Energy shift between two relativistic laser pulses copropagating in plasmas

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The interactive dynamics of two relativistic laser beams copropagating in underdense plasmas is studied using a coupled model equation for the relativistic laser propagation. It is shown that the relative phase difference between the two laser pulses plays a significant role in their interaction processes. When the relative phase varies, the two laser beams display different features, such as attraction, repulsion, and energy shift. In particular, energy flow from the phase-advanced beam to the spot domain of the phase-delayed beam is observed when the relative phase difference is between zero and π. When the relative phase is larger than $\pi/2$, repulsion is dominant and the interaction gradually becomes weak. When the relative phase difference is smaller than $\pi/2$, attraction becomes dominant and, as the phase difference decreases, the phase-advanced beam shifts most of its energy into the spot domain of the phase-delayed beam. These conclusions are verified by our three-dimensional particle-in-cell simulations. This provides an efficient way to manipulate the energy distribution of relativistically intense laser pulses in plasmas by adjusting their relative phase.

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I. INTRODUCTION

The interactive dynamics of laser beams copropagating in nonlinear media has stimulated great interest in a broad range of applications, including optical communications [1], long-range laser transport [2,3], energy transfer between lasers [4–6], laser-based particle acceleration [7,8], and inertial confinement fusion [9,10]. Recently, with the rapid development of laser technology, the interaction processes of laser pulses in a relativistic regime have attracted much attention. It is well known that in the nonrelativistic regime, when two laser beams propagate into nonlinear media, for example, in a Kerr media, whose refractive index is proportional to the laser intensity, the interference of the two laser pulses would induce the redistribution of the refractive index of the nonlinear media, leading to different interaction features [11–25] such as attraction, fusion, repulsion, and spiraling. In particular, the relative phase difference (ϕ) of the two copropagating laser beams can affect their interaction processes greatly. The influence of phase differences and individual powers of collapsing beams subject to nonlinear interaction was first analytically and numerically investigated by Bergé *et al.* [17]. It was shown that if the two laser beams are in phase ($\phi = 0$), they would collapse into a single beam under a critical distance, and if they are out of phase ($\phi = \pi$), they would repulse each other.

In the relativistic regime, similar interaction properties have also been found in plasmas [26–32]. Plasma can be regarded as a kind of nonlinear medium that is already in a stage of breakdown and in principle can tolerate ultrastrong laser fields. For a relativistically intense laser pulse propagating in the plasmas, the electron mass is corrected by the relativistic effect and the plasma density is modified by the laser ponderomotive force. These combined nonlinear effects induced by the relativistic laser pulse would modify the refractive index of the plasma. In particular, when two relativistic laser beams propagate in the plasmas, their interference leads to the redistribution of the refractive index of the plasma, and different interaction features have been observed in previous works. Dong *et al.* found that two laser beams in plasmas can merge into one beam or split into three beams in different cases [26]. Ren *et al.* observed the spiraling interaction of two laser beams with crossed polarization directions [27,28]. Mahdy studied the interaction process of two laser beams that are out of phase at different intensities and spot sizes [29]. However, so far, the effects of the relative phase difference (ϕ) between the two coherent laser beams have not been explored yet. Although $\phi = 0,\pi$ has been studied clearly, ϕ with arbitrary value has attracted little attention. In addition, it is noted that in previous works, a weak relativistic approximation ($a_0 < 1$) is considered, where a_0 refers to the normalized amplitude of the laser electric field. For the relativistic laser pulse with $a_0 > 1$, its ponderomotive force would lead to significant density depression along its propagation axis. In particular, when the power of each laser beam far exceeds the critical power for relativistic self-focusing, i.e., $P_{\mathrm{cr}} \approx 17.1(\omega_L/\omega_p)^2$ GW, where ω_L and ω_p refer to the laser and plasma frequency, respectively, these nonlinear effects cannot be neglected and the laser beams would first focus separately and then interact like solitons.

In this paper, we consider the interactive dynamics of two relativistically intense laser beams copropagating in underdense plasmas and study the effects of the relative phase difference (ϕ) between them. Based on the relativistic nonlinear Schrödinger equations (NLSEs) and three-dimensional (3D) particle-in-cell (PIC) simulations, it is found that when ϕ is between zero and π, an energy shift can be observed between the two laser beams. This phenomenon is different from the cases with $\phi = 0$ and $\phi = \pi$, because in this case interference

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of the two beams leads to the asymmetric distribution of the plasma refractive index and the laser energy selectively flows into the spot domain of the phase-delayed beam. Besides energy shift, attraction or repulsion is also found when $0 < \phi < \pi/2$ or $\pi/2 < \phi < \pi$, respectively. In addition, it is revealed that the energy-shift process corresponds to the redistribution of the energy of each laser beam: during the interaction the phase-delayed beam is affected less while the phase-advanced beam shifts most of its energy to the spot domain of the phase-delayed beam. It is suggested that it is possible to manipulate the interaction process of two relativistically intense laser beams by adjusting their relative phase.

The paper is organized as follows. In Sec. II, the physical model is discussed and theoretical analyses based on NLSEs are presented. In Sec. III, numerical simulations based on NLSEs are also presented. In Sec. IV, 3D PIC simulations are further employed to verify the conclusions that are obtained in Secs. II and III. Then a brief summary is given in the final section.

