Photodetachment of hydrogen negative ions in bichromatic oscillating electric fields

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The photodetachment of a hydrogen negative ion in the bichromatic oscillating electric field is investigated using the time-dependent closed orbit theory. The photodetachment cross section is specifically calculated in the bichromatic oscillating electric field with a 1:2 frequency ratio and various phase differences. It is found that the oscillatory structure appears in the photodetachment cross section, which depends sensitively on the strength and the phase difference of the bichromatic oscillating electric field. The time-dependent photodetachment cross section is nearly unaffected by a weak bichromatic oscillating electric field. However, with the increase of the oscillating electric field strength, four different types of the detached electron's closed orbits are identified, which makes the photodetachment cross section oscillate in a much more complicated way. In addition, the influence of the phase difference in the bichromatic oscillating electric field on the photodetachment cross section is also investigated quantitatively. The formula we put forward for the time-dependent photodetachment cross section is universal and can be used to calculate the cross section for other frequency ratios in the bichromatic oscillating electric field. On account of its generality, our work provides a clear and intuitive picture for the photodetachment processes of a negative ion in the bichromatic oscillating electric field, and may guide future experimental research on the photodetachment dynamics in the oscillating electric field.

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I. INTRODUCTION

With the development of strong field physics, photodetachment of negative ions in the external fields has undergone significant improvements and has opened the way to a wide range of research not accessible previously [1–12]. For the electron photodetached from negative ions, some theoretical studies have been carried out to investigate the oscillatory structure in the photodetachment cross section. Among these theoretical studies, the semiclassical closed orbit theory provides a clear physical description for the photodetachment process of negative ion in the external fields [13,14]. This theory gives very reliable results for the photodetachment cross section of the H^- ion in the electric field and is consistent with that of the quantum mechanical result or the experimental result [4,2,1]. Therefore, many studies have used this theory to study the photodetachment process of the H^- ion in other external fields, such as in a gradient electric field, in parallel electric and magnetic fields, in crossed electric and magnetic fields, etc. [15–20]. In all these previous studies, the external fields are time independent. As to the photodetachment process of the negative ion in the time-dependent external fields, the studies are relatively few. In the 1990s, Kuchiev and Ostrovsky studied the electron detachment from negative ions in a bichromatic laser field [21,22]. Spellmeyer *et al.* studied the photoabsorption spectra of a Rydberg atom in a static electric field plus an oscillating field [23,24]. Recently, Yang and Robicheaux calculated the photodetachment rate of H^- and F^- ions in the presence of a single-cycle terahertz pulse by extending the semiclassical closed orbit theory from the static electric field to the time-dependent situation [25,26]. They found their closed orbit theory result agrees well with the exact quantum simulations, which suggests the correctness of the time-dependent closed orbit theory. Inspired by their study, in this work, we investigate the photodetachment of the H^- ion in a bichromatic oscillating electric field based on the time-dependent closed orbit theory. An analytical formula for calculating the time-dependent photodetachment cross section of this system is put forward and calculations have been carried out for the case of a frequency ratio of 1:2. Especially, we investigate the influences of the strength and phase difference in the bichromatic oscillating electric field on the photodetachment cross section. Our work provides an intuitive understanding for the photodetachment dynamics of negative ions in the oscillating electric field from a time-dependent viewpoint, and may guide future experimental research on the photodetachment dynamics of a negative ion in the time-dependent electric field.

In the following section, we give the Hamiltonian and obtain the classical motion equations for the detached electron in the bichromatic oscillating electric field. In Sec. III, the time-dependent closed orbit theory for the photodetachment cross section is briefly summarized. Four different types of the detached electron's closed orbits are identified. In addition, the general formula for calculating the time-dependent photodetachment cross section is presented. Some calculations and discussions of the photodetachment cross section for different electric field strength and various phase difference in the bichromatic oscillating electric field are given in Sec. IV, followed by a brief summary of this paper in Sec. V. Atomic units (which are abbreviated as "a.u.") are used throughout this work unless indicated otherwise.

