Indistinguishability-induced classical-to-nonclassical transition of photon statistics

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Photon statistics is one of the key properties of the photon state for the study of quantum coherence and quantum information techniques. Here, we discuss the photon indistinguishability induced bunching effect which can significantly change photon statistics. Through the measurement of the second-order degree of coherence of a mixed photon state composed of a single-photon state and a weak coherent state, the statistical transition from a classical behavior to a nonclassical behavior is experimentally demonstrated by modifying the indistinguishability of the two-photon states. The study will help us to understand and control the photon statistics with a method for quantum optical coherence and quantum information applications. It also indicates that the photon indistinguishability is a key parameter for multipartite quantum coherence.

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Photon statistics is a fundamental property of the quantum optical field, which has been the basis of quantum coherence [1] and recently developed optical quantum information techniques [2–5]. It also has been applied in quantum superresolution microscopy [6,7] to achieve nanoscale resolution. Generally, photon statistics is mainly determined by the number of emitters and the process of photon-matter interaction. For example, a single photon [8–11] can be generated from a single quantum emitter, which is a key photon source for quantum communication [12–14] and quantum computation [3,4]. The multiphoton state from a nonlinear optical process has been applied to demonstrate quantum entanglement, quantum computation, and high-sensitivity quantum metrology [5,15–17]. In experiment, the statistics of a photon state can be modified by postselection measurement [18], interaction with atoms [19–21], and interference with another photon state [22–26]. In the interference process, besides the phase modulation, the indistinguishability of photons is also a key parameter. In principle, the indistinguishability of photons will induce photon bunching and stimulated emission [25,26]. It has been the basis of multiphoton interference [27,28] for scalable optical quantum information techniques, lasers, and stimulated emission depletion microscopy [29].

Experimentally, the photon statistics can be evaluated with the Hanbury-Brown–Twiss (HBT) interferometer [30] to get the second-order degree of coherence [1], $g^{(2)}(0)$. The values of $g^{(2)}(0)$ demonstrate different photon statistical behaviors. A coherent light source [1] with a Poissonian distribution of photon numbers has a $g^{(2)}(0)$ of 1. For a classical optical field, $g^{(2)}(0) \geqslant 1$. For example, a thermal state shows $g^{(2)}(0) = 2$, demonstrating a photon bunching behavior. However, with a photon antibunching behavior, $g^{(2)}(0) < 1$, it is a typical quantum optical field, such as a perfect single-photon source with $g^{(2)}(0) = 0$. For a nonclassical N-photon number state, $g^{(2)}(0) = (N-1)/N < 1$. However, it is much more complicated for a photon state composed of different photon number states where the photon-indistinguishability-induced bunching factor will modify the photon statistics. For an N-photon state, when they are indistinguishable, the photon bunching effect will show an $N!$ coefficient because of the permutation symmetry of the boson system [31]. For partial indistinguishable cases, the photon bunching coefficient will drop. For total distinguishable cases, there is no bunching effect. Therefore the indistinguishability-induced bunching factor will modify the amplitude of each N-photon state and significantly change the photon statistical behavior. In this work, we studied the photon statistics by changing the indistinguishability of photons based on the measurement of $g^{(2)}(0)$. With a photon interference process [22,23,25], we experimentally demonstrated that the photon statistics can be changed from bunching behavior [$g^{(2)}(0) > 1$] to antibunching behavior [$g^{(2)}(0) < 1$] by modifying the indistinguishability of photons from 0.86 to 0, realizing the transition from a classical optical field to a nonclassical optical field. This study will help us to understand and control the photon statistics with a method for quantum optical coherence and quantum information applications.