II. THEORETICAL ANALYSES BASED ON NLSE

The interactions of two copropagating laser beams can be described by two coupled relativistic NLSEs. In the slowly varying envelope approximation, the envelope evolution of a circularly polarized laser beam propagating through an underdense plasma can be written as [33,34]

$$2ik_L\frac{\partial}{\partial z}a_{1,2}+\nabla_\perp^2 a_{1,2}+k_p^2\left(1-\frac{n}{\gamma}\right)a_{1,2}=0 \qquad (1)$$

in the moving frame of the laser group velocity ($\tau = t - z/v_g$), where 1 and 2 represent the two laser beams, $k_L = \omega_L/c$ is the wave vector of the laser beams, ω_L is the laser frequency, $k_p = \omega_p/c$, $\omega_p = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$ is the plasma frequency, n_e is the plasma density, ϵ_0 is the vacuum permittivity, m_e is the electron mass, c is the vacuum speed of light, and z is the laser propagation direction. Here a is the slowly varying vector potential normalized as $a = eE/m_e\omega_L c$. The operator $\nabla_\perp^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the Laplace operator in the (x,y) plane, $n = n_e/n_0$ is the normalized electron density, n_0 is the background electron density, and $\gamma = \sqrt{1+|a_1+a_2|^2}$ is the Lorentz factor, which is coupled with the interference term of the two laser beams. When the relativistic laser pulse propagates into the plasmas, the laser ponderomotive force expels the electrons from the laser region and the plasma density profile will be strongly modified. On the slow time-space scales, by considering the balance between the laser ponderomotive force and the electrostatic force, the electron density can be given as $n = 1 + k_p^{-2}\nabla_\perp^2\gamma$, which is valid for $1 + k_p^{-2}\nabla_\perp^2\gamma > 0$. In order to avoid the nonphysical value of the plasma density [33], the expression of n can be rewritten as $n = \max\{0, 1 + k_p^{-2}\nabla_\perp^2\gamma\}$.

It is impossible to get exact analytical solutions for Eq. (1) because of the nonlinearity; however, one can use the global invariant Hamiltonian to elucidate some characteristics of the interaction process of two intense laser beams in plasmas. For Eq. (1), if one assumes there is no electron cavitation initially, i.e., $1 + k_p^{-2}\nabla_\perp^2\gamma > 0$, then the global Hamiltonian can be expressed as

$$H=\int\left[|\nabla_\perp(a_1+a_2)|^2-k_p^2(\gamma-1)^2-|\nabla_\perp\gamma|^2\right]dx\,dy, \qquad (2)$$

where the last two terms respectively result from the relativistic effect and the charge displacement driven by the laser ponderomotive force. In order to qualitatively describe the interaction process of two laser beams, one can define the overall mean-square radius as $\langle r^2\rangle \equiv \int (x^2+y^2)|a_1+a_2|^2\,dx\,dy/P$, where $P = \int(|a_1|^2+|a_2|^2)\,dx\,dy$ refers to the total input laser power. The evolution of $\langle r^2\rangle$ can be given by [35]

$$\frac{d^2\langle r^2\rangle}{dz^2}=\frac{2}{k_L^2P}\left[H+k_p^2\int\left(1-\frac{1}{\gamma}\right)(\gamma-1)^2\,dx\,dy+\int\left(\gamma-\frac{1}{\gamma}\right)(\nabla_\perp^2\gamma)\,dx\,dy\right]. \qquad (3)$$

It is noted that for the nonrelativistic case, i.e., $a_1,a_2 \ll 1$, Eq. (3) reduces to $\frac{d^2\langle r^2\rangle}{dz^2} = \frac{2H}{k_L^2P}$ with $H = \int[|\nabla_\perp(a_1+a_2)|^2 - \frac{k_p^2}{4}|a_1+a_2|^4]\,dx\,dy$, which is analogous to the case in a Kerr-type medium [17]. However, in the relativistic regime considered here, the above equation cannot be analytically solved because of the nonlinearity induced by the relativistic effect and plasma density depression. For two laser beams propagating in the plasmas, the expression of $\langle r^2\rangle$ can be written as $\langle r^2\rangle = \langle r_1^2\rangle + \langle r_2^2\rangle + \langle r_{12}^2\rangle$, where $\langle r_1^2\rangle = \int(x^2+y^2)|a_1|^2\,dx\,dy/P$, $\langle r_2^2\rangle = \int(x^2+y^2)|a_2|^2\,dx\,dy/P$, and $\langle r_{12}^2\rangle = \int(x^2+y^2)(a_1a_2^*+a_1^*a_2)\,dx\,dy/P$. Here a_1^* and a_2^* respectively correspond to the complex conjugates of a_1 and a_2. In this case, the evolution of $\langle r^2\rangle$ can be separated into three parts, $\frac{d^2\langle r^2\rangle}{dz^2} = \frac{d^2\langle r_1^2\rangle}{dz^2} + \frac{d^2\langle r_2^2\rangle}{dz^2} + \frac{d^2\langle r_{12}^2\rangle}{dz^2}$, where the first two terms describe the evolution process of each laser beam and the last term describes the interaction process of the two laser beams. In particular, when the separation distance d approaches infinity, then the last term is neglected and no interaction process occurs between the two laser beams. Thus, one can further get

$$\frac{d^2\langle r_{12}^2\rangle}{dz^2} = F - F|_{d\to\infty}, \qquad (4)$$

where F denotes the right-hand side of Eq. (3) and $F|_{d\to\infty}$ represents the value of the right-hand side of Eq. (3) when the separation distance of the two laser beams approaches infinity. From Eq. (4), one can get some characteristics of the interaction process of two relativistic laser beams in plasmas. When the value of the right-hand side of Eq. (4) is negative, then the two laser beams attract each other, and the two laser beams would repulse each other when the value of the right-hand side of Eq. (4) becomes positive.

