Monte Carlo wave-function description of losses in a one-dimensional Bose gas and cooling to the ground state by quantum feedback

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The effect of atom losses on a homogeneous one-dimensional Bose gas lying within the quasicondensate regime is investigated using a Monte Carlo wave-function approach. The evolution of the system is calculated, conditioned by the loss sequence, namely, the times of individual losses and the position of the removed atoms. We describe the gas within the linearized Bogoliubov approach. For each mode, we find that, for a given quantum trajectory, the state of the system converges towards a coherent state, i.e., the ground state, displaced in phase space. We show that, provided losses are recorded with a temporal and spatially resolved detector, quantum feedback can be implemented and cooling to the ground state of one or several modes can be realized.

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In [1], the effect of atom losses on a one-dimensional quasicondensate was investigated. The authors have shown that, within a linearized approach and for a large enough initial temperature, one expects the temperature of the low-lying modes to decrease in time, in agreement with recent experimental results [2]. The fluctuation induced by the loss process due to the discrete nature of atoms is, however, responsible for a heating, limiting the temperature which can be achieved. More precisely, one expects that the temperature asymptotically converges towards $g\rho(t)$ where g is the coupling constant and ρ is the linear atomic density [1,3]. In particular, excitations in the phononic regime, i.e., of frequency much smaller than $g\rho(t)/\hbar$, never enter the quantum regime: their mean occupation number stays very large such that they lie in the Raighley-Jeans regime. This heating only occurs if one ignores the results of the losses or, equivalently, if one takes the partial trace on the state of the reservoir in which losses occur, ending up with the master equation for the system's density matrix. If, on the other hand, one records the losses, more information is gained on the system and the analysis made in [1] is no longer sufficient.

In this paper, we assume the losses are monitored with a spatially and temporally resolved detector and we describe the evolution of the system using a Monte Carlo wave-function analysis. The measurement back action leads to an evolution of the system conditioned by the result of the loss process, namely, a given history of losses. Averaging over the different possible histories, the results of [1] are recovered. The analysis proposed in this paper, however, not only presents an alternative picture conveying more physical insight but also opens the road to the realization of measurement based quantum feedback: controlled dynamics, conditioned on the monitored losses, allows one to reach lower temperatures. In this paper, we show that feedback on a given mode of the system could in principle allow one to cool this mode to the ground state. In particular we show that phononic excitations can be brought to the quantum regime. State preparation using information inferred from losses has already been used to prepare a well-defined phase between two condensates [4], with a Monte Carlo wave-function approach providing a very clear understanding of the mechanism [5]. Manipulation of cold atomic clouds by quantum feedback has been proposed in many theoretical papers using dispersive light-atom interaction [6,7], while feedback has been implemented for internal degrees of freedom [8–10].

I. DISCRETIZATION OF THE PROBLEM

We consider a one-dimensional Bose gas with contact repulsive interactions of coupling constant g, such that the Hamiltonian writes, in second quantization,

$$H = -\frac{\hbar^2}{2m}\int dx\psi^+\partial^2\psi/\partial x^2 + \frac{g}{2}\int dx\psi^+(x)\psi^+(x)\psi(x)\psi(x).$$

We assume the gas is submitted to atom losses, the loss mechanism being a single atom process described by a loss rate Γ. For instance, magnetically trapped atoms could be submitted to a radio-frequency field that would transfer atoms to an untrapped state, as realized experimentally in [2]. Atoms could also be ionized by laser fields [11], or expelled by collision with fast electrons [12]. We moreover assume the lost atoms are detected one by one with position-resolved detectors, as sketched in Fig. 1. We denote $\bar{\rho}$ the mean linear density and we assume the gas lies within the quasicondensate regime such that the atomic density fluctuations are small and their characteristic length scale, equal to the healing length $l_h = \hbar/\sqrt{mg\bar{\rho}}$, is much larger than the mean interparticle distance [13]. We discretize space in N cells of length δx, containing a large mean atom number $\bar{n} = \delta x\bar{\rho}$ and with small relative fluctuations. We furthermore assume that δx is large enough such that the fluctuations are large compared to unity [14]. The state of the gas may be expanded (as long as one is not interested in length scales smaller than δx) on the Fock basis of each cell:

$$|\psi\rangle = \sum_{n_1,n_2,\ldots,n_N} c_{n_1,n_2,\ldots,n_N}|n_1,n_2,\ldots,n_N\rangle. \quad (1)$$

Time is also discretized in intervals Δt small compared to the time scales involved in the longitudinal dynamics of the gas. This allows one to consider, during Δt, the sole effect of

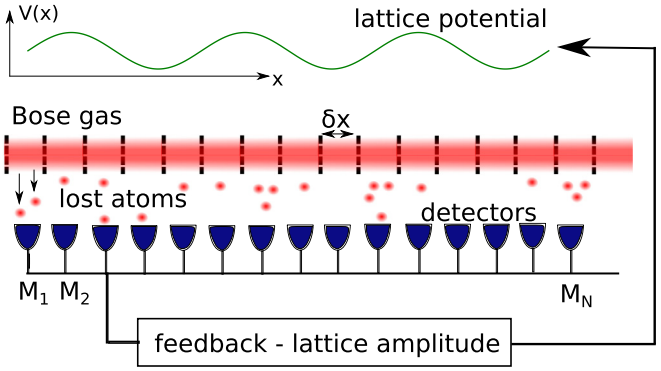

FIG. 1. A one-dimensional Bose gas with atom losses and a spatially resolved single atom detectors system. The information on the atom loss can be used to create a feedback loop on the atoms via a lattice potential. The amplitude of the lattice potential is controlled by a processing unit which uses the information gained from the atom losses.

losses first and then the effect of the free evolution. We will first concentrate on the effect of losses. Since losses do not introduce correlations between different cells, it is relevant to consider the case of a single cell first.

