Quantum dark solitons as qubits in Bose-Einstein condensates

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We study the possibility of using dark solitons in quasi-one-dimensional Bose-Einstein condensates to produce two-level systems (qubits) by exploiting the intrinsic nonlinear and coherent nature of the matter waves. We calculate the soliton spectrum and the conditions for a qubit to exist. We also compute the coupling between the phonons and the solitons and investigate the emission rate of the qubit in that case. Remarkably, the qubit lifetime is estimated to be of the order of a few seconds, being only limited by the dark-soliton "death" due to quantum evaporation.

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I. INTRODUCTION

Quantum effects strive to disappear for macroscopic objects. Typically, quantum effects become depreciated out into their classical averages, and therefore the manipulation of quantum states, relevant for quantum computation, becomes unsustainable at the macroscopic scale. However, Bose-Einstein condensates (BECs) constitute one important exception where quantum effects are perceptible on a macroscopic level. With the advent and rapid development of laser cooling and trapping of neutral atoms over the past decade, micron-sized atomic gases at ultralow temperatures are routinely formed in the laboratory [1–3]. Moreover, quantum optic techniques allow for an unprecedented versatile and precise control of internal degrees of freedom, putting cold atoms as one of the most prominent candidates to test the complex aspects of strongly correlated matter and to applications in quantum information processing [4–11].

Quantum information has been introduced in cold-atom systems at various levels [12,13]. One way consists in defining a qubit (a two-state system) via two internal states of an atom. This approach, however, requires each atom to be addressed separately. A similar problem appears when the qubit is introduced via a set of spatially localized states (e.g., in adjacent wells of an optical lattice potential) of an atom or a BEC. The complication is due to the fact that the number of atoms in a BEC experiment significantly fluctuates from run to run. As a result, any qubit system dependent on the number of atoms becomes problematic. A second way of producing qubits in these systems relies on the collective properties of ultracold atoms. Here, a two-level system can be formed by isolating a pair of macroscopic states that are set sufficiently far away from the multiparticle spectrum. At the same time, however, the energy gap between these lowest states must remain small enough to allow measurable dynamics [14]. Experiments performed in the double-well potential configuration are a pioneer example of such an approach [15,16]. Nevertheless, despite an appealing similarity with single-particle two-level states, double-well potentials are insufficient to achieve a macroscopic superposition allowing for a measurable dynamics [16]. To overcome the superposition issue, a more recent proposal based on BEC superfluid current states in the ring geometry, analogous to the superconducting flux qubit [17], has been discussed [18]. More recently, the concept of a *phononic* reservoir via the manipulation of the phononic degrees of freedom has pushed quantum information realizations to another level, comprising the dynamics of impurities immersed in BECs [19–23] and reservoir engineering to produce multipartite dark states [24–29]. Another important difference with respect to the quantum optical system is the possibility to use phononic reservoirs to test non-Markovian effects in many-body systems [30,31]. The implementation of quantum gates has been recently proposed in Ref. [32].

Another important family of macroscopic structures in BECs with potential applications in quantum information are the so-called dark solitons (DSs). They consist of nonlinear localized depressions in a quasi-one-dimensional (1D) BEC that emerge due to a precise balance between the dispersive and nonlinear effects in the system [33–36], being also ubiquitous in nonlinear optics [37], shallow liquids [38], and magnetic films [39]. Quasi-one-dimensional BECs with repulsive interatomic interactions are prone to the inception of dark solitons by various methods, such as imprinting the spatial phase distribution [35], inducing density defects in BEC [40], and by the collision of two condensates [41,42]. The stability and dynamics of DSs in BECs have been a subject of intense research over the last decade [43,44]. Recent activity in the field involve studies on the collective aspects of the so-called *soliton gases* [45], putting dark solitons as a good candidate to investigate many-body physics [46].