II. HAMILTONIAN

Assuming the H^- ion is interacting simultaneously with a weak laser field and a time-dependent electric field, which is constituted by a static electric field plus a bichromatic oscillating electric field. Both the weak laser field and the time-dependent electric field are pointing along the z axis. The specific envelope of the weak laser field has the following

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form [25,26]:

$$f_L(t) = \frac{1}{2}\left[\tanh\left(\frac{t-t_u}{t_L}\right) - \tanh\left(\frac{t-t_d}{t_L}\right)\right], \quad (1)$$

where t_u, t_d and t_L are the laser field parameters, $t_d = -t_u > 0$.

The time-dependent electric field is given by $F(t) = F_0 + F_{bo}(t)$, where F_0 is the static electric field and $F_{bo}(t)$ is a bichromatic oscillating electric field. In this work, we only consider the bichromatic oscillating electric field with a 1:2 frequency ratio, which is defined as follows:

$$F_{bo}(t) = F_0[\sin(\omega t) + \sin(2\omega t + \phi)]. \quad (2)$$

Here ω is the fundamental frequency of the oscillating electric field and ϕ is the phase difference of the bichromatic oscillating electric field.

As in the previous studies [4], the H^- ion is regarded as a single-electron system loosely bound by a short-range potential of the hydrogen atom. After the electron is photodetached from the hydrogen atom by laser light, the short-range potential can be neglected. Therefore, the Hamiltonian governing the motion of the detached electron in the time-dependent electric field is as follows (using cylindrical coordinates):

$$\begin{aligned} H(\rho,z,t,p_\rho,p_z,p_t) = {} & \tfrac{1}{2}p_\rho^2 + \tfrac{1}{2}p_z^2 + F_0 z \\ & + F_0[\sin(\omega t) + \sin(2\omega t + \phi)]z \\ & + p_t, \end{aligned} \quad (3)$$

where p_t is a conjugate momentum corresponding to the classical dynamical variable t. In order to solve the Hamiltonian canonical equations, we introduce an "evolution time" τ to describe the electron motion in the time-dependent electric field, $\tau = t - t_i$. Here, t_i is the initial time of the electron trajectory and t denotes the real, laboratory time. In addition, we add two motion equations related to the classical dynamical variable t and its conjugate momentum p_t: $dt/d\tau = 1$, $dp_t/d\tau = -\partial H/\partial t$.

Supposing the initial energy of the detached electron is E, its initial momentum is k_0; $k_0 = \sqrt{2E}$. The initial outgoing angle of the detached electron relative to the $+z$ axis is θ_i. By solving the Hamiltonian motion equations with the initial conditions $\rho(\tau = 0) = 0$, $z(\tau = 0) = 0$, $p_\rho(\tau = 0) = k_0 \sin\theta_i$, $p_z(\tau = 0) = k_0 \cos\theta_i$, $t(\tau = 0) = t_i$, and $p_t(\tau = 0) = -E$, we get the classical motion equations of the detached electron:

$$\begin{aligned} \rho(t) &= k_0 \sin\theta_i (t - t_i), \\ z(t) &= \left[k_0 \cos\theta_i + F_0 t_i - \frac{F_0}{\omega}\cos(\omega t_i) \right. \\ &\quad \left. - \frac{F_0}{2\omega}\cos(2\omega t_i + \phi)\right](t - t_i) \\ &\quad + \frac{F_0}{\omega^2}[\sin(\omega t) - \sin(\omega t_i)] \\ &\quad + \frac{F_0}{4\omega^2}[\sin(2\omega t + \phi) - \sin(2\omega t_i + \phi)] \\ &\quad - \frac{F_0}{2}\left(t^2 - t_i^2\right). \end{aligned} \quad (4)$$

III. CLOSED ORBIT THEORY FOR THE TIME-DEPENDENT PHOTODETACHMENT CROSS SECTION

Following the general picture depicted by the closed orbit theory [26], after the electron is photodetached from the hydrogen atom, a steady stream of outgoing electron waves with a fixed energy is produced. These waves are propagating along the classical trajectories. If the external electric field is strong enough, the detached electron wave can be driven back to the atom. This kind of electron trajectory is called the closed orbit. The interference between the returning wave with the outgoing wave produces oscillations in the photodetachment cross section. Each closed orbit of the detached electron corresponds to one sinusoidal term in the total photodetachment cross section.

