Retardation of quantum uncertainty of two radiative dipoles

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In this paper we consider the excitation of one quantum dipole by another in the deep quantum limit. We use a full quantum mechanical theory to describe the interaction of the dipoles through the electromagnetic field. Our nonperturbative analytical calculations result in the exact solution. We show that minimal quantum uncertainty of the dipole oscillation amplitudes, taken at different times, have a retarded character. It is demonstrated that the commutator of the dipole oscillation amplitudes becomes nonzero inside the light cone only. Moreover, due to radiation in free space the value of the commutator has a global maximum.

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I. INTRODUCTION

In classical electrodynamics it is impossible to have an interaction with a speed that is greater than the speed of light in a vacuum [1–4]. In quantum electrodynamics the problem is more difficult. Even considering electromagnetic waves in empty space one can find nonlocal quantities such as the correlation between the electric fields [5–9]. Nevertheless the existence of these quantities does not contradict causality insofar as physical values separated by spacelike intervals commute [5–9]. In terms of quantum mechanics this means that two measurements do not affect each other so long as they are separated by a spacelike interval [5–9].

When we consider the interaction of the electromagnetic field with a medium the situation becomes much more complicated. The causality problem in quantum electrodynamics was first considered by Fermi [10]. In his paper he calculated the dependence on time of the energy transfer probability between two atoms and obtained retardation of the energy transfer. However, during these calculations, performed according to perturbation theory, the rotating wave approximation (RWA) was used, the Coulomb interaction and the term proportional to the square of vector potential were omitted and the summation over the photon modes was extended to the negative region of the wave number's absolute values. More precise calculations were performed in [11]. Here the RWA was not used but rather the right Hamiltonian for a two-level system (TLS) interacting with an electromagnetic field. The Weisskopf-Vigner theory was applied and the wave number was extended to negative values. As a result the calculated energy transfer probability is zero outside the light cone. The weakest point of these calculations is an extension of the wave number's absolute values to the negative region. Regardless of the other approximations used, it is this extension which leads to the retarded probability of the energy transfer. For example, in [12] using the extension of the wave numbers to negative values and RWA, but not perturbation theory, it was shown that the energy transfer probability is retarded. However, in [13–16] it was shown that if one does not use the wave number extension to negative values then the resultant probability is not retarded.

The initial conditions in the mentioned works [10–16] are chosen such that there are no photons in the system, the first atom is in an excited state and the second atom is in its ground state. The final state is set to be with no photons in the system, the first atom in its ground state and the second atom in an excited state. In [14–16] it was suggested that consideration of a more general class of the final states can possibly regain the retardation, even when the wave number extension is not used. In Refs. [17,18] it was independently considered the final state to be with the second atom in the excited state, but with the photons and the first atom in arbitrary states. Summing over these final states, the retarded probability for the excitement of the second atom was obtained. In the mentioned works [17,18] this probability was calculated up to the fourth order of the interaction constant. In [18] it was proved that the retardation is conserved in all orders.

The exact solution of the problem concerning two TLSs that interact through the electromagnetic field is difficult; moreover, the exact excitation probability is obtained so far only in one dimension [19]. This is because the radiating TLS is a nonlinear system. This makes it necessary to use perturbation theory. However, if harmonic oscillators are considered instead of a TLS the whole system appears to be linear and the eigenstates can be found exactly. From the practical point of view a harmonic oscillator both quantum and classical can be treated as a model for the wide class of physical objects including the electric field of plasmonic nanoparticles [20–26], the electromagnetic field of plasmonic nanostructures [27] and the excitations of quantum wells [28]. At the same time, as far as we know, there is no exact solution of the Fermi problem concerning two linear emitters.

In this paper we consider the radiation of two dipole harmonic oscillators and the retardation character of the physical quantities of the system. We find that the quantity which preserves retardation and characterizes the interaction of two radiating quantum systems is the minimal quantum uncertainty for those systems. Applying the Fano diagonalization method [29–31], we find the eigenmodes of the system. Using the eigenmode representation, we introduce annihilation and creation operators for the eigenstates of the system. We prove that this quantization procedure is canonical. Then we consider the dynamics of the minimal quantum uncertainties of the physical quantities of the two harmonic oscillators. We show

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that these quantum uncertainties are zero when the dipoles are separated by a spacelike interval. Thus, the states of the two dipole harmonic oscillators cannot influence each other superluminally through the electromagnetic field and can be measured with arbitrary precision when they are separated by a spacelike interval. We also show that each of these minimal quantum uncertainties, as a function of time, has a global maximum. The obtained results may be useful in connection with recent works concerning the transfer of purely quantum quantities such as entanglement [32–50].

II. DETERMINATION OF THE DRESSED EIGENSTATES OF THE SYSTEM

Now we find the eigenmodes of the system consisting of two radiative dipoles. The dressed states of the system, as we will show below, include the electromagnetic field and the dipole oscillations. In the dressed state representation the Hamiltonian of the system takes the diagonal form. After the diagonalization procedure we will quantize the system and introduce the equal time commutation relations (ETCR) for collective annihilation and creation operators.

If the size of an object is much less than the characteristic wavelength then the dipole approximation gives the leading contribution to the radiated field (if the dipole transition is allowed). There are a lot of systems which correspond to this condition: atoms, plasmon nanoparticles, quantum dots with the optical transition frequency, etc.

We consider two pointlike dipole harmonic oscillators, interacting via the electromagnetic field. The first dipole is at $\mathbf{0}$ and the other is at $\mathbf{R}$. We will use the signature $(+---)$. The Lagrangian of the system is

$$L = L_{\mathrm{em}} + L_{\mathrm{int}} + L_{\mathrm{mat}}, \tag{1a}$$

where L_{em} is the Lagrangian of the electromagnetic field [1,2].

$$L_{\mathrm{em}} = -\frac{1}{16\pi}\int d^3\mathbf{r}\{F_{\mu\nu}F^{\mu\nu}\}, \tag{1b}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the tensor of the electromagnetic field. $A^\mu = (\varphi,\mathbf{A})$ is the 4-vector of the electromagnetic field where φ is the scalar potential and $\mathbf{A}$ is the vector potential. The Lagrangian L_{int} describes the coupling between the electromagnetic field and the dipoles [1,2],

$$L_{\mathrm{int}} = -\frac{1}{c}\int d^3\mathbf{r}\{j_\mu A^\mu\}, \tag{1c}$$

where $j^\mu = (c\rho,\mathbf{j})$ is the 4-vector of the current where ρ is the charge density and $\mathbf{j}$ is the charge current density. In our case the current density is $\mathbf{j}(\mathbf{r},t) = e\dot{\mathbf{R}}_1(t)\delta(\mathbf{r}) + e\dot{\mathbf{R}}_2(t)\delta(\mathbf{r}-\mathbf{R})$ and the charge density is $\rho(\mathbf{r},t) = -e\mathbf{R}_1(t)\cdot\nabla\delta(\mathbf{r}) - e\mathbf{R}_2(t)\cdot\nabla\delta(\mathbf{r}-\mathbf{R})$. Here $\mathbf{R}_1$ and $\mathbf{R}_2$ are the oscillation amplitudes of the first and the second dipole, respectively, e is charge. The mass of both dipoles is m and their transition frequency is ω_0. Then the nonrelativistic Lagrangian of the dipoles L_{mat} is

$$L_{\mathrm{mat}} = \frac{m\dot{\mathbf{R}}_1(t)^2}{2} - \frac{m\omega_0^2\mathbf{R}_1(t)^2}{2} + \frac{m\dot{\mathbf{R}}_2(t)^2}{2} - \frac{m\omega_0^2\mathbf{R}_2(t)^2}{2}. \tag{1d}$$

We introduce a dipole moment $\mathbf{d}$ and the dimensionless oscillation amplitudes of the dipoles r_1 and r_2 according to the expressions $\mathbf{d} = e\mathbf{l}$, $\mathbf{R}_1(t) = \mathbf{l}r_1(t)$ and $\mathbf{R}_2(t) = \mathbf{l}r_2(t)$. For our purposes it is more convenient to represent the Lagrangian (1a) in terms of scalar and vector potentials:

$$L = \int d^3\mathbf{r}\left\{\frac{[\nabla\varphi(\mathbf{r},t) + \dot{\mathbf{A}}(\mathbf{r},t)/c]^2}{8\pi} - \frac{[\nabla\times\mathbf{A}(\mathbf{r},t)]^2}{8\pi}\right\} + \frac{ml^2\dot{r}_1(t)^2}{2} - \frac{ml^2\omega_0^2 r_1(t)^2}{2} + \frac{ml^2\dot{r}_2(t)^2}{2} - \frac{ml^2\omega_0^2 r_2(t)^2}{2}$$
$$+ \int d^3\mathbf{r}\{\mathbf{A}(\mathbf{r},t)\cdot\mathbf{d}\dot{r}_1(t)\delta(\mathbf{r})/c + \varphi(\mathbf{r},t)r_1(t)\mathbf{d}\cdot\nabla\delta(\mathbf{r}) + \mathbf{A}(\mathbf{r},t)\cdot\mathbf{d}\dot{r}_2(t)\delta(\mathbf{r}-\mathbf{R})/c + \varphi(\mathbf{r},t)r_2(t)\mathbf{d}\cdot\nabla\delta(\mathbf{r}-\mathbf{R})\}. \tag{2}$$