II. MONTE CARLO DESCRIPTION OF LOSSES IN A SINGLE CELL

Considering a single cell, the initial state writes $|\psi(t)\rangle = \sum_n c_n|n\rangle$. Let us split Δt in elementary time steps of length dt, small enough so that the probability to have an atom lost during dt is small. According to the Monte Carlo wave-function procedure [15], if no atoms are detected during a time step dt, then the state of the system evolves according to the non-Hermitian Hamiltonian $H_{\rm eff} = -i\hbar\Gamma a^+ a/2$, which ensures the decrease of the probability of highly occupied states. If on the other hand a lost atom has been detected, the new state is obtained by the application of the jump operator $\hat{a}$, which annihilates an atom in the cell. Let us now assume M atoms have been lost from the cell between time t and time $t+\Delta t$, at times $t_1 < t_2 < \cdots < t_M$, as sketched in Fig. 2. By successively applying the procedure described above, we construct the quantum trajectory followed by the system and we find

$$|\psi(t+\Delta t)\rangle = e^{-iH_{\rm eff}(t+\Delta t-t_M)/\hbar}\hat{a}e^{-iH_{\rm eff}(t_M-t_{M-1})/\hbar} \ldots \hat{a}e^{-iH_{\rm eff}(t_1-t)/\hbar}|\psi(t)\rangle. \quad (2)$$

With the normalization chosen here, the probability of the loss sequence is $(dt\Gamma)^M\langle\psi(t+\Delta t)|\psi(t+\Delta t)\rangle/\langle\psi(t)|\psi(t)\rangle$. From Eq. (2), we find that the Fock state coefficients

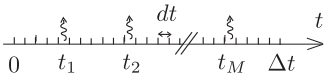

FIG. 2. A typical loss sequence during a time interval Δt, for a single cell. The associated quantum trajectory followed by the system's wave function is given by Eq. (2).

$c_n(t+\Delta t) = \langle n|\psi(t+\Delta t)\rangle$ write

$$c_n(t+\Delta t) = f_{\{t_i\}}(n+M)c_{n+M}(t), \quad (3)$$

where the function $f_{\{t_i\}}(n)$ depends on the loss sequence. Assuming Δt is small enough so that M is much smaller than the mean atom number in the cell, itself much larger than 1, $f_{\{t_i\}}(n)$, for a given M, becomes almost independent on the time sequence $\{t_i\}$ and can be approximated by

$$f_{\{t_i\}}(n) \simeq f_M(n) = n^{M/2}e^{-\Gamma nt/2}. \quad (4)$$

Within this approximation, the probability of the sequence is $\sum_n |c_n|^2(\Gamma n dt)^M e^{-\Gamma n\Delta t}$. Summing over all possible sequences with M lost atoms, we find that the probability to have M lost atoms is $P(M) = \sum_n |c_n|^2(\Gamma n\Delta t)^M e^{-\Gamma n\Delta t}/M!$. For a given initial atom number n, we recover the expected Poissonian distribution. In the limit $\bar{n}\Gamma\Delta t \gg 1$, the typical number of losses is much larger than 1 and the function $f_M(n)$ can be approximated by the Gaussian

$$f_M(n) \simeq \mathcal{A}_M e^{-\frac{[n-M/(\Gamma\Delta t)]^2(\Gamma\Delta t)^2}{4M}}, \quad (5)$$

where $\mathcal{A}_M$ is the normalization factor. Using the fact that the number of lost atoms M is typically equal to $\bar{n}\Gamma\Delta t$ and presents small relative fluctuations, Eq. (5) further approximates to

$$f_M(n) \simeq \mathcal{A}_M e^{-\frac{[n-M/(\Gamma\Delta t)]^2\Gamma\Delta t}{4\bar{n}}}. \quad (6)$$

The same approximations lead to a mean number of lost atoms $\langle M\rangle_{\rm tr} = \langle n\rangle\Gamma\Delta t$, with a variance $\langle M^2\rangle_{\rm tr} - \langle M\rangle^2_{\rm tr} = \Gamma\Delta t\bar{n}$, where the symbol tr indicates that averaging is done here over many different quantum trajectories.

III. GENERALIZATION TO ALL CELLS AND BOGOLIUBOV DECOMPOSITION

The results above can immediately be generalized to the case of several cells. If M_i denotes the number of lost atoms in the cell i, the probability amplitude of the Fock state $|n_1,n_2,\ldots,n_N\rangle$ is, up to a global normalization factor,

$$c_{n_1,n_2,\ldots,n_N}(t+\Delta t) = c_{n_1+M_1,n_2+M_2,\ldots,n_N+M_N}(t)\prod_i e^{-\frac{[n_i-M_i/(\Gamma\Delta t)]^2\Gamma\Delta t}{4\bar{n}}}. \quad (7)$$

Since the atom number per cell is typically very large and presents fluctuations large compared to unity, one can approximate discrete sums on n_i by continuous integrals and treat the n_i as continuous variables.

