Testing fundamental interactions on the helium atom

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We critically examine the current status of theoretical calculations of the energies, the fine structure, and the isotope shift of the lowest-lying states of helium, searching for unresolved discrepancies with experiments. Calculations are performed within the quantum electrodynamics expansion in powers of the fine structure constant α and the electron-to-nucleus mass ratio m/M. For energies, theoretical results are complete through orders $\alpha^6 m$ and $\alpha^6 m^2/M$, with the resulting accuracy ranging from 0.5 to 2 MHz for the $n = 2$ states. The fine-structure splitting of the $2\,^3P$ state is predicted with a much better accuracy, 1.7 kHz, as a consequence of a calculation of the next-order $\alpha^7 m$ effect. An excellent agreement of the theoretical predictions with the recent measurements of the fine structure provides one of the best tests of the bound-state QED in few-electron systems. The isotope shift between ^{3}He and ^{4}He is treated with a subkilohertz accuracy, which allows for a high-precision determination of the differences of the nuclear charge radii δr^2. Several such determinations, however, yield results that are in a 4σ disagreement with each other, which remains unexplained. Apart from this, we find no significant discrepancies between theory and experiment for the helium atom. A further calculation of the yet unknown $\alpha^7 m$ correction to energy levels will provide a sensitive test of universality in electromagnetic interactions of leptons by comparison of nuclear charge radii obtained by the helium and muonic helium spectroscopy.

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I. INTRODUCTION

Two-body bound systems such as hydrogen (H), hydrogenlike atoms ($\bar{\text{H}}$, He^+, μH), and pure leptonic atoms (positronium e^+e^- and muonium μ^+e^-) are commonly used for pursuing high-precision low-energy tests of the Standard Model. Comparisons of the measured transition frequencies with the theoretical calculations convey the extent to which the atomic energy level can be predicted by the Standard Model. If any discrepancy is found, it may be a signature of new physics or an indication that the values of physical constants are incorrect.

One of the best tests of fundamental physics in atomic systems is derived from the magnetic moment of the electron bound in the hydrogenlike carbon ion. The relative precision of the experiment of 3×10^{-11} [1] is matched by the complementary accuracy of *ab initio* theoretical calculations based on quantum electrodynamics (QED) [2,3]. Experiment and theory are in excellent agreement; their comparison is limited by the uncertainty of the electron mass, as taken from the best electron-trap measurement [4]. In practice, one reverses the problem and determines [5] the electron mass from the bound-electron g factor, gaining an improvement in accuracy by two orders of magnitude. So, this test of fundamental physics is presently limited by the accuracy of the electron-trap mass measurement [4].

Another prominent atomic test with a possible signature of new physics is based on the comparison of the Lamb shift of the muonic hydrogen μH [6] and the electronic hydrogen H (see Ref. [5] for a review). The lepton universality in the Standard Model states that the coupling constants of the electron and muon are equal, so one must use the same physical laws to predict the energy levels in H and μH and the same physical constants. What came out in practice, however, was a surprise. The proton root-mean-square charge radius, treated as an unknown parameter and extracted from the comparison of theory and experiment for the Lamb shift, turned out to be significantly different for the electronic [5] and muonic [6] spectra:

$$r_p(\text{H}) = 0.8770(45)\,\text{fm},$$
$$r_p(\mu\text{H}) = 0.8409(4)\,\text{fm}.$$

This discrepancy became known in the literature as the proton radius puzzle. It may signal the existence of interactions that are not accounted for, a lack of universality in the lepton-hadron interaction, or incorrect values of physical constants. Attempts to resolve the proton radius puzzle are currently being made in several experiments, such as measurements of the $2S$-$4P$ transition energy in H [7], the $1S$-$2S$ transition energy in He^+ [8,9], transitions between circular Rydberg states in heavy-H-like ions [10], and the direct comparison of the cross sections of the e-p versus μ-p elastic scattering [11]. Preliminary results from the H($2S$-$4P$) experiment [7] suggest that the presently accepted value of the Rydberg constant R_∞ might be incorrect, which would resolve the proton radius puzzle. Nevertheless, this suggestion needs to be checked by other experiments before definite conclusions can be drawn.

If the μH result for the proton radius is confirmed, a combination of the μH($2S$-$2P$) and H($1S$-$2S$) experiments will provide a much more accurate result for the R_∞ constant than the previously accepted value. Its accuracy will be limited by uncertainties in the two- and three-loop electron self-energy in H and the proton polarizability in μH. The electron self-energy can be improved by extensive calculations based on QED theory, whereas an improvement of the polarizability is less likely, because a part of it (the so-called subtraction term) cannot be deduced from inelastic electron-proton scattering data. Nonetheless, the prospective improvement of

the Rydberg constant will contribute to the extension of the atomic-physics tests of fundamental interactions.

The main goal of the present review is to demonstrate that accurate tests of fundamental physics can be obtained not only from the hydrogenic systems but also from the few-body atomic systems, such as He and He-like ions. In particular, the existing data of high-precision spectroscopy of helium can be used for setting constraints on spin-dependent forces between electrons [12], for the determination of the nuclear charge radius, and for the comparison with results obtained from muonic atoms and by the electron scattering. Such a determination would be of particular interest today, in the context of the proton radius puzzle and the ongoing experiment on muonic helium [13].

The accuracy achieved by the present-day theory is sufficient for accurate determinations of the *differences* of the nuclear radii from the isotope shift [14], but not the absolute values of radii. We demonstrate, however, that a calculation of the next-order $m\alpha^7$ QED correction to the energy levels will be sufficient for determination of the nuclear charge radii of helium isotopes on the level of 1% or better. Such a calculation is difficult but feasible, at least for the triplet states, and will provide a sensitive test of universality in electromagnetic interactions of leptons.

II. QUANTUM ELECTRODYNAMICS OF ATOMIC SYSTEMS

We now summarize the theoretical method for calculations of bound-state energy levels of light atoms. It is based on the Nonrelativistic QED (NRQED) expansion, originally introduced by Caswell and Lepage [15]. Although the method is applicable for an arbitrary light atom, we assume the simplest case of the two-electron atom in the formulas below.

