Generation of robust tripartite entanglement with a single-cavity optomechanical system

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(Received 2 December 2016; published 1 May 2017)

We present a proposal to generate robust tripartite optomechanical entanglement with a single-cavity optomechanical system driven by a single input laser field. The produced stationary tripartite entanglement among two longitudinal cavity modes and a mirror oscillation mode via radiation pressure force exhibits robustness to the variation of the environment temperature when the cavity free spectral range is close to the mechanical oscillation frequency. The present optomechanical system can serve as an alternative intermediary for quantum-state exchange between two microwave (or optical) fields as well as between photons and the macroscopic mechanical oscillator, and may be potentially useful for quantum information processing and quantum networks.

DOI: 10.1103/PhysRevA.95.052303

I. INTRODUCTION

Multipartite entanglement provides an essential resource for quantum computation, quantum networks, and quantum information processing [1–3]. The ways of conveniently and efficiently realizing the generation and manipulation of light-light, light-matter, and matter-matter entanglements become the basic requirements for quantum information processing protocols. The traditionally used schemes for producing light-light bipartite entanglements have been to employ parametric downconversion processes in nonlinear optical crystals and multiple entangled fields can be readily obtained by mixing the generated squeezed fields with polarizing beam splitters; however, the created entangled fields are normally degenerate, have large bandwidth, and suffer from short correlation time. The atomic system [4–9] or optomechanical system [10–30] provides an alternative avenue to the generation of multipartite entanglement. Recently, much attention has been paid to the optomechanical system due to the fact that, on the one hand, as in the atomic system, the generated entangled fields have the virtue of narrow bandwidth and the vibrating cavity mirror has a long decay time (microseconds or even seconds [10]), thereby providing a promising system for quantum memory required in a quantum repeater. On the other hand, unlike the atomic system where the generated fields normally rely on naturally existing resonances, mechanical oscillators can couple to light fields with any frequency, so, in principle, one can easily and conveniently produce entangled fields with any desired wavelengths (such as the near-infrared communication wavelengths). Apart from the light-light entanglement [13–15], the light-mirror [16–19], mirror-mirror [20–26], and light-mirror-light [27–30] entanglements—through radiation pressure force exerted on the mechanical oscillators—have also been extensively examined by using the cavity-free or cavity-assisted optomechanical systems. In the conventional cavity optomechanical system, the frequency separation between two neighboring longitudinal cavity modes is far larger than the oscillation frequency of the cavity mirror, so one only needs to consider the interaction of one cavity field mode with the mechanical oscillation mode. However, when the cavity free spectral range is of the order of the mechanical oscillation frequency, the simultaneous interaction of two longitudinal cavity field modes with the mechanical oscillation mode should be taken into account, and few studies have been performed in such a case [31,32].

In this paper, we present a convenient scheme to generate robust tripartite entanglement among two longitudinal modes of the cavity field and the mechanical oscillation mode of the vibrating cavity mirror via radiation pressure force by using a single-cavity optomechanical system driving by a single input laser field with the cavity free spectral range being close to the mechanical oscillation frequency. In addition, the produced tripartite optomechanical entanglement is quite robust to the environment temperature. Note that the present scheme for generating tripartite entanglement is quite distinct as compared to that in Refs. [27–30]. In Ref. [27], the light-mirror-light tripartite entanglement was produced through a dual-cavity optomechanical system with a common vibrating mirror, whereas in Ref. [30], the tripartite entanglement among two transverse cavity modes and vibrating mirror oscillation mode was generated in an optomechanical system. Though similar studies on entanglement among the Stokes and anti-Stokes sidebands of the driving field and the mechanical oscillation mode have been performed with cavity-assisted [28] and cavity-free [29] optomechanical systems, it is shown in Refs. [28,29] that the vibrating mirror mode is only entangled with the Stokes field and not entangled with the anti-Stokes field.