In this paper, we combine the intrinsic nonlinearity in quasi-one-dimensional BECs to construct two-level states (qubits) with dark solitons. As we will show, owing to the unique properties of the DS spectrum, perfectly isolated two-level states are possible to construct. As a result, a matter-wave qubit with a few kHz energy gap is achieved. Moreover, the phonons (quantum fluctuations around the background density) play the role of a proper quantum reservoir. Remarkably, due to their intrinsic slow-time dynamics, BEC phonons provide small

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decoherence rates of a few Hz, meaning that under typical experimental conditions, DS qubits have a lifetime comparable to the lifetime of the BEC, being only limited by the soliton quantum diffusion ("evaporation"). As we show below, this effect is not critical and the qubit is still robust within the 100 ms–0.1 s time scale.

The paper is organized as follows: In Sec. II, we discuss the properties of a single DS in a quasi-1D BEC immersed in a dilute gas of impurities. We start with a set of coupled Gross-Pitaevskii and Schrödinger equations and find under which conditions DSs can define a two-level atom (qubit). In Sec. III, we compute the coupling between phonons and DSs. Section IV discusses the Weisskopf-Wigner theory to determine the emission rate of the qubit. Some discussion and conclusions about the implications of our proposal in practical quantum information protocols are stated in Sec. V.

II. MEAN-FIELD EQUATIONS AND THE DARK-SOLITON QUBIT

We consider a quasi-1D BEC immersed in a dilute gas of impurities. The BEC and the impurity particles are described by the wave functions $\psi_1(x,t)$ and $\psi_2(x,t)$, respectively. A quasi-1D gas is produced when the transverse dimension of the trap is larger than or of the order of the s-wave scattering length and, at the same time, much smaller than the longitudinal extension [47,48]. At the mean-field level, the BEC is governed by the Gross-Pitaevskii equation

$$i\hbar\frac{\partial\psi_1}{\partial t} = -\frac{\hbar^2}{2m_1}\frac{\partial^2\psi_1}{\partial x^2} + g_{11}|\psi_1|^2\psi_1 + g_{12}|\psi_2|^2\psi_1, \quad (1)$$

while the impurities—which we consider to not interact—are governed by

$$i\hbar\frac{\partial\psi_2}{\partial t} = -\frac{\hbar^2}{2m_2}\frac{\partial^2\psi_2}{\partial x^2} + g_{21}|\psi_1|^2\psi_2. \quad (2)$$

Here, g_{11} is the one-dimensional coupling strength between particles in the BEC and $g_{12} = g_{21}$ is the interspecies coupling constant, $\hbar$ is the Planck constant, and m_i $(i = 1,2)$ denotes the mass of the species. We restrict the discussion to repulsive interactions, $g_{11} > 0$. In what follows, we assume that the dark soliton is not disturbed by the presence of impurities. This situation can be achieved if we choose the impurity gas to be sufficiently dilute, $|\psi_2|^2 \ll |\psi_1|^2$, and if the impurities are much less massive than the BEC particles (a discussion about the experimental realization can be found in Sec. IV). The impurities can therefore be regarded as free particles that feel the soliton as a potential (see Fig. 1). As such, Eq. (2) can be written as

$$i\hbar\frac{\partial\psi_2}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi_2}{\partial x^2} + g_{21}|\psi_{\rm sol}|^2\psi_2, \quad (3)$$

where the soliton profile, a singular nonlinear solution to Eq. (1), is given by [49–52]

$$\psi_{\rm sol}(x) = \sqrt{n_0}\tanh\left(\frac{x}{\xi}\right). \quad (4)$$

Here, n_0 is the background density which is typically of the order of $10^8\,m^{-1}$ in elongated ^{85}Rb BECs, and the healing length $\xi = \hbar/\sqrt{mn_0g_{11}}$ is of the order $(0.2–0.7)\,\mu$m. We also consider the experimentally accessible trap frequencies $\omega_r = 2\pi\times(1–5)$ kHz $\gg \omega_z = 2\pi\times(15–730)$ Hz and the corresponding length amount to be the value $l_z = (0.6–3.9)\,\mu$m [53]. More recent experiments produced much larger traps, $l_z \sim 100\,\mu$m [54], which led the eventual trap inhomogeneities to be much less critical. Notice that the previous results can be easily generalized for the case of gray solitons (i.e., solitons traveling with speed v) by replacing Eq. (4) with

