# Reply to "Comment on 'Spatial optical solitons in highly nonlocal media'"

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In their Comment, Petrović *et al.* claim that some of the results previously published by us on the use of the "accessible soliton" model of Snyder *et al.* are incorrect, and they claim that the correct results were published elsewhere. In order to give our perspective on the problem, we discuss and clarify some of the existing literature and our own work on the subject, underlining the importance of the accessible soliton approximation and its recent improvements towards enabling a general understanding of light self-confinement in highly nonlocal media, both quantitatively and qualitatively.

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Petrović *et al.* raise two main points about our paper [1] (and related papers) in their Comment: on the origin and credits of our results in Refs. [1,2] and on incongruities about the application of the Snyder-Mitchell model (SMM) [3]. They also raise technical questions about Ref. [1], stating that our model accounting for the longitudinal nonlocality is inconsistent. We first respond to the main points. In the second part of this Reply we provide detailed technical answers to Petrović *et al.*

## I. OVERALL REPLY

The claim of priority by Petrović *et al.* to the discovery of the quantitative inaccuracy of the SMM is in disagreement with the pertinent literature. To the best of our knowledge, this result was first published in Refs. [4,5] by Guo's group. Both these papers were cited in Ref. [2] (in Ref. [1] we cited Ref. [5] only) to clearly establish the credit for pinpointing the inaccuracy of the SMM when dealing with diffusive nonlinear media. In Ref. [6], Petrović *et al.* write explicitly for the first time that a factor 2 accounts for the discrepancy between solutions of the Schrödinger-Poisson equation and the SMM, but this can be easily derived using Guo's approach. Despite the statements by Petrović *et al.*, we never claimed this result as ours. With respect to the paper under Comment, Ref. [1] deals with a distinct topic, that is, the effect of longitudinal nonlocality on the propagation of spatial solitons governed by a Schrödinger-Poisson equation. Our paper [2] deals with the SMM. We wrote the following: "We explained on physical grounds why the SMM fails for any given amount of nonlocality" and "Finally, we derived an effective parabolic shaped photonic potential leading to an accurate description of solitons by means of simple analytical formulas."

The SMM is a fundamental tool for the investigation of spatial solitons in highly nonlocal media. It permits addressing all the main features of solitons in such media (stability, interaction, breathing). While these properties are now widely established, a couple of decades ago they were quite novel and surprising as most of the literature on spatial solitons (both theoretical and experimental) dealt with Kerr local media, including catastrophic collapse [(2+1)D case], inverse scattering [(1+1)D case], and so on [7]. As expected in science, since then, models and experiments have been greatly improved, including, e.g., nonlocal effects. In this course, Guo's group discovered the quantitative discrepancy between solitary wave propagation in real diffusive media and results provided by the SMM [4,5], as stated above. In this context, Conti *et al.* in Ref. [8] interpolated the breathing behavior of experimentally observed nematicons [solitons in nematic liquid crystals (NLCs)] with the sinusoidal behavior predicted by the SMM. A fitting procedure was necessary for two main reasons: (i) Some experimental parameters were not known (including the role of the NLC-air interface and the effective elastic constant) and (ii) the SMM is approximate, with accuracy being worse in voltage-biased than in bias-free cells [2]. In the past several years, relevant improvements have been made in both technology (better control of the input interface; see Ref. [9]) and modeling (accounting, e.g., for walk-off [10–12]). Our results establish a good quantitative agreement between experimental data and the modified SMM [2], as recently shown in Ref. [13]. Finally, the results from our group have been experimentally validated by several groups [14–23].

## II. POINT BY POINT RESPONSE

After these overall comments, let us now address the Comment by Petrović *et al.* in order of appearance of their main points.

### A. Application of the VA to the investigation of the role of boundary conditions for highly nonlocal responses

Petrović *et al.* claim the original introduction of a variational approximation (VA) to investigate the role of boundary conditions on the propagation of spatial solitons. In the literature, the interaction with boundaries was studied earlier

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than in Ref. [24], e.g., in lead glasses [25–29] and in NLCs [30,31]. The VA itself was applied to nematicons in a biased cell well before Petrović *et al.* (see Refs. [32,33]). It was even applied to the specific problem of the interaction of nematicons with boundaries in 2009 [34,35].

The role of noise on (the existence of) shape-preserving nematicons is the major issue in Ref. [24]. In Ref. [24] noise was inserted as a perturbation of the pretilt angle. This is quite a questionable assumption and affects the results, as the proper way to introduce molecular noise in NLCs is to consider noise both in time and in the transverse direction. When longitudinal random changes are applied exclusively to the pretilt angle, the consequence is a stochastic modulation of the nonlinearity in the system [12,36].

Nematicons tend to be robust and resilient to noise due to their high nonlocality (see the Supplemental Material in Ref. [37] and Fig. 6 in Ref. [2]). In actual experiments, noise is not the reason why shape-preserving solitons are not observed. Rather, the losses associated with strong scattering in NLCs [38] are the main cause of longitudinal changes in nematicons. Even with the assumption of negligible losses (only applicable over very short distances), launching shape-preserving nematicons is hindered by the presence of an input interface which breaks the symmetry along $z$, as also discussed in the paper under comment [1].