In the study of photon indistinguishability and statistics, we consider the interference of a single-photon state and a weak coherent state. Theoretically, the single-photon state should be $|1\rangle$. However, in a practical case with imperfect photon coupling and detection, the single-photon state with $g^{(2)}(0) = 0$ can be written as $\rho_s = (1-\eta)|0\rangle\langle 0| + \eta|1\rangle\langle 1|$, where $|0\rangle$ is the vacuum state and η is the mean photon number, and the weak coherent state with a mean photon number of $|\alpha|^2(\ll 1)$ is $|\alpha\rangle$ with $g^{(2)}(0) = 1$. Then the single-photon state and the weak coherent state is mixed with a mixing ratio of $r = |\alpha|^2/\eta$. The indistinguishability (K) [32] corresponds to the overlapping of the two-photon states. When the photons from these two sources are totally distinguishable ($K = 0$), the mixed photon state is

$$\begin{aligned}\rho_{k=0} &= \rho_s \otimes |\alpha\rangle\langle\alpha| \\ &= (1-\eta)|0\rangle\langle 0| \otimes |\alpha\rangle\langle\alpha| + \eta|1\rangle\langle 1| \otimes |\alpha\rangle\langle\alpha|. \quad (1)\end{aligned}$$

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The $g^{(2)}(0)$ of the mixed photon state is

$$g^{(2)}(0) = \frac{|\alpha|^2(2\eta + |\alpha|^2)}{(\eta + |\alpha|^2)^2} = \frac{r^2 + 2r}{(1+r)^2}. \quad (2)$$

For a classical mixing ($K = 0$) of the two-photon states with $g^{(2)}(0) = 1$ and $g^{(2)}(0) = 0$, we can always find that $g^{(2)}(0) < 1$, demonstrating a photon antibunching behavior. However, when the photons from these two sources are totally overlapping and indistinguishable ($K = 1$), the photon indistinguishability induces quite different result. When $K = 1$, the state of the mixed photons can be written as

$$\rho_{k=1} = \rho_s \otimes |\alpha\rangle\langle\alpha| = C[(1-\eta)|\alpha\rangle\langle\alpha| + \eta|\alpha'\rangle\langle\alpha'|], \quad (3)$$

where $|\alpha'\rangle = a^\dagger|\alpha\rangle$ and C is a normalization number. Here, the single-photon added coherent state $a^\dagger|\alpha\rangle$ [33] shows different amplitude with $|1\rangle\langle 1| \otimes |\alpha\rangle\langle\alpha|$ in Eq. (1) because of the photon-indistinguishability-induced bunching factor. Similarly, the value of $g^{(2)}(0)$ is

$$g^{(2)}(0) = \frac{(|\alpha|^4 + 4\eta|\alpha|^2 + 4\eta|\alpha|^4 + \eta|\alpha|^6)(1 + \eta|\alpha|^2)}{(\eta + |\alpha|^2 + 2\eta|\alpha|^2 + \eta|\alpha|^4)^2}. \quad (4)$$

When the mean photon numbers of both sources are much smaller than 1 ($\eta, |\alpha|^2 \ll 1$), Eq. (4) can be simplified as

$$g^{(2)}(0) = \frac{|\alpha|^4 + 4\eta|\alpha|^2}{(\eta + |\alpha|^2)^2} = \frac{r^2 + 4r}{(1+r)^2}. \quad (5)$$

It is easy to find that $g^{(2)}(0) > 1$ when $r > 0.5$. In these cases, the photon-indistinguishability-induced bunching effect significantly changes the photon statistics.

For partially indistinguishable cases with $0 < K < 1$, the mixed-photon state is

$$\rho_K \propto (1-\eta)|\alpha\rangle\langle\alpha| + \frac{\eta}{1+|\alpha|^2}|1\rangle\langle 1| + \frac{|\alpha|^2\eta(1+K)}{1+|\alpha|^2}|1,1'\rangle\langle 1,1'|\ldots, \quad (6)$$

where the coherent state is represented by number states with the higher-order terms dropped since $|\alpha|^2 \ll 1$. $|1,1'\rangle$ represents the state of two photons with partial indistinguishability with an amplitude enhancement of K over the distinguishable case. Also, the $g^{(2)}(0)$ of the mixed photon state can be deducted as

$$g^2(0) = \frac{r^2 + 2(1+K)r}{(1+r)^2}. \quad (7)$$

Here the photon statistics highly depends on the value of indistinguishability. Figure 1 shows the $g^{(2)}(0)$ of the mixed-photon state with different r and K. The transition from the photon antibunching behavior ($g^{(2)}(0) < 1$) to the bunching behavior ($g^{(2)}(0) > 1$) can be realized by increasing K with some r.