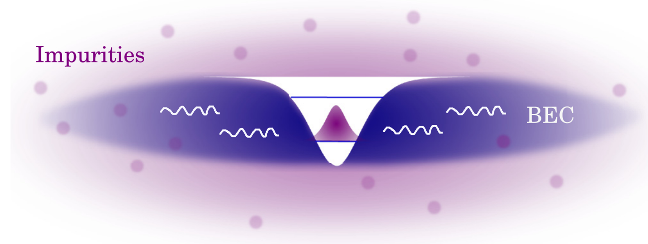

FIG. 1. Schematic representation of the system. A BEC contains a dark soliton, which acts as a potential to the impurity particles. Under certain circumstances, exactly two bound states can be formed. Due to quantum fluctuations, the BEC also support phonons (wiggly lines), which will interact with the dark soliton and, consequently, provide some dephasing.

$$\psi_{\rm sol}(x) = \sqrt{n_0}\left[i\theta + \frac{1}{\gamma}\tanh\left(\frac{x}{\xi\gamma}\right)\right], \quad (5)$$

where $\theta = v/c_s$, $\gamma = (1-\theta^2)^{-1/2}$, and $c_s = \sqrt{gn_0/m}$ is the BEC sound speed [46,55,56]. Therefore, the time-independent version of Eq. (3) reads

$$E'\psi_2 = -\frac{\hbar^2}{2m}\frac{\partial^2\psi_2}{\partial x^2} - g_{21}n_0\,{\rm sech}^2\left(\frac{x}{\xi}\right)\psi_2, \quad (6)$$

where $E' = E - g_{21}n_0$. Here, the dark soliton acts as a potential for the particles of the reservoir. Analytical solutions to Eq. (6) can be obtained by casting the potential term in the form

$$V(x) = -\frac{\hbar^2}{2m\xi^2}\nu(\nu+1){\rm sech}^2\left(\frac{x}{\xi}\right), \quad (7)$$

where $\nu = (-1+\sqrt{1+4g_{12}/g_{11}})/2$. The particular case of ν being a positive integer belongs to the class of *reflectionless* potentials [57], for which an incident wave is totally transmitted. For the more general case considered here, the energy spectrum associated with the potential in Eq. (7) reads

$$E_n^{'} = -\frac{\hbar^2}{2m\xi^2}(\nu-n)^2, \quad (8)$$

where n is an integer. The number of bound states is given by $n_{\rm bound} = \lfloor 1+\nu+\sqrt{\nu(\nu+1)}\rfloor$, where $\lfloor\cdot\rfloor$ denotes the integer part. A two-level system (qubit) can thus be perfectly isolated when the value of ν ranges as

$$\tfrac{1}{3} \leqslant \nu < \tfrac{4}{5}.$$

At the critical point $\nu = \frac{1}{2}$, the two-energy levels merge and the qubit is ill defined. For $\nu \geqslant \frac{4}{5}$, three-level systems (qutrits)

