PHYSICAL REVIEW A **95**, 060701(R) (2017)

Center-of-mass-momentum-dependent interaction between ultracold atoms

Jianwen Jie[1] and Peng Zhang[1,2,3,*]

[1] *Department of Physics, Renmin University of China, Beijing 100872, China*
[2] *Beijing Computational Science Research Center, Beijing 100084, China*
[3] *Beijing Key Laboratory of Opto-electronic Functional Materials & Micro-nano Devices, Renmin University of China, Beijing 100872, China*

(Received 21 September 2016; published 12 June 2017)

We show that a type of two-body interaction, which depends on the momentum of the center of mass (COM) of these two particles, can be realized in ultracold atom gases with a laser-modulated magnetic Feshbach resonance (MFR). Here the MFR is modulated by two laser beams propagating along different directions, which can induce Raman transition between two-body bound states. The Doppler effect causes the two-atom scattering length to be strongly dependent on the COM momentum of these two atoms. As a result, the effective two-atom interaction is COM-momentum dependent, while the one-atom free Hamiltonian is still the simple kinetic energy $\mathbf{p}^2/(2m)$.

DOI: 10.1103/PhysRevA.95.060701

Introduction. In most physical systems the interaction between two particles is a function of their relative position, and is independent of the two-body center-of-mass (COM) degree of freedom. In some other systems, e.g., ultracold atom gases with interatomic scattering length being modulated by an inhomogeneous magnetic or laser field [1], the two-body interaction can be dependent on the COM position. In this Rapid Communication we show that a type of interaction, which depends on the two-body COM *momentum*, can be realized in ultracold atom gases, while the one-body free Hamiltonian remains the simple kinetic energy. Explicitly, we propose an approach to realizing an ultracold atom gas where the Hamiltonian of every two atoms can be formally expressed as

$$H_{2b} = \frac{\mathbf{p}_1^2}{2m_1} + \frac{\mathbf{p}_2^2}{2m_2} + V_{\rm eff}(\mathbf{r}_1 - \mathbf{r}_2, \mathbf{p}_1 + \mathbf{p}_2), \qquad (1)$$

with m_i, $\mathbf{p}_i$, and $\mathbf{r}_i$ ($i = 1,2$) being the mass, momentum, and position of the ith atom, and $V_{\rm eff}$ being the COM-momentum dependent effective two-atom interaction. To our knowledge, this type of interaction, which couples the two-body COM motion and the relative motion without breaking the translational symmetry and changing the one-body dispersion relation, has not been discovered in any quantum system.

Our proposal is based on the magnetic Feshbach resonance (MFR) modulated by two Raman laser beams propagating along different directions, which couple the two-body bound states in the closed channel and are far off resonant for one-body transitions. When the interatomic interaction of an ultracold gas is controlled by such an approach, the two-atom scattering length is determined by the frequencies of these two laser beams. Furthermore, due to the Doppler effect, when the atoms are moving the frequency of the laser beams can be effectively shifted, and the magnitude of the frequency shift depends on the two-atom COM momentum. As a result, in this system the two-atom scattering length, which describes the effective two-atom interaction, becomes COM-momentum dependent. In addition, the two-body collisional loss induced by spontaneous emission from excited state atoms can be significantly suppressed by the molecular dark state effect [2].

*pengzhang@ruc.edu.cn

In all the previous research for the optical control of interaction between ultracold atoms [1–21], the Doppler effect has always been ignored. This can be explained as follows. In these control processes the laser beams should be far off resonant to the two-body transitions so that the collisional loss induced by atomic spontaneous emission can be suppressed. As a result, the Doppler shift of the laser frequency is much smaller than the detuning of the laser-induced two-body transitions, and thus the Doppler effect is negligible. However, in our system the two-atom scattering length depends on not only the one-photon detuning but also the two-photon detuning of the laser-induced two-body Raman transition. Since the two-photon detuning can be very small, it can be significantly changed by the Doppler frequency shift of the Raman laser beams. Therefore, the Doppler effect can be very important.