The starting point of this approach is the NRQED Lagrangian L,

$$L = \psi^+(i\,\partial_t - H)\psi, \tag{1}$$

where ψ is the nonrelativistic fermion field and H is the effective NRQED Hamiltonian, which is, in our implementation, derived from the Dirac equation by the Foldy-Wouthuysen (FW) transformation [16],

$$\begin{aligned} H = {} & eA_0 + \frac{\pi^2}{2m} - \frac{\pi^4}{8m^3} + \frac{\pi^6}{16m^5} - \frac{e}{8m^2}\,\vec{\nabla}\vec{E} \\ & + \frac{5\,i\,e}{128\,m^4}\,[\pi^2,\vec{\pi}\,\vec{E} + \vec{E}\vec{\pi}] + \frac{3}{64\,m^4}\{\pi^2,e\vec{\nabla}\vec{E}\} \\ & - \frac{e}{m}\,\vec{s}\,\vec{B} - \frac{e}{4\,m^2}\,\vec{s}\,(\vec{E}\times\vec{\pi} - \vec{\pi}\times\vec{E}) \\ & + \frac{e}{8\,m^3}\{\pi^i,\{\pi^i,\vec{s}\,\vec{B}\}\} - \frac{e}{8\,m^3}\nabla^2(\vec{s}\,\vec{B}) - \frac{e^2}{8\,m^3}\,\vec{B}^2 \\ & + \frac{3\,e}{32\,m^4}\,\vec{s}\,\{\pi^2,\vec{E}\times\vec{\pi} - \vec{\pi}\times\vec{E}\} + \frac{e^2}{8\,m^3}\,\vec{E}^2 + \cdots . \end{aligned} \tag{2}$$

Here, $\vec{\pi} = \vec{p} - e\vec{A}$, $\vec{s} = \vec{\sigma}/2$ is the electron spin operator, and e and m are the electron charge and mass, respectively. The dots in the above equation denote the higher-order terms that include, in addition to the FW Hamiltonian, local counterterms originating from virtual electron momenta of the order of the electron mass. The counterterms are derived by matching the NRQED and the full-QED scattering amplitudes.

Once the NRQED Lagrangian is obtained, the Feynman path-integral approach is used to derive various corrections to the nonrelativistic multielectron propagator $G(t-t')$, where t and t' are the common time of the *out* and the *in* electrons, correspondingly. The Fourier transform of the propagator is written as

$$G(E) = \frac{1}{E - H_0 - \Sigma(E)},$$

where H_0 is the Schrödinger-Coulomb Hamiltonian for N electrons. H_0 may also include the nucleus as a dynamic particle. The $\Sigma(E)$ operator incorporates corrections due to the photon exchange, the electron and photon self-energy, etc.

The energy of a bound state is obtained as a position of the pole of the matrix element of $G(E)$ between the nonrelativistic wave functions ϕ of the reference state,

$$\begin{aligned} \langle\phi|G(E)|\phi\rangle = {} & \frac{1}{E-E_0} + \frac{1}{(E-E_0)^2}\,\langle\phi|\Sigma(E)|\phi\rangle \\ & + \frac{1}{(E-E_0)^2}\,\langle\phi|\Sigma(E)\,\frac{1}{E-H_0}\Sigma(E)|\phi\rangle + \cdots \\ = {} & \frac{1}{E-E_0-\sigma(E)}, \end{aligned} \tag{3}$$

where

$$\sigma(E) = \langle\phi|\Sigma(E)|\phi\rangle + \langle\phi|\Sigma(E)\,\frac{1}{(E-H_0)'}\,\Sigma(E)|\phi\rangle + \cdots . \tag{4}$$

The resulting bound-state energy E (the position of the pole) is

$$E = E_0 + \sigma(E_0) + \sigma(E_0)\frac{\partial\sigma(E_0)}{\partial E_0} + \cdots . \tag{5}$$

The basic assumption of the NRQED is that E can be expanded in a power series of the fine-structure constant α,

$$\begin{aligned} E\left(\alpha,\frac{m}{M}\right) = {} & \alpha^2 E^{(2)}\left(\frac{m}{M}\right) + \alpha^4 E^{(4)}\left(\frac{m}{M}\right) + \alpha^5 E^{(5)}\left(\frac{m}{M}\right) \\ & + \alpha^6 E^{(6)}\left(\frac{m}{M}\right) + \alpha^7 E^{(7)}\left(\frac{m}{M}\right) + \cdots , \end{aligned} \tag{6}$$

where m/M is the electron-to-nucleus mass ratio and the expansion coefficients $E^{(n)}$ may contain finite powers of $\ln\alpha$. The coefficients $E^{(i)}(m/M)$ are further expanded in powers of m/M:

$$E^{(i)}\left(\frac{m}{M}\right) = E^{(i,0)} + \frac{m}{M}\,E^{(i,1)} + \left(\frac{m}{M}\right)^2 E^{(i,2)} + \cdots . \tag{7}$$

The expansion coefficients in Eqs. (6) and (7) can be expressed as expectation values of some effective Hamiltonians with the nonrelativistic wave function. The derivation of these effective Hamiltonians is the central problem of the NRQED method. While the leading-order expansion terms are simple, formulas becomes increasingly complicated for high powers of α.

The first term of the NRQED expansion of the bound-state energy, $E^{(2,0)} \equiv E$, is the eigenvalue of the Schrödinger-Coulomb Hamiltonian in the infinite nuclear mass limit,

$$H_0 \equiv H = \frac{p_1^2}{2} + \frac{p_2^2}{2} - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r}\,, \tag{8}$$

where $r \equiv r_{12}$. The finite nuclear mass corrections to $E^{(2,0)}$ can be obtained perturbatively:

$$E^{(2,1)} = \langle \delta_M H \rangle, \tag{9}$$

$$E^{(2,2)} = \left\langle \delta_M H \, \frac{1}{(E-H)'} \, \delta_M H \right\rangle, \tag{10}$$

$$E^{(2,3)} = \left\langle \delta_M H \frac{1}{(E-H)'} (\delta_M H - \langle \delta_M H \rangle) \frac{1}{(E-H)'} \delta_M H \right\rangle, \tag{11}$$

where

$$\delta_M H = \frac{\vec{P}^2}{2} \tag{12}$$

is the nuclear kinetic energy operator, with $\vec{P} = -\vec{p}_1 - \vec{p}_2$ being the nuclear momentum.