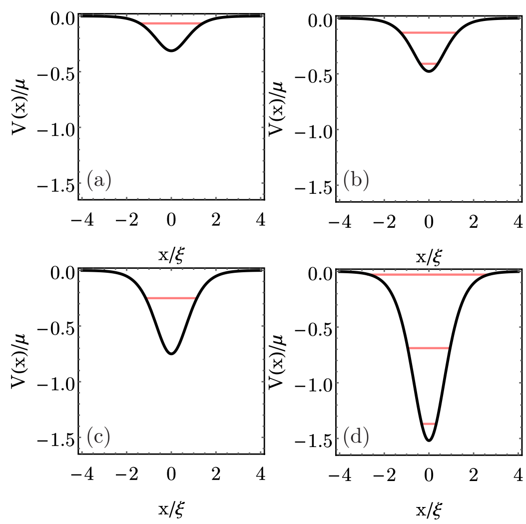

FIG. 2. Dark soliton acting as a binding potential for the free particles. (a) depicts a single bound state obtained for $\nu < \frac{1}{3}$, while (b) illustrates the case of a two-level system (qubit) case obtained for $\frac{1}{3} \leqslant \nu < \frac{4}{5}$. (c) illustrates the pathological case $\nu = \frac{1}{2}$ for which a degeneracy in the two-level system is obtained. A qutrit case is depicted in (d) for $\nu = 0.83$.

can also be formed, but this case is out of the scope of the present work and will be discussed in a separate publication. The features of the spectrum (8) are illustrated in Fig. 2.

III. QUBIT-PHONON INTERACTION

The mean-field soliton solution in Eq. (4) is accompanied by quantum fluctuations (phonons). In that case, the total wave function is given by $\psi_1(x) = \psi_{\rm sol}(x) + \delta\psi_1(x)$, where

$$\delta\psi_1(x) = \sum_k [u_k(x) b_k e^{ikx} + v_k(x)^* b_k^\dagger e^{-ikx}], \qquad (9)$$

with b_k denoting bosonic operators satisfying the commutation relation $[b_k, b_q^\dagger] = \delta_{kq}$. $u_k(x)$ and $v_k(x)$ are amplitudes verifying the normalization condition $|u_k(x)|^2 - |v_k(x)|^2 = 1$ and are explicitly given by [58]

$$u_k(x) = \sqrt{\frac{1}{4\pi\xi}\frac{\mu}{\epsilon_k}}\left[\left((k\xi)^2 + \frac{2\epsilon_k}{\mu}\right)\left[\frac{k\xi}{2} + i\tanh\left(\frac{x}{\xi}\right)\right] + \frac{k\xi}{\cosh^2\left(\frac{x}{\xi}\right)}\right],$$

and

$$v_k(x) = \sqrt{\frac{1}{4\pi\xi}\frac{\mu}{\epsilon_k}}\left[\left((k\xi)^2 - \frac{2\epsilon_k}{\mu}\right)\left[\frac{k\xi}{2} + i\tanh\left(\frac{x}{\xi}\right)\right] + \frac{k\xi}{\cosh^2\left(\frac{x}{\xi}\right)}\right].$$

Similarly, the impurity wave functions are eigenstates of Eq. (6), being therefore spannable in terms of the bosonic operators a_ℓ as

$$\psi_2(x) = \sum_{\ell=0}^{1} \varphi_\ell(x) a_\ell, \qquad (10)$$

where $\varphi_0(x) = \text{sech}(x/\xi)/(\sqrt{2\xi})$ and $\phi_1(x) = i\sqrt{3}\tanh(x/\xi)\,\varphi_0(x)$. The total Hamiltonian then reads

$$H = H_{\rm qubit} + H_{\rm ph} + H_{\rm int}. \qquad (11)$$

The first term $H_{\rm qubit}$ represents the dark-soliton (qubit) Hamiltonian

$$H_{\rm qubit} = \hbar\omega_0\sigma_z, \qquad (12)$$

where $\omega_0 = \hbar(2\nu - 1)/(2m\xi^2)$ is the qubit gap frequency and $\sigma_z = a_1^\dagger a_1 - a_0^\dagger a_0$ is the corresponding spin operator. The second term describes the phonon (reservoir) Hamiltonian

$$H = \sum_k \epsilon_k b_k^\dagger b_k, \qquad (13)$$

where the Bogoliubov spectrum is given by $\epsilon_k = \mu\xi\sqrt{k^2(\xi^2 k^2 + 2)}$, with $\mu = g n_0$ denoting the chemical potential.