Figure 1 shows the dependence of ΔF on different parameters for two parallel laser beams in plasmas, where $\Delta F = F - F|_{d\to\infty}$. It is shown from Fig. 1(a) that the absolute value of ΔF is reduced when the separation distance of the two laser beams is increased, suggesting that the interaction between the two laser beams gradually becomes weak. Figure 1(b) shows that for a relatively small separation distance, the value of ΔF depends strongly on the relative phase of the two beams and it

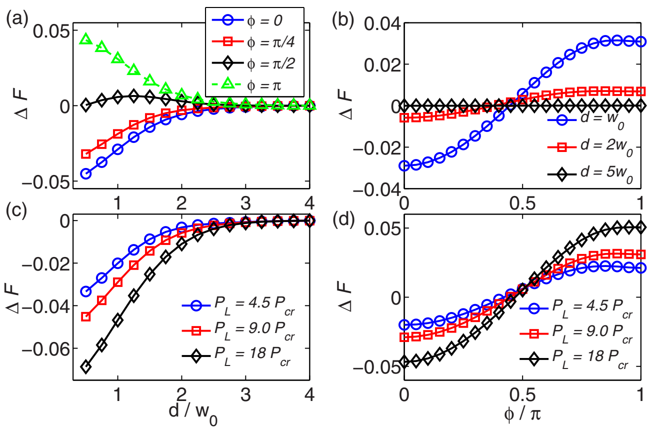

FIG. 1. The dependence of ΔF on different relative phases, separation distances, and input powers for the two laser beams, where $\Delta F = F - F|_{d\to\infty}$. (a) The dependence of ΔF on different separation distances for different relative phases, where $P_L = 9P_{\rm cr}$ is fixed. (b) The dependence of ΔF on different relative phases for different separation distances, where $P_L = 9P_{\rm cr}$ is fixed. (c) The dependence of ΔF on different separation distances for different input laser powers, where $\phi = 0$ is fixed. (d) The dependence of ΔF on different relative phases for different input laser powers, where $d = w_0$ is fixed. In the calculations, two relativistically intense laser beams are employed with the initial beam profiles of $a_1 = a_{10}\exp\{-[(x-d/2)^2 + y^2]/w_0^2\}$ and $a_2 = a_{20}\exp\{-[(x+d/2)^2 + y^2]/w_0^2\}\exp(i\phi)$, where d is the separation distance, w_0 is the beam radius, and ϕ is the relative phase. The value of $F|_{d\to\infty}$ is calculated at a large distance of $d = 10w_0$.

even changes signs when the relative phase is approximately larger than $\pi/2$. This indicates that the two laser beams would attract each other for a small phase difference (at least $< \pi/2$) and, in turn, would repulse each other for $\pi \geqslant \phi > \pi/2$. However, when the separation distance is large enough, the value of ΔF approaches zero and is independent of the relative phase of the two laser beams, as indicated by the black curve in Fig. 1(b). In this case, the well-separated two laser beams develop individually in plasmas without mutual interaction. In addition, it is noted from Fig. 1(a) the repulsion dominates the interaction process for $\phi = \pi/2$. Figures 1(c) and 1(d) show that the interaction process becomes stronger for a larger laser input power. For relativistically intense laser pulses with $P_L > P_{\rm cr}$ considered here, the condition between the separation distance and the input power cannot be analytically derived because of the strong nonlinearity. However, it can be seen from Figs. 1(a) and 1(c) that the separation distance impacts the interaction strength, but it does not affect the interaction features (repulsion vs attraction), which is only determined by the relative phase difference of the two laser beams.

III. NUMERICAL SIMULATIONS BASED ON NLSE

In order to demonstrate the evolution process of two relativistically intense laser beams in plasmas, the coupled equations (1) are solved numerically using a spectral split-step scheme [36]. The simulation box is set to be $40 \times 40\,\mu\text{m}^2$ in the x-y plane. In the simulations, two relativistically intense laser beams are employed with the initial beam profiles of $a_1 = a_{10}\exp\{-[(x-d/2)^2 + y^2]/w_0^2\}$ and $a_2 = a_{20}\exp\{-[(x+d/2)^2 + y^2]/w_0^2\}\exp(i\phi)$, where $a_{10} = a_{20} = 3.0$ represents the amplitude of the vector potential of the two beams, the separation distance is $d = 5\,\mu$m, and the transverse beam waist is chosen to be $w_0 = 2\,\mu$m. The laser wavelength is set as $\lambda_0 = 1\,\mu$m and plasma density is chosen to be $n_0 = 0.1n_c$, where n_c refers to the critical plasma density. For these parameters, the peak power of each beam corresponds to $P_L \approx 9P_{\rm cr}$. The phase difference ϕ is set to be $k\pi$, where k is between zero and 1. Here a_2 is regarded as the phase-advanced beam and a_1 is regarded as the phase-delayed beam.

Figure 2 shows the spatiotemporal intensity distribution for different values of ϕ. It is shown from Figs. 2(a) and 2(b) that the attraction is dominant for the interaction processes when the two laser beams are in phase ($\phi = 0$) and repulsion