The next term of the expansion, $E^{(4)}$, is the leading relativistic correction induced by the Breit Hamiltonian $H^{(4)}$ and the corresponding recoil addition $\delta_M H^{(4)}$:

$$E^{(4,0)} = \langle H^{(4)} \rangle, \tag{13}$$

$$E^{(4,1)} = 2 \left\langle H^{(4)} \, \frac{1}{(E-H)'} \, \delta_M H \right\rangle + \langle \delta_M H^{(4)} \rangle, \tag{14}$$

$$\begin{aligned} E^{(4,2)} = {} & 2 \left\langle \delta_M H^{(4)} \, \frac{1}{(E-H)'} \, \delta_M H \right\rangle + \left\langle \delta_M^2 H^{(4)} \right\rangle \\ & + 2 \left\langle H^{(4)} \, \frac{1}{(E-H)'} \, (\delta_M H - \langle \delta_M H \rangle) \, \frac{1}{(E-H)'} \, \delta_M H \right\rangle \\ & + \left\langle \delta_M H \, \frac{1}{(E-H)'} (H^{(4)} - \langle H^{(4)} \rangle) \, \frac{1}{(E-H)'} \, \delta_M H \right\rangle. \end{aligned} \tag{15}$$

The Breit Hamiltonian (without spin-dependent terms that do not contribute to the centroid energies) is given by

$$\begin{aligned} H^{(4)} = {} & -\frac{1}{8} \left(p_1^4 + p_2^4 \right) + \frac{Z\pi}{2} \left[\delta^3(r_1) + \delta^3(r_2) \right] + \pi \, \delta^3(r) \\ & - \frac{1}{2} \, p_1^i \left(\frac{\delta^{ij}}{r} + \frac{r^i \, r^j}{r^3} \right) p_2^j, \end{aligned} \tag{16}$$

$$\delta_M H^{(4)} = \frac{Z}{2} \left[p_1^i \left(\frac{\delta^{ij}}{r_1} + \frac{r_1^i \, r_1^j}{r_1^3} \right) + p_2^i \left(\frac{\delta^{ij}}{r_2} + \frac{r_2^i \, r_2^j}{r_2^3} \right) \right] P^j. \tag{17}$$

For a spinless nucleus, there is no nuclear Darwin correction and $\delta_M^2 H^{(4)} = 0$.

The leading QED correction $E^{(5)}$ is induced by the effective Hamiltonian $H^{(5)}$ [17,18] and the corresponding recoil addition $\delta_M H^{(5)}$,

$$\begin{aligned} H^{(5)} = {} & \sum_a \left(\frac{19}{30} + \ln(\alpha^{-2}) - \ln k_0 \right) \frac{4\,Z}{3} \, \delta^3(r_a) \\ & + \left(\frac{164}{15} + \frac{14}{3} \, \ln \alpha \right) \delta^3(r) - \frac{7}{6\,\pi} \, P\left(\frac{1}{r^3} \right), \\ \delta_M H^{(5)} = {} & \sum_a \left\{ \left(\frac{62}{3} + \ln(\alpha^{-2}) - 8 \, \ln k_0 - \frac{4}{Z} \, \delta_M \ln k_0 \right) \right. \\ & \left. \times \frac{Z^2}{3} \, \delta^3(r_a) - \frac{7\,Z^2}{6\,\pi} \, P\left(\frac{1}{r_a^3} \right) \right\}, \end{aligned} \tag{18}$$

where $\ln k_0$ is the Bethe logarithm

$$\ln k_0 = \frac{\left\langle \sum_a \vec{p}_a \, (H-E) \, \ln[2\,(H-E)] \, \sum_b \vec{p}_b \right\rangle}{2\,\pi \, Z \left\langle \sum_c \delta^3(r_c) \right\rangle}, \tag{19}$$

$P(1/r^3)$ denotes the Araki-Sucher term

$$\begin{aligned} \left\langle P\left(\frac{1}{r^3} \right) \right\rangle = {} & \lim_{\epsilon \to 0} \int d^3 r \, \phi^*(\vec{r}) \left[\frac{1}{r^3} \, \Theta(r-\epsilon) + 4\,\pi \, \delta^3(r) \right. \\ & \left. \times (\gamma + \ln \epsilon) \right] \phi(\vec{r}), \end{aligned} \tag{20}$$

and $\delta_M \ln k_0$ is a correction to Bethe logarithm $\ln k_0$ induced by the nonrelativistic kinetic energy $\delta_M H$ in Eq. (12).

The next expansion term $E^{(6)}$ is the higher-order QED correction, whose general form is

$$E^{(6)} = \langle H^{(6)} \rangle + \left\langle H^{(4)} \, \frac{1}{(E-H)'} \, H^{(4)} \right\rangle. \tag{21}$$

The complete derivation of $H^{(6)}$ was presented in the nonrecoil limit in Ref. [19], whereas the recoil correction $\delta_M H^{(6)}$ was obtained recently in Refs. [20,21]. The corresponding formulas are much too complicated to be presented here; we thus refer the reader to the original works. The main problem of the derivation of $E^{(6)}$ is that the first- and second-order matrix elements in Eq. (21) are divergent; the divergences cancel only when both terms are considered together. In order to subtract the singularities algebraically, one derives $H^{(n)}$ in $d = 3 - 2\epsilon$ dimensions, makes use of various commutator identities to eliminate divergences, and then takes the limit $\epsilon \to 0$.

The next term $E^{(7)}$ has the general form of

$$E^{(7)} = \langle H^{(7)} \rangle + 2 \left\langle H^{(4)} \, \frac{1}{(E-H)'} \, H^{(5)} \right\rangle. \tag{22}$$

So far $E^{(7)}$ has been calculated only for the fine structure of helium and heliumlike ions [22]. In the future it should be possible to extend this calculation to the energies of the triplet states of helium. The main problem would be the derivation of $H^{(7)}$ and the numerical calculation of relativistic corrections to the Bethe logarithm.