Since the gas lies in the quasicondensate regime, its Hamiltonian is well approximated by the Bogoliubov Hamiltonian [13]. For a homogeneous system, the Bogoliubov modes are obtained from the Fourier decomposition. More precisely, let us introduce the Fourier quantities

$$n_{k_i,c} = \sqrt{\frac{2}{N}}\sum_j \cos(k_i j\delta x)n_j$$
$$n_{k_i,s} = \sqrt{\frac{2}{N}}\sum_j \sin(k_i j\delta x)n_j\,. \quad (8)$$

Here $k_i = i2\pi/L$, where i is an integer taking values between 1 and $(N-1)/2$. We introduce in the same way the operator $\hat{n}_{k_i,c}$ and $\hat{n}_{k_i,s}$. The Bogoliubov Hamiltonian acts independently on

each Fourier mode and, for a given mode (k,r), where r stands for c or s, it writes, up to a constant term,

$$\hat{H}_{k,r} = A_k \hat{n}_{k,r}^2 + B_k \hat{\theta}_{k,r}^2, \tag{9}$$

where the phase operator $\hat{\theta}_{k,r}$ is the operator conjugated to $\hat{n}_k$ [16], $A_k = [g/2 + \hbar^2 k^2/(8m\bar{\rho})]/\delta x$, $B_k = \hbar k^2 \bar{\rho}\delta x/(2m)$, and the mean particle density $\bar{\rho} = \bar{n}/\delta x$. The frequency of the mode is $\omega_k = 2\sqrt{A_k B_k}/\hbar$.

Let us now investigate the effect of losses in the Bogoliubov basis. The state $|n_1,n_2,\ldots,n_N\rangle$ is also an eigenstate of each operator $\hat{n}_{k_i,r}$, where r stands for c or s, with eigenvalue $n_{k_i,r}$. We thus use the notation $|n_1,n_2,\ldots,n_N\rangle = |\{n_{k_i,r}\}\rangle$, where $\{n_{k_i,r}\}$ is a short notation for $n_{k_1,c},n_{k_1,s},n_{k_2,c},n_{k_2,s}\ldots,n_{k_N,c},n_{k_N,s}$. The state of the system then writes

$$|\psi\rangle = \int \prod_{i,r} dn_{k_i,r} \tilde{c}_{\{n_{k_i,r}\}} |\{n_{k_i,r}\}\rangle, \tag{10}$$

where $\tilde{c}_{\{n_{k_i,r}\}} = c_{n_1,n_2,\ldots,n_N}$. The modification of the state of the system after a time Δt due to atom losses is then, according to Eq. (7),

$$\tilde{c}_{\{n_{k_i,r}\}}(t+\Delta t) = \tilde{c}_{\{n_{k_i,r}+M_{k_i,r}\}}(t) \prod_i e^{-\frac{[n_{k_i}-M_{k_i,r}/(\Gamma\Delta t)]^2 \Gamma\Delta t}{4\bar{n}}}, \tag{11}$$

where $M_{k_i,c} = \sqrt{2/N}\sum_j M_j \cos(k_i x_j)$ and $M_{k_i,s} = \sqrt{2/N}\sum_j M_j \sin(k_i x_j)$. We used the facts that, here, on one hand the variances of each Gaussian in Eq. (7) are all equal and on the other hand the density profiles of Bogoliubov modes are orthogonal, namely, the transformation between the basis n_i and $n_{k,r}$ is orthogonal. The statistics of the different quantum trajectories gives a Gaussian distribution for $M_{k_i,r}$ with $\langle M_{k_i,r}\rangle_{\rm tr} = \Gamma\Delta t\langle n_{k,r}\rangle$ and $\langle M_{k_i,r}^2\rangle_{\rm tr} - \langle M_{k_i,r}\rangle_{\rm tr}^2 = \Gamma\Delta t\bar{n}$. Equation (11) shows that the losses affect each Fourier component, i.e., each Bogoliubov mode, independently.

If the initial state is at thermal equilibrium, different Bogoliubov modes are uncorrelated. The free evolution, under the Bogoliubov Hamiltonian, as well as the effect of losses, do not introduce correlations between modes and one can consider each mode independently. In the following, we consider a given mode of momentum k and we will omit the subscript c or s, since the upcoming considerations apply for both.

IV. EVOLUTION OF A GIVEN BOGOLIUBOV MODE: WIGNER REPRESENTATION

Here we consider a given mode, described by the two conjugate variables n_k and θ_k. A convenient representation of the state of the system is its Wigner function W, a two-dimensional real function, the expression of which, as a function of the density matrix D of the state, is

$$W(n_k,\theta_k) = \frac{1}{2\pi^2}\int da db e^{i(an_k - b\theta_k)} Tr(De^{(-ia\hat{n}_k + ib\hat{\theta}_k)}). \tag{12}$$

The effect of losses during Δt, in the n_k representation, is given by Eq. (11), and this effect transforms W into the new Wigner function function W' according to

$$W'(n_k,\theta_k) = \frac{\Gamma\Delta t}{2\pi^{3/2}\bar{n}}\int d\tilde{\theta} W(n_k + M_k,\tilde{\theta}) \times e^{-\frac{\Gamma\Delta t}{2\bar{n}}[n_k - M_k/(\Gamma\Delta t)]^2} e^{-\frac{2\bar{n}}{\Gamma\Delta t}(\tilde{\theta}-\theta)^2}. \tag{13}$$

The multiplication by a Gaussian function along the n_k axis shifts the distribution towards $M_k/(\Gamma\Delta t)$, the value for which $\langle M_k\rangle_{\rm tr}$ is equal to the recorder value M_k. It also decreases the width in n_k, which reflects the gain of knowledge acquired on n_k by the detection of the number of lost atoms. The associated convolution along the axis θ_k increases the width in θ_k, and ensures preservation of uncertainty relations.