The interaction Hamiltonian $H_{\rm int}$ between the qubit and the reservoir is defined as

$$H_{\rm int} = g_{12}\int dx\, \psi_2^\dagger \psi_1^\dagger \psi_1 \psi_2, \qquad (14)$$

which, with the prescriptions in Eqs. (9) and (10), can be decomposed as

$$H_{\rm int} = H_{\rm int}^{(0)} + H_{\rm int}^{(1)} + H_{\rm int}^{(2)}, \qquad (15)$$

respectively containing zero-, first-, and second-order terms in the operators b_k and $b_k^\dagger$. Owing to the small depletion of the condensate, and consistent with the Bogoliubov approximation performed in Eq. (9), we ignore the second-order term $H_{\rm int}^{(2)} \sim O(b_k^2)$. The first part of Eq. (15) corresponds to a Stark shift term of the type

$$H_{\rm int}^{(0)} = g_{12} n_0 \delta_{\ell\ell'} a_\ell^\dagger a_{\ell'} f_{\ell\ell'}, \qquad (16)$$

where $f_{\ell\ell'} = \int dx \varphi_\ell^\dagger(x)\varphi_{\ell'}(x)\tanh^2(x/\xi)$. The latter can be omitted by renormalizing the qubit frequency as $\tilde{\omega}_0 = \omega_0 + n_0 g_{12}$. In its turn, the first-order term $O(b_k)$ is given by

$$H_{\rm int}^{(1)} = \sum_k \sum_{\ell,\ell'} a_\ell^\dagger a_{\ell'}[b_k g_{\ell,\ell'}(k) + b_k^\dagger g_{\ell,\ell'}(k)^*] + \text{H.c.}, \qquad (17)$$

where

$$g_{\ell,\ell'}(k) = \sqrt{n_0} g_{12}\int dx \varphi_\ell^\dagger(x)\varphi_{\ell'}(x)\tanh\left(\frac{x}{\xi}\right)e^{ikx}u_k. \qquad (18)$$

As we can observe, Eq. (17) contains intraband ($\ell = \ell'$) and interband ($\ell \neq \ell'$) terms. However, for small values of the coupling between the system and the phonon reservoir, the qubit transition is driven by near-resonance phonons only, for which the interband coupling amplitude $|g_{01}(k)| = |g_{10}(k)^*|$ is much larger that the intraband terms $|g_{00}(k)|$ and $|g_{11}(k)|$ (see Fig. 3). As such, within the rotating-wave approximation (RWA), we can safely drop the intraband contribution and

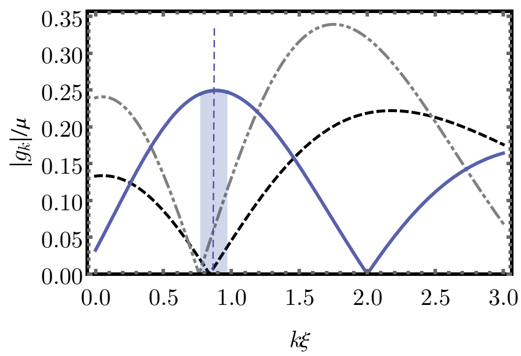

FIG. 3. Intraband $g_{00}(k)$ (dashed line) and $g_{11}(k)$ (dotted-dashed line) and interband $g_{10}(k)$ (solid line) coupling functions. Around the resonant values $k \sim \xi^{-1}$ (shadowed region), the interband term is dominant, allowing us to neglect the intraband contribution within the rotating-wave approximation.

obtain

$$H_{\text{int}}^{(1)} = -\sum_k g(k)\sigma_+ b_k - \sum_k g(k)^* \sigma_- b_k^\dagger + \text{H.c.}, \tag{19}$$

where $\sigma_+ = a_1^\dagger a_0$, $\sigma_- = a_0^\dagger a_1$, and the coupling constant $g_k \equiv g_{0,1}(k) = -g_{1,0}(k)$ is explicitly given by

$$g_k = \frac{i g_{12} k^2 \xi^{3/2}}{80\epsilon_k}\sqrt{\frac{n_0\pi}{6}}(2\mu + 8k^2\mu\xi^2 + 15\epsilon_k) \times (-4 + k^2\xi^2)\text{csch}\left(\frac{k\pi\xi}{2}\right).$$

We notice that invoking the RWA also implied the dropping of the counter-rotating terms proportional to $b_k\sigma_-$ and $b_k^\dagger\sigma_+$ that do not conserve the total number of excitations. The accuracy of such an approximation can be verified *a posteriori*, provided that the emission rate Γ, which we calculate below, is much smaller than the qubit transition frequency ω_0.

