term, $2\rho_{12} = u + iv$, can be considered as the components of an equivalent three-level system [see Fig. 1(a)]. With a counterintuitive pulse sequence in which the detuning pulse is turned on ahead of the pump pulse, the value of w ($w = -1$) will transfer to the real part of the coherence term u while the imaginary part of the coherence term v will stay at zero during the process. As a result, maximum coherence can be generated. During the process, the adiabatic condition $\int_{-\infty}^{\infty} \sqrt{\Delta^2 + \Omega^2} dt \gg \pi/2$ should be satisfied, in which Δ is the detuning and Ω is the Rabi frequency of the pump pulse. Since the process is realized under the assistance of a detuning pulse, here we call it detuning-induced STIRAP (D-STIRAP).

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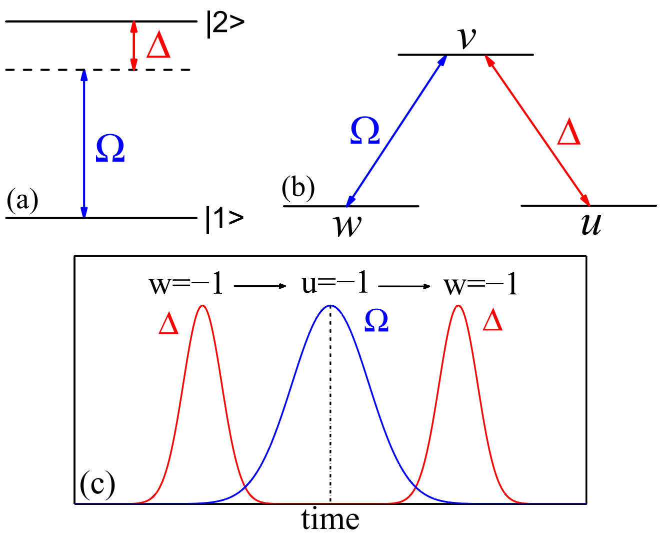

FIG. 1. (a) Prototype of double D-STIRAP scheme in an ideal two-level system. (b) Representation of D-STIRAP in analogy to the well-known three-level STIRAP. (c) The first detuning pulse and the left half of the pump pulse transfer the value of w to u. After that, the right half of the pump pulse and the second detuning pulse transfer the value of u back to w. When the process is finished, the two-level system is fully recovered.

D-STIRAP [24] can be used to induce transparency if it is applied twice [23], which we call double D-STIRAP. The authors of Ref. [23] showed theoretically that when the first D-STIRAP is completed, the real part of the coherence term u is -1 and the population inversion w is zero. Afterwards, the second D-STIRAP with the pump pulse ahead of the detuning pulse will again transfer the value of u back to that of w, as can be shown in Fig. 1(b). When the whole process is completed, the initial condition in the two-level system is fully recovered and alternatively the system is transparent to the pump pulse (double D-STIRAP induced transparency). In order to realize the double D-STIRAP induced transparency, two idealized detuning pulses and one pump pulse with flat top are employed [23]. It is found that, although the adiabatic condition is not satisfied, the double D-STIRAP can still induce transparency under the presence of Doppler broadening. Double D-STIRAP induced transparency in an ideal two-level system was also discussed in different systems such as quantum dot [25] or realized by specially chirped femtosecond pulse [26].

Knowing that the double D-STIRAP works to induce transparency in the simple two-level system, it is natural for us to wonder whether the transparency can be induced in a generalized two-level system where the upper and lower levels have hyperfine structures. It should be noted that we have studied the influence of hyperfine structure on generating coherence with single D-STIRAP [27] and found that the maximum coherence cannot be generated any more if the hyperfine structure is present. In order to obtain coherence as high as possible, polarization of the pump pulse should be chosen with care [27].

In this paper, we study the double D-STIRAP in the generalized two-level system by taking into account the influence of the hyperfine structure. Unlike the idealized detuning pulse and pump pulse with flat top [23], or the pump pulse with a special chirp rate [26], here we use Gaussian pulses without chirps for double D-STIRAP. The detuning is realized by far-off resonant Stark pulse and the pump pulse is near resonant with the two-level system. Moreover, since there may be time delay between the Stark and the pump pulse during the propagation and the time delay may ruin the adiabatic condition, we go beyond the single-atom response and use the full description of density-matrix equations combined with propagation equations. Following our previous work [27] by using states $3\,^2S_{1/2}$ and $3\,^2P_{1/2}$ of ^{23}Na as an example to investigate the single D-STIRAP, we carry out the analysis of double D-STIRAP in the same model. Since the pump pulse may be linearly or elliptically polarized, the generalized two-level system will then consist of several independent subsystems such as two-level, V, Λ, and double-Λ subsystems [27].

In the following sections we will first discuss the recovery of a generalized two-level system with hyperfine structure by double D-STIRAP in terms of single-atom response. It is found that full recovery of such a system can be realized only when the adiabatic condition is satisfied. It requires that the intensities of the Stark and pump pulses should be stronger than those used in the simple two-level system. Furthermore, the ratio of amplitude to time for the Rabi frequency of the pump pulse and the detuning induced by the Stark pulse had better be close to each other. After this, we will continue discussing the propagation effect of the pump pulse. The calculations show that the pump pulse will propagate transparently in the generalized two-level system with the speed of light in vacuum. Since the two far-off resonant Stark pulses also propagate in the system with the speed of light in vacuum, there is no time delay between the Stark, pump, and second Stark pulses. Hence the adiabatic condition can be preserved during the propagation and the transparency of the pump pulse is maintained in the generalized two-level system. The results indicate that, different from single D-STIRAP which is unable to produce maximum coherence in the generalized two-level system with hyperfine structure, double D-STIRAP induced transparency still works even in this case. More detailed calculations show that, as long as the adiabaticity is well maintained, the exact choice of parameters such as the pulse intensities, the time delay, and the initial detuning of the pump pulse does not influence the transparency. We also compare our results with SIT in the generalized two-level system with hyperfine structure to find that SIT does not work any more.

II. SINGLE-ATOM RESPONSE

In this section, we discuss the single-atom response of a two-level system with hyperfine structure under double D-STIRAP. As in our previous work [27], we employ the D_1 transition of ^{23}Na, which consists of $3\,^2S_{1/2}$ and $3\,^2P_{1/2}$, for double D-STIRAP. The hyperfine structure contained in the D_1 transition is shown in Fig. 2. As can be seen in this figure, when the near-resonant pump pulse is linearly or left-circularly polarized, the essential subsystems which are independent to each other are the two-level, V, Λ, and double-Λ subsystems. Subsystems established by the right-circularly polarized pump pulse is similar to those in the left-circular case and we do not