The thermal state of the Bogoliubov Hamiltonian has a Gaussian Wigner function. Since the Gaussian character is preserved by Eq. (13) and by the free evolution, the state of the system stays Gaussian. W is then completely determined by its center $R = (\langle n_k\rangle,\langle\theta_k\rangle)$ and its covariance matrix:

$$C = \begin{pmatrix} \langle n_k^2\rangle - \langle n_k\rangle^2 & \langle n_k\theta_k\rangle - \langle n_k\rangle\langle\theta_k\rangle \\ \langle n_k\theta_k\rangle - \langle n_k\rangle\langle\theta_k\rangle & \langle\theta_k^2\rangle - \langle\theta_k\rangle^2 \end{pmatrix}. \tag{14}$$

As shown in Appendix A, to first order in Δt, the transformation in Eq. (13) changes R and C to R' and C' with

$$C' = C + \frac{\Gamma\Delta t}{\bar{n}}\begin{pmatrix} -C_{11}^2 & -C_{11}C_{12} \\ -C_{11}C_{12} & -C_{12}^2 + \frac{1}{4} \end{pmatrix} \tag{15}$$

and

$$R' = R - \begin{pmatrix} \Gamma\Delta t\langle n_k\rangle \\ 0 \end{pmatrix} - d\xi\begin{pmatrix} 1 - C_{11}/\bar{n} \\ -C_{12}/\bar{n} \end{pmatrix}. \tag{16}$$

Here we introduced $d\xi = M_k - \Gamma\Delta t\langle n_k\rangle$. According to the statistic of trajectories, $d\xi$ is a Gaussian variable centered on zero and of variance $\langle d\xi^2\rangle_{\rm tr} = \Gamma\Delta t\bar{n}$. The above equations account for the evolution of the state under the sole effect of atom losses. One should then implement the evolution under the Hamiltonian (9), which amounts to a simple rotation of the Wigner function in phase space and acts independently on C and R [17]. Finally, one can compute the long term evolution iteratively following the procedure above, knowing, at each time interval Δt, the number of atoms lost in each cell, M_i, from which $d\xi$ is computed.

V. EVOLUTION OF THE CORRELATION MATRIX

Equation (15) shows that the evolution of the correlation matrix is the same for all possible quantum trajectories, and general statements can be made. Let us first consider a very slow mode such that one can ignore the free evolution. Then C_{12} stays at zero during the evolution and time integration of Eq. (15) on long times gives

$$C_{11} \simeq \bar{n}(t)/(1 - e^{-\Gamma t}), \quad C_{22} \simeq (1 - e^{-\Gamma t})/[4\bar{n}(t)] \tag{17}$$

where $\bar{n}(t) = \bar{n}(t=0)e^{-\Gamma t}$ is the time-dependent mean atom number per cell. The system thus goes towards a state of minimal uncertainty, as expected, since more and more information is acquired on the system. Let us now consider the other limit of a mode of very high frequency. Then the free evolution of the system ensures, at any time, $C_{12} \simeq 0$ and the equipartition of the energy between the two degrees

of freedom. Thus $A_k C_{11} \simeq B_k C_{22} \simeq \langle E_c \rangle/2$ where $\langle E_c \rangle = A_k C_{11} + B_k C_{22}$ is the contribution of the correlation matrix to the energy. We then find

$$\frac{d[\langle E_c \rangle/(\hbar\omega_k)]}{dt} = \Gamma_{\rm eff}\{-[\langle E_c \rangle/(\hbar\omega_k)]^2 + 1/4\}, \tag{18}$$

where $\Gamma_{\rm eff} = \Gamma/\sqrt{1 + 4mg\bar{\rho}/(\hbar^2 k^2)}$. $\Gamma_{\rm eff}$ depends on time via the exponential decrease of $\bar{\rho}$ due to losses. At long times, $\langle E_c \rangle/(\hbar\omega_k)$ goes to 1/2, such that the state of the system, as long as only the matrix C is concerned, evolves towards the ground state. If one assumes the excitation is initially in the phononic regime, however, we show in Appendix B that $\langle E_c \rangle$ approaches $\hbar\omega_k$ only once the decrease of $\bar{\rho}$ has already promoted the excitation to the particle regime. Thus phononic excitations cannot reach the quantum regime. The situation is different if the decrease of $\bar{\rho}$ is compensated by the following time dependence of g:

$$g(t) = g(t=0)e^{\Gamma t}. \tag{19}$$

Then $\Gamma_{\rm eff}$ and ω_k are constant and an excitation lying in the phononic regime stays in the phononic regime during the whole loss process and, as long as the C matrix is concerned, is cooled to the ground state. In the following, we will assume g is modified according to Eq. (19).