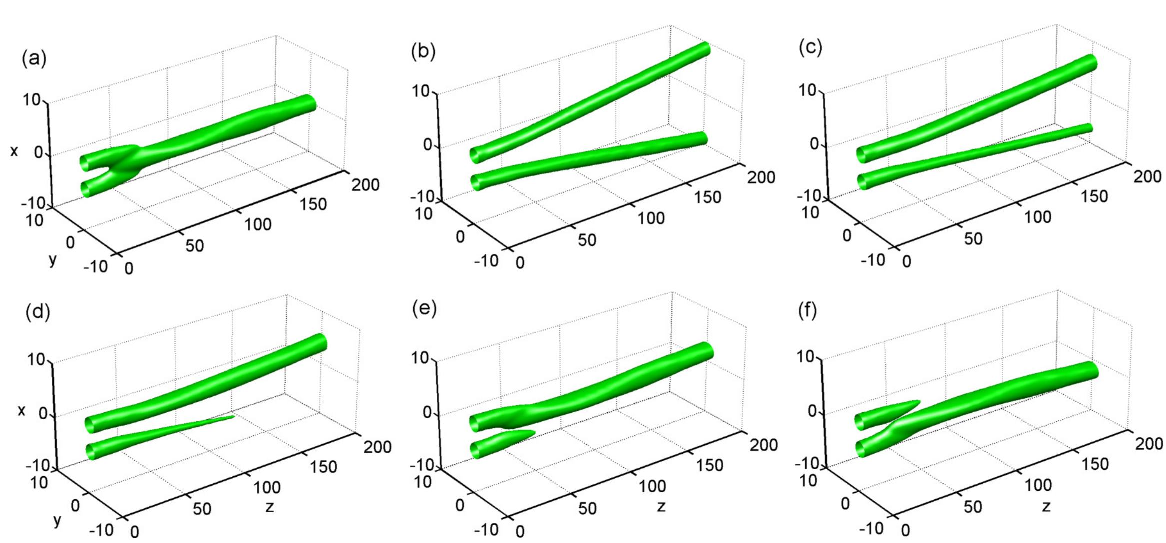

FIG. 2. The spatiotemporal intensity distribution of two interacting beams with different phase differences. (a)–(f) Simulation results for $\phi = 0$, π, $\pi/2$, $\pi/3$, $\pi/10$, and $-\pi/10$, respectively. The isovalue is set to be 1.0×10^{19} W/cm^2 in these simulations. The coordinate units are μm.

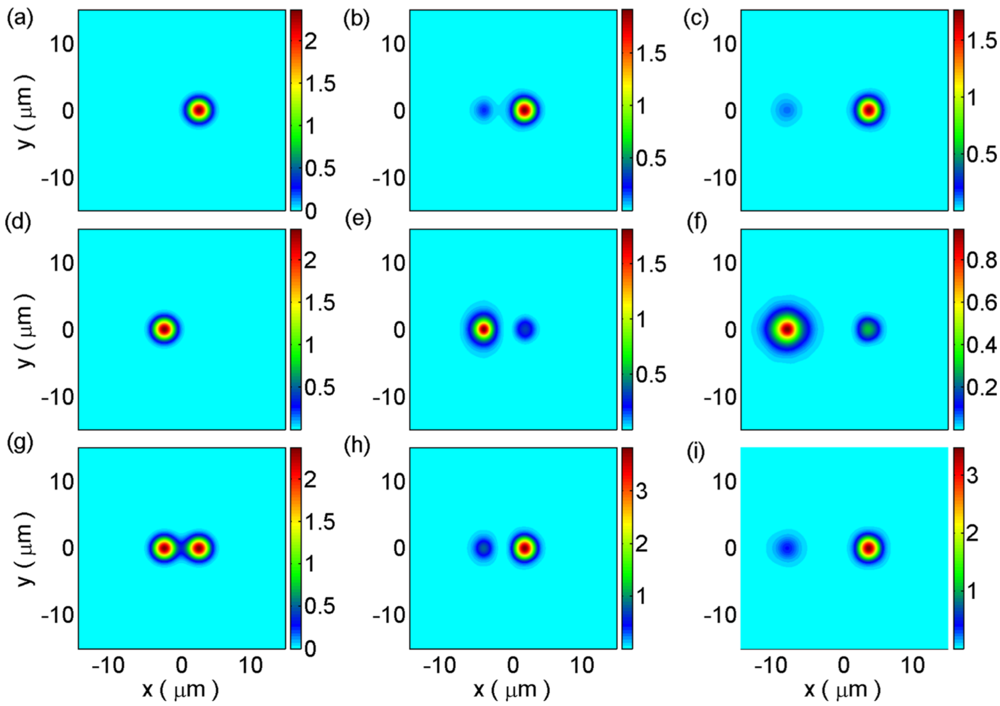

FIG. 3. The transverse intensity distribution of beam 1 ($I_1 \propto |a_1|^2$), beam 2 ($I_2 \propto |a_2|^2$), and the two coherent beams ($I \propto |a_1 + a_2|^2$) at different propagation distances z for $\phi = \pi/4$. (a)–(c) Results corresponding to beam 1, (d)–(f) results corresponding to beam 2, and (g)–(i) results for the two coherent beams. Simulation results at (a), (d), (g) $z = 0\ \mu$m; (b), (e), (h) $z = 100\ \mu$m; and (c), (f), (i) $z = 200\ \mu$m. Here the unit of laser intensity is normalized by 1.0×10^{19} W/cm^2.