III. BINDING ENERGIES AND TRANSITION FREQUENCIES

The numerical results of our calculations of the individual α and m/M-expansion contributions to the ionization energies of the lowest-lying states of the helium atom are listed in Table I.

TABLE I. Breakdown of theoretical contributions to the ionization (centroid) energies of the lowest-lying states of ^{4}He, in MHz. The nuclear parameters used in the calculations are $M/m = 7\,294.299\,541\,36\,(24)$, $r = 1.6755\,(28)$ fm. "NS" denotes the finite nuclear size contribution.

	$(m/M)^0$	$(m/M)^1$	$(m/M)^2$	$(m/M)^3$	Sum
$1\,^1S$					
α^2	−5 946 220 752.325	958 672.945	−209.270	0.049	−5 945 262 288.601
α^4	16 904.024	−103.724	0.028		16 800.327
α^5	40 506.158	−10.345			40 495.813
α^6	861.360	−0.348			861.012
α^7	−71. (36.)				−71. (36.)
NS	29.7 (1)				29.7 (1)
Total					−5 945 204 173. (36.)
$2\,^1S$					
α^2	−960 463 083.665	140 245.887	−37.131	0.010	−960 322 874.899
α^4	−11 971.453	−3.344	0.003		−11 974.795
α^5	2 755.761	−0.627			2 755.134
α^6	58.288	−0.022			58.267
α^7	−3.7 (1.9)				−3.7 (1.9)
NS	2.007 (7)				2.007 (7)
Total					−960 332 038.0 (1.9)
$2\,^3S$					
α^2	−1 152 953 922.421	164 775.354	−30.620	0.006	−1 152 789 177.680
α^4	−57 629.312	4.284	−0.001		−57 625.029
α^5	3 999.432	−0.800			3 998.632
α^6	65.235	−0.030			65.205
α^7	−5.2 (1.3)				−5.2 (1.3)
NS	2.610 (9)				2.610 (9)
Total					−1 152 842 741.4 (1.3)
$2\,^1P$					
α^2	−814 848 364.923	153 243.883	−47.514	0.016	−814 695 168.538
α^4	−14 024.044	−2.809	0.004		−14 026.850
α^5	38.769	0.470			39.240
α^6	8.818	−0.003			8.815
α^7	0.81 (40)				0.81 (40)
NS	0.064				0.064
Total					−814 709 146.46 (40)
$2\,^3P$					
α^2	−876 178 284.885	61 871.895	−25.840	0.006	−876 116 438.823
α^4	11 436.878	11.053	0.002		11 447.932
α^5	−1 234.732	−0.614			−1 235.346
α^6	−21.832	−0.001			−21.833
α^7	2.9 (0.7)				2.9 (0.7)
NS	−0.796 (3)				−0.796 (3)
Total					−876 106 246.0 (0.7)

These results mostly correspond to our calculations reported in Refs. [19,23,24], with two improvements: (i) we included the $\alpha^6\,m^2/M$ correction recently calculated in Refs. [20,21] and (ii) we added the higher-order recoil corrections $E^{(2,3)}$ and $E^{(4,2)}$ computed according to Eqs. (11) and (15). The uncertainty of the total energies is exclusively defined by the $\alpha^7\,m$ contribution, whose complete form is unknown at present. The approximate results for this correction listed in Table I were obtained by rescaling the hydrogenic values as described in Ref. [25], and the uncertainty is assumed to be 50% for the singlet states and 25% for the triplet states [due to vanishing of all terms proportional $\delta(r)$]. We note that in our previous compilation [23], the uncertainty was estimated as 50% for all states.

In Table II we compare our theoretical predictions for the ionization energy of the ground state and for transition energies of helium with the results of the most accurate measurements. Our results are in agreement with those of Drake [38] but are significantly more accurate. Our theoretical predictions agree well with the experimental results for the $2\,^1S$–$2\,^3S$, $2\,^3S$–$2\,^1P_1$, and $2\,^3P$–$2\,^3S$ transitions and for the $1\,^1S$ ground-state ionization energy. For the $1\,^1S$–$2\,^1S$ transition, however, we find a deviation of 180(36)(48) MHz from the measured value. Bearing in mind the good agreement observed for the other transitions involving the $1\,^1S$ and $2\,^1S$ states, we believe that the problem is likely to be on the experimental side. Almost equal differences between experimental and theoretical values are observed for different transitions involving the $3D$ states;

TABLE II. Comparison of the theoretical predictions for various transitions in ^{4}He with the experimental results, in MHz. IE denotes the ionization energy.

	Experiment/Theory/Difference	Ref.
$1\,^1S_0$ (IE)	5 945 204 212. (6)	[26]
	5 945 204 173. (36.)	
	39. (36.)	
$2\,^1S_0$ (IE)	960 332 041.01(15)	[27]
	960 332 038.0 (1.9)	
	3.0(1.9)	
$1\,^1S_0$–$2\,^1S_0$	4 984 872 315. (48.)	[28]
	4 984 872 135. (36.)	
	180. (60.)	
$2\,^3S_1$–$3\,^3D_1$	786 823 850.002 (56)	[29]
	786 823 848.4 (1.3)[a]	
	1.6 (1.3)	
$2\,^1S_0$–$2\,^1P_1$	145 622 892.886 (183)	[30]
	145 622 891.5(2.3)	
	1.4(2.3)	
$2\,^1P_1$–$3\,^1D_2$	448 791 399.113 (268)	[31]
	448 791 397.4(0.4)[b]	
	1.7(0.5)	
$2\,^3P_0$–$3\,^3D_1$	510 059 755.352 (28)	[32]
	510 059 754.0 (0.7)[a]	
	1.4 (0.7)[a]	
$2\,^3P$–$2\,^3S_1$	276 736 495.649 (2)[c]	[33]
	276 736 495.4 (2.0)	
	0.2 (2.0)	
$2\,^3S_1$–$2\,^1P_1$	338 133 594.4 (5)	[34]
	338 133 594.9 (1.4)	
	−0.5 (2.2)	
$2\,^1S_0$–$2\,^3S_1$	192 510 702.145 6 (18)	[35]
	192 510 703.4 (0.8)	
	−1.3 (0.8)	

[a]Using theoretical value $E(3\,^3D_1) = 366\,018\,892.97\,(2)$ MHz from Ref. [36].
[b]Using theoretical value $E(3\,^1D_2) = 365\,917\,749.02\,(2)$ MHz from Ref. [36].
[c]Using theoretical results for $2\,^3P$ fine structure from Ref. [37].

namely, 1.6 (1.3) MHz for the $2\,^3S$–$3\,^3D_1$ transition, 1.4 (0.7) MHz for the $2\,^3P_0$–$3\,^3D_1$, and 1.7 (0.5) MHz for the $2\,^1P_1$–$3\,^1D_2$ ones, which suggests a recalculation of the $3D$ energies (not calculated by us but taken from Ref. [36]).