Note that the first two terms inside the integral in (2) is $(\mathbf{E}^2-\mathbf{B}^2)/8\pi$ because $\mathbf{E}(\mathbf{r},t) = -\nabla\varphi(\mathbf{r},t) - \dot{\mathbf{A}}(\mathbf{r},t)/c$ and $\mathbf{B}(\mathbf{r},t) = \nabla\times\mathbf{A}(\mathbf{r},t)$. We will assume that $\hbar = c = 1$, and that the system parameter $ml^2 = 1$. We use the Euler-Lagrange equations,

$$\frac{d}{dt}\frac{\delta L}{\delta\dot{\mathbf{A}}(\mathbf{r},t)} = \frac{\delta L}{\delta\mathbf{A}(\mathbf{r},t)},\quad \frac{d}{dt}\frac{\delta L}{\delta\dot{\varphi}(\mathbf{r},t)} = \frac{\delta L}{\delta\varphi(\mathbf{r},t)},$$
$$\frac{d}{dt}\frac{\delta L}{\delta\dot{r}_1(t)} = \frac{\delta L}{\delta r_1(t)},\quad \frac{d}{dt}\frac{\delta L}{\delta\dot{r}_2(t)} = \frac{\delta L}{\delta r_2(t)},$$

and obtain the equations of motion from the Lagrangian (2) in the Coulomb gauge $\nabla\cdot\mathbf{A}(\mathbf{r},t) = 0$,

$$\ddot{r}_1(t) + \omega_0^2 r_1(t) + \mathbf{d}\cdot[\dot{\mathbf{A}}(0,t) + \nabla\varphi(0,t)] = 0,$$
$$\ddot{r}_2(t) + \omega_0^2 r_2(t) + \mathbf{d}\cdot[\dot{\mathbf{A}}(\mathbf{r},t) + \nabla\varphi(\mathbf{r},t)] = 0,$$
$$\ddot{\mathbf{A}}(\mathbf{r},t) - \Delta\mathbf{A}(\mathbf{r},t) - 4\pi\mathbf{d}\dot{r}_1\delta(\mathbf{r}) - 4\pi\mathbf{d}\dot{r}_2\delta(\mathbf{r}-\mathbf{R}) + \nabla\dot{\varphi}(\mathbf{r},t) = 0,$$
$$\Delta\varphi(\mathbf{r},t) - 4\pi r_1(t)\mathbf{d}\cdot\nabla\delta(\mathbf{r}) - 4\pi r_2(t)\mathbf{d}\cdot\nabla\delta(\mathbf{r}-\mathbf{R}) = 0. \tag{3}$$

Note that including the electromagnetic field degrees of freedom in consideration implies that the field is not the external force for the dipoles. Instead, we need to consider the dipole and field degrees of freedom on equal footing. To do this we go to the frequency-momentum domain. Then Eq. (3) takes the form,

$$(\omega_0^2-\omega^2)r_1(\omega) - i\mathbf{d}\cdot\int d^3\mathbf{k}(\omega\mathbf{A}(\mathbf{k},\omega) - \mathbf{k}\varphi(\mathbf{k},\omega)) = 0,$$
$$(\omega_0^2-\omega^2)r_2(\omega) - i\mathbf{d}\cdot\int d^3\mathbf{k}(\omega\mathbf{A}(\mathbf{k},\omega) - \mathbf{k}\varphi(\mathbf{k},\omega))e^{i\mathbf{k}\cdot\mathbf{R}} = 0,$$

$$(\mathbf{k}^2-\omega^2)\mathbf{A}(\mathbf{k},\omega)+\frac{i\omega\mathbf{d}r_1(\omega)}{2\pi^2}+\frac{i\omega\mathbf{d}r_2(\omega)}{2\pi^2}e^{-i\mathbf{k}\cdot\mathbf{R}}$$
$$+\mathbf{k}\omega\varphi(\mathbf{k},\omega)=0,$$
$$\mathbf{k}^2\varphi(\mathbf{k},\omega)+\frac{i\mathbf{k}\cdot\mathbf{d}r_1(\omega)}{2\pi^2}+\frac{i\mathbf{k}\cdot\mathbf{d}r_2(\omega)}{2\pi^2}e^{-i\mathbf{k}\cdot\mathbf{R}}=0. \tag{4}$$

The electromagnetic field equations obtained from (4) are

$$\varphi(\mathbf{k},\omega)=-i\frac{\mathbf{k}\cdot\mathbf{d}}{\mathbf{k}^2}\frac{r_1(\omega)}{2\pi^2}-i\frac{\mathbf{k}\cdot\mathbf{d}}{\mathbf{k}^2}\frac{r_2(\omega)}{2\pi^2}e^{-i\mathbf{k}\cdot\mathbf{R}},$$
$$(\mathbf{k}^2-\omega^2)\mathbf{A}(\mathbf{k},\omega)=-\frac{i\omega r_1(\omega)}{2\pi^2}\left(\mathbf{d}-\frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{d})}{\mathbf{k}^2}\right)-$$
$$-\frac{i\omega r_2(\omega)}{2\pi^2}\left(\mathbf{d}-\frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{d})}{\mathbf{k}^2}\right)e^{-i\mathbf{k}\cdot\mathbf{R}}. \tag{5}$$

As a result the vector potential is found to be

$$\mathbf{A}(\mathbf{k},\omega)=-\frac{i\omega}{\mathbf{k}^2-\omega^2-i0k\omega}\frac{r_1(\omega)}{2\pi^2}\left(\mathbf{d}-\frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{d})}{\mathbf{k}^2}\right)$$
$$-\frac{i\omega}{\mathbf{k}^2-\omega^2-i0k\omega}\frac{r_2(\omega)}{2\pi^2}\left(\mathbf{d}-\frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{d})}{\mathbf{k}^2}\right)e^{-i\mathbf{k}\cdot\mathbf{R}}$$
$$+\mathbf{Z}_\perp(\mathbf{k})\delta(k-\omega)+\mathbf{Z}^*_\perp(-\mathbf{k})\delta(k+\omega), \tag{6}$$

where $\mathbf{Z}_\perp(\mathbf{k})\delta(k-\omega)$ and $\mathbf{Z}^*_\perp(-\mathbf{k})\delta(k+\omega)$ are, respectively, the positive frequency and negative frequency solutions of the uniform wave equation in momentum space: $(\mathbf{k}^2-\omega^2)\mathbf{Z}_\perp(\mathbf{k})\delta(k-\omega)=0$. Note that these uniform solutions are transverse.

Next we obtain the closed equations for the dipole harmonic oscillators by inserting expressions (5) and (6) into the first and the second equation of (4),

$$D(\omega)r_1(\omega)+M(\omega)r_2(\omega)+i\omega\int d^3\mathbf{k}Z_\mathbf{d}(\mathbf{k})\delta(k-\omega)$$
$$+i\omega\int d^3\mathbf{k}Z^*_\mathbf{d}(\mathbf{k})\delta(k+\omega)=0,$$
$$D(\omega)r_2(\omega)+M(\omega)r_1(\omega)+i\omega\int d^3\mathbf{k}Z_\mathbf{d}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{R}}\delta(k-\omega)$$
$$+i\omega\int d^3\mathbf{k}Z^*_\mathbf{d}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{R}}\delta(k+\omega)=0. \tag{7}$$

Here we have introduced the functions,

$$\Omega(\omega)=\frac{2}{3\pi}\mathrm{P}\int_0^{+\infty}dk\frac{k^2d^2(k^2-3\omega^2)}{k^2-\omega^2-i0k\omega},$$
$$\Gamma(\omega)=\frac{2}{3}\mathbf{d}^2\omega^2,$$
$$D(\omega)=\omega^2+i\Gamma(\omega)\omega-\Omega(\omega)-\omega_0^2,$$
$$V(\omega)=\omega^3d_\perp^2\left(\frac{1}{\omega R}+\frac{i}{\omega^2R^2}-\frac{1}{\omega^3R^3}\right)$$
$$+\omega^3d_\parallel^2\left(-\frac{2i}{\omega^2R^2}+\frac{2}{\omega^3R^3}\right),$$
$$M(\omega)=V(\omega)e^{i\omega R},$$
$$Z_\mathbf{d}(\mathbf{k})=\mathbf{d}\cdot\mathbf{Z}_\perp(\mathbf{k}),\ Z^*_\mathbf{d}(\mathbf{k})=\mathbf{d}\cdot\mathbf{Z}^*_\perp(\mathbf{k}), \tag{8}$$

where $\Omega(\omega)$ is the Lamb's shift, $\Gamma(\omega)$ is the damping constant, $V(\omega)$ describes the interaction between dipoles, $D(\omega)$ is the inverse response function of a dipole harmonic oscillator in empty space (see Chap. 8 in [51] and Chap. 16 in [1]), $d_\parallel^2=(\mathbf{k}\cdot\mathbf{d})^2/\mathbf{k}^2$, and $d_\perp^2=\mathbf{d}^2-d_\parallel^2$.