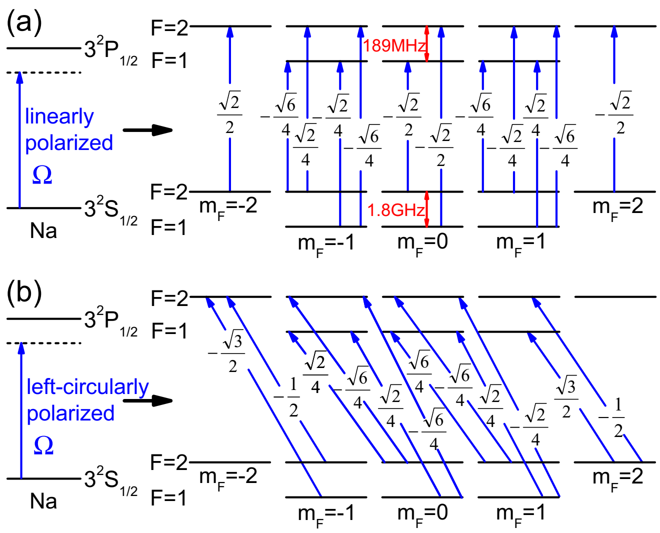

FIG. 2. Hyperfine structure of D_1 transition in ^{23}Na which is coupled by the near-resonant pump pulse with (a) linear and (b) left-circular polarization. Scheme (a) consists of two-level and double-Λ subsystems, while scheme (b) consists of V, Λ, and double-Λ subsystems. The numbers written in each scheme represent the relevant angular coefficients for the corresponding dipole moments.

discuss it here. The numbers written in Fig. 2 are the relevant angular coefficients for the corresponding dipole moments.

In order to describe the evolution of these subsystems, it is convenient to use a general four-level system which consists of the two hyperfine sublevels of $3\,^2S_{1/2}$ and those of $3\,^2P_{1/2}$ to represent all of them. The general four-level system is shown in Fig. 3. In this system, all possible dipole couplings and the corresponding detunings are taken into account. η_{ji} is the coupling coefficient for the dipole transition between the lower level $|i\rangle$ and upper level $|j\rangle$. By properly setting η_{ji} to zero or 1 we can reduce the four-level system to the two-level, V, Λ, or double-Λ subsystems.

The equation of motion of the general four-level system is governed by the density-matrix equations under rotating-wave

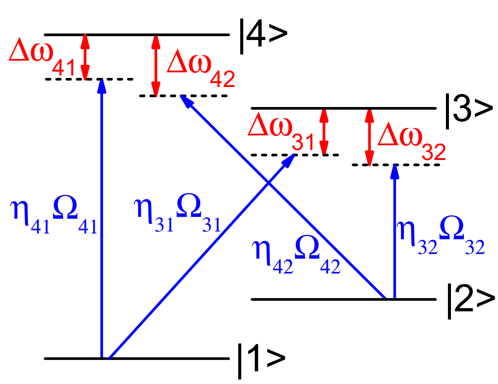

FIG. 3. Four-level system which is referred to as two-level system with two upper and two lower levels arising from hyperfine interactions. With the coupling coefficients, η_{ji}, being set as zero or 1, the system can be reduced to the two-level, V, Λ, and double-Λ subsystems.

approximation, which can be written in the following form:

$$
\begin{aligned}
\dot{\rho}_{11} &= i\eta_{31}A_{31}(\Omega_{\text{eff}}^{*}\rho_{31}-\Omega_{\text{eff}}\rho_{13})\\
&\quad + i\eta_{41}A_{41}(\Omega_{\text{eff}}^{*}\rho_{41}-\Omega_{\text{eff}}\rho_{14}),\\
\dot{\rho}_{22} &= i\eta_{32}A_{32}(\Omega_{\text{eff}}^{*}\rho_{32}-\Omega_{\text{eff}}\rho_{23})\\
&\quad + i\eta_{42}A_{42}(\Omega_{\text{eff}}^{*}\rho_{42}-\Omega_{\text{eff}}\rho_{24}),\\
\dot{\rho}_{33} &= i\eta_{31}A_{31}(\Omega_{\text{eff}}\rho_{13}-\Omega_{\text{eff}}^{*}\rho_{31})\\
&\quad + i\eta_{32}A_{32}(\Omega_{\text{eff}}\rho_{23}-\Omega_{\text{eff}}^{*}\rho_{32}),\\
\dot{\rho}_{44} &= i\eta_{41}A_{41}(\Omega_{\text{eff}}\rho_{14}-\Omega_{\text{eff}}^{*}\rho_{41})\\
&\quad + i\eta_{42}A_{42}(\Omega_{\text{eff}}\rho_{24}-\Omega_{\text{eff}}^{*}\rho_{42}),\\
\dot{\rho}_{12} &= i\omega_{21}\rho_{12}-i\eta_{32}A_{32}\Omega_{\text{eff}}\rho_{13}-i\eta_{42}A_{42}\Omega_{\text{eff}}\rho_{14}\\
&\quad + i\eta_{31}A_{31}\Omega_{\text{eff}}^{*}\rho_{32}+i\eta_{41}A_{41}\Omega_{\text{eff}}^{*}\rho_{42},\\
\dot{\rho}_{13} &= i\Delta\omega_{31}\rho_{13}+i\eta_{31}A_{31}\Omega_{\text{eff}}^{*}(\rho_{33}-\rho_{11})\\
&\quad - i\eta_{32}A_{32}\Omega_{\text{eff}}^{*}\rho_{12}+i\eta_{41}A_{41}\Omega_{\text{eff}}^{*}\rho_{43},\\
\dot{\rho}_{14} &= i\Delta\omega_{41}\rho_{14}+i\eta_{41}A_{41}\Omega_{\text{eff}}^{*}(\rho_{44}-\rho_{11})\\
&\quad - i\eta_{42}A_{42}\Omega_{\text{eff}}^{*}\rho_{12}+i\eta_{31}A_{31}\Omega_{\text{eff}}^{*}\rho_{34},\\
\dot{\rho}_{23} &= i\Delta\omega_{32}\rho_{23}+i\eta_{32}A_{32}\Omega_{\text{eff}}^{*}(\rho_{33}-\rho_{22})\\
&\quad - i\eta_{31}A_{31}\Omega_{\text{eff}}^{*}\rho_{21}+i\eta_{42}A_{42}\Omega_{\text{eff}}^{*}\rho_{43},\\
\dot{\rho}_{24} &= i\Delta\omega_{42}\rho_{24}+i\eta_{42}A_{42}\Omega_{\text{eff}}^{*}(\rho_{44}-\rho_{22})\\
&\quad - i\eta_{41}A_{41}\Omega_{\text{eff}}^{*}\rho_{21}+i\eta_{32}A_{32}\Omega_{\text{eff}}^{*}\rho_{34},\\
\dot{\rho}_{34} &= i\omega_{43}\rho_{34}-i\eta_{41}A_{41}\Omega_{\text{eff}}^{*}\rho_{31}+i\eta_{31}A_{31}\Omega_{\text{eff}}\rho_{14}\\
&\quad - i\eta_{42}A_{42}\Omega_{\text{eff}}^{*}\rho_{32}+i\eta_{32}A_{32}\Omega_{\text{eff}}\rho_{24}.
\end{aligned}
\tag{1}
$$