In the experimental demonstration, we applied a single-photon state heralded from spontaneous parametric down-conversion (SPDC). The coherent state was directly from the attenuated laser. As shown in Figure 2, the 780-nm pulsed laser beam was generated from a Ti:sapphire laser with a repetition frequency of 76 MHz and a pulse duration of 110 fs. A 390-nm laser was obtained through a second harmonic generation (SHG) process by a β-barium borate (BBO) crystal and served as the pump light for type-II SPDC. The parametric light was beamlike [34] and separated as the signal and idler beams. The signal single-photon state can be heralded by detecting the idler photon. Then, the single-photon state and the coherent state with orthogonal polarizations were mixed together by a polarized beam splitter. A Glan-Thompson prism was used as a polarizer to project the orthogonal polarized beams into a single polarization direction and remove the distinguishable polarization information. The polarizer also controlled the mixing ratio (r) with the rotation of the polarization direction. Three-nm-bandpass interference filters (IF) centered at 780 nm and single mode fibers were used to ensure the overlapping of both spatial and temporal modes for photon collection and interference.

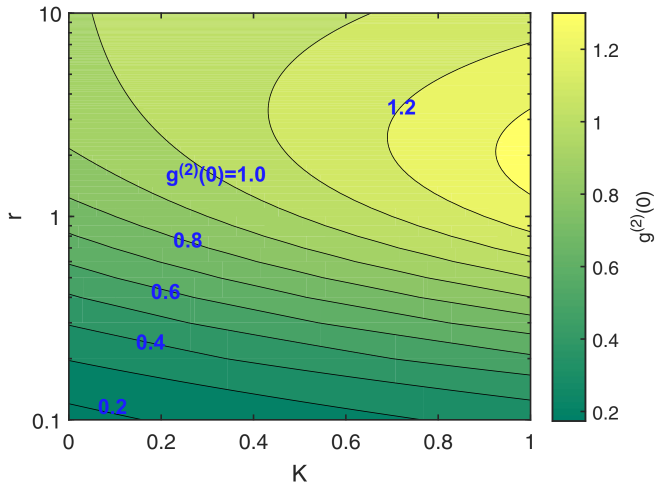

FIG. 1. $g^{(2)}(0)$ of the mixed-photon state versus mixing ratio r and photon indistinguishability K.

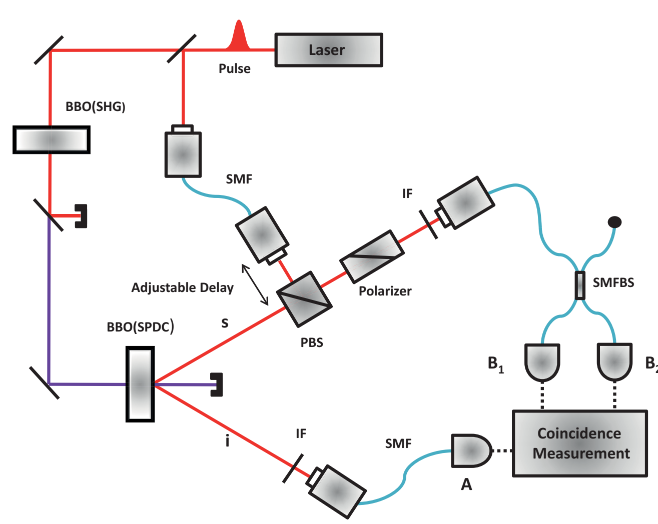

FIG. 2. Schematics for experimental setup to study the indistinguishability-induced photon statistical transition. SMF, single-mode fiber; IF, interference filter; PBS, polarization beam splitter; SMFBS, single-mode fiber beam splitter.