is dominant when the two beams are out of phase ($\phi = \pi$). However, for $\phi = \pi/2$, the repulsion is still dominant but the intensity evolution of the two beams is asymmetric, as indicated in Fig. 2(c). This is in good agreement with the theoretical predications in the above section. For a smaller phase difference with $\phi = \pi/3$, the asymmetry becomes more obvious and the phase-advanced beam seems to lose energy faster than the case of $\phi = \pi/2$, as shown in Fig. 2(d). In the NLSE system, it is noted that the power of each beam is conserved, i.e., $\partial P_{1,2}/\partial z = 0$, where $P_{1,2} = \int |a_{1,2}|^2\, dx\, dy$. Thus the asymmetric intensity distribution indicates the energy-shift process in these cases. In addition, as the energy shift continues, the two laser beams begin to repulse each other after an initial attraction process. In this case, the interaction process becomes weak and the energy shift becomes saturated. If ϕ is close to zero, for example, $\phi = \pi/10$, as shown in Fig. 2(e), more energy flows from the phase-advanced beam into the spot domain of the phase-delayed beam and the spot location of phase-delayed beam moves towards the center of the two beams, which indicates the dominant attraction process for a relatively small value of ϕ. When ϕ is negative, as shown in Fig. 2(f), beam 1 in the upper side becomes the phase-advanced beam, and the evolution process is reversed. The cases for $\pi/2 < |\phi| < \pi$ are also considered in our simulations (not shown here) and it is shown that the interaction processes in these cases are nearly the same with the case of $\phi = \pi/2$, where the repulsion dominates the interaction process, as predicted by the theoretical analyses. Once the repulsion process begins, the interaction force gradually becomes weak and can be neglected when the two laser beams are separated far enough. In addition, we have also considered the cases in which two laser beams have a common phase term but the phase difference is kept the same as that in Fig. 2; it is found that the simulation results make no difference. This can be understood in that the coupled equations are unchanged if the two vector potentials $a_{1,2}$ have a common phase term with $\exp(i\phi_0)$. Thus one can conclude that the interaction processes of the two laser beams in plasmas are only governed by their relative phase difference rather than the phases of each beam.

In order to investigate the mechanism of the energy-shift process, the evolution of the intensity profile of each beam is analyzed. Figure 3 gives the corresponding transverse intensity profiles of the two laser beams for $\phi = \pi/4$. It is shown that at the initial stage, their spots are located at $x = \pm 2.5\ \mu$m. When the interaction occurs, each beam would split into two parts: one stays at its original position and keeps on losing energy, while the split part is gradually transferred into the spot domain of the other beam. For the phase-delayed beam (beam 1), its profile changes little, as shown in Figs. 3(a)–3(c), while the peak intensity of the phase-advanced beam (beam 2) is decreased by a factor of 2, as indicated by Fig. 3(f). These results can be understood from the distribution of the refractive index of the plasma that is modified by the two relativistic laser beams. The plasma refractive index can be described as $\eta = \sqrt{1 - \frac{n_e}{\gamma n_c}}$, where $\gamma = \sqrt{1 + |a_1 + a_2|^2}$ is coupled with the interference term of the two laser beams. When the two laser beams propagate into the plasmas, the electrons in the spot domain of each beam would be expelled radially because of the laser ponderomotive force, and the gradient of η is negative in the regions of the two laser beams; thus each beam will deposit some energy

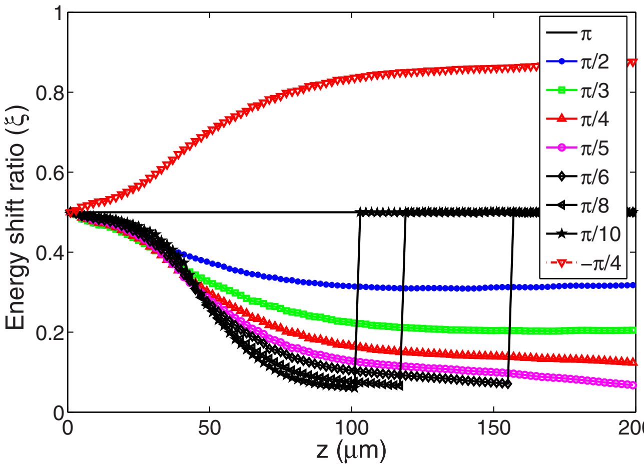

FIG. 4. The energy-shift ratio ξ vs the propagation distance z for different values of ϕ.

into the spot region of the other beam. In particular, the distribution of η is strongly dependent on the relative phase difference of the two laser beams. The Lorentz factor in the expression of η can be written as $\gamma = \sqrt{1+|a_1+a_2|^2} = \sqrt{1+|a_1|^2+|a_2|^2+2|a_1||a_2|\cos(\Delta kx-\phi)}$. Here Δk refers to the difference value of the transverse wave vectors of the two laser beams, which is induced by an attraction or repulsion interaction process. It is noted that for $\phi = 0$ or π, the distribution of η is symmetric along the x direction and no energy shift occurs in this case, whereas for $\phi \neq 0$ and π, the distribution of η becomes asymmetric, which would lead to the asymmetric energy flows from the two laser beams. Once the asymmetric shift process is triggered, this kind of asymmetry will be further intensified. As a result, the energy of the two beams would be redistributed, as shown in Fig. 3(i). In addition, it is noted from Figs. 3(g)–3(i) that at the beginning, attraction between the two beams is obvious while at a longer propagation distance $z = 200$ μm, repulsion becomes dominant. In this case, the interaction process becomes weak and the energy-shift process becomes saturated.