In these equations, $\dot{\rho}_{ij}(i,j=1,2,3,4)$ is the time derivative of ρ_{ij}. $\rho_{ii}(i=1,2,3,4)$ is the population of level $|i\rangle$ and $\rho_{ij}(i,j=1,2,3,4,i\neq j)$ is the coherence between level $|i\rangle$ and $|j\rangle$. ω_{21} and ω_{43} represent the hyperfine splitting of the lower and upper levels. $\Delta\omega_{ji}$ is the detuning of the pump pulse with respect to the transition frequency, ω_{ji}, for the dipole transition between $|j\rangle$ and $|i\rangle$. We also note that the Rabi frequency between levels $|j\rangle$ and $|i\rangle$, Ω_{ji}, in Fig. 3 is replaced by $\Omega_{ji}=A_{ji}\Omega_{\text{eff}}$ in Eqs. (1) with A_{ji} and Ω_{eff} being defined as

$$
\begin{aligned}
A_{ji} &= \frac{\mu_{ji}}{|\mu_{\text{eff}}|},\\
\Omega_{\text{eff}} &= \frac{|\mu_{\text{eff}}|E(t)}{2\hbar}.
\end{aligned}
\tag{2}
$$

In Eqs. (2), μ_{ji} is the dipole moment between levels $|j\rangle$ and $|i\rangle$, and $E(t)$ is the electric field envelope of the pump pulse. $|\mu_{\text{eff}}|=\sqrt{\sum_{j,i}(\eta_{ji}\mu_{ji})^2}$ is the effective dipole moment to connect the generalized two-level system (with hyperfine structure) and the simple two-level system [27]. Correspondingly, Ω_{eff} can be considered as the effective Rabi frequency between the upper and lower level manifolds of the generalized two-level system. The decay from the upper level manifold due to the spontaneous decay and ionization loss have been neglected here because, as we will show below, when the double D-STIRAP is completed and the system returns to its initial ground states, the decay does not play a role during such a short interaction time.

Now we turn to the discussion of single-atom response of all subsystems under double D-STIRAP by numerically solving Eqs. (1). The double D-STIRAP in this paper is realized by using two far-off resonant Stark pulses and one near-resonant pump pulse with all of them having the Gaussian shapes. The effective Rabi frequency $\Omega_{\rm eff}$ of the pump pulse and the detuning $\Delta\omega_{32}$ induced by the Stark pulse are given as

$$\Omega_{\rm eff} = \Omega_{\rm eff}^0 \exp\left[-\frac{\ln 4(t-t_1)^2}{\tau_{\rm p}^2}\right],$$

$$\Delta\omega_{32} = \Delta\omega_{32}^0\left\{\exp\left(-\frac{\ln 4t^2}{\tau_{\rm S}^2}\right) + \exp\left[-\frac{\ln 4(t-t_1-t_2)^2}{\tau_{\rm S}^2}\right]\right\} + \delta_0. \quad (3)$$

Accordingly, the other three detunings are $\Delta\omega_{31} = \Delta\omega_{32} + \omega_{21}$, $\Delta\omega_{42} = \Delta\omega_{32} + \omega_{43}$, and $\Delta\omega_{41} = \Delta\omega_{32} + \omega_{21} + \omega_{43}$. In Eqs. (3), $\Omega_{\rm eff}^0$ and $\Delta\omega_{32}^0$ are the amplitude of the effective Rabi frequency and the detuning. τ_p and $\tau_{\rm S}$ are the durations of the pump and Stark pulse. t_1 is the time delay between the first Stark pulse and the pump pulse, while t_2 is the time delay between the pump pulse and the second Stark pulse. δ_0 is the initial detuning of the pump pulse with respect to the transition between $|2\rangle$ and $|3\rangle$. In the following calculations, all the parameters are chosen with respect to $\tau_{\rm S}$. The time delays t_1, t_2 and the duration of the pump pulse $\tau_{\rm p}$ are defined in units of $\tau_{\rm S}$. The other parameters such as $\Omega_{\rm eff}^0$, $\Delta\omega_{32}^0$, δ_0, and also the hyperfine splittings ω_{21}, ω_{43}, are defined in units of $1/\tau_{\rm S}$. By setting $\tau_{\rm S} = 1$, we have $\tau_{\rm p} = 2\tau_{\rm S}$ and $t_1 = t_2 = 4\tau_{\rm S}$. Under the choice of these parameters, the two detunings induced by the two Stark pulses in $\Delta\omega_{32}$ are partially overlapped with the effective Rabi frequency. The two detunings, however, are well separated without overlapping [Fig. 1(b)]. With the absence of the Stark pulses, the pump pulse is assumed to be resonant with the transition ω_{32} by setting $\delta_0 = 0$.

A. Degenerate hyperfine structure case

Although none of the subsystems in Fig. 2 is degenerate, we start the discussion by assuming the degenerate subsystems, since it helps to make the underlying physics clearer. Accordingly we set $\omega_{21} = \omega_{43} = 0$ and $\Delta\omega_{31} = \Delta\omega_{32} = \Delta\omega_{41} = \Delta\omega_{42}$.

1. Transparency in double-Λ type subsystem

For the two-level, degenerate V, Λ, and double-Λ type subsystems, calculations show that it is very important to satisfy the adiabatic condition simultaneously for the realization of perfect transparency. It is also found that there is a more stringent requirement for pulses to meet the adiabaticity than in the simple two-level system. Here we use the double-Λ type subsystem as an example to explain how to meet the adiabaticity. From our previous work [27], we know that there exists a common term $B = A_{31}A_{42} - A_{32}A_{41}$ in the density-matrix equations only for the double-Λ subsystem. Depending on whether this term is equal to zero or nonzero, the different amount of coherence will be produced in the subsystem under single D-STIRAP. Hence particularly we choose double-Λ type subsystem with $B \neq 0$ for double

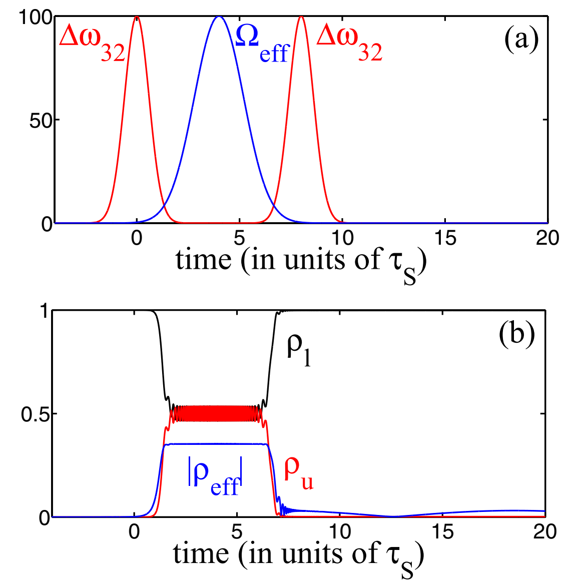

FIG. 4. (a) Pulse sequence of double D-STIRAP in double-Λ type subsystem. The common term $B = A_{31}A_{42} - A_{32}A_{41}$, which determines the amount of the generated transient coherence, is set to be nonzero. The detuning $\Delta\omega_{32}$ and the effective Rabi frequency $\Omega_{\rm eff}$ have the same amplitude $\Delta\omega_{32}^0 = \Omega_{\rm eff}^0 = 100/\tau_{\rm S}$ but different ratio of amplitude to time; (b) the corresponding evolution of the subsystem shows the imperfect transparency.