A. Closed orbits search

From Eq. (4), we find only the electron emitted from the origin with the initial outgoing angle $\theta_i = 0$ or $\theta_i = \pi$ can be drawn back to the origin to form a closed orbit. Let $z(t) = 0$; we get

$$\begin{aligned} &\left[k_0 \cos\theta_i + F_0 t_i - \frac{F_0}{\omega}\cos(\omega t_i) - \frac{F_0}{2\omega}\cos(2\omega t_i + \phi)\right](t - t_i) \\ &\quad + \frac{F_0}{\omega^2}[\sin(\omega t) - \sin(\omega t_i)] \\ &\quad + \frac{F_0}{4\omega^2}[\sin(2\omega t + \phi) - \sin(2\omega t_i + \phi)] - \frac{F_0}{2}\left(t^2 - t_i^2\right) = 0. \end{aligned} \quad (5)$$

By solving the above equation, we can obtain the initial time t_i and the returning time t for each closed orbit. Through numerical calculation, we find four types of closed orbits exist for the detached electron in the bichromatic oscillating electric field. These four types of closed orbit can be named according to their outgoing angle and returning direction. The first type of closed orbit starts with the initial outgoing angle $\theta_i = 0$, and returns with the returning angle $\theta_{\text{ret}} = \pi$. This type of closed orbit is called the up orbit. The second type of orbit is called the down orbit, which starts with the initial outgoing angle $\theta_i = \pi$, and returns with the returning angle $\theta_{\text{ret}} = 0$. The third type of closed orbit can be considered as a combination of the up orbit and the down orbit. Both the initial outgoing angle and the returning angle are equal to zero; $\theta_i = \theta_{\text{ret}} = 0$. We call this type of closed orbit the up-down orbit. The fourth type of closed orbit is called the down-up orbit, which is similar to the up-down orbit, but in reverse order; $\theta_i = \theta_{\text{ret}} = \pi$. Figure 1 shows the initial time t_i of each type of closed orbit as a function of the corresponding returning time t. The t–t_i curve for different types of closed orbit is distinguished by different lines. The graphic demonstrations of these four types of closed orbit for the detached electron in the bichromatic oscillating electric field are shown in Fig. 2.

B. Time-dependent photodetachment cross section

According to closed orbit theory, the photodetachment cross section for the negative ion in the time-dependent electric field

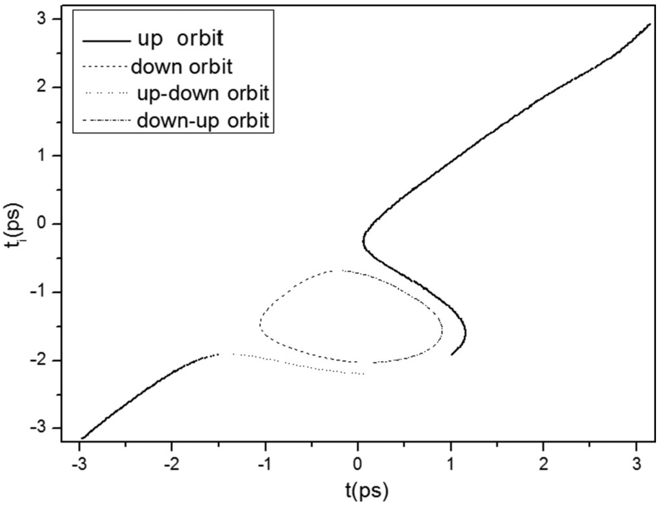

FIG. 1. The $t - t_i$ curve for the detached electron's closed orbit in the bichromatic oscillating electric field. The photon energy is $E_p = 1.0\,\text{eV}$; the strength of the electric field is $F_0 = 200\,\text{kV/cm}$; the frequency of the bichromatic oscillating electric field is 2.42×10^{-5} and 4.84×10^{-5} a.u., respectively. Different types of closed orbits are distinguished by different lines.