IV. SPONTANEOUS DECAY OF THE DARK-SOLITON QUBIT

Neglecting the effect of temperature and other external perturbations, the only source of decoherence of a dark-soliton qubit is the phonon bath. Because cold-atom experiments are typically very clean, and considering that the zero-temperature approximation is an excellent approximation for quasi-1D BECs [59,60], we employ the Wigner-Weisskopf theory in order to compute the lifetime of the qubit. We assume the qubit to be initially in its excited state and the field to be in the vacuum state. Under such conditions, the total system+reservoir wave function can be parametrized as

$$|\Psi(t)\rangle = \alpha(t)e^{-i\omega_0 t}|e,0\rangle + \sum_k \beta_k(t)e^{-i\omega_k t}|g,1_k\rangle, \tag{20}$$

where $\alpha(t)$ and $\beta_k(t)$ are the probability amplitudes. The Wigner-Weisskopf ansatz (20) is then allowed to evolve under the total Hamiltonian in Eq. (19), for which the corresponding Schrödinger equation yields

$$\dot{\alpha}(t) = \frac{i}{\hbar}\sum_k g_k e^{-i(\omega_k-\omega_0)t}\beta_k(t), \tag{21}$$

$$\beta_k(t) = \frac{i}{\hbar}g_k^*\int_0^t \alpha(t')e^{i(\omega_k-\omega_0)t'}dt'. \tag{22}$$

Due to separation of time scales between the phonons and the decay process, we may assume that the coefficient $\alpha(t)$ evolves much slower than $\beta_k(t)$, which allows us to invoke the Born approximation and write

$$\int_0^t \alpha(t')e^{-i(\omega_k-\omega_0)(t-t')}dt' \simeq \alpha(t)\int_0^t e^{-i(\omega_k-\omega_0)\tau}d\tau,$$

where $\tau = t - t'$. Moreover, since we expect $\alpha(t)$ to vary at a rate $\Gamma \ll \omega_0$, the relevant decay dynamics is expected to take place at times $t \gg \frac{1}{\omega_0}$, which allows us to take the upper limit of the above integral to ∞ (Markov approximation). Therefore, we have

$$\alpha(t)\int_0^\infty e^{-i(\omega_k-\omega_0)\tau}d\tau = \alpha(t)\pi\delta(\omega_k-\omega_0) - i\alpha(t)\wp\left(\frac{1}{\omega_k-\omega_0}\right), \tag{23}$$

where $\wp$ represents Cauchy's principal value describing an additional energy (Lamb) shift. Because it represents a small correction to the qubit energy ω_0, we do not compute its contribution explicitly. Thus, the excited state amplitude decays exponentially as

$$\alpha(t) = e^{-\Gamma t/2}, \tag{24}$$

where Γ is the population decay rate given as

$$\Gamma = \frac{L}{\sqrt{2}\hbar\xi}\int d\omega_k \frac{\sqrt{1+\eta_k}}{\eta_k}|g_k|^2\delta(\omega_k-\omega_0) \tag{25}$$

$$= \frac{\pi N_0 g_{12}^2}{76\,800\hbar\mu^5\xi^2\eta_0\sqrt{\frac{\mu+\eta_0}{\mu}}}(-\mu+\eta_0)(-5\mu+\eta_0)^2 \times\left[8\eta_0 + 3\mu\left(-2+5\xi\sqrt{\frac{\hbar^2\omega_0^2}{\mu^2\xi^2}}\right)\right]^2 \times \text{csch}^2\left(\frac{\pi\sqrt{-\mu+\eta_0}}{2\sqrt{\mu}}\right), \tag{26}$$