In all cases except for the $1\,^1S$–$2\,^1S$ transition theoretical predictions are less accurate than the experimental results, which indicates the importance of the yet unknown $\alpha^7 m$ correction. As can be seen from Table I, a calculation of this correction would improve the theoretical precision up to the level of about 10 kHz. Combining such theory with the available experimental result for the $2\,^3S$–$2\,^3P$ transition [33], one will be able to determine the nuclear charge radius with a sub-1% accuracy, which is comparable with the expected precision of the radius from the μHe experiment [13]. Specifically, the finite nuclear size contribution to the $2\,^3S$–$2\,^3P$ transition energy is $E_{\rm fs} = 3\,450$ kHz$\times h$. Since $E_{\rm fs}$ is proportional to r^2, the assumed 10-kHz theoretical error corresponds to the following error of r:

$$\frac{\Delta r}{r} = \frac{1}{2}\frac{\delta E_{\rm fs}}{E_{\rm fs}} \approx \frac{1}{2}\frac{10}{3\,450} \approx 1.5\times 10^{-3}.$$

A similar determination of the charge radii of nuclei of light elements should be possible from the $2\,^3S$–$2\,^3P$ transitions in He-like ions. Measurements at the required level of accuracy are planned for He-like boron and carbon [39]. On the theoretical side, the higher-order relativistic effects in He-like ions become much more important than in the helium atom, and thus the NRQED approach will need to be combined with the fully relativistic calculations based on the $1/Z$ expansion [40], as already demonstrated by Drake in Ref. [25]. It is important that a determination of the charge radius of one stable isotope of a light element will give us access to the whole chain of radii of other isotopes, because the differences of the radii are nowadays very efficiently extracted from the isotope shift measurements [41].

IV. FINE STRUCTURE OF $2\,^3P_J$ STATE

The fine-structure transition frequencies between the $2\,^3P_J$ levels are presently the most accurately known transition frequencies in helium. On the theoretical side, these transitions are calculated rigorously within NRQED up to order $\alpha^7 m$ [22,37,42–44], with a resulting theoretical accuracy of about 1 kHz. A summary of the theoretical results for individual α and m/M expansion fine-structure contributions is presented in Table III. The small deviations from the values reported in our original calculation [22] are due to the updated value of α.

TABLE III. Breakdown of theoretical contributions to the energies of the $2\,^3P_J$ fine-structure levels of ^{4}He, with respect to the centroid $2\,^3P$ energy, in MHz. The value of α used in calculations is $\alpha^{-1} = 137.035\,999\,139\,(31)$.

	$(m/M)^0$	$(m/M)^{1+}$	Sum
$2\,^3P_0$			
α^4	27 566.991 8	0.934 0	27 567.925 9
α^5	36.102 4	−0.001 6	36.100 8
α^6	−5.043 6	−0.003 8	−5.047 4
α^7	0.077 9		0.077 9
α^8	0.000 0 (8)		0.000 0 (8)
Total			27 599.057 2 (8)
$2\,^3P_1$			
α^4	−1 997.604 0	1.764 2	−1 995.839 8
α^5	−18.605 4	0.002 2	−18.603 2
α^6	−3.436 1	0.006 2	−3.429 9
α^7	−0.022 4		−0.022 4
α^8	0.000 0 (8)		0.000 0 (8)
Total			−2 017.895 3 (8)
$2\,^3P_2$			
α^4	−4 314.835 9	−1.245 3	−4 316.081 3
α^5	3.942 8	−0.001 0	3.941 8
α^6	3.070 4	−0.003 0	3.067 4
α^7	−0.002 1		−0.002 1
α^8	0.000 0 (8)		0.000 0 (8)
Total			−4 309.074 2 (8)

TABLE IV. Comparison of the theoretical predictions for the $2\,^3P$ fine-structure intervals of ^{4}He with the experimental results, in kHz.

	$2\,^3P_0$–$2\,^3P_2$	$2\,^3P_1$–$2\,^3P_2$	$2\,^3P_0$–$2\,^3P_1$
Theory			
Pachucki and Yerokhin [22]	31 908 131.4 (1.7)	2 291 178.9 (1.7)	29 616 952.5 (1.7)
Experiment			
Zheng *et al.* [46]	31 908 130.98 (13)	2 291 177.56 (19)	
Feng *et al.* [47]		2 291 177.69 (36)	
Smiciklas *et al.* [48]	31 908 131.25 (30)		
Smiciklas *et al.* [48] reevaluated in Ref. [45]	31 908 131.25 (32)		
Borbely *et al.* [49]		2 291 177.53 (35)	
Borbely *et al.* [49] reevaluated in Ref. [45]		2 291 177.55 (35)	
Zelevinsky *et al.* [50]	31 908 126.8 (0.9)	2 291 175.6 (0.5)	29 616 951.7 (0.7)
Zelevinsky *et al.* [50] reevaluated in Ref. [45]	31 908 126.8 (3.0)	2 291 176.8 (1.1)	29 616 951.7 (3.0)
Guisfredi *et al.* [51]			29 616 952.7 (1.0)
Guisfredi *et al.* [51] reevaluated in Ref. [45]			29 616 953. (10.0)
George *et al.* [52]			29 616 950.9 (0.9)
George *et al.* [52] reevaluated in Ref. [45]			29 616 950.8 (0.9)
Castillega *et al.* [53]		2 291 175.9 (1.0)	
Castillega *et al.* [53] reevaluated in Ref. [45]		2 291 177.1 (1.0)	

The uncertainty of the theoretical predictions is fully defined by the unknown higher-order $\alpha^8 m$ contribution. The uncertainties listed in the table are obtained by multiplying the corresponding values for the $\alpha^6 m$ corrections by $(Z\alpha)^2$.