### B. Discovery of the quantitative inaccuracy for the accessible soliton model in diffusive media

To the best of our knowledge, the first quantitative and qualitative demonstration of the inaccuracy of the SSM was provided in Ref. [4]. Its abstract reads as follows: "We show that for the nonlocal case of an exponential-decay type nonlocal response the Gaussian-function-like soliton solutions cannot describe the nonlocal soliton states exactly even in the strongly nonlocal case." In the body of Ref. [4], Figs. 6–9 show the quantitative results. Additionally, it is clearly explained that the problem stems from the fact that a harmonic potential does not provide an accurate approximation of the actual nonlinear index well. Petrović *et al.* in their Comment state that 'the first published accurate quantitative correction to AS" was presented in Ref. [24]. In that paper [24], however, the only reference to SMM appears to be a sentence qualitatively stating that the shape-preserving soliton is almost Gaussian, except for the tails. The latter result was reported in Ref. [39], Sec. 2.5.

### C. Complaint on lack of credit

In their Comment, Petrović *et al.* criticize our lack of references to their work in our earlier paper [2]. As stated above, the inaccuracy of the SMM was first emphasized by Guo's group [4,5,40]. In the introduction of Ref. [4], Ouyang *et al.* state the following: "On the other hand, even though a convenient method has been introduced in Refs. [3,5,13,14] to study the propagation of light beams in the strongly nonlocal case or even in the sub-strongly nonlocal case, to employ this method efficiently the nonlocal response function must be twice differentiable at its center. As will be shown this method cannot deal with the nonlocal case of an exponential-decay type nonlocal response function that is not differentiable at its center." In the conclusions of Ref. [4] they write the following: "For the nonlocal case of the exponential-decay type nonlocal response, the Gaussian-function-like soliton solution cannot describe the fundamental soliton state of the NNLSE exactly even in the strongly nonlocal case, that greatly differs from the nonlocal case of the Gaussian function type nonlocal response."

The goal of Ref. [2]—as clearly stated throughout the Letter—was to explain on physical grounds why this discrepancy exists, why it takes a certain numerical value, and why solitons do not exist below a power threshold. Conversely, Petrović and co-workers in their papers retrieve this discrepancy as a result of computations.

Petrović *et al.* state that the findings of Ref. [4] are not relevant as they provide an approximate solution stemming from a perturbative correction to the SMM and that they do not mention a factor 2 or $\sqrt{2}$. First, the VA used by Petrović *et al.* *is an approximate method* as well. Noteworthy, for a Gaussian input the director distribution can be computed exactly [30]. Second, the analytical expression of Ouyang *et al.* is much closer to the exact one than the solution from the VA (see Fig. 6 in Ref. [4]). Third, the scaling factor connecting exact and approximate solutions can even be computed from the closed-form solutions presented in Ref. [4] [for instance, the existence curve power versus soliton width is expressed by Eq. (42) and plotted in Fig. 9 of Ref. [4]]. The statement that the numerical correction cannot be found in the framework of perturbation theory is simply mathematically unsubstantiated.

### D. Physical reason behind the inaccuracy of the accessible soliton model in diffusive media

Petrović *et al.* wonder what is the reason behind the SMM inaccuracy, hinting at an "inconsistency" in our scientific approach. The singularity in the response function is the mathematical reason for the quantitative inaccuracy in the SMM. When the response function is differentiable to quadratic order, the SMM is valid for large powers [3,4]. From a physical point of view, the quantitative inaccuracy stems from the fact that the nonlinear perturbation is governed by a Poisson equation, leading to an infinitely extended range of nonlocality, as phrased in Ref. [25]. In other words, the pointwise solution of an elliptic equation depends on the solution in the whole domain [41]. Thus, the width of the nonlinear response is inherently related to the size of the integration domain, and the spatial overlap between the input beam and the anharmonic components of the self-induced index well does not vanish as power is increased [30]. This is due to the boundary conditions and is the physical reason for the quantitative (not qualitative) inaccuracy of the original SMM when dealing with diffusive media [2]. Summarizing, the two explanations are equivalent, describing the same effect on different grounds.

### E. Factor 2 missing in the paper under Comment

In Ref. [1] we mistakenly defined the soliton width according to the modified SMM, previously found in Ref. [2] (see the Erratum in Ref. [61]). As for the correction factor, it comes from Ref. [2], in which the background physics is discussed in detail. Our results are also in agreement with those presented by Petrović *et al.* and based on the VA [6].