Taking the inverse transformation of the solution of Eq. (7) we obtain

$$r_1(t)=-i\int d^3\mathbf{k}\frac{D(k)-M(k)e^{i\mathbf{k}\cdot\mathbf{R}}}{D^2(k)-M^2(k)}kZ_\mathbf{d}(\mathbf{k})e^{-ikt}+\text{H.c.},$$
$$r_2(t)=-i\int d^3\mathbf{k}\frac{D(k)e^{i\mathbf{k}\cdot\mathbf{R}}-M(k)}{D^2(k)-M^2(k)}kZ_\mathbf{d}(\mathbf{k})e^{-ikt}+\text{H.c.} \tag{9}$$

Note that the functions D and M in Eq. (9) have the argument k instead of ω. This happens because the right-hand parts of Eq. (7) include the delta functions $\delta(k+\omega)$ and $\delta(k-\omega)$. Note that Eq. (9) does not include a homogeneous part (the solution with $\mathbf{Z}=0$) of Eq. (7). This is because the eigenfrequencies of the homogeneous part of (7) are complex [in this regard see the discussion after Eq. (76) in [29]].

We note that all expressions of (9) have poles both below and above the real frequency axes. The pole that has the positive imaginary part leads to the unphysical self-excitation of the system. In addition, the Lamb's shift $\Omega(k)$ is equal to infinity unless we cut off the integral in momentum space. These problems of classical electrodynamics have been known for a long time. They arise even in classical electrodynamics with one dipole harmonic oscillator in empty space [2]. The standard solution to the problem is to set the function $\Gamma(k)$ to a constant $\Gamma(\omega_0)$, the Lamb's shift $\Omega(k)$ to zero (see Chap. 8 in [51], Chap. 16 in [1], and Chap. 9 in [2]). Setting the Lamb's shift to constant is also possible [52] (see also Chap. 16 in [1] and Appendix B in [51]), but we assume that the value is small compared with ω_0 and can be neglected. We will use the same simplifications for the system under consideration. In addition we set the function $M(k)$ to the function $\mathrm{Re}M(\omega_0)+i\mathrm{Im}M(\omega_0)k/\omega_0$ [53].

After these replacements the denominator of the expressions (9) takes the form,

$$\left(k^2+i\Gamma(\omega_0)k-\omega_0^2\right)^2-(\mathrm{Re}M(\omega_0)+i\mathrm{Im}M(\omega_0)k/\omega_0)^2. \tag{10}$$

Expression (10) as a function of k has only two pairs of roots with a negative imaginary part $\mathrm{Im}k<0$. We also assume that the real part of the roots are close to ω_0 which leads to the conditions $\Gamma(\omega_0)\ll\omega_0$ and $\mathrm{Re}M(\omega_0)\ll\omega_0^2$. The last inequality can be rewritten in the form $R\gg\lambda(\Gamma(\omega_0)/\omega_0)$. Thus, the inequality $\Gamma(\omega_0)\ll\omega_0$ determines the upper bound of the dipole momentum [see Eq. (8)] and inequality $R\gg\lambda(\Gamma(\omega_0)/\omega_0)$ determines the lower bound of the distances between the dipoles. The four roots of Eq. (10) correspond to two oscillating modes with decay rates $[\Gamma(\omega_0)+\mathrm{Im}M(\omega_0)/\omega_0]/2$ and $[\Gamma(\omega_0)-\mathrm{Im}M(\omega_0)/\omega_0]/2$, respectively (Fig. 1). The difference between this and the decay rate of one oscillator $\Gamma(\omega_0)$ occurs because of the dipole-dipole interaction through the electromagnetic field [53]. One can see from Fig. 1 that when $R=0$ the decay rate of one of the modes is zero. In this case the homogeneous solution of Eq. (7) exists. However, the case $R=0$ is beyond our assumption ($R\gg\lambda(\Gamma(\omega_0)/\omega_0)$) and we do not consider it here.

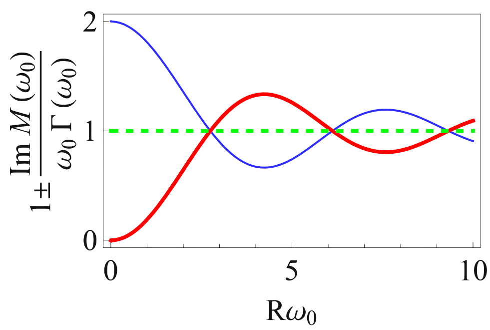

FIG. 1. The dependence of the radiation decay rates for $\mathbf{d} \perp \mathbf{R}$ of two modes (thick solid red line and solid blue line) of the two dipole system on the distance R between these dipoles in units of the radiation decay rate of one radiative dipole in empty space (dashed green line).

From the Lagrangian (2) one can obtain the canonical conjugated momentums for the amplitudes of the dipole harmonic oscillators,

$$p_1(t) = \dot r_1(t) + \mathbf{d} \cdot \mathbf{A}(0,t),$$
$$p_2(t) = \dot r_2(t) + \mathbf{d} \cdot \mathbf{A}(\mathbf{R},t). \quad (11)$$

This can be expressed in terms of new variables $Z_{\mathbf{d}}(\mathbf{k})e^{-ikt}$ and $Z^*_{\mathbf{d}}(\mathbf{k})e^{ikt}$ by using (6) and (9) as

$$\begin{aligned} p_1 &= \int d^3\mathbf{k} \frac{D(0)(D(k) - M(k)e^{i\mathbf{k}\cdot\mathbf{R}})}{D^2(k) - M^2(k)} Z_{\mathbf{d}}(\mathbf{k})e^{-ikt} \\ &\quad + \int d^3\mathbf{k} \frac{M(0)(D(k)e^{i\mathbf{k}\cdot\mathbf{R}} - M(k))}{D^2(k) - M^2(k)} Z_{\mathbf{d}}(\mathbf{k})e^{-ikt} + \text{H.c.} \\ p_2 &= \int d^3\mathbf{k} \frac{D(0)(D(k)e^{i\mathbf{k}\cdot\mathbf{R}} - M(k))}{D^2(k) - M^2(k)} Z_{\mathbf{d}}(\mathbf{k})e^{-ikt} \\ &\quad + \int d^3\mathbf{k} \frac{M(0)(D(k) - M(k)e^{i\mathbf{k}\cdot\mathbf{R}})}{D^2(k) - M^2(k)} Z_{\mathbf{d}}(\mathbf{k})e^{-ikt} + \text{H.c.} \end{aligned} \quad (12)$$

In the same way one can use Eqs. (5), (6), and (9) to obtain the expressions for the scalar potential $U(\mathbf{r},t)$, the vector potential $\mathbf{A}(\mathbf{r},t,)$ and the canonically conjugated momentum to the vector potential $\Pi(\mathbf{r},t) = (\dot{\mathbf{A}}(\mathbf{r},t) + \nabla\varphi(\mathbf{r},t))/4\pi$ in terms of new variables $Z_{\mathbf{d}}(\mathbf{k})e^{-ikt}$ and $Z^*_{\mathbf{d}}(\mathbf{k})e^{ikt}$ (see Appendix A).

The quantities $Z_{\mathbf{d}}(\mathbf{k})e^{-ikt}$ and $Z^*_{\mathbf{d}}(\mathbf{k})e^{ikt}$ can be regarded as new dynamical variables which transform the Hamiltonian of the system into the diagonal form [31]. Thus Eqs. (9), (12), (A2), and (A3) describe the transformation from the old variables r_1, r_2, p_1, p_2, $\mathbf{A}$, and Π to the new ones $Z_{\mathbf{d}}(\mathbf{k})e^{-ikt}$ and $Z^*_{\mathbf{d}}(\mathbf{k})e^{ikt}$.