On the experimental side, there were numerous results for the fine-structure intervals of helium obtained during the last decades, some of them contradicting each other. Recently, it was pointed out [45] that the effect of the quantum-mechanical interference between neighboring resonances (even if such neighbors are separated by thousands of natural widths) can cause significant shifts of the line center. The reexamination of existing measurements presented in Ref. [45] improved the overall agreement of the experimental results with the theoretical predictions, whereas the two latest measurements [46,47] are in excellent agreement with the theory. The comparison of the present theory with the experimental results for the fine-structure intervals of helium are presented in Table IV and Fig. 1.

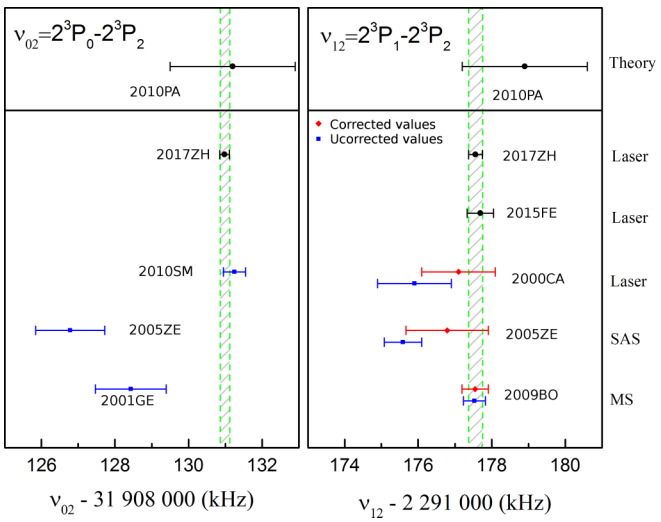

FIG. 1. Comparison of the theoretical and experimental results for the $2\,^3P_0$–$2\,^3P_2$ and $2\,^3P_1$–$2\,^3P_2$ intervals of the helium fine structure based on figure from Ref. [46]. Abbreviations are as follows: 2017ZH, Ref. [46]; 2015FE, Ref. [47]; 2010SM, Ref. [48]; 2009BO, Ref. [49]; 2005ZE, Ref. [50]; 2001GE, Ref. [52]; 2000CA, Ref. [53]; and 2010PA, Ref. [22].

A combination of the experimental and theoretical results for the fine structure of helium can be used in order to determine the fine-structure constant α with an accuracy of 31 ppb [22,37], which is about two orders of magnitude less precise than the current best determination of α [5]. Further improvement of theory by calculating the next-order $\alpha^8 m$ correction appears to be too complicated to be accomplished in the near future. However, an identification of the $\alpha^8 m$ contribution from experiments on light He-like ions and rescaling it to helium could provide an improvement of the theoretical precision and, therefore, the accuracy of the helium α determination.

V. ISOTOPE SHIFT

The isotope shift is defined, for the spinless nuclei, as the difference of the transition frequencies of different isotopes of the same element. For the ^{4}He and ^{3}He isotopes, however, the comparison of the spectra is complicated by the presence of the nuclear spin in ^{3}He and, as a consequence, by a large mixing of the fine and hyperfine sublevels. In order to separate out the effects of the nuclear spin in ^{3}He, the isotope shift of the $2S$ and $2P$ levels is defined [14] as the shift of the centroid energies, which are the average over all fine and hyperfine energy sublevels,

$$E(2\,^{2S+1}L) = \frac{\sum_{J,F}(2F+1)\,E(2\,^{2S+1}L_{J,F})}{(2I+1)(2S+1)(2L+1)}, \qquad (23)$$

where $^{2S+1}L_{J,F}$ denotes the state with electron angular momentum L, spin S, and total momentum J, whereas F is the total momentum of the atom. The theory of the helium isotope shift was described in detail in our recent investigation [14], so it is not repeated here.

TABLE V. Determinations of the nuclear charge difference of ^{3}He and ^{4}He, $\delta r^2 \equiv r^2(^3\mathrm{He}) - r^2(^4\mathrm{He})$ from different measurements. Units are kHz if not stated otherwise. δE is the part of the isotope shift induced by the finite nuclear size, represented as $\delta E = C\,\delta r^2$, with C being the coefficient calculated from theory.

Determination from Rooij *et al.* [35]		
$E(^3\mathrm{He}, 2\,^1S^{F=1/2}\text{–}2\,^3S^{F=3/2}) - E(^4\mathrm{He}, 2\,^1S\text{–}2\,^3S)$	−5 787 719.2(2.4)	Expt. [35]
$\delta E_{\mathrm{hfs}}(2\,^3S^{3/2})$	−2 246 567.059(5)	Expt. [57,58]
$-\delta E_{\mathrm{iso}}(2\,^1S\text{–}2\,^3S)$ (point nucleus)	8 034 065.91 (19)	Theory [20,21]
δE	−220.4(2.4)	
C	−214.66 (2) kHz/fm^2	[14]
δr^2	1.027 (11) fm^2	[21]
Determination from Cancio Pastor *et al.* [33,55]		
$E(^3\mathrm{He}, 2\,^3P\text{–}2S)$ (centroid)	276 702 827 204.8 (2.4)	Expt. [55]
$-E(^4\mathrm{He}, 2\,^3P\text{–}2S)$ (centroid)	−276 736 495 649.5 (2.1)	Expt. [33,48][a]
$-\delta E_{\mathrm{iso}}(2\,^3P\text{–}2\,^3S)$ (point nucleus)	33 667 149.3 (0.9)	Theory [20,21]
δE	−1 295.4 (3.3)	
C	−1212.2 (1) kHz/fm^2	[14]
δr^2	1.069 (3) fm^2	[20]
Determination from Shiner *et al.* [56]		
$E(^3\mathrm{He}, 2\,^3P_0{}^{1/2}\text{–}2\,^3S_1{}^{3/2}) - E(^4\mathrm{He}, 2\,^3P_2\text{–}2\,^3S_1)$	810 599.0 (3.0)	Expt. [56]
$\delta E_{\mathrm{hfs}}(2\,^3S_1{}^{3/2})$	−2 246 567.059 (5)	Expt. [57,58]
$\delta E_{\mathrm{fs}}(2\,^3P_2)$	−4 309 074.2 (1.7)	Theory [22]
$-\delta E_{\mathrm{fs,hfs}}(2\,^3P_0{}^{1/2})$	−27 923 393.7 (1.7)	Theory [20,21]
$-\delta E_{\mathrm{iso}}(2\,^3P\text{–}2\,^3S)$ (point nucleus)	33 667 149.3 (0.9)	Theory [20,21]
δE	−1286.7 (3.5)	
C	−1212.2 (1) kHz/fm^2	[14]
δr^2	1.061 (3) fm^2	[20]