II. THEORETICAL MODEL AND HEISENBERG-LANGEVIN EQUATIONS

The considered single-cavity optomechanical system driven by a single input laser field is shown in Fig. 1(a), which is composed of a vibrating mirror with oscillation frequency ω_m and mechanical damping rate γ_m simultaneously coupled to two neighboring longitudinal modes of the cavity field due to radiation pressure. As displayed in Fig. 1(b), we assume that the cavity free spectral range ω_{fsr} is equal to twice of

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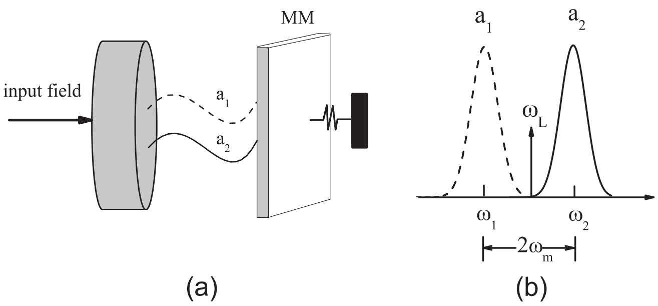

FIG. 1. (a) The single-cavity optomechanical system with a movable mirror (MM) driven by a single input laser field with frequency ω_L, where a_1 and a_2 represent the two longitudinal cavity field modes 1 and 2 with frequencies ω_1 and ω_2, respectively. (b) The relevant frequencies of the driving field and two cavity modes 1 and 2, where the cavity free spectral range is equal to twice the oscillating frequency of the mirror and the input field is tuned around the midfrequency of the two cavity modes.

the oscillation frequency of the mirror, and the input laser field frequency ω_L is tuned around the midfrequency of two longitudinal cavity modes. In this case, the single input laser field drives the two cavity modes simultaneously, and the Hamiltonian of the system can be written as [31,33,34]

$$\begin{aligned} H_v &= \hbar\omega_1 a_1^+ a_1 + \hbar\omega_2 a_2^+ a_2 + \hbar\omega_m(p^2+q^2) \\ &\quad - \hbar g_0 (a_1+a_2)^+(a_1+a_2)q \\ &\quad + i\hbar\eta_1(a_1^+ e^{-i\omega_l t} - a_1 e^{i\omega_l t}) \\ &\quad + i\hbar\eta_2(a_2^+ e^{-i\omega_l t} - a_2 e^{i\omega_l t}), \end{aligned} \tag{1}$$

where a_1 (a_1^+) and a_2 (a_2^+) are the annihilation (creation) operators of the cavity field modes 1 and 2 with frequencies ω_1 and ω_2 and decay rates k_1 and k_2, respectively; q and p are the dimensionless position and momentum operators of the vibrating mirror; $g_0 = \sqrt{\hbar\omega_1\omega_2/m\omega_m}/L$ is the optomechanical coupling coefficient of the radiation pressure with L being the cavity length, and m the effective mass of the mechanical oscillator. The last two terms in Eq. (1) describe the input driving laser field and its interaction with the two cavity modes; $\eta_{1(2)}$ is related to the input field power P with $\eta_{1(2)} = \sqrt{2Pk/\hbar\omega_{1(2)}}$ (here we assume $k_1 = k_2 = k$ for simplicity). In the frame rotating at the input field frequency ω_L, the Heisenberg-Langevin equations can be written as [16,31]

$$\dot{q} = \omega_m p, \tag{2a}$$

$$\dot{p} = -\omega_m q - \gamma_m p + g_0(a_1+a_2)^+(a_1+a_2) + \xi, \tag{2b}$$

$$\dot{a}_1 = -(i\Delta_1 + k_1)a_1 + i g_0(a_1+a_2)q + \eta_1 + \sqrt{2k_1}a_{\rm in}, \tag{2c}$$

$$\dot{a}_2 = -(i\Delta_2 + k_2)a_2 + i g_0(a_1+a_2)q + \eta_2 + \sqrt{2k_2}a_{\rm in}, \tag{2d}$$

where $\Delta_{1,2} = \omega_{1,2} - \omega_L$ is the frequency detuning of the cavity field 1 (2) with respect to the input laser field, $a_{\rm in}(t)$ and $\xi(t)$ are the optical and mechanical noise operators with the relevant nonzero correlation functions $\langle a_{\rm in}(t)a_{\rm in}^+(t')\rangle = \delta(t-t')$ and $\langle \xi(t)\xi(t') + \xi(t')\xi(t)\rangle/2 = \gamma_m(2\bar{n}+1)\delta(t-t')$ in the limit of the large mechanical quality factor (i.e., $Q_m = \omega_m/\gamma_m \gg 1$ [35]), where $\bar{n} = 1/[\exp(h\omega_m/k_B T) - 1]$ is the mean thermal phonon number with k_B being the Boltzmann constant and T the mirror temperature. By writing each Heisenberg operator as the sum of its steady-state mean value and a small fluctuation operator with zero-mean value, $a_{1,2} = \alpha_{1,2} + \delta a_{1,2}$, $q = q_s + \delta q$, and defining the cavity field quadratures $\delta X_{1,2} = (\delta a_{1,2} + \delta a_{1,2}^+)/\sqrt{2}$ and $\delta Y_{1,2} = (\delta a_{1,2} - \delta a_{1,2}^+)/\sqrt{2}i$, and the corresponding Hermitian input noise operators $X_{\rm in} = (a_{\rm in} + a_{\rm in}^+)/\sqrt{2}$ and $Y_{\rm in} = (a_{\rm in} - a_{\rm in}^+)/\sqrt{2}i$, we can obtain the quantum Langevin equations for the fluctuation operators,