can be written as [27]

$$\sigma(E,t) = \sigma_0(E,t) + \sigma_{\text{osc}}(E,t), \tag{6}$$

where $\sigma_0(E,t) = \sigma_0(E) f_L^2(t)$. $f_L(t)$ is the laser field given by Eq. (1). $\sigma_0(E) = \frac{16\sqrt{2}B^2\pi^2 E^{3/2}}{3cE_p^3}$ is the photodetachment cross section of the H^- ion without the external electric field, c is the speed of light, and E_p denotes the photon energy, $E_p = E + E_b$. E is the energy of the detached electron and E_b is the binding energy of the H^- ion, $E_b = 0.754\,\text{eV}$. $B = 0.315\,52$ is a normalization constant, which is related to the initial bound state wave function of the H^- ion: $\phi_i(r) = Be^{-k_b r}/r$ with $k_b = \sqrt{2E_b}$. The detailed derivation of the value of B can be found in Ref. [3].$\sigma_{\text{osc}}(E,t)$ is the oscillating part of the cross section related to the detached electron's closed orbits:

$$\sigma_{\text{osc}}(E,t) = -\frac{4\pi E_p}{c}\text{Im}\langle I(t)|\psi_{\text{ret}}(\boldsymbol{r},t)\rangle, \tag{7}$$

in which $I(t)$ describes the interaction of the H^- ion with the laser field [24]:

$$I(t) = f_L(t)e^{-iEt}D\phi_i, \tag{8}$$

where D is the dipole operator. Suppose the laser light is polarized along the z axis; then $D = Z$.

$\psi_{\text{ret}}(\boldsymbol{r},t)$ is the returning electron wave function, where $\boldsymbol{r}$ is the position vector of the electron relative to the origin. $\psi_{\text{ret}}(\boldsymbol{r},t)$ can be written as

$$\psi_{\text{ret}}(\boldsymbol{r},t) = \sum_v \psi_{\text{ret}}^v(\boldsymbol{r},t), \tag{9}$$

where the sum includes all the electron's closed orbits in the bichromatic oscillating electric field. Each closed orbit is denoted by the symbol v, $v = 1, 2, 3$. $\psi_{\text{ret}}^v(\boldsymbol{r},t)$ corresponds to the returning electron wave function traveling along the vth closed orbit of the detached electron, which can be constructed using the semiclassical approximation method:

$$\psi_{\text{ret}}^v(\boldsymbol{r},t) = f_L(t_i)\psi_{\text{out}}(R,\theta_i,\varphi_i)A_v \times \exp\,[i(S_v - Et_i - \lambda_v\pi/2)]. \tag{10}$$

Here, $f_L(t_i)$ is the laser field at the initial instant t_i. $\psi_{\text{out}}(R,\theta_i,\varphi_i)$ is the initial outgoing electron wave function from the negative ion. A_v is the amplitude; S_v is the action along the vth trajectory. λ_v is the Maslov index. In order to get the returning wave function, we draw a small spherical surface with radius R around the negative ion, $R \approx 10$ a.u. The outgoing wave emitted from this small sphere surface can be written as

$$\psi_{\text{out}}(R,\theta_i,\varphi_i) = C(k_0)Y_{lm}(\theta_i,\varphi_i)\frac{e^{ik_0R}}{R}, \tag{11}$$

where $k_0 = \sqrt{2E}$ denotes the initial momentum of the detached electron. $C(k_0)$ is a coefficient related to the detached electron's energy:

$$C(k_0) = \text{i}\sqrt{\frac{4\pi}{3}}\frac{4Bk_0}{(k_b^2 + k_0{}^2)^2}. \tag{12}$$

$Y_{lm}(\theta_i,\varphi_i)$ is a harmonic function. For the photodetachment of the H^- ion, the initial bound state is an s state; then we choose a p wave as the outgoing detached electron wave source with $l = 1, m = 0$.