The transition to the quantum description can be done in the standard canonical way [54]. First we find the Hamiltonian of the system (2) in terms of the old variables,

$$\begin{aligned} H &= \frac{(p_1 - \mathbf{d}\cdot\mathbf{A}(0))^2}{2} - \frac{D(0)r_1^2}{2} \\ &\quad + \frac{(p_2 - \mathbf{d}\cdot\mathbf{A}(\mathbf{R}))^2}{2} - \frac{D(0)r_2^2}{2} - M(0)r_1r_2 \\ &\quad + \int d^3\mathbf{r}\left\{\frac{(4\pi\Pi - \nabla\varphi)^2}{8\pi} + \frac{(\nabla\times\mathbf{A})^2}{8\pi}\right\}. \end{aligned} \quad (13)$$

The first four terms represent the dipole oscillators' energy, the fifth term is their electrostatic interaction energy, and the last two terms are the electromagnetic field energy. One can obtain the Hamiltonian of the system in terms of the new variables $\mathbf{Z}_\perp(\mathbf{k})$ and $\mathbf{Z}^*_\perp(\mathbf{k})$ by substituting the expressions (9), (12), (A2), and (A3) into (13):

$$H = 4\pi^2 \int d^3\mathbf{k}(\mathbf{k}^2\mathbf{Z}^*_\perp(\mathbf{k})\cdot\mathbf{Z}_\perp(\mathbf{k})). \quad (14)$$

Next we replace the variables $\mathbf{Z}_\perp(\mathbf{k})$ and $\mathbf{Z}^*_\perp(\mathbf{k})$ with quantum operators $\hat{\mathbf{Z}}_\perp(\mathbf{k})$ and $\hat{\mathbf{Z}}^+_\perp(\mathbf{k})$ and set the equal time commutation relations,

$$\begin{aligned} [\hat Z_{\mathbf{d}}(\mathbf{k}), \hat Z^+_{\mathbf{d}'}(\mathbf{k}')] &= [\mathbf{d}\cdot\hat{\mathbf{Z}}_\perp(\mathbf{k}), \mathbf{d}'\cdot\hat{\mathbf{Z}}^+_\perp(\mathbf{k})] \\ &= \frac{1}{4\pi^2 k}\left(\mathbf{d}\cdot\mathbf{d}' - \frac{(\mathbf{k}\cdot\mathbf{d})(\mathbf{k}\cdot\mathbf{d}')}{\mathbf{k}^2}\right)\delta(\mathbf{k}-\mathbf{k}'). \end{aligned} \quad (15)$$

Having quantized the system we obtain from expressions (9) and (12) the quantum amplitudes,

$$\begin{aligned} \hat r_1(t) &= -i\int d^3\mathbf{k}\frac{D(k) - M(k)e^{i\mathbf{k}\cdot\mathbf{R}}}{D^2(k) - M^2(k)} k\hat Z_{\mathbf{d}}(\mathbf{k})e^{-ikt} + \text{H.c.}, \\ \hat r_2(t) &= -i\int d^3\mathbf{k}\frac{D(k)e^{i\mathbf{k}\cdot\mathbf{R}} - M(k)}{D^2(k) - M^2(k)} k\hat Z_{\mathbf{d}}(\mathbf{k})e^{-ikt} + \text{H.c.}, \end{aligned} \quad (16)$$

and canonical conjugated momentums of these dipoles,

$$\begin{aligned} \hat p_1 &= \int d^3\mathbf{k} \frac{D(0)(D(k) - M(k)e^{i\mathbf{k}\cdot\mathbf{R}})}{D^2(k) - M^2(k)} \hat Z_{\mathbf{d}}(\mathbf{k})e^{-ikt} \\ &\quad + \int d^3\mathbf{k} \frac{M(0)(D(k)e^{i\mathbf{k}\cdot\mathbf{R}} - M(k))}{D^2(k) - M^2(k)} \hat Z_{\mathbf{d}}(\mathbf{k})e^{-ikt} + \text{H.c.}, \\ \hat p_2 &= \int d^3\mathbf{k} \frac{D(0)(D(k)e^{i\mathbf{k}\cdot\mathbf{R}} - M(k))}{D^2(k) - M^2(k)} \hat Z_{\mathbf{d}}(\mathbf{k})e^{-ikt} \\ &\quad + \int d^3\mathbf{k} \frac{M(0)(D(k) - M(k)e^{i\mathbf{k}\cdot\mathbf{R}})}{D^2(k) - M^2(k)} \hat Z_{\mathbf{d}}(\mathbf{k})e^{-ikt} + \text{H.c.} \end{aligned} \quad (17)$$

In a similar way one can obtain the quantum vector potential $\hat{\mathbf{A}}(\mathbf{r},t)$ and its quantum canonical conjugated momentum $\hat\Pi(\mathbf{r},t) = (\dot{\hat{\mathbf{A}}}(\mathbf{r},t) + \nabla\hat\varphi(\mathbf{r},t))/4\pi$ in terms of new operators $\hat{\mathbf{Z}}_\perp(\mathbf{k})$ and $\hat{\mathbf{Z}}^+_\perp(\mathbf{k})$ (see Appendix A).

The geometric structure of the ETCR (15) provides the right ETCR of the operators (16) and (17) and of the electromagnetic field variables. Indeed, using the expressions,

$$\begin{aligned} \int d^3\mathbf{k}\left(\mathbf{d}^2 - \frac{(\mathbf{k}\cdot\mathbf{d})^2}{\mathbf{k}^2}\right) &= \frac{2\pi}{i}\int_0^{+\infty} dk\frac{D(k) - D^*(k)}{k}, \\ \int d^3\mathbf{k}\left(\mathbf{d}^2 - \frac{(\mathbf{k}\cdot\mathbf{d})^2}{\mathbf{k}^2}\right)e^{i\mathbf{k}\cdot\mathbf{R}} &= \frac{2\pi}{i}\int_0^{+\infty} dk\frac{M(k) - M^*(k)}{k}, \end{aligned} \quad (18)$$

one can obtain the ETCR for the dipole variables (see Appendix B),

$$[\hat r_j(t), \hat p_{j'}(t)] = i\delta_{jj'}, \quad [\hat r_j(t), \hat r_{j'}(t)] = 0, \quad [\hat p_j(t), \hat p_{j'}(t)] = 0, \quad (19)$$

and for the electromagnetic field variables,

$$[\hat{A}_\alpha(\mathbf{r},t),\hat{\Pi}_\beta(\mathbf{r}',t)] = i\delta^{\perp}_{\alpha\beta}(\mathbf{r}-\mathbf{r}'), \tag{20}$$

where $\delta^{\perp}$ is a transverse delta function [1] and α and β are the components of the field. From (19) and (20) one can conclude that the transition defined by expressions (16), (17), (A2), and (A3) is a canonical transformation.

Introducing the annihilation and creation operators of dipole excitation in the standard way,

$$\begin{aligned} \hat{a}_j^{+} &= \frac{1}{\sqrt{2\omega_0}}(\omega_0\hat{r}_j + i\hat{p}_j),\\ \hat{a}_j &= \frac{1}{\sqrt{2\omega_0}}(\omega_0\hat{r}_j - i\hat{p}_j), \end{aligned} \tag{21}$$

and substituting (16) and (17) into (21), we obtain the expression for the creation and annihilation operators of the dipoles in terms of new collective quantum operators,

$$\begin{aligned} \hat{a}_1(t) = {}& \frac{-i}{\sqrt{2\omega_0}}\int d^3\mathbf{k}\frac{(D(0)+\omega_0 k)(D - Me^{i\mathbf{k}\cdot\mathbf{R}})}{D^2-M^2}\hat{Z}_{\mathbf{d}}(\mathbf{k})e^{-ikt}\\ &+\frac{-i}{\sqrt{2\omega_0}}\int d^3\mathbf{k}\frac{M(0)(De^{i\mathbf{k}\cdot\mathbf{R}}-M)}{D^2-M^2}\hat{Z}_{\mathbf{d}}(\mathbf{k})e^{-ikt}\\ &+\frac{-i}{\sqrt{2\omega_0}}\int d^3\mathbf{k}\frac{(D(0)-\omega_0 k)(D^*-M^*e^{-i\mathbf{k}\cdot\mathbf{R}})}{D^{*2}-M^{*2}}\\ &\times\hat{Z}^{+}_{\mathbf{d}}(\mathbf{k})e^{ikt}\\ &+\frac{-i}{\sqrt{2\omega_0}}\int d^3\mathbf{k}\frac{M(0)(D^*e^{-i\mathbf{k}\cdot\mathbf{R}}-M^*)}{D^{*2}-M^{*2}}\hat{Z}^{+}_{\mathbf{d}}(\mathbf{k})e^{ikt}. \end{aligned} \tag{22}$$

$$\begin{aligned} \hat{a}_2(t) = {}& \frac{-i}{\sqrt{2\omega_0}}\int d^3\mathbf{k}\frac{(D(0)+\omega_0 k)(De^{i\mathbf{k}\cdot\mathbf{R}} - M)}{D^2-M^2}\hat{Z}_{\mathbf{d}}(\mathbf{k})e^{-ikt}\\ &+\frac{-i}{\sqrt{2\omega_0}}\int d^3\mathbf{k}\frac{M(0)(D-Me^{i\mathbf{k}\cdot\mathbf{R}})}{D^2-M^2}\hat{Z}_{\mathbf{d}}(\mathbf{k})e^{-ikt}\\ &+\frac{-i}{\sqrt{2\omega_0}}\int d^3\mathbf{k}\frac{(D(0)-\omega_0 k)(D^*e^{-i\mathbf{k}\cdot\mathbf{R}}-M^*)}{D^{*2}-M^{*2}}\\ &\times\hat{Z}^{+}_{\mathbf{d}}(\mathbf{k})e^{ikt}\\ &+\frac{-i}{\sqrt{2\omega_0}}\int d^3\mathbf{k}\frac{M(0)(D^*-M^*e^{-i\mathbf{k}\cdot\mathbf{R}})}{D^{*2}-M^{*2}}\hat{Z}^{+}_{\mathbf{d}}(\mathbf{k})e^{ikt}. \end{aligned} \tag{23}$$

Here and throughout the following, if the argument of the functions D and M is not written, the argument is assumed to be k.