$$\begin{aligned} \delta\dot{X}_1 &= -k\delta X_1 + (\Delta_1 - g_0 q_s)\delta Y_1 - g_0 q_s \delta Y_2 \\ &\quad - \sqrt{2}g_0 {\rm Im}(\alpha_1+\alpha_2)\delta q + \sqrt{2k}X_{\rm in}, \end{aligned} \tag{3a}$$

$$\begin{aligned} \delta\dot{Y}_1 &= -k\delta Y_1 - (\Delta_1 - g_0 q_s)\delta X_1 + g_0 q_s \delta X_2 \\ &\quad + \sqrt{2}g_0 {\rm Re}(\alpha_1+\alpha_2)\delta q + \sqrt{2k}Y_{\rm in}, \end{aligned} \tag{3b}$$

$$\begin{aligned} \delta\dot{X}_2 &= -k\delta X_2 + (\Delta_2 - g_0 q_s)\delta Y_2 - g_0 q_s \delta Y_1 \\ &\quad - \sqrt{2}g_0 {\rm Im}(\alpha_1+\alpha_2)\delta q + \sqrt{2k}X_{\rm in}, \end{aligned} \tag{3c}$$

$$\begin{aligned} \delta\dot{Y}_2 &= -k\delta Y_2 - (\Delta_2 - g_0 q_s)\delta X_2 + g_0 q_s \delta X_1 \\ &\quad + \sqrt{2}g_0 {\rm Re}(\alpha_1+\alpha_2)\delta q + \sqrt{2k}Y_{\rm in}, \end{aligned} \tag{3d}$$

$$\delta\dot{q} = \omega_m \delta p, \tag{3e}$$

$$\begin{aligned} \delta\dot{p} &= -\omega_m\delta q - \gamma_m\delta p + \sqrt{2}g_0 {\rm Re}(\alpha_1+\alpha_2)(\delta X_1 + \delta X_2) \\ &\quad + \sqrt{2}g_0 {\rm Im}(\alpha_1+\alpha_2)(\delta Y_1 + \delta Y_2) + \xi. \end{aligned} \tag{3f}$$

In the above equations, the steady-state mean values can be easily obtained by letting the time derivatives be equal zero and neglecting the noise operators in Eqs. (2a)–(2d); thus one can get $\alpha_2 = \alpha_1(k+i\Delta_1)/(k+i\Delta_2)$, $q_s = g_0|\alpha_1+\alpha_2|^2/\omega_m$, and obtain α_1 by substituting α_2 and q_s into Eq. (2c), where we have chosen the phase reference of the driving field so that α_1 is real and positive. As analyzed in Ref. [16], by solving the linearized quantum Langevin equations Eqs. (3a)–(3f) for the fluctuation operators, the steady-state covariance matrix V can be obtained. We use the logarithmic negativity E_N defined in Ref. [36] to investigate the entanglement features among two longitudinal modes of the cavity field and mechanical oscillation mode of the vibrating cavity mirror, where $E_N \geqslant 0$ means the generation of genuine entanglement, and the larger the value of the logarithmic negativity E_N, the stronger the entanglement can be attained. In the following, the relevant parameters of the optomechanical system are set as $L = 0.01$ m, $T = 0.1$ K, $\omega_m = \pi c/2L$, $k = 100\gamma_m = 1$ MHz, $\lambda = 1.33\,\mu$m, $m = 5\times10^{-9}$ kg, $P = 20$ mW, $\Delta_1 = -\Delta_2 = -\omega_m$, which are experimentally feasible with current technology [31,37–39].