Since the external electric field is time dependent, then the amplitude factor A_v and the classical action S_v along the vth closed orbit are all time dependent. The amplitude factor A_v is calculated by $A_v = |\frac{J_v(\tau=0)}{J_v(\tau=t-t_i)}|^{1/2}$. Here, $J(\tau)$ is the Jacobian at time τ [26]:

$$J(\tau) = \rho\,\det\left[\frac{\partial(\rho,z,t)}{\partial(t_i,\theta_i,\tau)}\right]. \tag{13}$$

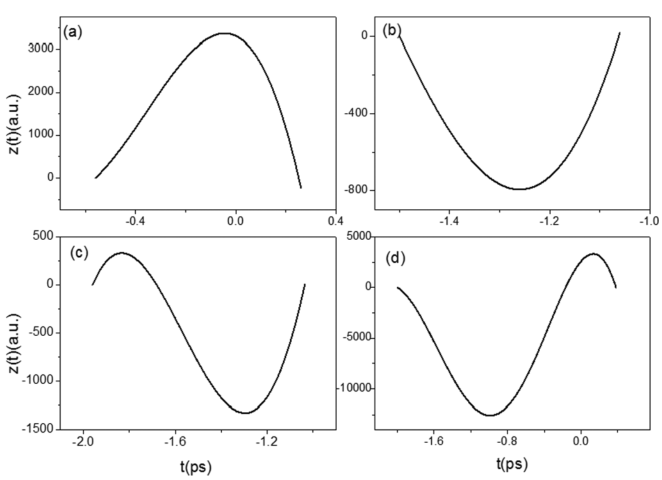

FIG. 2. Graphic demonstrations of four types of closed orbits for the detached electron in the bichromatic oscillating electric field. The photon energy is $E_p = 1.0\,\text{eV}$; the strength of the electric field is $F_0 = 200\,\text{kV/cm}$; the frequency of the bichromatic oscillating electric field is 2.42×10^{-5} and 4.84×10^{-5} a.u., respectively. (a) The up closed orbit; (b) the down closed orbit; (c) the up-down closed orbit; (d) the down-up closed orbit.

Using the classical motion equations of the detached electron [Eq. (4)], we obtain

$$A_v = \frac{R}{k_0(t-t_i)}\left|\frac{k_0}{k_0 - F(t_i)\cos\theta_i(t-t_i)}\right|^{1/2}, \tag{14}$$

where $F(t_i) = F_0[1+\sin(\omega t_i)+\sin(2\omega t_i+\phi)]$.

The classical action S_v for the detached electron traveling along the vth closed orbit is defined as follows:

$$\begin{aligned} S_v &= \int pdq = \int_{t_i}^{t} p_\rho d\rho + \int_{t_i}^{t} p_z dz + \int_{t_i}^{t} p_t dt \\ &= \left[\frac{k_0^2}{2} + \frac{1}{2}A^2(t_i) - A(t_i)\cos\theta_i\right](t-t_i) \\ &\quad -\frac{1}{2}\int_{t_i}^{t} A^2(t')dt'. \end{aligned} \tag{15}$$

Here $A(t) = -F_0 t + \frac{F_0}{\omega}[\cos(\omega t) + \frac{1}{2}\cos(2\omega t+\phi)]$.

The Maslov index λ_v can be calculated by counting the number of the singularities along the vth closed orbit.