VI. AVERAGING OVER TRAJECTORIES

If the loss events are not recorded, then only the quantities averaged over all possible trajectories are meaningful. If the Wigner distribution is initially centered around zero, it will stay centered at zero. Let us investigate its evolution over a time Δt. For a given quantum trajectory, i.e., a given value $d\xi$, the losses modify the correlation matrix according to Eq. (15) as well as the center R, which acquires the nonzero value $R = -d\xi(1 - C_{11}/\bar{n}; -C_{12}/\bar{n})$. One then has

$$\langle n_k^2 \rangle_{\rm st}(t+\Delta t) = \langle n_k^2 \rangle(t) - \Gamma\Delta t/\bar{n} C_{11}^2 + d\xi^2\left(1 - \frac{\langle n_k^2 \rangle}{\bar{n}}\right)^2 \tag{20}$$

and

$$\langle \theta_k^2 \rangle_{\rm st}(t+\Delta t) = \langle \theta_k^2 \rangle(t) + \Gamma\Delta t/\bar{n}\left(1/4 - C_{12}^2\right) + (d\xi C_{12}/\bar{n})^2, \tag{21}$$

where the subscript st specifies that this holds for a single trajectory. Averaging over all possible trajectories, we then find, using $\langle d\xi^2 \rangle_{\rm tr} = \Gamma\Delta t\bar{n}$, that losses modify the variances according to

$$\langle n_k^2 \rangle(t+\Delta t) - \langle n_k^2 \rangle(t) = -2\Gamma\Delta t\langle n_k^2 \rangle(t) + \Gamma\Delta t\bar{n}(t) \tag{22}$$

and

$$\langle \theta_k^2 \rangle(t+\Delta t) - \langle \theta_k^2 \rangle(t) = \Gamma\Delta t/(4\bar{n}). \tag{23}$$

As expected, Eqs. (22) and (23) are equal to those obtained using a master equation description of the loss process [1,18]. Due to the diffusive process experienced by R, an increased rate in both equations limits the decrease of the mode energy. For phonons, and assuming the loss rate is small compared to the mode frequency, we show in Appendix C that the temperature asymptotically goes towards $g\bar{\rho}(t)/2$.

This value of the asymptotic temperature T_∞ is particular to the case of a homogeneous gas with a coupling constant evolving according to Eq. (19). For constant g the asymptotic temperature is $T_\infty = g\bar{\rho}$ [1,18]. For a gas trapped in a harmonic potential we expect T_∞ to scale as $g\rho_p$, where ρ_p is the peak density. The proportionality factor has not been derived yet, but since the averaged density is smaller than ρ_p, one naively expects that T_∞ is smaller than $g\rho_p$. Experimentally T_∞ has not been identified, while temperatures as low as $0.25\, g\rho_p$ have been reported for a harmonically confined gas [2].

VII. USING INFORMATION RETRIEVED FROM LOSSES DETECTION: QUANTUM FEEDBACK

If the losses are recorded, such that at each time interval Δt the values M_j are recorded, the trajectory followed by the center of the Wigner distribution, R, can be computed exactly, and the heating associated to the diffusion process seen in Eqs. (23) and (22) can be compensated for. One strategy is to perform, during the whole time evolution, a quantum feedback on the system, based on the knowledge acquired via the atom losses, in order to prevent the center of the Wigner distribution from drifting away from the phase-space center. Let us here, as an illustration, assume one is interested in a given mode k,c. The most simple back action is to submit the atomic cloud to a potential $V(x) = A(t)\cos(kx)$, where the computed amplitude $A(t)$ depends on the recorded history of the losses. Such a potential could be realized, for instance, using the dipole potential experienced by atoms in a laser field. The cosine modulation of the laser intensity can be realized using an optical lattice, or using a spatial light modulator. The contribution to the Hamiltonian of this potential is $\hat{H}_{\rm fb} = A(t)\sqrt{N/2}\hat{n}_{k,c}$. In order to counteract the diffusion process of R due to the loss process, one could adjust $A(t)$ such that the feedback Hamiltonian is

$$\hat{H}_{\rm fb} = -\hbar\nu\langle\theta_{k,c}\rangle\hat{n}_{k,c} \tag{24}$$

where, at each time interval, $\langle\theta_{k,c}\rangle$ is computed by integrating the equations of motion including the effect of losses, the free evolution, and the feedback process. This Hamiltonian acts as an active damping, the damping rate ν preventing θ_k from drifting far from the phase-space center. The free-evolution Hamiltonian, by coupling the two degrees of freedom, will ensure that neither θ_k nor n_k drifts away. For a large enough damping rate ν, the contribution of R to the energy of the mode is expected to be negligible compared to the contribution of the covariance matrix C and, according to Eq. (18), one expects to reach the ground state. We present numerical results illustrating such a scenario below.