Three-dimensional (3D) s-wave scattering length. We consider the s-wave scattering of two ultracold alkali atoms in the ground electronic orbital state (i.e., the S state). As shown in Fig. 1, we assume these two atoms are incident from the open channel O corresponding to one specific two-atom hyperfine state. The threshold of this channel is near resonant to a bound state $|\phi_\alpha\rangle$ in the closed channel C, which corresponds to another hyperfine state of these two S-state atoms. The energy difference between $|\phi_\alpha\rangle$ and the threshold of channel O can be controlled by the magnetic field. Thus, a MFR [22] can be induced by the hyperfine coupling $V_{\rm hf}$ between the open channel O and $|\phi_\alpha\rangle$. We further assume that a laser beam α with wave vector $\mathbf{k}_\alpha$ is applied to couple $|\phi_\alpha\rangle$ to another two-body bound state $|\phi_e\rangle$ in an excited channel F where one atom is in the S state and another atom is in the excited electronic orbital state (i.e., the P state). The excited molecular state $|\phi_e\rangle$ can decay via the atomic spontaneous emission process. In addition, $|\phi_e\rangle$ is also coupled to a bound state $|\phi_\beta\rangle$ in the closed channel C, by another laser beam β with wave vector $\mathbf{k}_\beta$. As mentioned above, we assume the laser beams α and β are far off resonant for all the one-atom transition processes. As a result, they only induce two-body transitions and do not change the one-atom Hamiltonian.

In our system the scattering length a can be controlled by both the magnetic field and the laser beams α and β. As we will show below, when these two beams propagate along the same direction, the Doppler effect is negligible and the scattering length is still independent of the two-atom COM

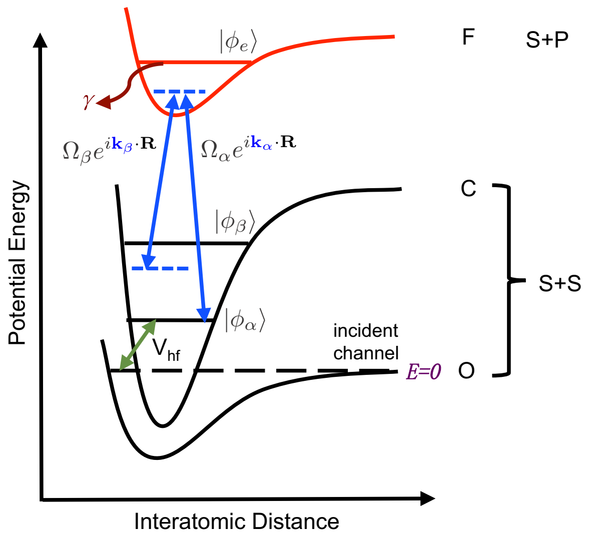

FIG. 1. Schematic diagram for the MFR modulated by Raman laser beams propagating along different directions (i.e., $\mathbf{k}_\alpha \neq \mathbf{k}_\beta$).

momentum. The control of the interatomic interaction for this case was proposed by Wu and Thomas in Refs. [2,3] in 2011, where the wave vectors $\mathbf{k}_{\alpha,\beta}$ were reasonably ignored. It was experimentally realized by Jagannathan *et al.* in 2016 [4].

In this Rapid Communication we consider the case where the two laser beams are propagating along different directions, i.e., $\mathbf{k}_\alpha \neq \mathbf{k}_\beta$. We will show that in this case the Doppler effect can be very important. As a result, the scattering length becomes significantly dependent on the two-atom COM momentum.

Now we calculate the scattering length a. Here we first ignore the spontaneous decay of the excited molecular state $|\phi_e\rangle$ and illustrate the approach of our calculation. Then we will take this decay into account and derive the explicit expression for a. In the absence of the spontaneous decay, in the Schrödinger picture the two-body Hamiltonian can be written as ($\hbar = 1$)

$$H_S = \frac{\mathbf{P}^2}{2M} + H_{\rm MFR} + E_\beta|\phi_\beta\rangle\langle\phi_\beta| + E_e|\phi_e\rangle\langle\phi_e| + \sum_{l=\alpha,\beta}\Omega_l e^{i(\mathbf{k}_l\cdot\mathbf{R}-\omega_l t)}|\phi_e\rangle\langle\phi_l| + \text{H.c.}, \tag{2}$$

with $H_{\rm MFR}$ being defined as

$$H_{\rm MFR} = \left[\frac{\mathbf{p}^2}{2\mu} + V_{\rm bg}(r)\right]|O\rangle_I\langle O| + E_\alpha|\phi_\alpha\rangle\langle\phi_\alpha| + V_{\rm hf}(r)|O\rangle_I\langle C| + \text{H.c.} \tag{3}$$