III. RESULTS AND DISCUSSION

Figure 2 shows the evolution of entanglement among the two cavity fields and the vibrating mirror mode tested by logarithmic negativity E_N as a function of the normalized detuning Δ_2/ω_m of the cavity field 2 with respect to the input laser field. It is clear that $E_N^{a_1a_2}$ has very large nonzero positive values in a

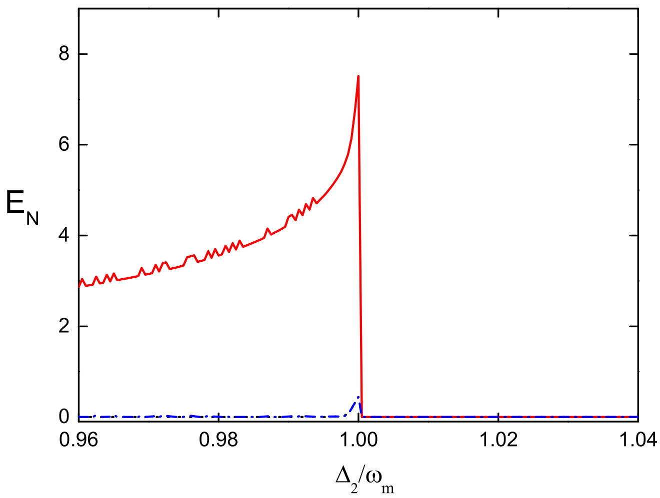

FIG. 2. The evolution of the logarithmic negativity $E_N^{a_1 a_2}$ (solid line), $E_N^{a_1 c}$ (dotted line), and $E_N^{a_2 c}$ (dotted-dashed line) as a function of the normalized detuning Δ_2/ω_m of the cavity field 2 with respect to the input laser field, where $E_N^{a_1 a_2}$ presents for the entanglement between the two cavity fields 1 and 2, and $E_N^{a_1 c}$ ($E_N^{a_2 c}$) for that between the cavity field 1 (2) and the vibrating mirror, respectively. The relevant parameters are $L = 0.01$ m, $T = 0.1$ K, $\omega_m = \pi c/2L$, $k = 100\gamma_m = 1$ MHz, $\lambda = 1.33\,\mu$m, $m = 5 \times 10^{-9}$ kg, $P = 20$ mW.

wide range ($0 \sim \omega_m$) of the detuning Δ_2, which demonstrates that a high degree of bipartite entanglement between the two neighboring longitudinal cavity fields is generated; however, as seen from $E_N^{a_1 c}$ and $E_N^{a_2 c}$ in Fig. 2, the bipartite optomechanical entanglement between one of the cavity fields 1 (or 2) and the mirror vibrational mode can exist only in a limited range of detuning around $\Delta_2 = \omega_m$; in addition, in this small detuning range, $E_N^{a_1 a_2}$, $E_N^{a_1 c}$, and $E_N^{a_2 c}$ are all larger than zero, which means that the two neighboring longitudinal cavity fields and the vibrating cavity mirror mode are genuinely entangled with each other, where the strongest tripartite optomechanical entanglement is obtained when the input field is tuned to the middle frequency of the two neighboring cavity fields (i.e., $\Delta_2 = \omega_m$). Note that the evolution of E_N as a function of Δ_2 looks zigzag for $\Delta_2/\omega_m < 1$. This is due to the following fact. In the present scheme, the two cavity fields are generated with a single-cavity optomechanical system driven by a single input laser field, where we have chosen the phase reference of the driving field to ensure the steady-state mean value α_1 to be real and positive, so the steady-state mean value α_2 [$\alpha_2 = \alpha_1(k + i\Delta_1)/(k + i\Delta_2)$] is a complex number; subsequently, the steady-state mean value q_s [$q_s = g_0|\alpha_1 + \alpha_2|^2/\omega_m$, which determines the effective detunings and the coupling coefficient of the two cavity fields, as seen from Eqs. (3a)–(3d)] would exhibit small oscillation as a function of Δ_2, which leads to the zigzag of the logarithmic negativity with respect to Δ_2.

The robustness of the tripartite optomechanical entanglement among the two longitudinal cavity fields and the mirror mode with respect to the environmental temperature and the quality factor (Q_m) of the vibrating mirror mode is displayed in Fig. 3, where we assume $k = 100\gamma_m$, so the quality factor (Q) of the cavity will simultaneously vary with the variation of Q_m. As seen from the three-dimensional (3D) plots of the evolution of logarithmic negativity E_N, all of $E_N^{a_1 a_2}$, $E_N^{a_1 c}$, and $E_N^{a_2 c}$ would increase with the increase of the quality factors of the cavity and vibrating mirror mode; that is, the degree of the bipartite entanglement between the cavity fields 1 and 2 as well as between the cavity field 1 (or 2) and the mirror would be strengthened, which means that the tripartite optomechanical entanglement among the two cavity fields and the vibrating mirror mode would be enhanced by increasing the two quality factors Q_m and Q. Though the bipartite optomechanical entanglement between the cavity field 1 (or 2) and the mirror mode would be weakened with the increase of the mirror temperature, the bipartite entanglement between the two cavity fields exhibits strong robustness to the environmental temperature, and even at room or even higher temperature, a high degree of entanglement between the two cavity fields can still be obtained, which provides a convenient way to generate two entangled fields.