[a]The centroid energy E is obtained as $E = (6\,E_0 + 3\,E_1 - 5\,E_{02})/9$, where $E_{0,1} \equiv E(2\,^3S_1\text{–}2\,^3P_{0,1})$ from Ref. [33] and $E_{02} \equiv E(2\,^3P_0\text{–}2\,^3P_2)$ from Ref. [48].

A remarkable feature of the isotope shift is that the relative contribution of the finite nuclear size effect to it is much larger than that to the transition energies. In particular, for the $2\,^3S\text{–}2\,^3P$ transition energy, the finite nuclear size correction is only a 5×10^{-9} effect, while for the isotope shift it becomes as large as 4×10^{-5}. Because of this, the $2\,^3S\text{–}2\,^3P$ transition is particularly suitable for determinations of the nuclear radii differences from the isotope shift.

The present theoretical accuracy for the isotope shift of the $2S$ and $2P$ states of helium is at a subkilohertz level [14,20,21], which enables precise determinations of the nuclear charge radius difference of ^{4}He and ^{3}He isotopes. Because the theory is supposed to be so very accurate, the precision of these determinations is orders of magnitude higher than the traditional determinations by means of the electron scattering [54], and it is limited only by the uncertainty of the frequency measurements.

Table V reports the present status of the determinations of the ^{3}He-^{4}He nuclear charge radii difference δr^2 from the isotope shift of the $2\,^3S\text{–}2\,^3P$ and $2\,^3S\text{–}2\,^1S$ transitions, as obtained in different experiments. Surprisingly, these two transitions lead [20,21] to contradicting results for δr^2; i.e., 1.069 (3) fm^2 and 1.061 (3) fm^2 from the $2\,^3S\text{–}2\,^3P$ transition versus 1.027 (11) fm^2 from the $2\,^3S\text{–}2\,^1S$ transition. Obviously, the nuclear charge radius has to be the same, provided that no new physics is involved. The numerically dominating part of the theoretical predictions for the isotope shift is verified by checking against independent calculations by Drake and co-workers [36,59] (see the comparison in Tables 1 and 2 of Ref. [14]). The difference in the calculations is 2 kHz for the $2\,^3S\text{–}2\,^1S$ and 3 kHz for the $2\,^3S\text{–}2\,^3P$ calculations and cannot explain the 4σ discrepancy between the results for δr^2. This unexplained discrepancy calls for the verification of the experimental results, first of all, the $2\,^3S\text{–}2\,^1S$ transition, for which only one measurement has been reported in the literature.

VI. SUMMARY

We have examined the current status of the theory and the experiment for the energies, the fine structure, and the isotope shift of the lowest-lying states of helium. The comparison of theoretical predictions and experimental results does not indicate significant discrepancies, apart from the one for the isotope shifts between ^{4}He and ^{3}He, which remains to be confirmed. With a further improvement of theory, i.e., with a calculation of the $\alpha\,^7m$ correction, it will be possible to determine the nuclear charge radii in He and He-like ions. Such a determination, combined with a complementary determination from muonic atoms, will provide a sensitive test of universality in electromagnetic interactions of leptons.

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[1] S. Sturm, F. Köhler, J. Zatorski, A. Wagner, Z. Harman, G. Werth, W. Quint, C. H. Keitel, and K. Blaum, Nature (London) **506**, 467 (2014).

[2] K. Pachucki, U. D. Jentschura, and V. A. Yerokhin, Phys. Rev. Lett. **93**, 150401 (2004); **94**, 229902(E) (2005).

[3] K. Pachucki, A. Czarnecki, U. D. Jentschura, and V. A. Yerokhin, Phys. Rev. A **72**, 022108 (2005).

[4] D. L. Farnham, R. S. V. Dyck, Jr., and P. B. Schwinberg, Phys. Rev. Lett. **75**, 3598 (1995).

[5] P. J. Mohr, D. B. Newell, and B. N. Taylor, Rev. Mod. Phys. **88**, 035009 (2016).

[6] R. Pohl *et al.*, Nature (London) **466**, 213 (2010).

[7] L. Maisenbacher (unpublished).

[8] K. Eikema (private communication).

[9] T. Udem (private communication).

[10] J. N. Tan, S. M. Brewer, and N. D. Guise, Phys. Scr., **T144**, 014009 (2011).

[11] R. Gilman *et al.*, arXiv:1303.2160.

[12] F. Ficek, D. F. J. Kimball, M. G. Kozlov, N. Leefer, S. Pustelny, and D. Budker, Phys. Rev. A **95**, 032505 (2017).

[13] A. Antognini *et al.*, Can. J. Phys. **89**, 47 (2011).

[14] K. Pachucki and V. A. Yerokhin, J. Phys. Chem. Ref. Data **44**, 031206 (2015).

[15] W. E. Caswell and G. P. Lepage, Phys. Lett. B **167**, 437 (1986).

[16] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).

[17] H. Araki, Prog. Theor. Phys. **17**, 619 (1957).

[18] J. Sucher, Phys. Rev. **109**, 1010 (1958).

[19] K. Pachucki, Phys. Rev. A **74**, 022512 (2006).

[20] V. Patkóš, V. A. Yerokhin, and K. Pachucki, Phys. Rev. A **94**, 052508 (2016).

[21] V. Patkóš, V. A. Yerokhin, and K. Pachucki, Phys. Rev. A **95**, 012508 (2017).