D-STIRAP. The double-Λ type subsystem with $B \neq 0$ can be realized by choosing $|3\,^2S_{1/2}\ F=1, m_F=-1\rangle$, $|3\,^2S_{1/2}\ F=2, m_F=-1\rangle$, $|3\,^2P_{1/2}\ F=1, m_F=-1\rangle$, and $|3\,^2P_{1/2}\ F=2, m_F=-1\rangle$ as $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$ in Fig. 2(a), for which $A_{31} = -\frac{\sqrt{2}}{4}$, $A_{32} = -\frac{\sqrt{6}}{4}$, $A_{41} = -\frac{\sqrt{6}}{4}$, and $A_{42} = \frac{\sqrt{2}}{4}$, and consequently $B = -\frac{1}{2}$.

By assuming all the population equally populated in the two ground sublevels $|1\rangle$ and $|2\rangle$, we first choose $\Delta\omega_{32}^0 = \Omega_{\rm eff}^0 = 100/\tau_{\rm S}$ for the detuning and the effective Rabi frequency to carry out the double D-STIRAP, which is stronger than what we used to realize single D-STIRAP in the simple two-level system [27]. The calculation results are shown in Fig. 4. Note that in Fig. 4(b), $|\rho_{\rm eff}| = |\sum_{j,i}\eta_{ji}A_{ji}\rho_{ji}|$ is the absolute value of the effective coherence between the lower and upper level manifolds, while $\rho_{\rm l}$ and $\rho_{\rm u}$ indicate the population in the lower and upper level manifolds, respectively. Although the much more intense pulses are employed for the double D-STIRAP [Fig. 4(a)], the double-Λ type subsystem with $B \neq 0$ does not return to the initial state, as we see in Fig. 4(b). The reason is that the ratio of amplitude-to-time for the detuning and effective Rabi frequency, which is critical for the adiabatic condition, is different during the process. The duration of the pump pulse is twice longer than that of the Stark pulse, and hence the ratio of amplitude to time for the detuning is twice larger than that of the effective Rabi frequency. In order to obtain the perfect transparency in the double-Λ type subsystem, we may adjust the ratio of amplitude to time for the detuning and effective Rabi frequency to close to each other by changing the Stark pulse intensity or the pump pulse intensity.

Accordingly, in the following, we repeat the calculations by reducing the amplitude of the detuning to $\Delta\omega_{32}^0 = 50/\tau_{\rm S}$ with

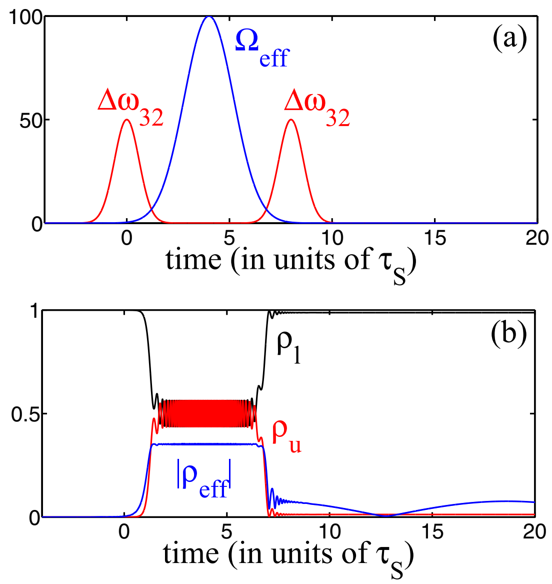

FIG. 5. (a) Same pulse sequence as shown in Fig. 4(a) but with different amplitudes of $\Delta\omega_{32}^0 = 50/\tau_S$ and $\Omega_{\rm eff}^0 = 100/\tau_S$. Now the ratio of amplitude to time for effective Rabi frequency and detuning are equal to each other; (b) the corresponding evolution of the double-Λ type subsystem with $B \neq 0$. Still we cannot obtain the perfect transparency.

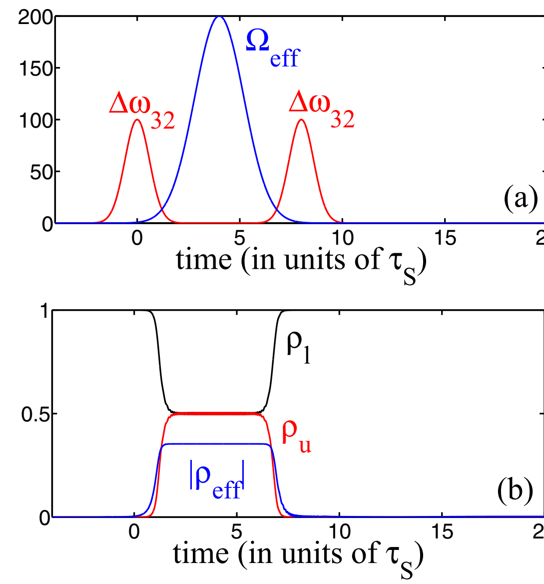

FIG. 6. (a) Same pulse sequence as shown in Fig. 5(a) but with stronger amplitudes of $\Delta\omega_{32}^0 = 100/\tau_S$ and $\Omega_{\rm eff}^0 = 200/\tau_S$; (b) the corresponding evolution of the double-Λ type subsystem with $B \neq 0$. Now the adiabatic condition is well satisfied and the perfect transparency is obtained.

other parameters unchanged so that the ratio of amplitude to time for the detuning and effective Rabi frequency becomes the same. The results are shown in Fig. 5. With the pulse sequence shown in Fig. 5(a), the complete recovery of the double-Λ type subsystem is not yet attained as we see in Fig. 5(b). This may be because the employed pulse intensities are not sufficiently strong to satisfy the adiabatic condition.

Therefore, we increase the amplitude of the employed parameters to $\Omega_{\rm eff}^0 = 200/\tau_S$ and $\Delta\omega_{32}^0 = 100/\tau_S$, as shown in Fig. 6(a), and repeat the calculations with all other parameters kept the same. The results are shown in Fig. 6(b). In this figure, we can clearly see that with the same ratio of amplitude to time and stronger pulse intensities used, the adiabatic condition is well satisfied and, as a result, the perfect transparency in the double-Λ type subsystem is realized. Thus, from Figs. 4–6 we can say the following: for the double-Λ type subsystem with $B \neq 0$ to be perfectly transparent through the double D-STIRAP, the adiabatic condition has to be satisfied. That is, the effective Rabi frequency and detuning must have sufficiently strong amplitudes with comparable amplitude-to-time ratios.