Here $|j\rangle_I$ ($j = O,C,F$) denote the two-body internal state corresponding to channel j [23], M is the total mass and μ is the two-atom reduced mass, E_l ($l = \alpha,e,\beta$) is the energy of the bound state $|\phi_l\rangle$, ω_l ($l = \alpha,\beta$) is the frequency of laser beam l, and Ω_l ($l = \alpha,\beta$) is the strength of the laser-induced coupling between $|\phi_l\rangle$ and $|\phi_e\rangle$. We have chosen the zero-energy point as the threshold of the open channel O. In Eqs. (2) and (3) $\mathbf{R}$ and $\mathbf{P}$ are the coordinate and momentum of the COM, while $\mathbf{r}$ and $\mathbf{p}$ are those of the two-atom relative motion. The interaction in the open channel O is $V_{\rm bg}(r)$, and the hyperfine coupling between channels O and C is described by $V_{\rm hf}(r)$.

We can simplify our problem and remove the phase factor $e^{\pm i(\mathbf{k}_l\cdot\mathbf{R}-\omega_l t)}$ ($l = \alpha,\beta$) by introducing a rotated frame (interaction picture) where quantum state $|\Psi\rangle_{\rm rot}$ is related to the state $|\Psi\rangle_S$ in the Schrödinger picture via the relation $|\Psi\rangle_{\rm rot} = \mathcal{U}|\Psi\rangle_S$, Here the unitary transformation $\mathcal{U}$ is given by

$$\mathcal{U} = e^{i(\omega_\alpha t-\mathbf{k}_\alpha\cdot\mathbf{R})|\phi_e\rangle\langle\phi_e|}e^{i[(\omega_\alpha-\omega_\beta)t-(\mathbf{k}_\alpha-\mathbf{k}_\beta)\cdot\mathbf{R}]|\phi_\beta\rangle\langle\phi_\beta|}. \tag{4}$$

In this rotated frame, the two-body Hamiltonian becomes

$$H_{\rm rot} = \frac{\mathbf{P}^2}{2M} + H_{\rm MFR} + \sum_{l=\alpha,\beta}\Omega_l|\phi_e\rangle\langle\phi_l| + \text{H.c.} + \Delta_{\rm 1p}(\mathbf{P})|\phi_e\rangle\langle\phi_e| + \Delta_{\rm 2p}(\mathbf{P})|\phi_\beta\rangle\langle\phi_\beta|, \tag{5}$$

where

$$\Delta_{\rm 1p}(\mathbf{P}) = \Delta_{\rm 1p}^{(0)} + \frac{|\mathbf{k}_\alpha|^2}{2M} + \frac{\mathbf{k}_\alpha\cdot\mathbf{P}}{M}; \tag{6}$$

$$\Delta_{\rm 2p}(\mathbf{P}) = \Delta_{\rm 2p}^{(0)} + \frac{|\mathbf{k}_\alpha-\mathbf{k}_\beta|^2}{2M} + \frac{(\mathbf{k}_\alpha-\mathbf{k}_\beta)\cdot\mathbf{P}}{M}, \tag{7}$$