The quantum expression for (14) is

$$\hat{H} = 4\pi^2\int d^3\mathbf{k}\big(\mathbf{k}^2\hat{\mathbf{Z}}^{+}_{\perp}(\mathbf{k})\cdot\hat{\mathbf{Z}}_{\perp}(\mathbf{k})\big). \tag{24}$$

This Hamiltonian appears to be the Hamiltonian of the set of independent harmonic oscillators with unusual factor $4\pi^2\mathbf{k}^2$. This factor is connected to the factor of the ETCR (15). One can see that if we redefine the operators $\hat{\mathbf{Z}}_{\perp}(\mathbf{k})$ and $\hat{\mathbf{Z}}^{+}_{\perp}(\mathbf{k})$, multiplying them by $2\pi\sqrt{k}$, then the resulting operators will obey the standard bosonic ETCR and will be described by the standard bosonic Hamiltonian. The properties of the ground state of the system are discussed in Appendix C.

The outcome of the above is that we have found the canonical transformation (16)–(17) that leads the Hamiltonian to the diagonal form. We have derived the eigenstates of the system which are the dressed states and include the dipoles and the electromagnetic field excitations.

III. RETARDATION OF THE QUANTUM UNCERTAINTY

Now we show that the quantum amplitudes of the dipoles commute if they are divided by a spacelike interval. In other words the commutator is a retarded function.

The minimal quantum uncertainty of the dipole amplitudes and their velocities can be calculated from the commutators by the following equations [55]:

$$\begin{aligned} \min\Delta r_i(t)\Delta r_j(0) &= \tfrac{1}{2}|\langle[\hat{r}_i(t),\hat{r}_j(0)]\rangle|,\\ \min\Delta r_i(t)\Delta\dot{r}_j(0) &= \tfrac{1}{2}|\langle[\hat{r}_i(t),\hat{\dot{r}}_j(0)]\rangle|,\\ \min\Delta\dot{r}_i(t)\Delta\dot{r}_j(0) &= \tfrac{1}{2}|\langle[\hat{\dot{r}}_i(t),\hat{\dot{r}}_j(0)]\rangle|, \end{aligned} \tag{25}$$

where brackets $\langle\rangle$ indicate to averaging over a given state of the system. Expressions (16) and (17) and the ETCR (15) allow us to find the commutators on the right-hand side of (25):

$$\begin{aligned} [\hat{r}_2(t),\hat{r}_1(0)] &= \frac{1}{2\pi i}\left\{\int_{-\infty}^{+\infty}dk\left[\frac{Ve^{ik(R-t)}}{D^2-M^2}\right] - \int_{-\infty}^{+\infty}dk\left[\frac{Ve^{ik(R+t)}}{D^2-M^2}\right]\right\},\\ [\hat{\dot{r}}_2(t),\hat{r}_1(0)] &= \frac{1}{2\pi i}\left\{\int_{-\infty}^{+\infty}dk\left[\frac{kVe^{ik(R-t)}}{D^2-M^2}\right] + \int_{-\infty}^{+\infty}dk\left[\frac{kVe^{ik(R+t)}}{D^2-M^2}\right]\right\},\\ [\hat{\dot{r}}_2(t),\hat{\dot{r}}_1(0)] &= -\frac{1}{2\pi i}\left\{\int_{-\infty}^{+\infty}dk\left[\frac{k^2Ve^{ik(R-t)}}{D^2-M^2}\right] + \int_{-\infty}^{+\infty}dk\left[\frac{k^2Ve^{ik(R+t)}}{D^2-M^2}\right]\right\}. \end{aligned} \tag{26}$$

The commutators (26) are c numbers. Therefore during the calculations of the minimal quantum uncertainties (25) one can omit the averaging over a given state $\langle\rangle$. One can see that (26) displays the same problems as the classical expressions (9): an infinitely large Lamb's shift and a pole with a positive imaginary part. Using the same methods to address these problems we obtain that all of the poles of the right-hand side of (26) can be found to be

$$\begin{aligned} k_{1,2} &= -i\frac{\Gamma(\omega_0)}{2}\left[1+\frac{\mathrm{Im}M(\omega_0)}{\omega_0\Gamma(\omega_0)}\right]\\ &\quad\pm\sqrt{\omega_0^2+\mathrm{Re}M(\omega_0)-\frac{(\Gamma(\omega_0)+\mathrm{Im}M(\omega_0)/\omega_0)^2}{4}},\\ k_{3,4} &= -i\frac{\Gamma(\omega_0)}{2}\left[1-\frac{\mathrm{Im}M(\omega_0)}{\omega_0\Gamma(\omega_0)}\right]\\ &\quad\pm\sqrt{\omega_0^2-\mathrm{Re}M(\omega_0)-\frac{(\Gamma(\omega_0)-\mathrm{Im}M(\omega_0)/\omega_0)^2}{4}}. \end{aligned} \tag{27}$$

The sum under the square root in (27) is positive because of the assumptions $R \gg \lambda(\Gamma(\omega_0)/\omega_0)$ and $\Gamma(\omega_0) \ll \omega_0$. These

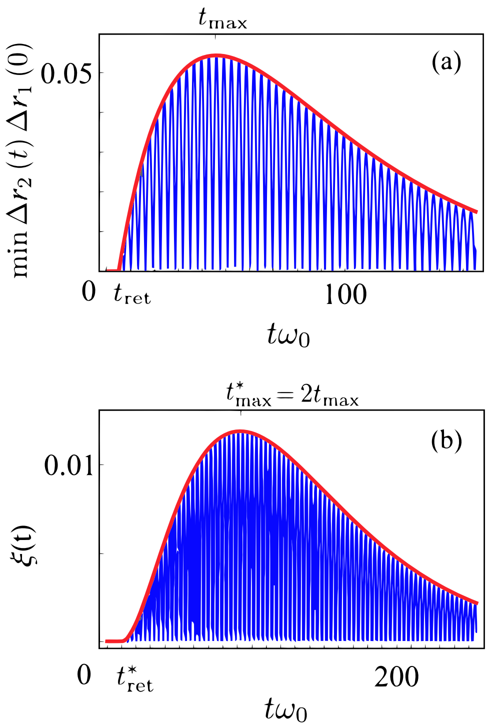

FIG. 2. (a) The dependence on time of the minimal quantum uncertainty of the dipole amplitudes (blue line) and its envelope (thick red line) for $\mathbf{d}\perp\mathbf{R}$. The parameters of the system are $R\omega_0 = 5$ and $\Gamma/\omega_0 = 0.05$. The retardation time $t_{\rm ret}$ is equal to R/c. At times $0 \leqslant t \leqslant t_{\rm ret}$ the minimal quantum uncertainty $\Delta r_2(t)\Delta r_1(0)$ is zero. (b) The dependence on time of the difference $\xi(t)$ between the minimal quantum uncertainties [see Eq. (33)] of the dipole amplitudes in the cases of one dipole in empty space [$\min \Delta r(t)\Delta r(0)$] and two dipoles separated by distance R [$\min \Delta r_1(t)\Delta r_1(0)$] (blue line) for $\mathbf{d}\perp\mathbf{R}$. The envelope of this quantity (thick red line) is given by (34). The parameters of the system are $R\omega_0 = 5$ and $\Gamma/\omega_0 = 0.05$. The retardation time $t^*_{\rm ret}$ is equal to $2R$. At times $0 \leqslant t \leqslant t^*_{\rm ret}$ the difference $\xi(t)$ is equal to zero.

assumptions were discussed after the expression (10). Thus the poles have a negative imaginary part which is defined by the first term in each expression of (27) (Fig. 1). When dipoles are separated by a spacelike interval $|t| < R$ the contour of the integrals (26) must lie in the upper half of the space. Thus the integrals equal zero. This means that the minimal quantum uncertainties caused by radiation is retarded with retardation time $t_{\rm ret}$ [Fig. 2(a)]. Therefore the two quantum dipole oscillators do not affect each other at times $t \leqslant t_{\rm ret}$.

Figure 2(a) depicts the minimal quantum uncertainty of the amplitudes of the two dipoles taken at different times. The quantity is equal to zero when $t < t_{\rm ret}$. After time $t = t_{\rm ret}$ the minimal quantum uncertainty starts to oscillate with the period π/ω_0.

From (25) and (26) the envelope of the minimal quantum uncertainty $\min \Delta r_2(t)\Delta r_1(0)$ at times $|t| > t_{\rm ret}$ can be obtained [thick red line in Fig. 2(a)]:

$$\min \Delta r_2(t)\Delta r_1(0) \propto e^{-\frac{\Gamma(\omega_0)}{2}(t-t_{\rm ret})}\,\mathrm{sh}\left(\frac{\mathrm{Im}M(\omega_0)}{2\omega_0}(t-t_{\rm ret})\right). \tag{28}$$

It is interesting to note that the minimal quantum uncertainty $\min \Delta r_2(t)\Delta r_1(0)$ has a global maximum at the time $t_{\max}$ [Fig. 2(a)]. The time $t_{\max}$ can be derived from (28)

$$t_{\max} = R + \frac{2\omega_0}{\mathrm{Im}M(\omega_0)}\,\mathrm{arcth}\left(\frac{\mathrm{Im}M(\omega_0)/\omega_0}{\Gamma(\omega_0)}\right). \tag{29}$$

The existence of the global maximum of the minimal quantum uncertainty of the amplitudes of the two dipole oscillators can be explained by considering two opposing processes. On one hand the resonant interaction of the dipoles through the radiation increase the minimal quantum uncertainty with time [the second factor in (28)]. On the other hand the radiation dissipation of the dipole energy reduces the quantum uncertainty [the first factor in (28)].