### F. On the proper application of the paraxial approximation when dealing with nonlocal spatial solitons

Petrović *et al.* doubt the self-consistency of the approximations in Ref. [1], with particular reference to the paraxial approximation. For electromagnetic waves, the paraxial approximation breaks down when the wave-packet size is comparable to, or smaller than, the wavelength. A first-order correction then requires a non-negligible longitudinal field [42]. The corrections due to the second derivative of the field along the propagation direction are second order [42] (the application of these results to nematicons can be found, e.g., in Ref. [43]). The longitudinal second derivative of the field must be accounted for when dealing with solitons propagating at large angles ($\approx 30^\circ$ with respect to $z$) [44]. Fast variations along $z$ imply a change in the refractive index of the carrier. On the one hand, this effect does not affect transverse confinement. On the other hand, the change is adiabatic on the wavelength scale, both in typical experiments [12,45] and in numerical simulations [1,46]. Importantly, this needs to be accounted for even when the second derivative of the nonlinear index well along the propagation direction is not considered. Analogously, back reflections are neglected whenever light propagation is described by a unidirectional paraxial Helmholtz equation [that is, a nonlinear Schröödinger equation (NLSE) in the nonlinear case], corresponding numerically to the use of a unidirectional beam propagation method.

After these considerations on well-known results, let us discuss in detail the model we employed in Ref. [1]. For light propagation in the presence of a highly nonlocal response, the latter smooths out the longitudinal variations in the light-induced index well, thus minimizing back reflections. The inclusion of longitudinal nonlocality improves the agreement between the mathematical model and the physical system [47,48]. In essence, our overall model (i.e., including light evolution and light-matter interaction) satisfies the paraxial approximation *better* than standard ones (i.e., when longitudinal nonlocality is neglected). Our group members have never observed any back reflection when light is self-trapped in experiments with undoped NLCs. Finally, in the framework of classical optics, a backscattered wave cannot be generated without an input wave.

### G. Alleged inconsistency of experimental results published by our group

Petrović *et al.* claim an inconsistency in the experimental data reported in Ref. [8]. Let us first—and foremost—stress that the SMM is approximate. Thus, it cannot (and it is not meant to) match perfectly real experiments. Incidentally, even in the first (theoretical) paper about accessible solitons in NLCs [49], corrections to the SMM were discussed [see Eq. (16) in Ref. [49]]. Second, the observations in Ref. [8] were performed in a biased cell, for which the nonlinear index well is governed by a screened Poisson equation. As we showed in Ref. [2], the SMM is thus even more inaccurate than in bias-free cells (in unbiased cells the nonlinear index well obeys a Poisson equation in the perturbative regime [12]). In fact, in Ref. [8], Conti *et al.* wrote that Eqs. (6) in Ref. [8] are derived from Eq. (4) in Ref. [8] using an approximation. Third, in biased cells nematicons propagate in a wide linear index well and possess walk-off in the vertical plane $(x,z)$: The model used in Ref. [8] does not account for this dynamics, which was addressed later in Refs. [11,15]. Reality is far more complex than a single Schrödinger-Poisson equation: (i) In real samples there are interfaces at finite distances, as we discussed in Ref. [1]; (ii) several nonlinear effects act together, thus models considering only the reorientational response are approximations [50,51]; (iii) the actual reorientation is driven by a more complicated equation than a single Poisson equation, even in the perturbative regime [38,52]; (iv) NLCs described by a molecular director field are a simplification of an underlying many-body system which is subject to continuous temporal fluctuations [38,53]. The elegant SMM, despite its mathematical simplicity, explains qualitatively nonlinear light propagation, predicting an oscillatory (breathing) behavior qualitatively different from the breather dynamics in a standard NLSE [54–56], stability in $(2+1)$D [57], and interaction between solitons [17,58,59]. In short, all the main features of nematicons are well described by the accessible soliton model. With respect to the results in Ref. [8], just before the statement Petrović *et al.* cited, we read the following: "The best fit is obtained from Eq. (7) by introducing as a parameter the coupling efficiency $\alpha$ of the laser power $P_{in}$ into the soliton-trapped power $P$ (i.e., $\alpha P = P_{in}$)." This is the best-fit coefficient $\alpha$ accounting for all factors discussed above (in Ref. [8], $\alpha \approx 7\%$, meaning that all these factors were acting simultaneously). Thus, the scaling constant stemming from the SMM (which is not 2, as in Ref. [8] the cell was biased; see Ref. [2]) was simply included in the coefficient $\alpha$.

## III. CONCLUSIONS

In conclusion, we have shown that no systematic errors were made by us or members of our group with reference to the results discussed in the Comment by Petrović *et al.*. The correction to the SMM was not published for the first time by Petrović *et al.* We have clarified that there are no conceptual deficiencies in the paper under Comment [1]. With respect to Petrović's *et al.* final sentence, "This model is just a linear approximation to a highly nonlocal nonlinear problem," we would like to stress that, as physicists, our primary goal is understanding the main physical mechanisms and describing them in the simplest way, including the use of approximate methods. Better models and approximations can be implemented subsequently. Along this path, we recently elaborated a corrected SMM able to model quantitatively nonlinear light propagation [13,60]. In doing so, we also tried to understand the limits of this approximation by a direct comparison with experiments. We thank Petrović *et al.* for spotting a wrong factor in Ref. [1], which we have amended in Ref. [61].

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