[22] K. Pachucki and V. A. Yerokhin, Phys. Rev. Lett. **104**, 070403 (2010).

[23] V. A. Yerokhin and K. Pachucki, Phys. Rev. A **81**, 022507 (2010).

[24] K. Pachucki, Phys. Rev. A **74**, 062510 (2006); **76**, 059906(E) (2007).

[25] G. W. F. Drake, Can. J. Phys. **66**, 586 (1988).

[26] D. Z. Kandula, C. Gohle, T. J. Pinkert, W. Ubachs, and K. S. E. Eikema, Phys. Rev. A **84**, 062512 (2011).

[27] W. Lichten, D. Shiner, and Z.-X. Zhou, Phys. Rev. A **43**, 1663 (1991).

[28] S. D. Bergeson, A. Balakrishnan, K. G. H. Baldwin, T. B. Lucatorto, J. P. Marangos, T. J. Mcllrath, T. R. O'Brian, S. L. Rolston, C. J. Sansonetti, J. Wen, N. Westbrook, C. H. Cheng, and E. E. Eyler, Phys. Rev. Lett. **80**, 3475 (1998).

[29] C. Dorrer, F. Nez, B. de Beauvoir, L. Julien, and F. Biraben, Phys. Rev. Lett. **78**, 3658 (1997).

[30] P.-L. Luo, J.-L. Peng, J.-T. Shy, and L.-B. Wang, Phys. Rev. Lett. **111**, 013002 (2013); **111**, 179901(E) (2013).

[31] P.-L. Luo, Y.-C. Guan, J.-L. Peng, J.-T. Shy, and L.-B. Wang, Phys. Rev. A **88**, 054501 (2013).

[32] P.-L. Luo, J.-L. Peng, J. Hu, Y. Feng, L.-B. Wang, and J.-T. Shy, Phys. Rev. A **94**, 062507 (2016).

[33] P. C. Pastor, G. Giusfredi, P. D. Natale, G. Hagel, C. de Mauro, and M. Inguscio, Phys. Rev. Lett. **92**, 023001 (2004); **97**, 139903(E) (2006).

[34] R. P. M. J. W. Notermans and W. Vassen, Phys. Rev. Lett. **112**, 253002 (2014).

[35] R. van Rooij, J. S. Borbely, J. Simonet, M. D. Hoogerland, K. S. E. Eikema, R. A. Rozendaal, and W. Vassen, Science **333**, 196 (2011).

[36] D. C. Morton, Q. Wu, and G. W. F. Drake, Can. J. Phys. **84**, 83 (2006).

[37] K. Pachucki and V. A. Yerokhin, J. Phys.: Conf. Ser. **264**, 012007 (2011).

[38] G. W. F. Drake, in *Handbook of Atomic, Molecular, and Optical Physics*, edited by G. W. F. Drake (Springer, Berlin, 2005).

[39] W. Nörtershäuser (private communication).

[40] A. N. Artemyev, V. M. Shabaev, V. A. Yerokhin, G. Plunien, and G. Soff, Phys. Rev. A **71**, 062104 (2005).

[41] W. Nörtershäuser, D. Tiedemann, M. Žáková, Z. Andjelkovic, K. Blaum, M. L. Bissell, R. Cazan, G. W. F. Drake, C. Geppert, M. Kowalska, J. Krämer, A. Krieger, R. Neugart, R. Sánchez, F. Schmidt-Kaler, Z.-C. Yan, D. T. Yordanov, and C. Zimmermann, Phys. Rev. Lett. **102**, 062503 (2009).

[42] G. W. F. Drake, Can. J. Phys. **80**, 1195 (2002).

[43] K. Pachucki, Phys. Rev. Lett. **97**, 013002 (2006).

[44] K. Pachucki and V. A. Yerokhin, Phys. Rev. A **79**, 062516 (2009) **80**, 019902(E) (2009) **81**, 039903(E) (2010).

[45] A. Marsman, M. Horbatsch, and E. A. Hessels, J. Phys. Chem. Ref. Data **44**, 031207 (2015); Phys. Rev. A **91**, 062506 (2015).

[46] X. Zheng, Y. R. Sun, J.-J. Chen, W. Jiang, K. Pachucki, and S.-M. Hu, Phys. Rev. Lett. **118**, 063001 (2017).

[47] G. P. Feng, X. Zheng, Y. R. Sun, and S. M. Hu, Phys. Rev. A **91**, 030502(R) (2015).

[48] M. Smiciklas and D. Shiner, Phys. Rev. Lett. **105**, 123001 (2010).

[49] J. S. Borbely, M. C. George, L. D. Lombardi, M. Weel, D. W. Fitzakerley, and E. A. Hessels, Phys. Rev. A **79**, 060503 (2009).

[50] T. Zelevinsky, D. Farkas, and G. Gabrielse, Phys. Rev. Lett. **95**, 203001 (2005).

[51] G. Guisfredi, P. C. Pastor, P. D. Natale, D. Mazzotti, C. de Mauro, L. Fallani, G. Hagel, V. Krachmalnicoff, and M. Inguscio, Can. J. Phys. **83**, 301 (2005).

[52] M. C. George, L. D. Lombardi, and E. A. Hessels, Phys. Rev. Lett. **87**, 173002 (2001).

[53] J. Castillega, D. Livingston, A. Sanders, and D. Shiner, Phys. Rev. Lett. **84**, 4321 (2000).

[54] I. Sick, Phys. Rev. C **84**, 024307 (2011).

[55] P. C. Pastor, L. Consolino, G. Giusfredi, P. D. Natale, M. Inguscio, V. A. Yerokhin, and K. Pachucki, Phys. Rev. Lett. **108**, 143001 (2012).

[56] D. Shiner, R. Dixson, and V. Vedantham, Phys. Rev. Lett. **74**, 3553 (1995).

[57] H. A. Schluesser, E. N. Fortson, and H. G. Dehmelt, Phys. Rev. **187**, 5 (1969); Phys. Rev. A **2**, 1612(E) (1970).

[58] S. D. Rosner and F. M. Pipkin, Phys. Rev. A **1**, 571 (1970); **3**, 521 (1971).

[59] G. W. F. Drake (private communication), as cited in Ref. [35].