with $\Delta_{\rm 1p}^{(0)} = E_e - \omega_\alpha$ and $\Delta_{\rm 2p}^{(0)} = E_\beta - (\omega_\alpha - \omega_\beta)$. The physical meaning of $\Delta_{\rm 1p}(\mathbf{P})$ and $\Delta_{\rm 2p}(\mathbf{P})$ can be understood as follows. Scattering length a is determined by the multiorder transition process from the open channel O to the bound state $|\phi_\alpha\rangle$ (induced by hyperfine coupling) and then to the excited molecular state $|\phi_e\rangle$ (induced by laser α), and finally to the bound state $|\phi_\beta\rangle$ (induced by laser β). This is essentially a Raman process induced by the laser beams α and β. $\Delta_{\rm 1p}(\mathbf{P})$ given by Eq. (6) is the one-photon detuning of this Raman process (i.e., the detuning of the transition $O \to |\phi_\alpha\rangle \to |\phi_e\rangle$). Similarly, $\Delta_{\rm 2p}(\mathbf{P})$ is the two-photon detuning of the complete Raman process from O to $|\phi_\beta\rangle$. Moreover, in Eqs. (6) and (7) $\Delta_{\rm 1p}^{(0)}$ and $\Delta_{\rm 2p}^{(0)}$ can be understood as the bare value of these detunings, $|\mathbf{k}_\alpha|^2/(2M)$ and $|\mathbf{k}_\alpha - \mathbf{k}_\beta|^2/(2M)$ are the shifts induced by the momentum-recoil effects, and the $\mathbf{P}$-dependent terms are the Doppler shifts.

According to Eq. (4), we have $\mathcal{U}|O\rangle_I\langle O|\mathcal{U}^\dagger = |O\rangle_I\langle O|$. Thus, all the operators for the open channel O are unchanged under the frame transformation. Therefore, we can calculate the scattering length of two atoms incident from channel O, which is determined by the behavior of the low-energy scattering wave function in this channel in the limit $r \to \infty$, by solving the scattering problem in the rotated frame, which is governed by $H_{\rm rot}$. Furthermore, Eq. (5) shows that in this frame the COM momentum $\mathbf{P}$ is conserved. Thus, the scattering length a is a function of $\mathbf{P}$.

Now we consider the spontaneous decay of the excited molecular state $|\phi_e\rangle$. We can theoretically take into account this decay by introducing an auxiliary scattering channel which is coupled to $|\phi_e\rangle$ [24]. With this approach we derive the exact analytical expression of the scattering length [25]:

$$a(\mathbf{P}) = a_{\rm bg} - \frac{(\delta\mu)a_{\rm bg}\Delta_B\left[\Delta_{\rm 1p}(\mathbf{P}) - i\frac{\gamma}{2} - \frac{|\Omega_\beta|^2}{\Delta_{\rm 2p}(\mathbf{P})}\right]}{(\delta\mu)(B-B_0)\left[\Delta_{\rm 1p}(\mathbf{P}) - i\frac{\gamma}{2} - \frac{|\Omega_\beta|^2}{\Delta_{\rm 2p}(\mathbf{P})}\right] - |\Omega_\alpha|^2}. \tag{8}$$

Here a_{bg}, B_0, and Δ_B are the background scattering length, the resonance position, and the width of the MFR. Explicitly, in the absence of the Raman laser beams we have $a = a_{bg}[1 - \Delta_B/(B - B_0)]$. $\delta\mu$ is the magnetic moment difference between the channels O and C, and γ is the spontaneous decay rate of $|\phi_e\rangle$. The energy of the two-body bound state can also be calculated via the same approach.

It is clear that the scattering length $a(\mathbf{P})$ depends on the COM momentum $\mathbf{P}$ via the one-photon and two-photon detuning $\Delta_{1p}(\mathbf{P})$ and $\Delta_{2p}(\mathbf{P})$. In a realistic system, to suppress the collisional loss induced by the spontaneous decay of $|\phi_e\rangle$, one usually sets the bare value $\Delta_{1p}^{(0)}$ of the one-photon detuning to be very large. As a result, the $\mathbf{P}$ dependence of $\Delta_{1p}(\mathbf{P})$ is usually negligible. However, the bare two-photon detuning $\Delta_{2p}^{(0)}$ can be very small, and thus the variation of $\Delta_{2p}(\mathbf{P})$ with $\mathbf{P}$ can be very significant. Therefore, $a(\mathbf{P})$ can be strongly dependent on $\mathbf{P}$ via $\Delta_{2p}(\mathbf{P})$.