Figure 4 shows the energy-shift ratio ξ as a function of the propagation distance z for different values of ϕ. Here, ξ is defined as $\xi = P_2/(P_1+P_2)$, where $P_1 = \int_{|x-x_1|<w_0} |a_1+a_2|^2\,dr$ is the power in the spot domain of beam 1 (phase delayed), $P_2 = \int_{|x-x_2|<w_0} |a_1+a_2|^2\,dr$ is the power in the spot domain of beam 2 (phase advanced), x_1 and x_2 refer to the positions of the two beams, respectively, which correspond to the maximum intensity of each beam. It is supposed that the main spot sizes of beam 1 and beam 2 change little during the interaction. This is reasonable because it is known that the matched spot size of the light bullet in underdense plasma is about $r_0 \approx 2c/\omega_{pe}$ [36], and in our simulations, $w_0 \approx r_0$. Thus one can employ the value of ξ to describe the energy-shift ratio and characterize the interaction strength of the two laser beams. For $\phi = \pi$ or zero, the two laser beams transport symmetrically and no energy shift is observed; in this case, the value of ξ is strictly equal to 0.5, as shown in Fig. 4. For other cases with $\phi \leqslant \pi/2$, as the two laser beams propagate, the value of ξ is decreased at the beginning and then gradually becomes saturated, indicating that the interaction process becomes weak. In addition, it is noted that for $0 < \phi \leqslant \pi/2$, ξ is much decreased when the phase difference gets smaller. This implies that more energy is shifted from the phase-advanced beam into the phase-delayed beam and the interaction process becomes stronger when the phase difference is decreased. In particular, for $\phi < \pi/5$, the energy-shift process becomes rather strong and the curves in Fig. 4 change dramatically at a special distance z_d, for example, $z_d \approx 100$ μm for $\phi = \pi/10$. At this distance, most of the laser energy in the phase-advanced beam is shifted into the region of the phase-delayed beam; as a result, $P_1 = P_2$ and the value of ξ jumps to 0.5. For $\phi = \pi/10$, more than 90% of the laser energy is deposited into the region of the phase-delayed beam. In addition, in these cases with $\phi < \pi/5$, the value of z_d is decreased as the phase difference is decreased, suggesting a stronger and faster energy-shift process. Figure 4 also shows the case with negative phase difference with $\phi = -\pi/4$. It is shown that $\xi(\phi = -\pi/4) + \xi(\phi = \pi/4) = 1$. Similarly, one can get the relation with $\xi(\phi) + \xi(-\phi) = 1$. This indicates that the interaction processes of two relativistic laser beams in plasmas are phase reversible.

To demonstrate the robustness of the energy-shift process, the asymmetric cases where the two laser beams have different initial intensities are also considered in our simulations. In the simulations, the initial relative phase difference is set as $\pi/4$, and only the parameters of a_{10} and a_{20} are changed. Other parameters are the same as in Fig. 2. The corresponding simulation results are shown in Fig. 5. It is shown that the energy-shift process still occurs for different initial laser intensities. Even when the phase-advanced beam has a larger intensity initially, its energy can still be efficiently shifted into the region of the phase-delayed beam. The cases for $\phi = \pi/6$ are also considered and it is shown that the results are similar to the cases of $\phi = \pi/4$. For the cases with $\phi = \pi/2$, repulsion is dominant and the energy shift is not so obvious as indicated above. Thus one can conclude that the energy-shift process is rather robust for two relativistic laser beams propagating in plasmas, which is independent of their initial laser intensities and is mainly governed by the relative phase difference of the two laser beams.

IV. 3D PIC SIMULATIONS

In this section, a three-dimensional PIC simulation method is employed to investigate the energy-shift process discussed above using the EPOCH code [37]. In the simulations, two circularly polarized intense laser pulses with a relative phase difference are incident into a uniform underdense plasma from the left boundary. The simulation box is $2 \times 20 \times 80$ μm^3 with a grid of $200 \times 200 \times 800$ cells and eight particles for both electrons and ions per cell. The laser wavelength is set as $\lambda = 1$ μm and the durations of the two laser pulses are set as $20T_0$, where T_0 refers to the laser period. The initial transverse profiles of the two laser pulses are $a_1 = a_{10}\exp\{-[(x-d/2)^2+y^2]/w_0^2\}$ and $a_2 = a_{20}\exp\{-[(x+d/2)^2+y^2]/w_0^2\}\exp(i\phi)$, respectively, where $a_{10} = a_{20} = 10$, $d = 4$ μm, $w_0 = 2$ μm, and ϕ is the relative phase term. The plasma density is $n_0 = 0.3n_c$, where $n_c = 1.1 \times 10^{21}$ cm^{-3} is the critical plasma density. It is noted that compared with the simulation setup in Fig. 2, here two stronger laser pulses and larger plasma density

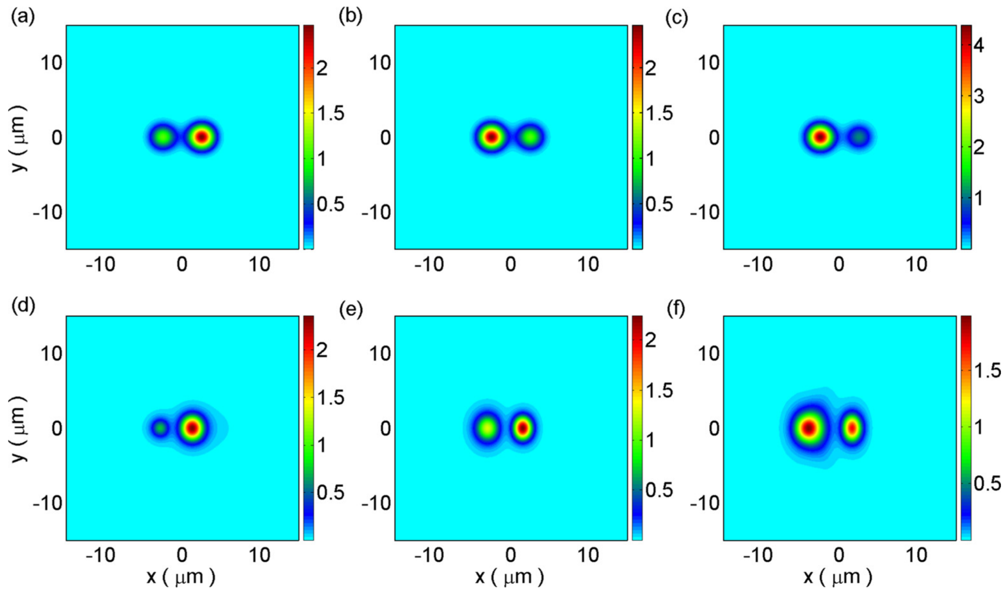

FIG. 5. The transverse intensity distribution of the two beams at different propagation distances z for $\phi = \pi/4$. Simulation results for (a), (d) $a_1 = 3, a_2 = 2$; (b), (e) $a_1 = 2, a_2 = 3$; and (c), (f) $a_1 = 2, a_2 = 4$. Simulation results at (a)–(c) $z = 0\ \mu$m and (d)–(f) $z = 60\ \mu$m. Here the laser intensity is in units of 1.0×10^{19} W/cm^2.

are considered in order to avoid the scattering instabilities in underdense plasmas [38].