To get physical insight into the strong tripartite optomechanical entanglement among the two cavity fields and the mechanical oscillator, it is instructive to consider the interaction between the cavity fields and cavity mirror. As shown in Fig. 1(a), the radiation pressure of the input laser beam, impinging on the mechanical oscillator, produces optomechanical coupling between the vibrational mode and two optical sideband modes (i.e., Stokes mode and anti-Stokes mode in the backscattered field), where the generated Stokes and anti-Stokes modes can be equivalently viewed as scattering the input laser field off the mechanical oscillator, which acts

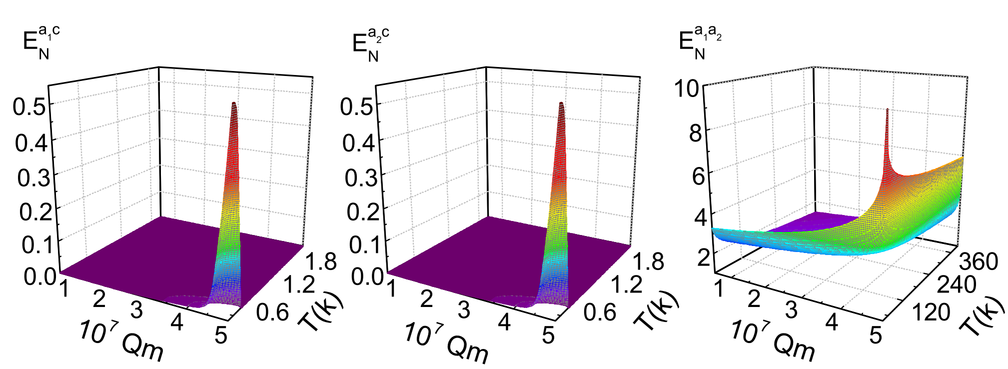

FIG. 3. The 3D plots of the evolution of logarithmic negativity $E_N^{a_1 a_2}$, $E_N^{a_1 c}$, and $E_N^{a_2 c}$ with respect to the environmental temperature and the quality factor Q_m of the vibrating mirror, where $\Delta_1 = -\Delta_2 = -\omega_m$ and we set $k = 100\gamma_m$ (so the quality factor Q of the cavity simultaneously varies with Q_m), and the other parameters are the same as those in Fig. 2.

as a frequency converter with frequency equal to its oscillation frequency. When the input field is tuned to the middle frequency of the two neighboring longitudinal cavity modes, as seen from the relative frequencies of the input field and the two cavity fields in Fig. 1(b), the Stokes and anti-Stokes modes, which just correspond to the two cavity modes 1 and 2, are simultaneously resonant inside the cavity and coherently build up, so strong three-mode resonant interactions would take place; this is analogous to both the one-photon and two-photon resonant excitation in the Λ-type three-level atomic system studied in Ref. [9]. Since every Stokes photon generation is accompanied by absorbing one input field photon and emitting one mirror oscillation phonon, whereas every anti-Stokes photon generation is accompanied by absorbing both one input field photon and one mirror oscillation phonon, the generated two longitudinal cavity modes can be equivalently regarded as the result of the frequency downconversion (or upconversion) process, which has a similar feature as that for generating anti-Stokes-Stokes-atom tripartite entanglement via atomic spin coherence in atomic system in Refs. [7,8]. Therefore, strong tripartite optomechanical entanglement among the two cavity fields and the mechanical oscillator can be established. Note that although similar studies on tripartite optomechanical entanglement among the Stokes and anti-Stokes sidebands of the driving field and the mechanical oscillation mode via radiation pressure force have been reported in Refs. [28,29], the mechanism for generating tripartite entanglement in the present scheme is essentially different from that in Refs. [28,29]. In Refs. [28,29], the effective interaction Hamiltonian can be described by two separate two-mode interactions, where one for generating the Stokes field is a parametric-type interaction resulting in bipartite entanglement between the Stokes field and vibrational mode, and the other for generating the anti-Stokes field is a beam-splitter-type interaction leading to no entanglement between the anti-Stokes field and vibrational mode with classical input laser field. This may be the reason of why the anti-Stokes field is not entangled with the vibrating mirror mode in Refs. [28,29]. However, in our present case, the effective interaction Hamiltonian is described by a three-mode interaction, as shown in Eq. (1), and the entanglements between the two cavity modes as well as between the two cavity modes and the mechanical oscillator can be realized with a single-cavity optomechanical system driven by a single input laser field.