Before presenting numerical results, let us identify the relevant quantities governing the dynamics. Introducing the reduced variable $\tilde{n}_k = \bar{n}^{-1/2} n_k$ and $\tilde{\theta}_k = \bar{n}^{1/2}\theta_k$, as shown in Appendix D, we find as expected that the cell size δx drops out of the problem and, provided time is rescaled by $1/\Gamma$, the dynamics of the mode of wave vector k is solely governed by the dimensionless parameters ω_k/Γ, $\hbar^2k^2/(mg\bar{\rho})$ and ν/Γ. The relevant measurement signal, for the time interval Δt, is then $\tilde{M}_k = \sqrt{2/N_{\rm at}}\int dx m(x)\cos(kx)$, where $N_{\rm at}$ is the total atom number and $m(x)$ is the number of lost atoms per unit length. Figure 3 shows the phase-space evolution of a

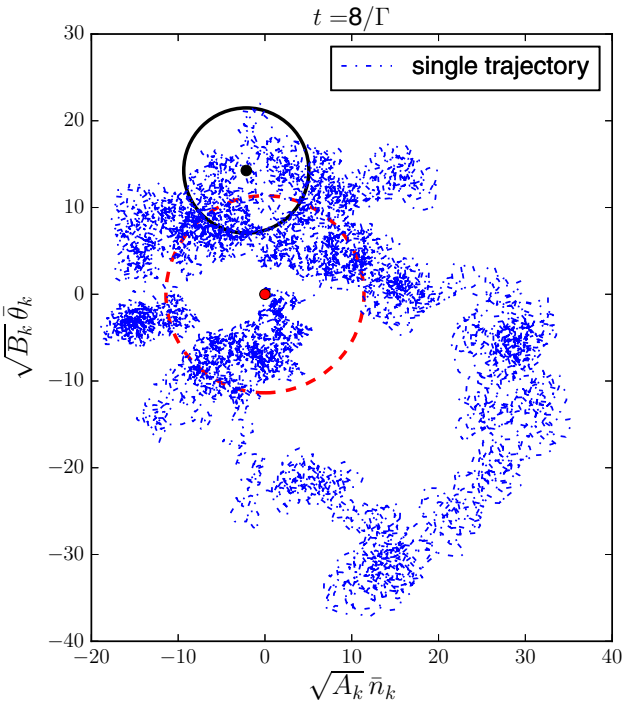

FIG. 3. Evolution of the phase-space distribution for a single trajectory from time $t = 0$ to $8/\Gamma$ in the absence of feedback. The mode $k = 0.1\sqrt{mg\bar{\rho}}/\hbar$ is considered, with the initial temperature $T_i = 1.1g\bar{\rho}$ and a loss rate $\Gamma = \omega_k/400$. Scattered blue points give the evolution of the center R. We verified that C_{12} stays small while, at any time, $A_kC_{11} \simeq B_kC_{22} \simeq E_c/2$, as expected for such a large ω_k. The black solid circle, of radius $\sqrt{E_C}$, represents the final rms width of the Wigner function. For comparison, the red dashed circle, of radius $\sqrt{E}$, where E is the energy obtained after averaging over 100 trajectories, gives the rms width of the averaged phase-space distribution. The coordinates are given in the frame rotating according to the free evolution: namely, the plotted quantity is $\bar{R} = (\bar{n}_k, \bar{\theta}_k) = \mathfrak{R}^{-1}(\omega_k t)R$ [17].

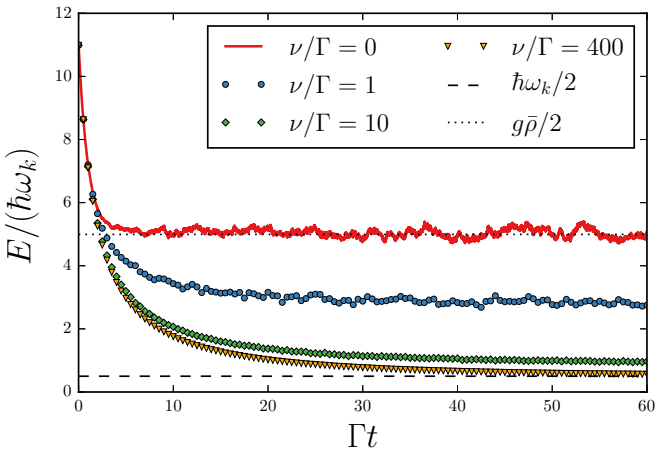

FIG. 4. Simulation of the loss process for different feedback strengths ν. Plotted is the time evolution of the energy in the mode k, averaged over 500 quantum trajectories, the parameters being those of Fig. 3. Without feedback the energy converges to $g\bar{\rho}/2$ (horizontal dotted line); lower energies are obtained with feedback, and the ground state, of energy $\hbar\omega_k/2$ (horizontal dashed line), is reached for large enough ν.

single quantum trajectory, for a mode lying in the phononic regime, in the absence of quantum feedback. Figure 4 shows the time evolution of the energy in this mode, averaged over quantum trajectories, both in the absence and in the presence of feedback. In the absence of feedback, the energy converges towards the expected value $g\rho/2$. If the feedback scheme is implemented, we observe that the energy in the mode reaches much smaller values. For a large feedback strength ν, the drift of the center is almost completely prohibited and the mode is cooled to its ground state.