Furthermore, according to Eq. (7), when the two Raman beams are propagating along the same direction (i.e., $\mathbf{k}_\alpha \approx \mathbf{k}_\beta$), $\Delta_{2p}(\mathbf{P})$ takes a $\mathbf{P}$-independent value $\Delta_{2p}^{(0)}$, and thus the Doppler effect can be ignored. For this case it was shown that the two-body loss can be suppressed by the molecular dark state effect, provided that $\Delta_{2p}^{(0)}$ is small enough [2]. Since we can reobtain the scattering length for this case [2] by replacing $\Delta_{2p}(\mathbf{P})$ in Eq. (8) with $\Delta_{2p}^{(0)}$, for our system the two-body loss can also be suppressed when $\Delta_{2p}(\mathbf{P})$ is small enough. This suppression can also be understood from the fact $\lim_{\Delta_{2p}(\mathbf{P})\to 0} \mathrm{Im}[a(\mathbf{P})] \propto O(\Delta_{2p}[\mathbf{P}]^2)$.

3D ultracold Fermi gas. As an example, we consider a 3D ultracold gas of two-component ^{40}K atoms in the lowest two hyperfine states $|\uparrow\rangle \equiv |F = 9/2, m_F = -9/2\rangle$ and $|\downarrow\rangle \equiv |F = 9/2, m_F = -7/2\rangle$. Here we focus on the MFR with $B_0 = 202.2$ G, $\Delta_B = 8$ G, and $a_{bg} = 174 a_0$ [14,22], with a_0 being the Bohr radius, and assume that this MFR modulated by two Raman beams as discussed above. We take these two beams to be counterpropagating along the x axis. Thus, the scattering length a only depends on the x component P_x of $\mathbf{P}$. In the ultracold Fermi gas P_x is mainly in the region between $-2k_F$ and $2k_F$, with k_F being the Fermi momentum. We consider an ultracold gas with Fermi temperature $T_F = 0.5\ \mu$K (corresponding to $k_F = 9.1 \times 10^6\ \mathrm{m}^{-1}$).

In Figs. 2(a) and 2(b) we illustrate the real part of a given by Eq. (8) for $B - B_0 = \mp 0.07\Delta_B$. It is shown that in these cases Re[a] is always positive or always negative for $P_x \in [-2k_F, 2k_F]$, and can change by about $2500 a_0$ with P_x, ranging from the region with $1/(k_F|\mathrm{Re}[a]|) < 1$ to the region with $1/(k_F|\mathrm{Re}[a]|) > 1$. Direct calculations show that these results are robust with respect to the uncertainties of B_0 and Δ_B. For the case of Fig. 2(a) we further calculate the energy E_b of the most shallow two-body bound state, as a function of the COM momentum P_x. In Fig. 2(c) we show the total dimer energy $E_{\mathrm{dim}}(P_x) \equiv \frac{P_x^2}{2M} + \mathrm{Re}[E_b(P_x)]$ as a function of P_x, i.e., the dispersion relation of the shallow dimer. It is clear that if the E_b were independent of P_x, the minimum point of E_{dim} appears at $P_x = 0$. As shown in Fig. 2(c), due to the P_x dependence of E_b, in our system $E(P_x)$ takes its minimum value when $P_x = -0.66 k_F$.

Now we investigate the two-atom collisional loss. If the scattering length a was independent of the COM momentum, the two-body collisional loss rate is $K_2 \equiv -8\pi\hbar\,\mathrm{Im}[a]/m$, with m being the single-atom mass. Accordingly, the lifetime of the ultracold gas can be defined as $\tau = 1/[K_2 n_0]$, where n_0 is the initial atom density. When a is P_x dependent, it is difficult to exactly calculate the two-body loss rate and lifetime for the gas. Nevertheless, we can still estimate the loss effect via the parameter $K_2(P_x) \equiv -8\pi\hbar\,\mathrm{Im}[a(P_x)]/m$. As shown in Fig. 2(d), $K_2(P_x)$ is below $2 \times 10^{-12}\ \mathrm{cm}^3/\mathrm{s}$ for the systems studied in Figs. 2(a) and 2(b). Using the atom density $n_0 = 1.28 \times 10^{13}/\mathrm{cm}^3$ corresponding to $T_F = 0.5\ \mu$K, we can obtain $1/[K_2(P_x)n_0] > 0.04$ s. In addition, in Fig. 2(d) we illustrate the lifetime $\tau_b \equiv 1/(\mathrm{Im}[E_b(P_x)])$ of the shallow dimer. It is shown that τ_b is about 0.1 s at the minimum point of the dimer energy.