The tripartite optomechanical entanglement can be well understood as well by comparing the above Eqs. (2a)–(2d) to Eqs. (2)–(4) in Ref. [7] for describing the generation of multipartite entanglement via atomic spin coherence in the Λ-type electromagnetically induced transparency (EIT) configuration. As seen clearly from Eqs. (2a)–(2d), the two cavity field modes 1 and 2 are both optomechanically coupled to the mechanical oscillation mode due to radiation pressure, and subsequently interact with each other. If there is no mechanical excitation, then the two cavity modes would have no mutual coupling, and no correlation would exist between the two cavity modes as well as between the two cavity modes and the mechanical oscillator. In this regard, the mechanical excitation plays the same role as the atomic spin coherence for creating multipartite entanglement in the Λ-type atomic system [7,8], which is the origin of the generation of tripartite optomechanical entanglement.

In order to experimentally observe the tripartite optomechanical entanglement, one can use an additional adjacent cavity sharing the common vibrating mirror and a much weaker probe field with frequency tuned also to the midfrequency of the two cavity modes 1 and 2 incident on the second cavity; in this case, the annihilation operators of the two cavity modes a_1' and a_2' of the additional adjacent cavity obey equations analogous to the linearized version of Eqs. (2a)–(2d). As done in Ref. [16], in the frame rotating at the mirror oscillation frequency ω_m and under rotating-wave approximation, the annihilation operators of the cavity modes a_1' and a_2' can be expressed as $\delta\dot{a}_1' = -k\delta a_1' + \frac{ig_0(\alpha_1'+\alpha_2')}{\sqrt{2}}\delta c' + \sqrt{2k}a_{\rm in}'$, and $\delta\dot{a}_2' = -k\delta a_2' + \frac{ig_0(\alpha_1'+\alpha_2')}{\sqrt{2}}\delta c' + \sqrt{2k}a_{\rm in}'$, respectively. Since $k \gg g_0|\alpha_1' + \alpha_2'|/\sqrt{2}$ in this case, the two cavity modes adiabatically follow the vibrating mirror dynamics [16,23], and we get ${a_1'}^{\rm out} = \frac{ig_0(\alpha_1'+\alpha_2')}{\sqrt{k}}\delta c' + a_{\rm in}'$, or ${a_2'}^{\rm out} = \frac{ig_0(\alpha_1'+\alpha_2')}{\sqrt{k}}\delta c' + a_{\rm in}'$. It can be seen that either of the outputs of the additional cavity modes has the same expression as Eq. (15) in Ref. [16], which exactly characterizes the property of the mechanical oscillation mode. Therefore, the quadrature of the oscillation mode of the vibrating mirror can be detected by homodyning either of the outputs of the additional second cavity modes. In the case of the two longitudinal cavity field quadratures, they can be directly measured through homodyne detection of the corresponding cavity outputs.

IV. CONCLUSIONS

In conclusion, we have proposed a simple and convenient way to produce tripartite entanglement among two photon modes and a macroscopic oscillator via radiation pressure force with a single-cavity optomechanical system driven by a single driving laser field. The generated tripartite optomechanical entanglement is quite robust to the variation of the environmental temperature. As the vibrating mirror can couple to a light field of any frequency, the present cavity optomechanical system provides a type of light-light as well as light-matter quantum interface and may find potential applications in quantum information processing and quantum networks.

ACKNOWLEDGMENTS

This work is supported by National Science Foundation of China (Grants No. 11574195, No. 61435007, and No. 11274225), Innovation Action Plan of International Academic Cooperation and Exchange Program of Shanghai Municipal Science and Technology Commission (Grant No. 16520720800), Key Basic Research Program of Shanghai Municipal Science and Technology Commission (Grant No. 14JC1402100), and Shanghai Natural Science Foundation (Grant No. 14ZR1415400). X.Y. would like to thank Wenjie Nie at East China Jiaotong University for helpful discussions.

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