2. *Perfect transparency in other subsystems*

Now we turn to the cases of other kinds of subsystems of two-level, V, Λ, and double-Λ type with $B = 0$ with degenerate hyperfine structure under double D-STIRAP. Perfect transparency in all the subsystems should be realized under the same pulse sequence and adiabatic condition. Otherwise the entire system with all subsystems will not be transparent. Thus we employ the parameters used for Fig. 6 in the following calculations. Furthermore, we suppose that all the population is initially in level $|2\rangle$ in the two-level and degenerate V type subsystems, or equally populated in levels $|1\rangle$ and $|2\rangle$ in the degenerate Λ and double-Λ type subsystems with $B = 0$. The calculation results are shown in Fig. 7. We can see that all these subsystems also exhibit perfect transparency after the double D-STIRAP. The only difference is that transient coherence behaves in different ways during the pulse sequence in different subsystems. In the two-level and degenerate V type subsystems, maximum coherence $|\rho_{\rm eff}| = 0.5$ is obtained as shown in Fig. 7(a), while in the degenerate Λ and double-Λ type subsystems with $B = 0$, a moderate amount of coherence of $|\rho_{\rm eff}| = 0.25$ is produced, as shown in Fig. 7(b).

By combining the results of Figs. 6 and 7, we can see that perfect transparency can be realized in all kinds of subsystems through double D-STIRAP if the pulse intensities and the amplitude-to-time ratios of the effective Rabi frequency and the detuning are appropriately chosen. However, these results are not enough to comfirm that the double D-STIRAP works to attain perfect transparency in the two-level systems with degenerate hyperfine structure, which is different from the double D-STIRAP in the simple two-level system (Fig. 1). This is because the pump pulse may have different propagation speed in these subsystems and the relative time delay between the pump and Stark pulses may be altered. The mismatch of the propagation speeds of the resonant pump and far-off resonant Stark pulses will destroy the adiabatic condition and accordingly the transparency will be lost. Therefore, we need further investigations in Sec. III by taking into account the propagation effect.

B. Nondegenerate hyperfine structure case

In this section we will consider the evolution of all subsystems with nondegenerate hyperfine structure under double D-STIRAP, which is indeed the case in the D_1 transition

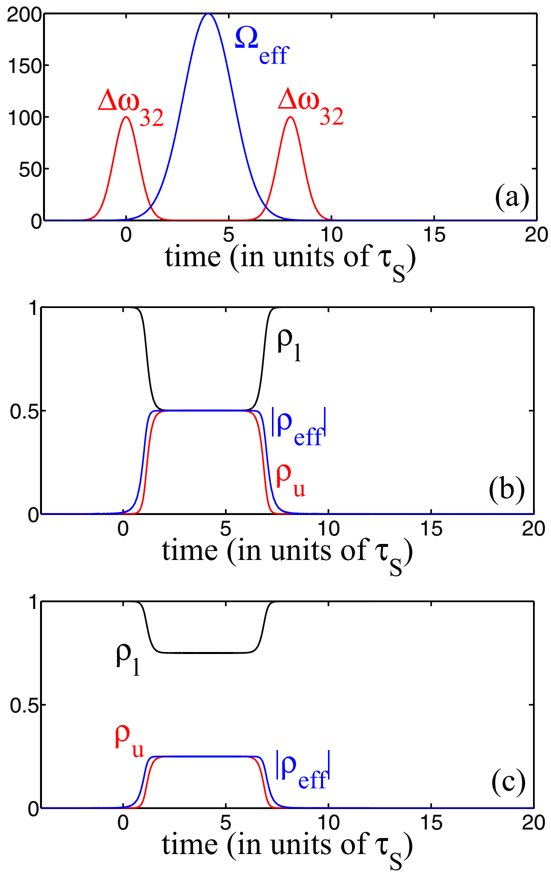

FIG. 7. (a) Same pulse sequence of the detuning and pump pulse, and same pulse intensities as those used in Fig. 6(a) are employed again for the double D-STIRAP, in which $\Delta\omega_{32}$ and $\Omega_{\rm eff}$ have the units of $1/\tau_{\rm S}$. The evolution of two-level, degenerate V type subsystems in (b) and degenerate Λ, double-Λ type subsystems with $B = 0$ in (c) are given. Perfect transparency is obtained.

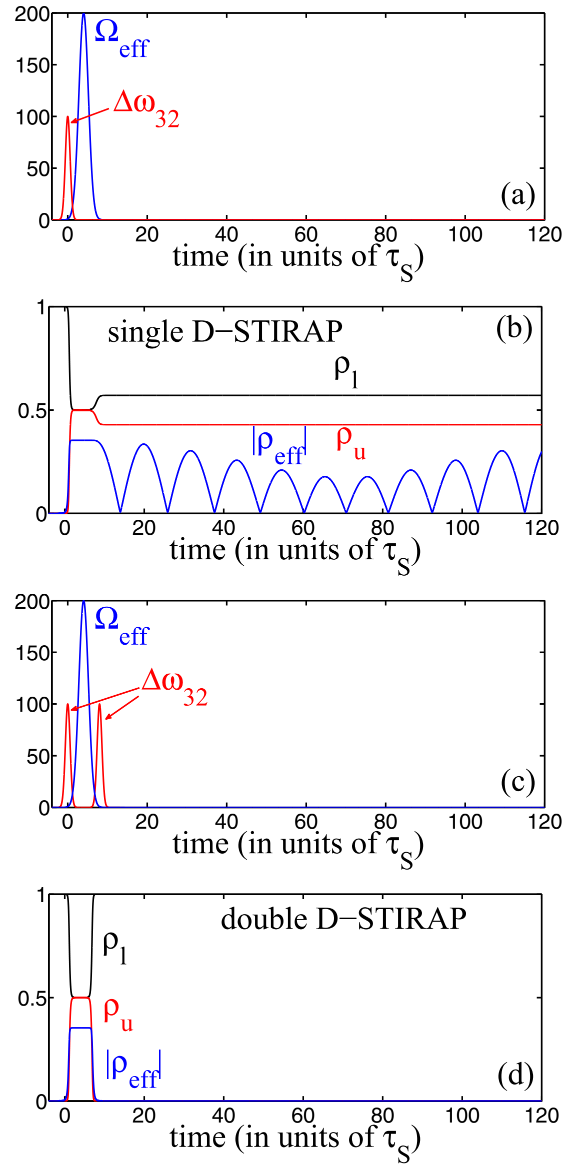

FIG. 8. (a) Single D-STIRAP and (c) double D-STIRAP in nondegenerate double-Λ type subsystem with $B \neq 0$ realized by different pulse sequence. All the parameters are kept the same as in Fig. 6 except that the hyperfine splittings are chosen as $\omega_{21} = 0.5/\tau_{\rm S}$ and $\omega_{43} = 0.05/\tau_{\rm S}$, while $\Delta\omega_{32}$ and $\Omega_{\rm eff}$ have the units of $1/\tau_{\rm S}$. The corresponding evolution of the subsystem is shown in (b) and (d), respectively.

of ^{23}Na. As we have recently shown [27], nondegenerate hyperfine structure in these subsystems results in the transient oscillation of coherence under single D-STIRAP. This is practically due to the nonzero detuning of the pump pulse with respect to the hyperfine sublevels, and we first examine whether similar undesired oscillation occurs for the double D-STIRAP under the presence of nondegenerate hyperfine structure.