The minimal quantum uncertainty also describes the influence of one dipole on another. To show this we consider the quantum uncertainty of the dipole amplitude taken at different times. Without loss of generality we consider the first dipole. As mentioned earlier the quantum uncertainty of two measurable quantities is expressed through the commutator in the following way:

$$\min \Delta r_1(t)\Delta r_1(0) = \tfrac{1}{2}|\langle[\hat r_1(t),\hat r_1(0)]\rangle|. \tag{30}$$

The first expression of (16) can be used to find the commutator on the right-hand side of Eq. (30),

$$\begin{aligned}[\hat r_1(t),\hat r_1(0)] = \frac{1}{2\pi i}\Bigg\{&\int_{-\infty}^{+\infty} dk\left[\frac{e^{ikt}}{D}\right] - \int_{-\infty}^{+\infty} dk\left[\frac{e^{-ikt}}{D}\right]\\ &+ \int_{-\infty}^{+\infty} dk\left[\frac{DV^2 e^{ik(2R+t)}}{D(D^2-M^2)}\right]\\ &- \int_{-\infty}^{+\infty} dk\left[\frac{DV^2 e^{ik(2R-t)}}{D(D^2-M^2)}\right]\Bigg\}.\end{aligned} \tag{31}$$

The commutator of the amplitudes taken at different times for one dipole in empty space can be derived in the similar way as we derived Eq. (32) for the two dipoles in empty space. It has a form,

$$[\hat r(t),\hat r(0)] = \frac{1}{2\pi i}\left\{\int_{-\infty}^{+\infty} dk\left[\frac{e^{ikt}}{D}\right] - \int_{-\infty}^{+\infty} dk\left[\frac{e^{-ikt}}{D}\right]\right\}. \tag{32}$$

Figure 2(b) depicts the difference between the dipole amplitude minimal quantum uncertainties in the case of one dipole in empty space and two dipoles separated by distance R,

$$\xi(t) = |\min \Delta r_1(t)\Delta r_1(0) - \min \Delta r(t)\Delta r(0)|. \tag{33}$$

Using the pole analysis of (31) similar to that given in relation to (9) it is possible to distinguish some features of the dynamics of this minimal quantum uncertainty (30). Up until the retardation time $t^*_{\rm ret} = 2R$, the minimal quantum uncertainty in the case of two dipoles behaves exactly as for one dipole in empty space [Fig. 2(b)]. Indeed altering the dynamics would require the radiated photon to travel from the first dipole to the second one and back which would take time $t^*_{\rm ret}$. After the retardation time $t^*_{\rm ret}$ the quantity starts to oscillate with frequency ω_0. The envelope of this minimal uncertainties difference $\xi(t)$ at times $t > t^*_{\rm ret}$ [Fig. 2(b)] can be obtained

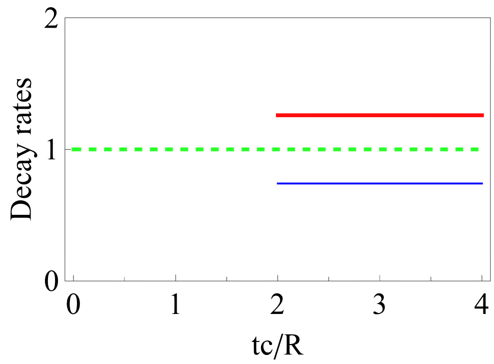

FIG. 3. The imaginary parts of the poles of the commutator (31) normalized on $\Gamma(\omega_0)/2$ for $\mathbf{d}\perp\mathbf{R}$ and the distance between two dipoles is $R\omega_0 = 5$. The imaginary parts of the poles contributes to the decay rates of the corresponding modes.

from expressions (30)–(33).

$$\xi(t) \propto e^{-\frac{\Gamma(\omega_0)}{2}(t-t^*_{\rm ret})}\left[\mathrm{ch}\left(\frac{\mathrm{Im}M(\omega_0)}{2\omega_0}(t-t^*_{\rm ret})\right)-1\right]. \quad (34)$$

This envelope has a global maximum at time $t^*_{\max} = 2t_{\max}$, where $t_{\max}$ is defined by (29).

The influence of one dipole by another may be illustrated in another way. The poles of Eq. (31) determine the decay of the commutator of the coordinate of the first dipole at different times. The imaging parts of these poles are illustrated in Fig. 3. We see that after time $t = 2R/c$ the decay rates which correspond to collective radiation modes appear. These are the modes for which decay rates are shown in Fig. 1. We see that collective quantum dynamics turns on when the electromagnetic signal from the first dipole is reemitted by another dipole and returns to the first one.

IV. CONCLUSION

The excitation of one quantum dipole by another is considered in this paper. Employing the Fano diagonalization method we pass from the classical Lagrangian of two dipoles interacting via electromagnetic field to the Hamiltonian for an infinite set of noninteracting oscillators. After the standard quantization procedure we transform the dipole moment and electromagnetic field operators to the creation and annihilation operators of the eigenstates of the Hamiltonian. Using this, the exact expressions for the commutators of the dipole amplitudes are obtained. We characterize the influence of one radiative quantum dipole on the other by the commutator of the dipole amplitude operators taken at different time moments.

We prove that the commutators at the spacelike interval are exactly equal to zero, e.g., the commutators are retarded quantities. Since the expected values of the commutators correspond to minimal quantum uncertainty for the dynamic variables of the dipoles [56], the minimal quantum uncertainty of these variables has a retarded character as well.

Thus, the ideal measurement of the amplitudes of two dipoles can be performed with arbitrary accuracy only if these measurements are separated by a spacelike interval [56].

It has been demonstrated that the minimal quantum uncertainty of the oscillation amplitudes of the two radiative dipoles taken at different moments has a global maximum. The maximum appears because of the two opposing processes. The first process is the influence of one dipole on the other through the electromagnetic field which increases the oscillation amplitude of the second dipole. The second process is the energy dissipation through radiation of the dipoles into the free space, which reduces the oscillation amplitudes of both dipoles.

The model considered in the paper appears to have an exact solution. The solution can be useful for the consideration of the nonretarded effects in quantum electrodynamics, for example, entanglement.

ACKNOWLEDGMENT

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APPENDIX A: THE EXPRESSIONS FOR THE ELECTROMAGNETIC FIELD IN TERMS OF NEW COLLECTIVE VARIABLES

Here we state the expressions for vector potential $\mathbf{A}(\mathbf{k},t)$ and its canonically conjugated variable,

$$\Pi(\mathbf{r},t) = \frac{\delta L}{\delta\dot{\mathbf{A}}(\mathbf{r},t)} = \frac{\dot{\mathbf{A}}(\mathbf{r},t)+\nabla\varphi(\mathbf{r},t)}{4\pi}, \quad (A1)$$

in momentum representation in terms of the new variables $\mathbf{Z}_\perp(\mathbf{k})$ and $\mathbf{Z}^*_\perp(\mathbf{k})$. The derivation is lengthy, but straightforward. It requires the substitution of expression (9) into (6). As a result one can obtain

$$\begin{aligned}\mathbf{A}(\mathbf{k},t) = {} & \left(\frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{d})}{\mathbf{k}^2}-\mathbf{d}\right)\int d^3\mathbf{s}\frac{D(\mathbf{s})(1+e^{i(\mathbf{s}-\mathbf{k})\cdot\mathbf{R}})}{D^2(\mathbf{s})-M^2(\mathbf{s})}\frac{\mathbf{s}^2Z_{\mathbf{d}}(\mathbf{s})e^{-ist}}{\mathbf{k}^2-\mathbf{s}^2-i0ks}\\ & +\left(\mathbf{d}-\frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{d})}{\mathbf{k}^2}\right)\int d^3\mathbf{s}\frac{M(\mathbf{s})(e^{-i\mathbf{k}\cdot\mathbf{R}}+e^{i\mathbf{s}\cdot\mathbf{R}})}{D^2(\mathbf{s})-M^2(\mathbf{s})}\frac{\mathbf{s}^2Z_{\mathbf{d}}(\mathbf{s})e^{-ist}}{\mathbf{k}^2-\mathbf{s}^2-i0ks}\\ & +\left(\frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{d})}{\mathbf{k}^2}-\mathbf{d}\right)\int d^3\mathbf{s}\frac{D^*(\mathbf{s})(1+e^{-i(\mathbf{s}+\mathbf{k})\cdot\mathbf{R}})}{D^{*2}(\mathbf{s})-M^{*2}(\mathbf{s})}\frac{\mathbf{s}^2Z^*_{\mathbf{d}}(\mathbf{s})e^{ist}}{\mathbf{k}^2-\mathbf{s}^2+i0ks}\\ & +\left(\mathbf{d}-\frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{d})}{\mathbf{k}^2}\right)\int d^3\mathbf{s}\frac{M^*(\mathbf{s})(e^{-i\mathbf{k}\cdot\mathbf{R}}+e^{-i\mathbf{s}\cdot\mathbf{R}})}{D^{*2}(\mathbf{s})-M^{*2}(\mathbf{s})}\frac{\mathbf{s}^2Z^*_{\mathbf{d}}(\mathbf{s})e^{ist}}{\mathbf{k}^2-\mathbf{s}^2+i0ks}+\mathbf{Z}_\perp(\mathbf{k})e^{-ikt}+\mathbf{Z}^*_\perp(-\mathbf{k})e^{ikt}. \end{aligned} \quad (A2)$$