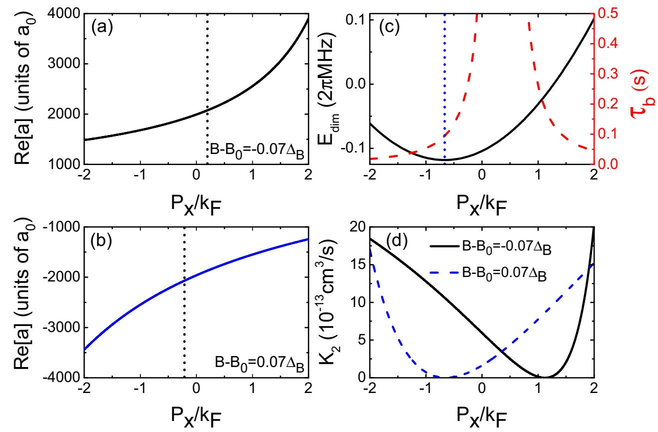

FIG. 2. (a) and (b) Re[$a(P_x)$] of ^{40}K atoms with $B - B_0 = \mp 0.07\Delta_B$. The black dotted line indicates the point where $1/(k_F|\mathrm{Re}[a]|) = 1$. (c) The dispersion relation $E_{\mathrm{dim}}(P_x) \equiv \frac{P_x^2}{4m} + \mathrm{Re}[E_b(P_x)]$ (black solid line) and lifetime $\tau_b(P_x)$ (red dashed line) of the most shallow dimer for the case in (a). The blue dotted line indicates the point with minimum total dimer energy. (d) The loss rate $K_2(P_x)$ for the cases in (a) and (b). In the calculations we take $\Delta_{1p}^{(0)} + |\mathbf{k}_\alpha|^2/(2M) = 2\pi \times 400$ MHz while $\Delta_{2p}^{(0)} + |\mathbf{k}_\alpha - \mathbf{k}_\beta|^2/(2M) = -2\pi \times 2.1 \times 10^4$ Hz for (a) and $2\pi \times 1.2 \times 10^4$ Hz for (b). In the experiment of laser-modulated MFR of ^{40}K atoms [14], the laser-induced coupling intensity between bound states can be as large as 30 MHz. Thus, here we choose $\Omega_\alpha = 2\pi \times 50$ MHz while $\Omega_\beta = 2\pi \times 10$ MHz for (a) and $2\pi \times 12$ MHz for (b). We further choose $\omega_a \approx \omega_b = 2\pi \times 3.9 \times 10^{14}$ Hz and $\gamma = 2\pi \times 6$ MHz, respectively [14]. Other parameters are given in the main text.

Quasi-one-dimensional (quasi-1D) ultracold Fermi gas. Now we consider an ultracold two-component Fermi gas in an axially symmetric two-dimensional harmonic potential in the y-z plane, with trapping frequency $\omega_\perp$. When the atomic transverse motion is frozen in the ground state of this harmonic potential, the ultracold gas becomes a quasi-1D system. For this system the effective low-energy 1D interaction between two atoms in different components can be expressed as

$$V_{1D} = g_{1D}\delta(x) \equiv -\frac{1}{\mu a_{1D}}\delta(x). \tag{9}$$

Here a_{1D} is the effective 1D scattering length. It can be controlled by both the 3D scattering length and the transverse