Again, we first investigate the nondegenerate double-Λ type subsystem with $B \neq 0$. According to the level structure shown in Fig. 2, we set the hyperfine splitting of the lower and upper levels as $\omega_{21} = 0.5/\tau_{\rm S}$ and $\omega_{43} = 0.05/\tau_{\rm S}$, respectively. The initial detuning of the pump pulse with respect to the transition between $|2\rangle$ and $|3\rangle$ is adjusted to be $\delta_0 = -\frac{1}{2}(\omega_{21} + \omega_{43}) = -0.275/\tau_{\rm S}$, although this is not a mandatory choice. All the other parameters are kept the same with those in Figs. 6 and 7. According to the analysis in Ref. [27], if we choose the duration of the Stark pulse as $\tau_S = 10$ ps, then the relative value of the duration of the pump pulses is $\tau_p = 20$ ps, and the time delay between them is $t_1 = t_2 = 40$ ps. The amplitudes of the Rabi frequency $\Omega_{\rm eff}$ and the detuning $\Delta\omega_{32}$ are $\Omega^0_{\rm eff} = 20 \times 10^{12}$ rad/s and $\Delta\omega^0_{32} = 10 \times 10^{12}$rad/s, respectively. The wavelength of the pump pulse is 589 nm and Stark pulse with wavelength of 1064 nm is used to induce the detuning. This corresponds to the intensities of the pump pulse and Stark pulse as $I_{\rm pump} = 0.59$ GW/cm^2 and $I_{\rm Stark} = 50$ GW/cm^2.

As an example, we compare the performance of the double D-STIRAP and single D-STIRAP in the double-Λ type subsystem with $B \neq 0$. The pulse sequence we employ for the single D-STIRAP is shown in Fig. 8(a). The corresponding evolution of the subsystem is shown in Fig. 8(b). We observe the transient oscillation of coherence after the pulse sequence is over. The oscillation period is determined by the

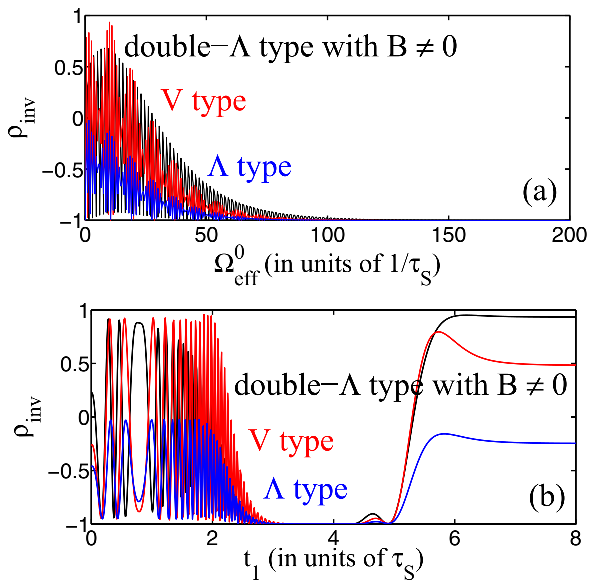

FIG. 9. Population inversion $\rho_{\rm inv} = \rho_{\rm u} - \rho_{\rm l}$ at $t = 20\tau_{\rm S}$ after the double D-STIRAP as a function of (a) the amplitude of the effective Rabi frequency, $\Omega^0_{\rm eff}$, and (b) the time delay between the first Stark pulse and the pump pulse, t_1, in nondegenerate V, Λ, and double-Λ type subsystems with $B \neq 0$. In (a), the amplitude of the detuning $\Delta\omega^0_{32}$ changes linearly with $\Omega^0_{\rm eff}$, which is $\Delta\omega^0_{32} = \frac{1}{2}\Omega^0_{\rm eff}$, while in (b), the time delay between the pump pulse and the second Stark pulse t_2 changes accordingly with t_1, which is $t_2 = t_1$. The other parameters are kept to be the same as those in Fig. 8.

hyperfine splittings ω_{21} and ω_{43}. In contrast, for the double D-STIRAP with the pulse sequence shown in Fig. 8(c), we observe no transient oscillation of coherence, as shown in Fig. 8(d). This is simply because all the populations in the subsystem goes back to the initial ground state by applying the pulse sequence, and hence coherence remains zero after the pulse sequence is over. From the results shown in Fig. 8, we can say that the single D-STIRAP does not work to induce perfect coherence if the system is not the simple two-level but generalized two-level system with hyperfine structure. However, the double D-STIRAP does work even in such a case.

We repeat calculations for other kinds of subsystems with nondegenerate hyperfine structure under double D-STIRAP while all parameters are kept to be the same with those used in Fig. 8. The results in these subsystems with nondegenerate hyperfine structure are the same as those with degenerate hyperfine structure, as shown in Fig. 7.

Without the loss of generality, it is necessary to point out that the parameters employed in the above figures do not need to be so specific. Therefore, we continue investigating the robustness of the double D-STIRAP in two-level systems with nondegenerate hyperfine structure. In Fig. 9, by defining $\rho_{\rm inv} = \rho_{\rm u} - \rho_{\rm l}$ with the initial value of -1 as the population inversion in the generalized two-level systems, we calculate its value at $t = 20\tau_{\rm S}$ after the double D-STIRAP in all kinds of subsystems as a function of the amplitude of the effective Rabi frequency, $\Omega^0_{\rm eff}$, and the time delay between the first Stark pulse and the pump pulse, t_1. The results indicate that the curves for the two-level subsystem and double-Λ type subsystem with $B = 0$ are quite similar to those of the V type subsystem and Λ type subsystem, respectively; thus they are not shown in this figure. In Fig. 9(a), since the ratio of amplitude to time for the effective Rabi frequency and the detuning had better be comparable to each other to realize the double D-STIRAP, the amplitude of the detuning varies proportionally to that of the effective Rabi frequency, which is $\Delta\omega^0_{32} = \frac{1}{2}\Omega^0_{\rm eff}$. The other parameters are the same as those in Fig. 8. From this figure we can see that the population inversion in all the subsystems will go back to its initial value as long as the pulses are strong enough to satisfy the adiabatic condition. Moreover, the overlapped region of these curves suggests that all the subsystems can show transparency simultaneously. In Fig. 9(b), due to the symmetry of the double D-STIRAP, the time delay between the pump pulse and the second Stark pulse t_2 changes the same with t_1, which is $t_2 = t_1$. The other parameters are also the same as those in Fig. 8. The figure shows that the population inversion in these subsystems can return to its initial value for a certain region of the time delay. The overlapped region of the curves also suggests that these subsystems can be transparent simultaneously. However, when comparing Fig. 9(b) with 9(a), we can see that double D-STIRAP is more sensitive to the time delay.

As for the influence of the initial detuning of the pump pulse on double D-STIRAP, calculations show that double D-STIRAP works for either near-resonant or far-off resonant pump pulse. Since we focus on the transparency of the near-resonant pump pulse, the impact of the initial detuning of the pump pulse is not a main consideration in our study.

Once again, from Figs. 8 and 9 we cannot yet say with confidence that double D-STIRAP works for the two-level systems with nondegenerate hyperfine structure to achieve perfect transparency. The reason is the same as that we have mentioned at the end of Sec. IIA2.