and $\eta_{0,k} = \sqrt{\mu^2 + \hbar^2\omega_{0,k}^2}$. As depicted in Fig. 4, the decay rate Γ is orders of magnitude smaller than the qubit gap ω_0, confirming the validity of both RWA and Born-Markov approximations. Remarkably, for a quasi-1D of a chemical potential of a few kHz, we can obtain a qubit lifetime $\tau_{\text{qubit}} \sim 1/\Gamma$ of the order of a second, a time comparable to the BEC lifetime itself. Notice that the value of g_{12} (and, consequently, the qubit natural frequency ω_0 and lifetime τ_{qubit}) can be experimentally tuned with the help of Feshbach resonances. The only immediate limitation to the performance of our proposal may be related to the dark-soliton quantum diffusion [58]. Since they interact with the background

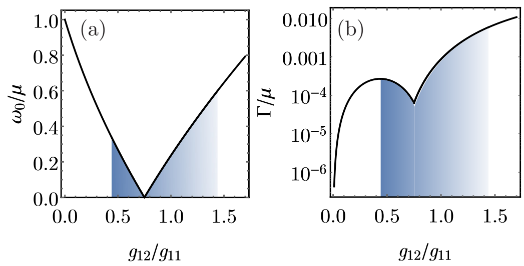

FIG. 4. Dependence of (a) the transition frequency ω_0 and (b) decay rate Γ on the coupling constant g_{12}. The shadowed region corresponds to the range $\frac{1}{3} \leqslant \nu < \frac{4}{5}$ for which the qubit can be exactly defined. The case $\nu = \frac{1}{2}$ produces a degenerate two-level system.

phonons, they are expected to evaporate within the time scale $\tau_{\text{diffusion}} = 8\xi/c_s\sqrt{3n_0\xi/2}$. For typical 1D BECs with $\xi \sim 0.7$–$1.0\,\mu$m and $c_s \sim 1.0$ mm/s, we estimate $\tau_{\text{diffusion}} \sim 0.05$–$0.1$ s, which reduces τ_{qubit} by about 20%. Additional experimental limitations may be associated with the soliton radiation in the presence of a trap. Although the soliton-phonon interaction has been shown to be balanced if the trap is harmonic (i.e., that does not lead to the decay of the soliton oscillations [61,62]), the ensuing dynamics are sensitive to trap anharmonicities [63], for which the soliton oscillations may be destroyed after a few hundred milliseconds. However, for the experimental condition of Ref. [62], the observed lifetime of the solitons goes up to 2.8 s for sufficiently shallow traps. In any case, soliton oscillations are not expected to be relevant here, since we are considering the dark-soliton limit $v \sim 0$. To circumvent this problem, one may consider loading the BEC in box-shaped potentials, for which the condensate is homogeneous along a trap of size $l_z \sim 70\,\mu$m, largely exceeding the soliton core ξ and therefore rendering finite-size effects to be less important [64]. The advantage offers additional advantages regarding the scalability (i.e., in a multiple-soliton quantum computer), as a long-range phonon mediated soliton-soliton interaction appears when inhomogeneities exist [65]. Another important effect that may hinder the stability of the soliton is the scattering with the impurities, leading, for example, to Brownian diffusion [66]. In this case, ^{87}Rb dark solitons live up to 1 s in the presence of an impurity concentration of 6.3%. We expect this effect to be further reduced if we consider ^{174}Yb BECs immersed in dilute ^{7}Li impurities, since the mass imbalance $m_{\text{Li}}/m_{\text{Yb}} \ll 1$ quenches the value of the friction coefficient. The caveat is that the qubit potential depth also depends on this ratio, so a delicate balance between the soliton diffusion and qubit energy split is necessary. Moreover, if the concentration of (bosonic) impurities is sufficiently high, impurity condensation on the bottom of the soliton may occur, leading to a spurious qubit energy shift and the single-particle assumption to break down. This can be avoided if fermionic impurities are used instead, e.g., ^{6}Li [67].