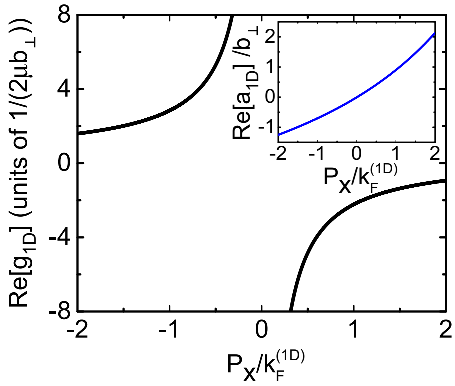

FIG. 3. Re[$g_{1D}(P_x)$] and Re[$a_{1D}(P_x)$] (inset) of the quasi-1D ultracold gas of ^{40}K atoms. We consider the system where the one-body momentum k_x along the x direction is in the region between $\pm k_F^{(1D)} \equiv \pm 3/(4b_\perp)$ with $b_\perp = \sqrt{2/(m\omega_\perp)}$. Thus we have $P_x \in [-2k_F^{(1D)}, 2k_F^{(1D)}]$. In our calculation we take $\Delta_{1p}^{(0)} + |\mathbf{k}_\alpha|^2/(2M) = 2\pi \times 100$ MHz, $\Delta_{2p}^{(0)} + |\mathbf{k}_\alpha - \mathbf{k}_\beta|^2/(2M) = 2\pi \times 1.1 \times 10^4$ Hz, $\Omega_\alpha = 2\pi \times 50$ MHz, $\Omega_\beta = 2\pi \times 3.5$ MHz, $b_\perp = 4000a_0$, and $B - B_0 = -0.07\Delta_B$. Other parameters are the same as in Fig. 2.

trapping frequency $\omega_\perp$ via the confinement-induced resonance (CIR) effect [26]. As shown above, when the two-atom interaction is controlled by a MFR modulated by two Raman beams counterpropagating along the x axis, the 3D scattering length becomes a function of the COM momentum P_x. As a result, both a_{1D} and the effective 1D interaction intensity g_{1D} become P_x dependent.

In particular, when $a_{1D}(P_x = 0)$ is tuned to the CIR point, i.e., when $a_{1D}(P_x = 0) = 0$, one can even obtain a dramatic quasi-1D system with

$$g_{1D}(P_x)\begin{cases} <0 & \text{for } P_x > 0 \\ =\infty & \text{for } P_x = 0 \\ >0 & \text{for } P_x < 0. \end{cases} \tag{10}$$

Namely, in this system the atoms have attractive and repulsive 1D interactions when the COM momentum is along the $+x$ and $-x$ direction, respectively, and have an infinitely strong interaction when the COM momentum is zero. In Fig. 3 we illustrate such a case for a quasi-1D ultracold gas of ^{40}K atoms in hyperfine states $|\uparrow\rangle$ and $|\downarrow\rangle$. The lifetime of this quasi-1D gas is estimated to be larger than 0.04 s [25].

Summary and discussion. In common quantum systems, the two-body COM and relative motion can be coupled by a nonharmonic confinement potential, a spin-orbit coupling, or a COM-position-dependent two-body interaction. Nevertheless, these approaches either destroy the translational symmetry or change the one-body dispersion relation. Here we show that a COM-momentum-dependent interaction between ultracold atoms can be realized via a laser-modulated MFR. This interaction can couple the COM and relative motion without breaking translational symmetry or changing the one-body dispersion relation. Thus, various new effects can be induced by this interaction. For instance, for a 3D two-component Fermi gas with COM-momentum-dependent positive scattering length $a(\mathbf{P})$, the dimers are possible to condense in a superfluid state with nonzero momentum. When $a(\mathbf{P})$ is negative, it is possible that the minimum energy of a Cooper pair appears in the region with nonzero COM momentum, and thus the Fulde-Ferrell phase [27] can be induced. In addition, the single-atom momentum distribution or contact relation can also be qualitatively modified by a COM-momentum-dependent interaction [28]. Furthermore, the ultracold quasi-1D gas with this type of interaction can be used to realize the 1D anyon model [29] or other high-order nonlinear Schrödinger models [30–32].

Acknowledgments. We thank Paul Julienne, Haibin Wu, and Hui Zhai for helpful discussions. We also thank the referees for improving the quality of this manuscript. This work has been supported by the Natural Science Foundation of China under Grants No. 11434011 and No. 11674393, NKBRSF of China under Grant No. 2012CB922104, the Fundamental Research Funds for the Central Universities, and the Research Funds of Renmin University of China under Grants No.16XNLQ03 and No. 17XNH054.