III. PROPAGATION EFFECT

In this section we discuss the propagation effect of the pump pulse under the presence of the two Stark pulses by solving the density-matrix equations together with the propagation equation to see whether the two-level system with degenerate or nondegenerate hyperfine structure is transparent for the pump pulse.

A. Double D-STIRAP induced transparency

The propagation equation of the near-resonant pump pulse in the general four-level system can be written in the form of

$$\frac{d}{dz}E(z,t) + \frac{1}{c}\frac{d}{dt}E(z,t) = \frac{i\omega}{2\epsilon_0 c}P(z,t), \tag{4}$$

in which c is the speed of light in vacuum, ϵ_0 is the vacuum permittivity, and ω is the central frequency of the pump pulse. $P(z,t)$ is the macroscopic polarization of the system, and it is directly connected to the off-diagonal elements of density matrix, i.e.,

$$P(z,t) = N\sum_{j,i}\eta_{ji}\mu_{ji}\rho_{ji}(z,t) \quad (i,j = 1,2,3, \text{ or } 4), \tag{5}$$

with N being the atomic number density. For the Stark pulses, they will propagate in the system with the speed of light in vacuum, since they are far-off resonant.

Equation (4) can be rewritten, with the aid of Eqs. (2) and (5), as

$$\left(\frac{d}{dz}+\frac{1}{c}\frac{d}{dt}\right)\Omega_{\text{eff}}(z,t)=\frac{iN\omega|\mu_{\text{eff}}|^2}{4\hbar\epsilon_0 c}\sum_{j,i}\eta_{j,i}A_{j,i}\rho_{j,i}(z,t), \tag{6}$$

which becomes the propagation equation in terms of the effective Rabi frequency. Thus the equations we have to simultaneously solve are Eqs. (1) and (6), for the atom and field, respectively.

For the convenience of numerical calculations, we can write the density-matrix equations (1) and the propagation equation (6) in the local frame. By introducing $\xi = z$, and $\tau = t - z/c$, Eq. (6) becomes

$$\frac{d}{d\xi}\Omega_{\text{eff}}(\xi,\tau)=\frac{iN\omega|\mu_{\text{eff}}|^2}{4\hbar\epsilon_0 c}\sum_{j,i}\eta_{j,i}A_{j,i}\rho_{j,i}(\xi,\tau). \tag{7}$$

Correspondingly, the time derivative of ρ_{ij} in Eqs. (1) changes from $\partial/\partial t$ to $\partial/\partial\tau$.

The calculation detail for the propagation of the pump pulse in all kinds of the subsystems with degenerate or nondegenerate hyperfine structure under double D-STIRAP is given in Fig. 10. In the calculations, the parameters used in the density-matrix equations are the same as those in Figs. 6–8, for which we have already seen the perfect recovery of all subsystems within the framework of single-atom response after the pulse sequence has been applied. The multiplication factor $N\omega|\mu_{\text{eff}}|^2/4\hbar\epsilon_0 c$ in Eq. (7) is a constant and for simplicity it has been set to unity throughout this paper. From Fig. 10 we see that the temporal pump pulse profile in the local frame remains identical at any propagation depth, for example, $\xi = 0$ and 30. Note that the product of ξ and $N\omega|\mu_{\text{eff}}|^2/4\hbar\epsilon_0 c$ is in units of $1/\tau_S$. First this figure means that the pump pulse undergoes no distortion during the propagation. Secondly, since the relative time delay between the Stark and pump pulses does not change at all, the speed of the pump pulse in the system is found to be the speed of light in vacuum. The fact that the pulse timing in the local frame does not change during the propagation implies that the adiabatic condition is well-maintained during the propagation. Hence we can say that double D-STIRAP can induce perfect transparency for the near-resonant pump pulse in the two-level system with hyperfine structure.

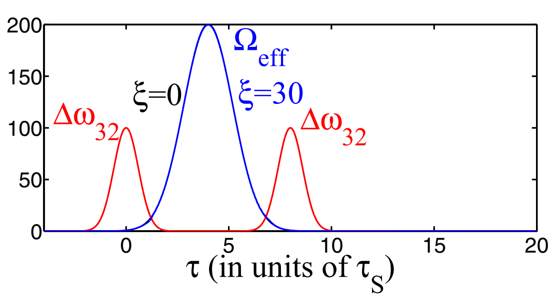

FIG. 10. Shape of the near-resonant pump pulse and the relative time delay with respect to the far-off resonant Stark pulse at different distances of $\xi = 0$ and $\xi = 30$ in the local frame. For all kinds of subsystems with degenerate or nondegenerate hyperfine structure, the calculation results are the same. Note that $\Delta\omega_{32}$ and Ω_{eff} are in units of $1/\tau_S$, while the product of ξ and $N\omega|\mu_{\text{eff}}|^2/4\hbar\epsilon_0 c$ is also in units of $1/\tau_S$.

Further calculations reveal (the results not shown here) that the different choice of time delay and intensities for the pulses hardly influences the transparency if these parameters are chosen appropriately according to Fig. 9. From all the above facts, we can say that double D-STIRAP is a robust method to induce transparency.

B. SIT in two-state system with hyperfine structure

SIT is a very well-known mechanism to induce transparency in the ideal two-level system. In order to highlight the significance of double D-STIRAP to induce transparency, we now study the behavior of the pump pulse without the Stark pulse in the two-level system with hyperfine structure.

One of the properties of SIT in the ideal two-level system is that, in the steady state, resonant hyperbolic scent pulse with an area of 2π becomes transparent through the medium of such atoms. This, however, does not necessarily mean that SIT is also guaranteed for the generalized two-level system with hyperfine structure. We now introduce a hyperbolic secant pulse with an area of 2π. It reads

$$\Omega'_{\text{eff}} = \Omega'^{0}_{\text{eff}}\,\text{sech}\left(\frac{1.76t}{\tau'_{\text{p}}}\right), \tag{8}$$

where the pulse duration is chosen as $\tau'_p = \tau_S$, and the effective Rabi frequency is $\Omega'^{0}_{\text{eff}} = 1.76A_p/2\pi\tau'_p$ with the area A_p equal to 2π.

To start with we let the above pump pulse propagate in the atomic medium which consists of subsystems described by the two-level, V, Λ, and double-Λ type subsystems with degenerate hyperfine structure. Calculation results based on Eqs. (1) and (7) after neglecting the terms associated with the Stark pulse are shown in Fig. 11. From Figs. 11(a) and 11(b), we can see that SIT is still realized in the two-level, V, Λ, and double-Λ type subsystems with $B = 0$. However, the propagation speed in the two-level, V type subsystem is faster than that in the Λ and double-Λ type subsystem with $B = 0$. As for the double-Λ type subsystem with $B \neq 0$, it is apparent from Fig. 11(c) that the pulse undergoes distortion during the propagation and consequently SIT is not realized. Clearly, SIT cannot be realized in the generalized two-level system with degenerate hyperfine structure.