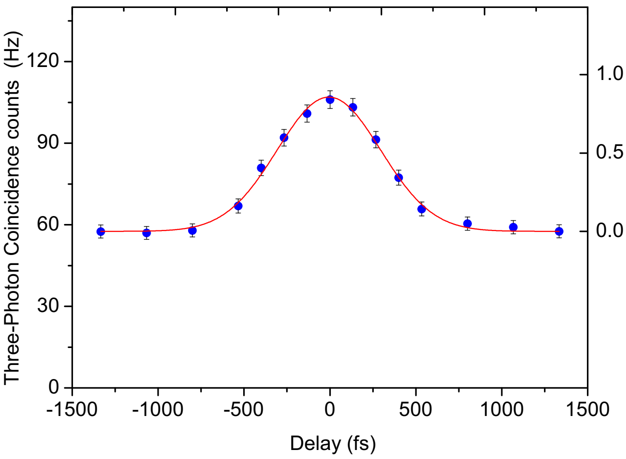

FIG. 3. Three-photon coincidence counts registered with different delays. Dots are the experimental data. The error bars are given based on the total coincidence counts. The solid line is the Gaussian fitting of the data.

When indistinguishable photons from the coherent state and the single-photon state arrive at the PBS simultaneously, the photon bunching effect will happen with more photon counts after the polarizer. Then, the indistinguishability of the coherent state and the single-photon state can be measured from the enhancement of three-photon coincidence due to the constructive interference [22,25]. By changing the relative delay between the two mixed-photon states, we can measure the three-photon coincidence at zero delay [$N(0)$] and the delay time much longer than the pulse duration [$N(\infty)$], where the two-photon states are well separated. Therefore the value of indistinguishability is

$$K = \frac{N(0)}{N(\infty)} - 1 = 0.86 \pm 0.02. \tag{8}$$

The reason of $K < 1$ may come from the imperfect overlapping of the spatial and frequency modes and the distinguishability of the single-photon state from SPDC [32]. Such a value can be further enhanced by narrower interference filters. Also, by changing the delay time to control the temporal overlapping between the two photon states, we were able to modify K in a simple way with $K(\tau) = N(\tau)/N(\infty) - 1$, where $K(\tau)$ and $N(\tau)$ represent the indistinguishability and the three-photon counts at the delay of τ. The experimental result is shown in Fig. 3. We can apply Gaussian distribution to fit the data as

$$K(\tau) = K \exp\left[-\left(\frac{\tau}{\tau_0}\right)^2\right], \tag{9}$$

where $\tau_0 = 425.1 \pm 11.6$ fs, which is determined by the duration of the pump pulse, the bandwidth of the interference filter and the properties of the SPDC process in the BBO crystal, such as the thickness and the phase-matching condition [32,35]. When $\tau \gg \tau_0$, the two-photon state was temporally well separated, $K(\tau) \to 0$. Therefore we can modify the value of indistinguishability from 0 to K to study the photon indistinguishability and statistics.

The $g^{(2)}(0)$ of the mixed-photon state can be measured with the HBT interferometer, as shown in Fig. 2. Since the single-photon state was heralded by detecting the idler state, the $g^{(2)}(0)$ of the mixed photon state at the signal path can be obtained as

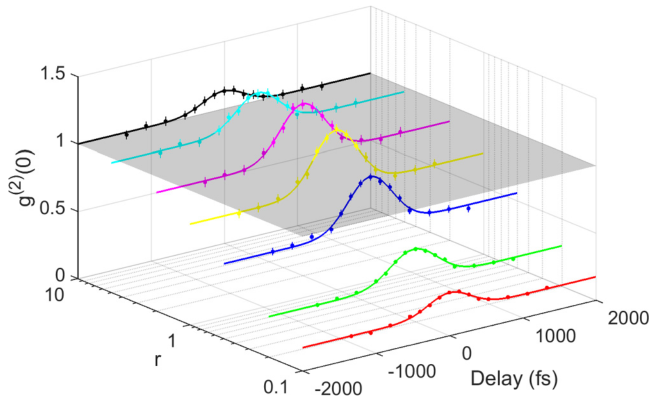

FIG. 4. The experimental values of $g^{(2)}(0)$ of the mixed-photon state with mixing ratio r. Each colored curve is the Gaussian fitting of the same color dotted data.