VIII. DISCUSSION

In conclusion, we proposed a description of the effect of losses in a many-body system through a Monte Carlo wave-function approach, and we showed that quantum feedback by monitoring losses could be used to cool down selected modes of a quasicondensate to vanishing temperatures. This work could be extended in many directions. In view of practical implementation, the sensitivity of the feedback mechanism on the exact knowledge of the system parameters should be investigated. Assuming, as is done in this paper, the system parameters are known exactly, the larger the feedback strength the better the cooling. In the presence of uncertainties, a too large feedback strength will induce heating as it will not match the exact dynamics. Additionally, in most experimental situations the quasicondensates, trapped in a shallow longitudinal potential, are nonhomogeneous. Then, the effect of losses depends on the spatial coordinate. Moreover, the linearized description should use, instead of the sinusoidal modes, the spatial density profiles of the Bogoliubov modes, which are not necessarily orthogonal. These issues complicate the picture. Losses might then induce correlations between modes [7]. Another concern is the coupling between modes, which exists beyond the linearized approach considered here. Such coupling is present, for instance, in the Gross-Pitaevskii equation, which is a classical field approximation of the Lieb-Liniger model. However, long-lived nonthermal states with different Bogoliubov modes experiencing long lifetimes [19] have been reported, which indicates small coupling between modes and the possibility to cool down a particular Bogoliubov mode. Finally, note that this cooling process is not limited to one-dimensional systems.

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APPENDIX A: EFFECT OF LOSSES ON THE WIGNER REPRESENTATION

Here we consider a given mode and we will omit the subscripts k, r to make our notations lighter. We also introduce $\sigma^2 = \bar{n}/(\Gamma\Delta t)$ and $q_0 = M_k/(\Gamma t)$. Equation (12) writes, in

representation n,

$$W(n,\theta) = \frac{1}{\pi}\int du \langle u+n|D|u-n\rangle e^{-2iu\theta}. \tag{A1}$$

The effect of losses given by Eq. (11) transforms the Wigner function of the mode to

$$W'(n+M_k,\theta) = \frac{1}{2\pi^2\sigma^2}\int du \langle u+n|D|u-n\rangle e^{-2iu\theta} \times e^{-(n-q_0+u)^2/(4\sigma^2)} e^{-(n-q_0-u)^2/(4\sigma^2)}. \tag{A2}$$

Injecting $\langle u+n|D|u-n\rangle = \int d\tilde{\theta} W(n,\tilde{\theta}) e^{i2\tilde{\theta}u}$, we then find Eq. (13).

Let us now consider a Gaussian state. Its Wigner function writes

$$W(n,\theta) = \frac{1}{2\pi\sqrt{det(C)}} e^{-\frac{1}{2}[(X-R)^t B(X-R)]} \tag{A3}$$

where $X = \binom{n}{\theta}$, R is the center of the distribution, C is the covariant matrix, and $B = C^{-1}$. The transformation in Eq. (13) transforms the Gaussian state into a new Gaussian state centered on R' and of covariance C'. The convolution on the axis θ does not change R and changes C in $\tilde{C}$ according to

$$\tilde{C} = C + \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{4\sigma^2} \end{pmatrix}. \tag{A4}$$

Let us now consider the effect of the multiplication of W by $e^{-\frac{1}{2\sigma^2}(n-q_0)^2}$, as well as the shift along n by M_k. From Eq. (A3), we find

$$B' = \begin{pmatrix} 1/\sigma^2 & 0 \\ 0 & 0 \end{pmatrix} + \tilde{B} \tag{A5}$$

and

$$\tilde{B}R + \frac{q_0}{\sigma^2}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = B'\left[R' + \begin{pmatrix} M_k \\ 0 \end{pmatrix}\right] \tag{A6}$$

where $\tilde{B} = \tilde{C}^{-1}$ and $B' = C'^{-1}$. From Eq. (A5), we obtain

$$C' = \left[Id + \tilde{C}\begin{pmatrix} 1/\sigma^2 & 0 \\ 0 & 0 \end{pmatrix}\right]^{-1} C_2. \tag{A7}$$

Injecting $\sigma = \sqrt{\bar{n}/(\Gamma\Delta t)}$ and expanding to first order in Δt, one gets

$$C' \simeq \left[Id - \frac{\Gamma\Delta t}{\bar{n}}\begin{pmatrix} C_{11} & 0 \\ C_{12} & 0 \end{pmatrix}\right]\tilde{C}. \tag{A8}$$

Here we used the fact that $\tilde{C}_{11} = C_{11}$ and $\tilde{C}_{12} = C_{12}$. This equation also takes the form of Eq. (15). Let us now consider the center of the distribution. Multiplying the left- and right-hand parts of Eq. (A6) by C' and injecting Eq. (A8), we deduce

$$R' = \left[Id - \frac{\Gamma\Delta t}{\bar{n}}\begin{pmatrix} C_{11} & 0 \\ C_{12} & 0 \end{pmatrix}\right]\left[R + \frac{M_k}{\bar{n}}\begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix}\right] - \begin{pmatrix} M_k \\ 0 \end{pmatrix}. \tag{A9}$$

Neglecting terms beyond first order in Δt, we obtain

$$R' = R + \left(\frac{M_k}{\bar{n}} - \frac{\Gamma\Delta t}{\bar{n}}\langle n_k\rangle\right)\begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} - \begin{pmatrix} M_k \\ 0 \end{pmatrix}. \tag{A10}$$

Injecting $M_k = \Gamma\Delta\langle n_k\rangle + d\xi$, we recover Eq. (16).