Substitution of (A2), (5), and (9) into (A1) leads to the expression,

$$\begin{aligned}\Pi(\mathbf{k},t) = &\int d^3\mathbf{s}\frac{D(\mathbf{s})(1+e^{i(\mathbf{s}-\mathbf{k})\cdot\mathbf{R}})}{D^2(\mathbf{s})-M^2(\mathbf{s})}\frac{\mathbf{d}\mathbf{s}^2-\mathbf{k}(\mathbf{k}\cdot\mathbf{d})}{\mathbf{k}^2-\mathbf{s}^2-i0ks}\frac{isZ_{\mathbf{d}}(\mathbf{s})}{4\pi}e^{-ist}+\int d^3\mathbf{s}\frac{M(\mathbf{s})(e^{-i\mathbf{k}\cdot\mathbf{R}}+e^{i\mathbf{s}\cdot\mathbf{R}})}{D^2(\mathbf{s})-M^2(\mathbf{s})}\frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{d})-\mathbf{d}\mathbf{s}^2}{\mathbf{k}^2-\mathbf{s}^2-i0ks}\frac{isZ_{\mathbf{d}}(\mathbf{s})}{4\pi}e^{-ist}\\ &-\int d^3\mathbf{s}\frac{D^*(\mathbf{s})(1+e^{-i(\mathbf{s}+\mathbf{k})\cdot\mathbf{R}})}{D^{*2}(\mathbf{s})-M^{*2}(\mathbf{s})}\frac{\mathbf{d}\mathbf{s}^2-\mathbf{k}(\mathbf{k}\cdot\mathbf{d})}{\mathbf{k}^2-\mathbf{s}^2+i0ks}\frac{isZ^*_{\mathbf{d}}(\mathbf{s})}{4\pi}e^{ist}\\ &-\int d^3\mathbf{s}\frac{M^*(\mathbf{s})(e^{-i\mathbf{k}\cdot\mathbf{R}}+e^{-i\mathbf{s}\cdot\mathbf{R}})}{D^{*2}(\mathbf{s})-M^{*2}(\mathbf{s})}\frac{\mathbf{k}(\mathbf{k}\cdot\mathbf{d})-\mathbf{d}\mathbf{s}^2}{\mathbf{k}^2-\mathbf{s}^2+i0ks}\frac{isZ^*_{\mathbf{d}}(\mathbf{s})}{4\pi}e^{ist}-\frac{ik}{4\pi}\mathbf{Z}_\perp(\mathbf{k})e^{-ikt}+\frac{ik}{4\pi}\mathbf{Z}^*_\perp(-\mathbf{k})e^{ikt}.\end{aligned} \tag{A3}$$

The quantum expressions for the fields $\mathbf{A}(\mathbf{k},t)$ and $\Pi(\mathbf{k},t)$ can be obtained directly form (A2) and (A3) by replacing the variables $\mathbf{A}(\mathbf{k},t)$, $\Pi(\mathbf{k},t)$, $\mathbf{Z}_\perp(\mathbf{k})$, and $\mathbf{Z}^*_\perp(\mathbf{k})$ with the operators $\hat{\mathbf{A}}(\mathbf{k},t)$, $\hat{\Pi}(\mathbf{k},t)$, $\hat{\mathbf{Z}}_\perp(\mathbf{k})$, and $\hat{\mathbf{Z}}^+_\perp(\mathbf{k})$.

The vector potential $\hat{\mathbf{A}}(\mathbf{k},t)$ is transverse whereas the field $\hat{\Pi}(\mathbf{k},t)$ is not transverse. Nevertheless the commutation relation in direct space remains the same as for the electromagnetic field in empty space,

$$[\hat{A}_\alpha(\mathbf{r},t),\hat{\Pi}_\beta(\mathbf{r}',t)] = i\delta^\perp_{\alpha\beta}(\mathbf{r}-\mathbf{r}'), \tag{A4}$$

where $\delta^\perp$ is a transverse delta function [1] and α and β denote the components of the fields.

APPENDIX B: CONSERVATION OF THE COMMUTATION RELATIONS

Here we present the explicit derivation of the ETCR (19) from the Eqs. (15)–(17).We explicitly derive the equation $[\hat{r}_1(t),\hat{p}_1(t)] = i$ but all the others equalities (19) have been checked as well.

First, we substitute explicit expressions (16) and (17) for $\hat{r}_1(t)$ and $\hat{p}_1(t)$ in the left-hand side of (19) and obtain

$$\begin{aligned}[\hat{r}_1(t),\hat{p}_1(t)] = &\Bigg[-i\int d^3\mathbf{k}\frac{D(k)-M(k)e^{i\mathbf{k}\cdot\mathbf{R}}}{D^2(k)-M^2(k)}k\hat{Z}_{\mathbf{d}}(\mathbf{k})e^{-ikt},\\ &\int d^3\mathbf{k}\frac{D(0)(D^*(k)-M^*(k)e^{-i\mathbf{k}\cdot\mathbf{R}})+M(0)(D^*(k)e^{-i\mathbf{k}\cdot\mathbf{R}}-M^*(k))}{D^{*2}(k)-M^{*2}(k)}\hat{Z}^+_{\mathbf{d}}(\mathbf{k})e^{ikt}\Bigg]\\ &+\Bigg[i\int d^3\mathbf{k}\frac{D^*(k)-M^*(k)e^{i\mathbf{k}\cdot\mathbf{R}}}{D^{*2}(k)-M^{*2}(k)}k\hat{Z}^+_{\mathbf{d}}(\mathbf{k})e^{ikt},\\ &\int d^3\mathbf{k}\frac{D(0)(D(k)-M(k)e^{i\mathbf{k}\cdot\mathbf{R}})+M(0)(D(k)e^{i\mathbf{k}\cdot\mathbf{R}}-M(k))}{D^2(k)-M^2(k)}\hat{Z}_{\mathbf{d}}(\mathbf{k})e^{-ikt}\Bigg]\\ =&\frac{-i}{2\pi^2}\int d^3\mathbf{k}\left(d^2-\frac{(\mathbf{k}\cdot\mathbf{d})^2}{k^2}\right)\frac{(D(k)-M(k)e^{i\mathbf{k}\cdot\mathbf{R}})(D(0)(D^*(k)-M^*(k)e^{-i\mathbf{k}\cdot\mathbf{R}})+M(0)(D^*(k)e^{-i\mathbf{k}\cdot\mathbf{R}}-M^*(k)))}{(D^2(k)-M^2(k))(D^{*2}(k)-M^{*2}(k))}.\end{aligned} \tag{B1}$$

We use Eq. (18) and arrive at

$$\begin{aligned}[\hat{r}_1(t),\hat{p}_1(t)] &= -i\int_0^{+\infty}dk\frac{(D^*(k)D(0)-M^*(k)M(0))\big(D^2(k)-M^2(k)\big)-(D(k)D(0)-M(k)M(0))\big(D^{*2}(k)-M^{*2}(k)\big)}{\pi ik\big(D^2(k)-M^2(k)\big)\big(D^{*2}(k)-M^{*2}(k)\big)} =\\ &= \frac{1}{\pi}\int_0^{+\infty}\frac{dk}{k}\left(\frac{D(k)D(0)-M(k)M(0)}{D^2(k)-M^2(k)}-\frac{D^*(k)D(0)-M^*(k)M(0)}{D^{*2}(k)-M^{*2}(k)}\right)\\ &= \frac{1}{\pi}\mathrm{p.v.}\int_{-\infty}^{+\infty}\frac{dk}{k}\left(\frac{D(k)D(0)-M(k)M(0)}{D^2(k)-M^2(k)}\right).\end{aligned} \tag{B2}$$

Then we use the equation $\mathrm{p.v.}\frac{1}{x} = \frac{1}{x+i0}+i\pi\delta(x)$ and obtain

$$\begin{aligned}[\hat{r}_1(t),\hat{p}_1(t)] &= \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{dk}{k+i0}\left(\frac{D(k)D(0)-M(k)M(0)}{D^2(k)-M^2(k)}\right)-\frac{1}{\pi}\left(-i\pi\frac{D(0)D(0)-M(0)M(0)}{D^2(0)-M^2(0)}\right)\\ &= \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{dk}{k+i0}\left(\frac{D(k)D(0)-M(k)M(0)}{D^2(k)-M^2(k)}\right)+i.\end{aligned} \tag{B3}$$

The first integral on the right-hand side of the last equation is zero because all the poles are below the real axis. The discussion of the poles is presented in relation to Eq. (9) in the manuscript. As a result, we prove that

$$[\hat{r}_1(t),\hat{p}_1(t)] = i. \tag{B4}$$

All the others equalities in (19) can be checked in a similar way.