Finally, by putting Eqs. (22) and (24) together, we can evaluate the evolution of the amplitude coefficient $\beta_k(t)$ as

$$\beta_k(t) = \frac{i}{\hbar} g_k^* \int_0^t e^{-[\frac{\Gamma}{2} - i(\omega_k - \omega_0)]t} dt, \tag{27}$$

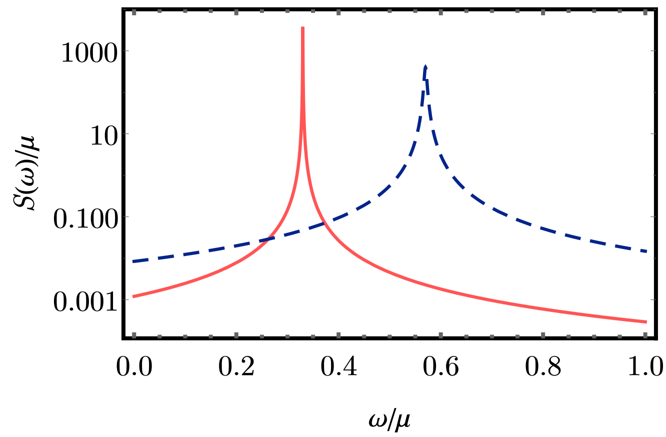

FIG. 5. Emission spectrum of a soliton qubit due to the interaction with the background phonons. Red (solid) and blue (dashed) curves are respectively obtained for $\nu = 0.33$ and $\nu = 0.79$.

which yields the following Lorentzian spectrum,

$$S(\omega_k) = \lim_{t\to\infty} |\beta_k(t)|^2 = \frac{1}{\hbar^2} \frac{|g_k|^2}{\frac{\Gamma^2}{4} + (\omega_k - \omega_0)^2}, \tag{28}$$

as depicted in Fig. 5. As expected, it is observed that the Lorentzian spectrum is narrower for a weak-coupling constant g_{12}.

V. CONCLUSION

In conclusion, we have shown that a dark soliton in a quasi-one-dimensional Bose-Einstein condensate can produce a well-isolated two-level system, which can act as a matter-wave qubit with an energy gap of a few kHz. This feature is intrinsic to the nonlinear nature of Bose-Einstein condensates and does not require manipulation of the internal degrees of freedom of the atoms. We observe that the decoherence induced by the quantum fluctuations (phonons) produces a finite qubit lifetime. Quite remarkably, leading calculations indicate a qubit lifetime of the order of a few seconds, a time scale comparable to the duration of state-of-the-art cold atomic traps. A major limitation to the qubit robustness is the quantum diffusion of the soliton, which is estimated to reduce the qubit lifetime to around 20% its value, and an eventual phonon-soliton interaction due to the presence of inhomogeneities introduced by the trap. Moreover, impurity scattering could, in principle, decrease the qubit coherence, but a clever choice of BEC particle and impurity mass ratio may circumvent this problem. With all these limitations in mind, we believe that qubits made of dark solitons are excellent candidates to store information for large times ($\sim$0.01 – 1s), offering an appealing alternative to quantum optical solid-state platforms. While dark solitons may not compete in terms of quantum scalability (the number of solitons in a typical elongated BEC is not expected to surpass a few tens), their unprecedented coherence and lifetime will certainly make them attractive for the design of new quantum memories and quantum gates. Moreover, due to the possibility of interfacing cold atomic clouds with solid-state and optical systems, our findings may inspire further applications in hybrid quantum computers. We notice that the present dark-soliton qubits could also be considered in a pure one-dimensional case, i.e., for a Tonks-Girardeau gas. However, quantum fluctuations are more important and the analysis based on the Bogoliubov theory fails.

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