[1] L. W. Clark, L. C. Ha, C. Y. Xu, and C. Chin, Phys. Rev. Lett. **115**, 155301 (2015).
[2] H. Wu and J. E. Thomas, Phys. Rev. Lett. **108**, 010401 (2012).
[3] H. Wu and J. E. Thomas, Phys. Rev. A **86**, 063625 (2012).
[4] A. Jagannathan, N. Arunkumar, J. A. Joseph, and J. E. Thomas, Phys. Rev. Lett. **116**, 075301 (2016).
[5] P. O. Fedichev, Y. Kagan, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **77**, 2913 (1996).
[6] J. L. Bohn and P. S. Julienne, Phys. Rev. A **56**, 1486 (1997).
[7] F. K. Fatemi, K. M. Jones, and P. D. Lett, Phys. Rev. Lett. **85**, 4462 (2000).
[8] M. Theis, G. Thalhammer, K. Winkler, M. Hellwig, G. Ruff, R. Grimm, and J. H. Denschlag, Phys. Rev. Lett. **93**, 123001 (2004).
[9] G. Thalhammer, M. Theis, K. Winkler, R. Grimm, and J. H. Denschlag, Phys. Rev. A **71**, 033403 (2005).
[10] K. Enomoto, K. Kasa, M. Kitagawa, and Y. Takahashi, Phys. Rev. Lett. **101**, 203201 (2008).
[11] R. Yamazaki, S. Taie, S. Sugawa, K. Enomoto, and Y. Takahashi, Phys. Rev. A **87**, 010704(R) (2013).
[12] D. M. Bauer, M. Lettner, C. Vo, G. Rempe, and S. Durr, Phys. Rev. A **79**, 062713 (2009).
[13] D. M. Bauer, M. Lettner, C. Vo, G. Rempe, and S. Durr, Nat. Phys. **5**, 339 (2009).
[14] Z. Fu, P. Wang, L. Huang, Z. Meng, H. Hu, and J. Zhang, Phys. Rev. A **88**, 041601(R) (2013).
[15] L. Zhang, Y. Deng, and P. Zhang, Phys. Rev. A **87**, 053626 (2013).
[16] L. Dong, L. Jiang, H. Hu, and H. Pu, Phys. Rev. A **87**, 043616 (2013).
[17] D. M. Kurkcuoglu and C. A. R. Sáde Melo, Phys. Rev. A **93**, 023611 (2016).

[18] R. A. Williams, M. C. Beeler, L. J. LeBlanc, K. Jiménez-García, and I. B. Spielman, Phys. Rev. Lett. **111**, 095301 (2013).

[19] R. Qi and H. Zhai, Phys. Rev. Lett. **106**, 163201 (2011).

[20] Y. C. Zhang, W. M. Liu, and H. Hu, Phys. Rev. A **90**, 052722 (2014).

[21] P. Zhang, P. Naidon, and M. Ueda, Phys. Rev. Lett. **103**, 133202 (2009).

[22] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010).

[23] In our two-body problem the Hilbert space is $\mathcal{H}_{\rm COM}\bigotimes\mathcal{H}_{\rm rel}\bigotimes\mathcal{H}_{\rm internal}$, where $\mathcal{H}_{\rm internal}$ is the space for the two-atom internal state, while $\mathcal{H}_{\rm COM}$ and $\mathcal{H}_{\rm rel}$ are the spaces for the spatial states of COM motion and relative motion, respectively. Here we use $|\cdot\rangle_I$ to denote the states in $\mathcal{H}_{\rm internal}$ and $|\cdot\rangle$ for those in $\mathcal{H}_{\rm rel}\bigotimes\mathcal{H}_{\rm internal}$.

[24] J. L. Bohn and P. S. Julienne, Phys. Rev. A **60**, 414 (1999).

[25] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.95.060701 for the detail of the calculations for the scattering length for 3D and 1D cases, as well as the discussions for the robustness of our results.

[26] M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998).

[27] P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964).

[28] X. Cui and H. Dong, Phys. Rev. A **94**, 063650 (2016).

[29] M. T. Batchelor, X-W. Guan, and A. Kundu, J. Phys. A: Math. Theor. **41**, 352002 (2008).

[30] A. G. Shnirman, B. A. Malomed, and E. Ben-Jacob, Phys. Rev. A **50**, 3453 (1994).

[31] E. Gutkin, Phys. Rep. **167**, 1 (1988).

[32] E. Gutkin, Ann. Phys. **176**, 22 (1987).