APPENDIX C: THE GROUND STATE OF THE SYSTEM

We define the vacuum state of the system $|\text{vac}\rangle$ as

$$\hat{\mathbf{Z}}_{\perp}(\mathbf{k})|\text{vac}\rangle = 0. \tag{C1}$$

The vacuum state $|\text{vac}\rangle$ is expressed in terms of the sum to infinity of the initial Fock's states of the dipoles and the electromagnetic field. The physical reason why the vacuum state $|\text{vac}\rangle$ contains an infinite number of initial Fock's states is that we do not exclude the counter-rotating terms. Indeed one can see from the expressions (22), (23), and (C1) that

$$[\hat{a}_1(t)]^n|\text{vac}\rangle \neq 0, \quad [\hat{a}_2(t)]^n|\text{vac}\rangle \neq 0, \tag{C2}$$

for any n. Therefore the vacuum state $|\text{vac}\rangle$ that we define is a so-called "dressed state" [57]. It is easy to see from (24) and (C1) that vacuum $|\text{vac}\rangle$ obeys the equality,

$$\hat{H}|\text{vac}\rangle = 0, \tag{C3}$$

which means that this state is the ground state of the system. Consequently the vacuum state defined at any time, continues to be the vacuum of the operator $\hat{\mathbf{Z}}_{\perp}(\mathbf{k})$ at any other time. In other words,

$$\hat{\mathbf{Z}}_{\perp}(\mathbf{k})e^{-i\hat{H}t}|\text{vac}\rangle = 0, \tag{C4}$$

for any time t. We note that the state $|\text{vac}\rangle$ is not a vacuum state for either the dipoles or the electromagnetic field considered separately [see Eq. (C2)].

[1] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1999).
[2] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, Oxford, 1971).
[3] R. P. Feynman, R. B. Leighton, and M. L. Sands, *The Feynman Lectures on Physics: Vol. 2: The Electromagnetic Field* (Addison-Wesley, Reading, 1965).
[4] W. K. Panofsky and M. Phillips, *Classical Electricity and Magnetism* (Courier Corporation, North Chelmsford, 2005).
[5] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory (Frontiers in Physics)* (Westview Press, Boulder, 1995).
[6] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics* (Butterworth-Heinemann, Oxford, 1982), Vol. 4.
[7] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 2000), Vol. 1.
[8] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (Courier Corporation, North Chelmsford, 2006).
[9] N. Bogolyubov and D. Shirkov, *Introduction to Quantum Fields Theory*, 2nd ed. (Nauka, Moscow, 1973).
[10] E. Fermi, Rev. Mod. Phys. **4**, 87 (1932).
[11] P. Milonni and P. L. Knight, Phys. Rev. A **10**, 1096 (1974).
[12] D. Kaup and V. Rupasov, J. Phys. A: Math. Gen. **29**, 6911 (1996).
[13] M. Shirokov, Sov. Phys. Usp. **21**, 345 (1978).
[14] A. Valentini, Phys. Lett. A **156**, 5 (1991).
[15] M. H. Rubin, Phys. Rev. D **35**, 3836 (1987).
[16] D. Craig and T. Thirunamachandran, Chem. Phys. **167**, 229 (1992).
[17] P. R. Berman and B. Dubetsky, Phys. Rev. A **55**, 4060 (1997).
[18] E. Power and T. Thirunamachandran, Phys. Rev. A **56**, 3395 (1997).
[19] C. Sabín, M. Del Rey, J. J. García-Ripoll, and J. León, Phys. Rev. Lett. **107**, 150402 (2011).
[20] H. Kuang, H. Yin, L. Liu, L. Xu, W. Ma, and C. Xu, ACS Appl. Mater. Interf. **6**, 364 (2013).
[21] E. S. Andrianov, A. A. Pukhov, A. V. Dorofeenko, A. P. Vinogradov, and A. A. Lisyansky, Phys. Rev. B **85**, 035405 (2012).
[22] V. Bordo, Phys. Rev. B **93**, 155421 (2016).
[23] M. I. Stockman, J. Opt. **12**, 024004 (2010).
[24] D. Li and M. I. Stockman, Phys. Rev. Lett. **110**, 106803 (2013).
[25] S. Bakhti, N. Destouches, and A. V. Tishchenko, ACS Photon. **2**, 246 (2015).
[26] M. Richter, M. Gegg, T. S. Theuerholz, and A. Knorr, Phys. Rev. B **91**, 035306 (2015).
[27] R. Taubert, D. Dregely, T. Stroucken, A. Christ, and H. Giessen, Nat. Commun. **3**, 691 (2012).
[28] A. N. Poddubny, A. V. Poshakinskiy, B. Jusserand, and A. Lemaître, Phys. Rev. B **89**, 235313 (2014).
[29] T. G. Philbin, New J. Phys. **14**, 083043 (2012).
[30] B. Huttner and S. M. Barnett, Phys. Rev. A **46**, 4306 (1992).
[31] T. G. Philbin, New J. Phys. **12**, 123008 (2010).
[32] J. Franson, J. Mod. Opt. **55**, 2117 (2008).
[33] M. Borrelli, C. Sabín, G. Adesso, F. Plastina, and S. Maniscalco, New J. Phys. **14**, 103010 (2012).
[34] J. León and C. Sabín, Phys. Rev. A **78**, 052314 (2008).
[35] J. León and C. Sabín, Phys. Rev. A **79**, 012301 (2009).
[36] J. León and C. Sabín, Phys. Rev. A **79**, 012304 (2009).
[37] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[38] W.-J. Yang and X.-B. Wang, Sci. Rep. **5**, 10110 (2015).
[39] X. Yang and M. Xiao, Sci. Rep. **5**, 13609 (2015).
[40] C. Sabín, J. J. García-Ripoll, E. Solano, and J. León, Phys. Rev. B **81**, 184501 (2010).
[41] H. de Riedmatten, I. Marcikic, W. Tittel, H. Zbinden, D. Collins, and N. Gisin, Phys. Rev. Lett. **92**, 047904 (2004).
[42] J. Yin, J.-G. Ren, H. Lu, Y. Cao, H.-L. Yong, Y.-P. Wu, C. Liu, S.-K. Liao, F. Zhou, Y. Jiang *et al.*, Nature (London) **488**, 185 (2012).
[43] J.-W. Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, and A. Zeilinger, Nature (London) **403**, 515 (2000).
[44] C.-Z. Peng, T. Yang, X.-H. Bao, J. Zhang, X.-M. Jin, F.-Y. Feng, B. Yang, J. Yang, J. Yin, Q. Zhang *et al.*, Phys. Rev. Lett. **94**, 150501 (2005).
[45] H. Buhrman, R. Cleve, S. Massar, and R. de Wolf, Rev. Mod. Phys. **82**, 665 (2010).
[46] D. Salart, A. Baas, C. Branciard, N. Gisin, and H. Zbinden, Nature (London) **454**, 861 (2008).
[47] J. Yin, Y. Cao, H.-L. Yong, J.-G. Ren, H. Liang, S.-K. Liao, F. Zhou, C. Liu, Y.-P. Wu, G.-S. Pan *et al.*, Phys. Rev. Lett. **110**, 260407 (2013).

[48] H. Takesue, N. Matsuda, E. Kuramochi, and M. Notomi, Sci. Rep. **4**, 3913 (2014).
[49] Y. Sun, X. Song, H. Qin, X. Zhang, Z. Yang, and X. Zhang, Sci. Rep. **5**, 9175 (2015).
[50] S. Dong, L. Yu, W. Zhang, J. Wu, W. Zhang, L. You, and Y. Huang, Sci. Rep. **5**, 9195 (2015).
[51] L. Novotny and B. Hecht, *Principles of Nano-Optics* (Cambridge University Press, Cambridge, 2012).
[52] J. P. Dowling, Found. Phys. **28**, 855 (1998).
[53] P. R. Berman and C. H. Raymond Ooi, Phys. Rev. A **93**, 013804 (2016).
[54] P. A. M. Dirac, *The Principles of Quantum Mechanics*, International Series of Monographs on Physics, Book 27 (Oxford University Press, Oxford, 1981).
[55] E. Schrödinger, *Zum heisenbergschen unschärfeprinzip* (Akademie der Wissenschaften, Vienna, 1930).
[56] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).
[57] S. M. Barnett and P. M. Radmore, *Methods in Theoretical Quantum Optics* (Oxford University Press, Oxford, 2002), Vol. 15.