Figures 6 and 7 show the corresponding simulation results for different values of ϕ. It is shown from Figs. 6(a) and 6(b) that when the two laser beams are in phase with $\phi = 0$, they would attract each other and then collapse into a single beam. In this process, the peak intensity is almost increased by a factor of 2. In particular, when the relative phase difference is in the range $0 < \phi < \pi$, the energy-shift process can be clearly observed, as indicated in Figs. 6(c)–6(f), which is in good agreement with the NLSE simulation results. In addition, for $\phi = \pi/2$, the repulsion process is dominant and the interaction gradually becomes weak, as shown in Fig. 6(f), whereas for $\phi = \pi/4$, attraction is dominant and the energy-shift process is much faster than the case with $\phi = \pi/2$, as shown in Fig. 6(d). The corresponding transverse intensity distribution in the x-y plane is shown in Fig. 7 for different values of ϕ. It is shown that when the two beams are out of phase ($\phi = \pi$), the repulsion process is dominant and no energy shift occurs, as shown in Fig. 7(b), whereas for the cases with $0 < \phi < \pi$, the laser energy is shifted from the phase-advanced beam into the phase-delayed beam and, in these cases, the peak laser intensity is much larger than the case with $\phi = \pi$. In addition, for $0 < \phi < \pi/2$, when the phase difference is decreased, the interaction process would become stronger and more energy can be shifted into the phase-delayed beam, as indicated from Figs. 7(d) and 7(e). The case with a negative phase difference $\phi = -\pi/4$ is also considered, as shown in Fig. 7(f). It can be seen that the results are reversible compared with the results in Fig. 7(e) except for some quantitative fluctuations. These conclusions agree well with the theoretical predications and the NLSE simulation results. The PIC simulations with two linearly polarized lasers are also conducted under the same parameters, and it is found that the energy-shift process is identical to the circularly polarized cases, which indicates that the energy-shift process is independent of the polarizations of the laser beams.

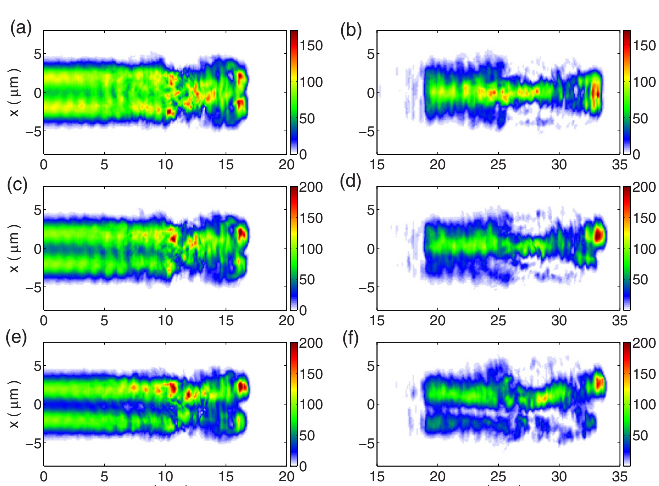

FIG. 6. The laser intensity distribution along the $y = 0$ plane for two copropagating laser beams with relative phase difference in 3D PIC simulations. Simulation results for $\phi = 0$ at (a) $T = 20T_0$ and (b) $T = 40T_0$; for $\phi = \pi/4$ at (c) $T = 20T_0$ and (d) $T = 40T_0$; and for $\phi = \pi/2$ at (e) $T = 20T_0$ and (f) $T = 40T_0$. The laser intensity is in units of 2.76×10^{18} W/cm^2. Here the beam in the upper side is referred to as the phase-delayed beam and the beam in the lower side is referred to as the phase-advanced beam.

Figure 8 shows the laser electric field distribution for the two laser pulses with different relative phases. The energy-shift process can also be observed in the electric field distribution, as shown in Figs. 8(a) and 8(d). It is shown that energy in the leading edge of the phase-advanced laser pulse is almost completely transferred into the phase-delayed pulse. During the energy-shift process, the strength of the phase-advanced