$$g^{(2)}(0) = \frac{N_{A,B_1,B_2}}{(N_{A,B_1}N_{A,B_2}/N_A^2)N_A} = \frac{N_{A,B_1,B_2}N_A}{N_{A,B_1}N_{A,B_2}}, \tag{10}$$

where N_A, $N_{A,B_1(B_2)}$, and N_{A,B_1,B_2} represent the single-photon counts of detector A, the two-photon coincidence counts of detectors A and $B_1(B_2)$, and the three-photon coincidence counts of detectors A, B_1, and B_2, respectively. By changing the photon indistinguishability [$K(\tau)$] with relative delay time and mixing ratio (r) with the rotation of the polarization direction, we can modify the value of $g^{(2)}(0)$ and the photon statistical behavior. Figure 4 depicts the results of a series of measurement results. In each measurement, r was fixed, and the $g^{(2)}(0)$ of the mixed photon state was measured at different delays. The curves are the Gaussian fittings. Each of them represents the value of $g^{(2)}(0)$ as a function of K with a certain mixed-photon state. As the plane of $g^{(2)}(0) = 1$, which is the boundary between the classical field and the nonclassical field, is also presented in the figure, we can observe that some of the curves lie across this plane. The experimental result manifests that the indistinguishability is one of the main parameters of photon statistics and can lead to the transition from classical [$g^{(2)}(0) > 1$] to nonclassical [$g^{(2)}(0) < 1$] regions.

By converting the time delay (τ) to the photon indistinguishability (K) with Eq. (9), we can demonstrate the photon statistics behavior with different K and r. Figures 5(a) and 5(b) shows the value of $g^{(2)}(0)$ with fixed photon indistinguishability K and mixing ratio r, respectively. Each sequence of colored dots represents the measured value and the colored line is the theoretical result from Eq. (7). The nonclassical photon statistics with $g^{(2)}(0) < 1$ is shown in the gray area. In Fig. 5(a), when r is much smaller than 1, the single-photon state from SPDC dominates the photon statistics, demonstrating the nonclassical behavior. When r is much larger than 1, the coherent state dominates the photon statistics with $g^{(2)}(0) = 1$. For these two states, the values of $g^{(2)}(0)$ are never larger than 1. However, when they are mixed with some ratios, the photon-indistinguishability-induced bunching significantly changes the photon statistics, demonstrating the transition to a classical behavior with

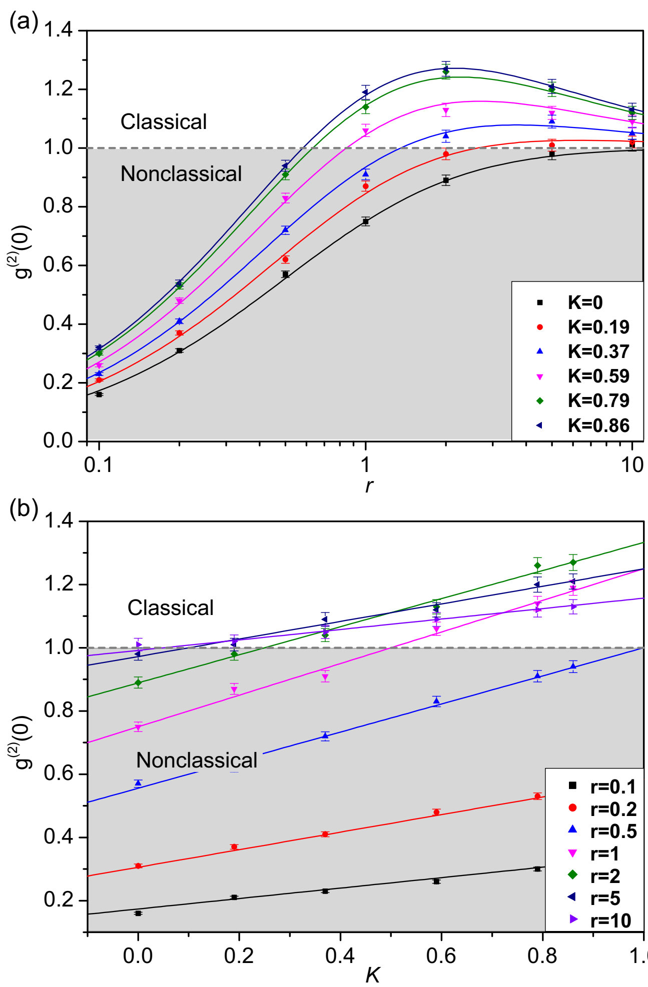

FIG. 5. The experimental value of $g^{(2)}(0)$ with fixed photon indistinguishability K (a) and mixing ratio r (b). Experimental results are denoted by color dots with error bars and solid color lines are fittings with Eq. (7).