APPENDIX B: EVOLUTION OF E_C FOR CONSTANT g

We assume $\omega_k \gg \Gamma$ such that Eq. (18) is valid. Note that the condition $\omega_k \gg \Gamma$ also ensures adiabatic following, namely, the time evolution of the Hamiltonian parameters A_k and B_k preserves the ratio $E_C/(\hbar\omega_k)$, such that Eq. (18) holds both for a constant g and a time-varying g. Let us introduce the variable $y = \frac{\langle E_C\rangle}{g\bar{\rho}}$ and rewrite Eq. (18) in the form

$$y'(t) = y\Gamma\left\{1 - \frac{g\rho}{2g\bar{\rho} + \hbar^2k^2/(2m)}(1+y)\right\} + \frac{\Gamma\hbar^2k^2}{8mg\bar{\rho}}. \tag{B1}$$

For $y = 1$ we see that $y'(t) \geqslant 0$ and therefore $y(t)$ has to be an increasing function at $y = 1$. It follows that for all initial conditions $y(0) \geqslant 1$ the energy E_C stays greater than $g\bar{\rho}$. This implies in particular that, as long as an excitation stays in the phononic regime (i.e., its frequency stays much smaller than $g\bar{\rho}/\hbar$), it stays in the high-temperature regime, namely, $E_C/(\hbar\omega_k) \gg 1$.

APPENDIX C: ASYMPTOTIC TEMPERATURE FOR NONRECORDED LOSSES

We consider a mode k (we omit the index r for simplicity) and we assume averaging is done over trajectories. Then evolution of the variances of n_k and θ_k due to the loss process is given in Eqs. (22) and (23). Let us consider the quantity $\tilde{E} = \langle H_k\rangle/(\hbar\omega_k)$. We assume the loss rate is small enough so that the free evolution under the Hamiltonian (9) ensures equipartition of the energy between the two quadratures, namely, at each time $A_k\langle n_k^2\rangle = B_k\langle\theta_k^2\rangle = E/2$. Note that this is equivalent to the condition of adiabatic following. Then the modification of $\tilde{E}$ under the loss process is

$$\frac{1}{\Gamma}\frac{d\tilde{E}}{dt} = -\tilde{E} + (K + 1/K)/4 \tag{C1}$$

where $K = 4\bar{n}A_k/\omega_k = 2\bar{n}\sqrt{A_k/B_k}$. The evolution under the Hamiltonian (9), provided the adiabatic following condition is satisfied, does not modify $\tilde{E}$. Thus Eq. (C1) gives the total time evolution of $\tilde{E}$, and it is valid both when A_k and B_k depend on time and when they do not depend on time. In this paper, we consider the situation given by Eq. (19), where the exponential decrease of $\bar{\rho}$ is compensated by a time dependence of g such that K is time independent. Then Eq. (C1) evolves at long

times to

$$\tilde{E} \underset{t\to\infty}{\to} (K + 1/K)/4. \tag{C2}$$

For phononic modes, for which $k^2 \ll g\rho$, one has $K \simeq 2\sqrt{g\rho}/k$. Then $\tilde{E}$ goes to $\sqrt{g\rho}/(2k)$ at long times, which gives

$$E \underset{t\to\infty}{\to} \frac{1}{2} g\rho. \tag{C3}$$

This energy is very large compared to ω_k. Thus the excitation lies in the high-temperature limit and its temperature is $T \simeq E \simeq g\rho/2$. Note that, in the case g is constant, then K depends on time and solving Eq. (C1) with the time-dependant value of K gives that E converges to $g\bar{\rho} = g\bar{\rho}_0 e^{-\Gamma t}$, as derived in [1].

APPENDIX D: EQUATIONS IN REDUCED VARIABLES

Here we derive the evolution equations for the reduced variables $\tilde{n}_k = n_0^{-1/2} n_k$ and $\tilde{\theta}_k = n_0^{1/2}\theta_k$. We denote $\tilde{R}$ and $\tilde{C}$ the associated mean vector and covariant matrix. Taking into account the exponential decrease of n_0, Eqs. (15) and (16) give

$$\tilde{C}' = \tilde{C} + \Gamma\Delta t \begin{pmatrix} -\tilde{C}_{11}^2 + \tilde{C}_{11} & -\tilde{C}_{11}\tilde{C}_{12} \\ -\tilde{C}_{11}\tilde{C}_{12} & -\tilde{C}_{12}^2 + \frac{1}{4} - \tilde{C}_{22} \end{pmatrix} \tag{D1}$$

and

$$\tilde{R}' = \tilde{R} + d\tilde{\xi} \begin{pmatrix} \tilde{C}_{11} - 1 \\ \tilde{C}_{12} \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 3\Gamma\Delta t\langle\tilde{n}_k\rangle \\ -\Gamma\Delta t\langle\tilde{\theta}_k\rangle \end{pmatrix}. \tag{D2}$$

Here $d\tilde{\xi} = \tilde{M}_k - \Gamma\Delta t\langle\tilde{n}_k\rangle$ where $\tilde{M}_k = M_k/\sqrt{n_0}$. The statistic of trajectory implies that $\tilde{M}_k$ follows a Gaussian statistic with $\langle\tilde{M}_k\rangle = \Gamma\Delta t\langle\tilde{n}_k\rangle$ and $\mathrm{Var}\tilde{\mathrm{M}}_\mathrm{k} = \Gamma\delta\mathrm{t}$.

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