Similar investigation for the propagation of a 2π pulse in all kinds of subsystems with nondegenerate hyperfine structure is shown in Fig. 12. The hyperfine splitting is set to $\omega_{21} = 0.5/\tau_S$ and $\omega_{43} = 0.05/\tau_S$, and we choose the detuning of the pulse with respect to the $|2\rangle - |3\rangle$ transition as $\delta_0 = -\frac{1}{2}(\omega_{21} + \omega_{43}) = -0.275/\tau_S$ for all subsystems. Compared with Fig. 11 for the generalized two-level subsystems with degenerate hyperfine structure, the attenuation and distortion of the pump pulse is even worse for those with nondegenerate hyperfine

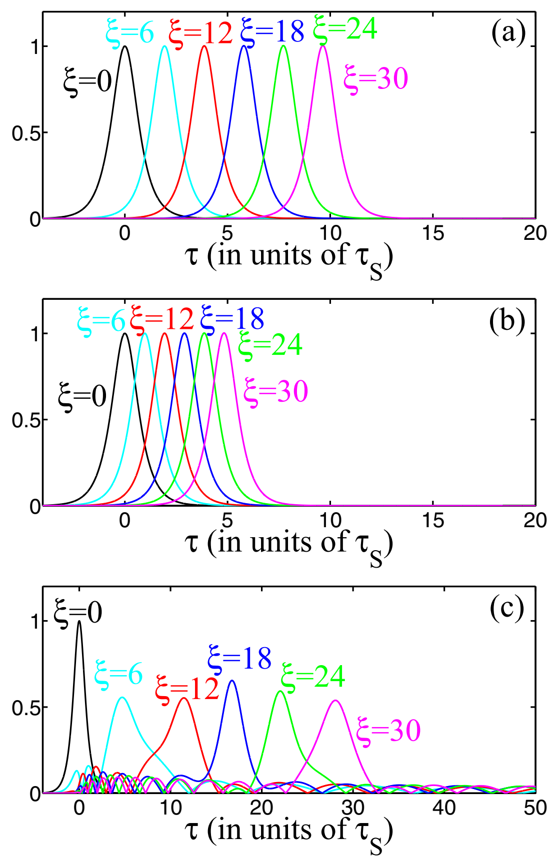

FIG. 11. Propagation of resonant pulse with an area of 2π and steady hyperbolic scent shape in subsystems of (a) two-level and V, (b) Λ and double-Λ type with $B = 0$, and (c) double-Λ type subsystems with $B \neq 0$, which are all the possible subsystems in the two-level system with degenerate hyperfine structure.

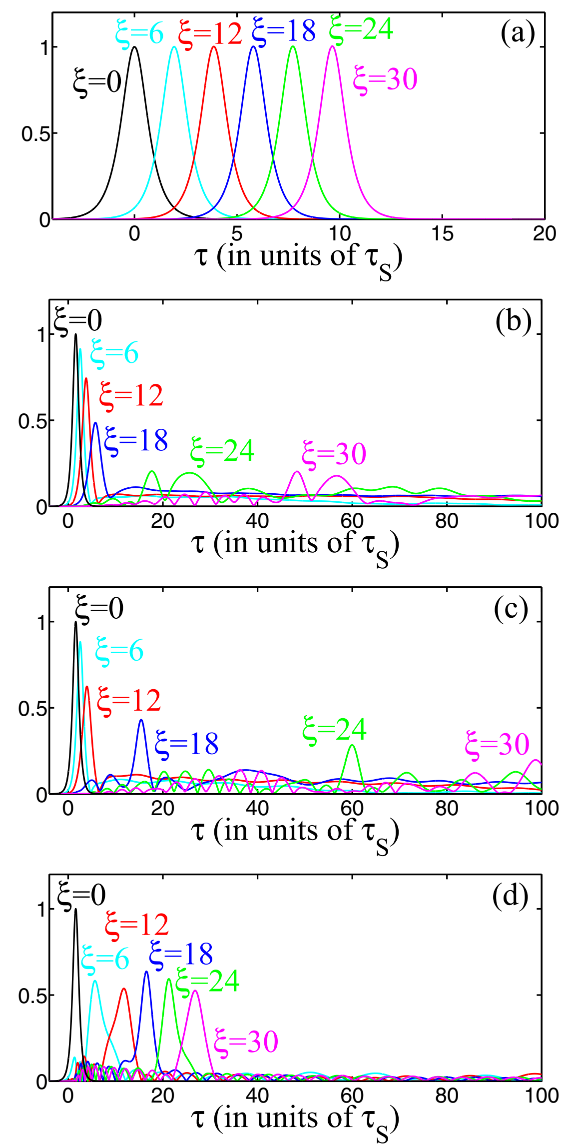

FIG. 12. Propagation of pulse with area of 2π and steady hyperbolic scent shape in subsystems of (a) V, (b) Λ, (c) double-Λ type with $B = 0$, and (d) double-Λ type with $B \neq 0$ in two-level system with nondegenerate hyperfine structure.

structure. SIT can only be realized in a nondegenerate V type subsystem, as shown in Fig. 12(a). This is because the upper level splitting is quite small compared with the bandwidth of the pulse we have chosen. For other kinds of nondegenerate subsystems such as nondegenerate Λ, double-Λ type subsystems, SIT does not come up. Our results are consistent with the experiment reported in Ref. [28]. In the experiment [28], nanosecond pulse is employed for SIT. Thus only one ground level $|3\,^2S_{1/2}\,F=2\rangle$ is coupled to the two upper levels $|3\,^2P_{1/2}\,F=1\rangle$ and $|3\,^2P_{1/2}\,F=2\rangle$. The three levels consist of two-level and nondegenerate V type subsystems, depending on the detuning. Hence the experiments confirm the area theorem. In contrast, in this work the other ground level $|3\,^2S_{1/2}\,F=1\rangle$ is also coupled to the two upper levels. From the results in Fig. 12 we can say that SIT cannot be realized in the generalized two-level system with nondegenerate hyperfine structure.

The results in Figs. 11 and 12 show that the perfect transparency cannot be realized if the pump pulse alone propagates in the generalized two-level system with hyperfine structure whether it is degenerate or nondegenerate, while double D-STIRAP can lead to perfect transparency, as we have shown in Fig. 10.

IV. CONCLUSIONS

In conclusion, we have studied the double D-STIRAP induced transparency beyond the ideal two-level system, i.e., generalized two-level systems with degenerate and nondegenerate hyperfine structures. Not only from the viewpoint of single-atom response but macroscopic response which includes propagation effects we have carried out the detailed study by simultaneously solving the density-matrix and propagation equations. From the calculation results we have found that the SIT cannot be achieved in the generalized two-level system with hyperfine structure. Using double D-STIRAP, however, we can obtain perfect transparency. Unlike the ideal two-level system, the transparency in the generalized two-level system with hyperfine structure has more stringent requirements. That is, in order to satisfy the adiabatic condition, the Stark and pump pulses should have stronger intensities and similar amplitude-to-time ratio between the effective Rabi frequency of the pump pulse and the detuning induced by the Stark pulse. During the propagation, the near-resonant pump pulse has the speed of light in vacuum, which ensures that the adiabatic condition is well preserved and the pulse remains transparent in the system. We close this paper by emphasizing that double D-STIRAP induced transparency is a robust method and it is not very sensitive to the specific choice of pulse intensities, time delay, and initial detuning of the pump pulse.

ACKNOWLEDGMENTS

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