$g^{(2)}(0) > 1$. Figure 5(b) clearly shows the contribution of the photon indistinguishability to the photon statistics. When the photons are totally distinguishable ($K = 0$), the $g^{(2)}(0)$ of the mixed state is always less than 1. However, when K increases, the value of $g^{(2)}(0)$ increases to be larger than 1 and a transition from the nonclassical to the classical field happens when $r > 0.5$.

In conclusion, we have studied the photon-indistinguishability-induced bunching effect to modify the photon statistics. In photon interference, the photon indistinguishability is changed by the temporal overlapping of the single-photon state and the weak coherent state. The transition from a classical to a nonclassical photon statistical behavior was experimentally demonstrated by changing the indistinguishability of photons. It provides a method to manipulate the photon statistics for the study of quantum coherence. Besides the quantum phase, the study also indicates that the photon indistinguishability is a key parameter for quantum coherence, especially for multipartite quantum coherence.

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APPENDIX: THE CALCULATION OF $g^2(0)$

In this Appendix, we show the calculation of $g^2(0)$. The coherent state is described as

$$|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_{n'=0}^{\infty}\frac{\alpha^{n'}}{(n'!)^{1/2}}|n'\rangle. \tag{A1}$$

To distinguish the photons from the single-photon state and the coherent state, here the number state from the coherent state is written as $|n'\rangle$. Therefore, the single-photon added coherent state can be described as in Eq. (6) with $e^{-|\alpha|^2} \approx 1/(1+|\alpha|^2) \approx 1$ when $|\alpha|^2 \ll 1$. The amplitude $1+K$ is proportional to the two-photon counts of $|1,1'\rangle\langle 1,1'|$ [32]:

$$\begin{aligned} C_2 &= \langle 1,1'|a^\dagger a^\dagger aa|1,1'\rangle \\ &= 2 + 2\mathrm{Tr}(|1\rangle\langle 1||1'\rangle\langle 1'|) \\ &= 2(1+K). \end{aligned} \tag{A2}$$

If $K = 1$ with $|1\rangle \equiv |1'\rangle$, the indistinguishable two-photon state is $2|2\rangle\langle 2|$, showing an indistinguishability-induced perfect photon bunching effect with $(a^\dagger)^2|0\rangle = \sqrt{2}|2\rangle$. However, if $|1\rangle \perp |1'\rangle$, $K = 0$, $a^\dagger a'^\dagger|0\rangle = |1\rangle \otimes |1'\rangle$, the two-photon state is $|1\rangle\langle 1| \otimes |1'\rangle\langle 1'|$. For partially distinguishable cases with $0 < K < 1$, the enhancement of K in two-photon counts describes an imperfect two-photon bunching effect [32].

When $|\alpha|^2 \ll 1$, we can omit high-order terms in the calculation of the second correlation function. Based on Eq. (A2), we get

$$\begin{aligned} g^{(2)}(0) &= \frac{\langle a^\dagger a^\dagger aa\rangle}{\langle a^\dagger a\rangle^2} \\ &= \frac{\mathrm{Tr}(a^\dagger a^\dagger aa\rho_K)}{\mathrm{Tr}(a^\dagger a\rho_K)^2} \\ &= \frac{(1-\eta)\alpha^4 + 2\eta(1+K)\alpha^2}{[(1-\eta)\alpha+\eta]^2} \\ &= \frac{(1-\eta)r^2 + 2(1+K)r}{[(1-\eta)r+1]^2}, \end{aligned} \tag{A3}$$

where $r \equiv \alpha^2/\eta$.

Under the assumption that $\eta \ll 1$ and $1-\eta \approx 1$, Eq. (A3) can be simplified as Eq. (7):

$$g^2(0) = \frac{r^2 + 2(1+K)r}{(1+r)^2}. \tag{A4}$$

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