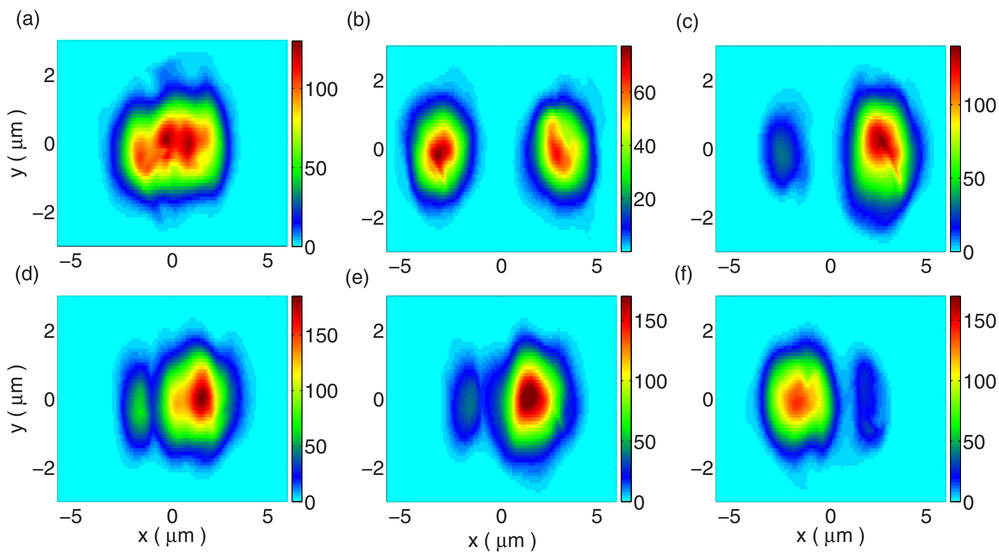

FIG. 7. The corresponding transverse laser intensity distribution along the plane of $z = 33\ \mu$m at $T = 40T_0$ for different values of ϕ. (a)–(f) Simulation results for $\phi = 0, \pi, \pi/2, \pi/6, \pi/4$, and $-\pi/4$, respectively. The laser intensity is in units of 2.76×10^{18} W/cm^2.

beam is gradually decreased. In this case, the phase velocity of the phase-advanced beam in plasmas is increased due to the weaker relativistic effect. This can be understood from the dependence of the phase velocity on the laser intensity, which can be written as $v_{\rm ph} = c/\eta = c/\sqrt{1 - \frac{n_e}{\gamma_L n_c}}$. Such an effect will induce the increase of the phase difference between the two laser pulses. Figures 8(b), 8(c), 8(e), and 8(f) show the longitudinal profiles of the laser electric field in a localized region at different interaction times. It can be seen from Figs. 8(b) and 8(e) at an early stage that the energy shift between the two laser pulses is negligible and the relative phase difference remains as the initial value. At a later stage, the energy shift becomes remarkable, as shown in Figs. 8(a) and 8(d). In this case, the phase velocity of the phase-advanced beam is larger than the phase-delayed beam; thus the relative phase difference between the two laser pulses is increased, as indicated in Figs. 8(c) and 8(f). In particular, for the initial phase difference with $\phi < \pi/2$, the increase of ϕ may induce different interaction features; as shown in Fig. 8(a), the repulsion becomes dominant at the later stage. Once the repulsion process begins, the interaction force becomes weak and can be neglected when the two laser beams are well separated. In this case, the energy shift gradually becomes saturated.

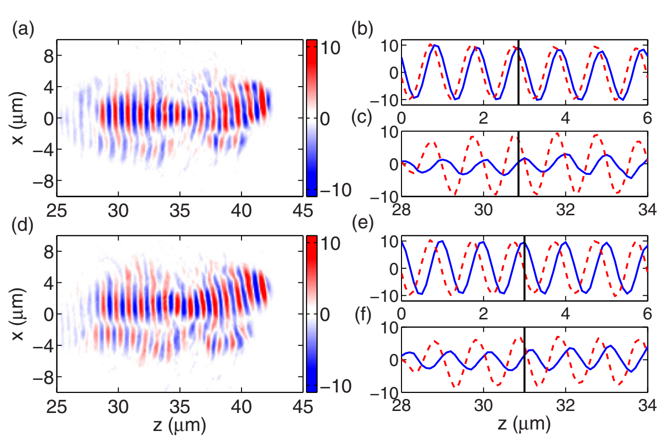

FIG. 8. The normalized laser electric field ($eE_y/m_e\omega_L c$) distribution along the plane $y = 0\ \mu$m at $T = 50T_0$ for (a)–(c) $\phi = \pi/4$ and (d)–(f) $\phi = \pi/2$. Localized laser electric field distribution along the line $x = \pm 2\ \mu$m at (b) $T = 20T_0$ for $\phi = \pi/4$, (c) $T = 50T_0$ for $\phi = \pi/4$, (e) $T = 20T_0$ for $\phi = \pi/2$, and (f) $T = 50T_0$ for $\phi = \pi/2$. In (b), (c), (e), and (f), the solid blue line corresponds to the phase-advanced beam along $x = -2\ \mu$m, the dashed red line corresponds to the phase-delayed beam along $x = +2\ \mu$m, and the solid black line marks the position of one wave crest for the phase-advanced beam at the initial stage.

V. SUMMARY

In summary, the energy-shift process between two relativistically intense laser pulses copropagating in underdense plasmas is investigated. It is shown that the refractive index of the plasma is strongly dependent on the relative phase difference (ϕ) of the two laser pulses, which leads to different interaction features, including the attraction, repulsion, and energy shift. In particular, when the relative phase difference is between zero and π, the laser energy is shifted from the phase-advanced beam into the phase-delayed beam. The repulsion process becomes dominant for $\pi/2 < \phi < \pi$ and the attraction process is dominant for $0 < \phi < \pi/2$. In addition, a phase offset of $\pi/2$ favors repulsion and asymmetric distribution of energy between the two beams. When $0 < \phi < \pi/2$, the interaction becomes stronger and more energy is shifted into the phase-delayed beam as the relative phase difference is decreased. During the interaction process, the relative phase difference is gradually increased due to the different phase velocities. It is further demonstrated that the energy-shift process is independent of the initial laser intensities and also the polarizations of the laser pulses. It is possible to manipulate the interaction process of ultraintense laser pulses in plasmas by controlling their relative phase difference. In this way, the controllable laser energy redistribution and reallocation become feasible, which would simulate great interest in a broad range of applications, including laser-driven plasma accelerators, laser-based radiation sources, and inertial confinement fusion.

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