After the electron wave travels in the bichromatic oscillating electric field for a period of time, it will return to the source region. Around the source region (usually a few atomic units in size, $r < 10a_0$ [4]), from Eq. (3), we find that the influence of the electric field potential $F(t)z$ on the detached electron is small and can be neglected, which makes the returning wave function behave like a plane wave and can be approximated as

$$\psi_{\rm ret}^{v}(\mathrm{r},t) = f_L(t_i)e^{-iEt}\tilde{\psi}_{\rm ret}^{v}(\boldsymbol{r},t), \tag{16}$$

where $\tilde{\psi}_{\rm ret}^{v}(\boldsymbol{r},t)$ denotes the reduced returning electron wave function:

$$\tilde{\psi}_{\rm ret}^{v}(\boldsymbol{r},t) = C(k_0)N_{co}^{v}Y_{lm}(\theta_i,\varphi_i)e^{\pm ik_{\rm ret}z}. \tag{17}$$

In the above equation, N_{co}^{v} is a matching factor, which is related to the amplitude and phase of the returning wave along each closed orbit:

$$N_{co}^{v} = \frac{A_v}{R}\exp\left[i\left(\tilde{S}_v - \lambda_v\frac{\pi}{2}\right)\right], \tag{18}$$

where $\tilde{S}_v$ is a redefined action function [26]:

$$\tilde{S}_v = S_v + E(t-t_i). \tag{19}$$

The symbol "±" in the phase factor in Eq. (17) is related to the direction of the returning wave. For the up or down-up closed orbits, the returning electron wave travels along the $-z$ axis, so we choose "–" in the phase factor; however, for the down or up-down closed orbit, the returning wave travels along the $+z$ axis, and we choose the "+" symbol instead. $k_{\rm ret}$ in the phase factor represents the returning electron momentum:

$$\begin{aligned} k_{\rm ret} &= k_0\cos\theta_i - F_0(t-t_i) \\ &\quad + \frac{F_0}{\omega}\left[\cos(\omega t) + \frac{1}{2}\cos(2\omega t+\phi)\right] \\ &\quad - \frac{F_0}{\omega}\left[\cos(\omega t_i) + \frac{1}{2}\cos(2\omega t_i+\phi)\right]. \end{aligned} \tag{20}$$

From the above equation, we find that due to the influence of the time-dependent electric field, $k_{\rm ret}$ is usually different from the initial momentum k_0 of the detached electron.

Substituting the reduced returning wave $\tilde{\psi}_{\rm ret}^{v}(\mathrm{r},t)$ into Eq. (16), we can obtain the returning electron wave function. After carrying out the overlap integral $\langle I(t)|\psi_{\rm ret}(\mathrm{r},t)\rangle$ in Eq. (7), we obtain the oscillating part in the photodetachment cross section:

$$\begin{aligned} \sigma_{\rm osc}(E,t) &= \sigma_0\sum_v 3g_v f_L(t)f_L(t_i)\frac{C(k_{\rm ret})}{C(k_0)}\frac{A_v}{k_0R} \\ &\quad \times \sin\left(\tilde{S}_v - \lambda_v\frac{\pi}{2}\right), \end{aligned} \tag{21}$$

where g_v is a factor. For the up or down closed orbit, the outgoing and returning directions are different; $g_v = -1$. However, for the up-down or down-up closed orbit, the outgoing and returning directions are the same; then $g_v = 1$ [26].

By combining Eqs. (21) and (6), we derive the total time-dependent photodetachment cross section of the H^- ion in the bichromatic oscillating electric field:

$$\begin{aligned} \sigma(E,t) &= \frac{16\sqrt{2}B^2\pi^2E^{3/2}}{3cE_p^3}f_L^2(t) \\ &\quad + \frac{16\sqrt{2}B^2\pi^2E^{3/2}}{cE_p^3}\sum_v g_v f_L(t)f_L(t_i)\frac{C(k_{\rm ret})}{C(k_0)}\frac{A_v}{k_0R} \\ &\quad \times \sin\left(\tilde{S}_v - \lambda_v\frac{\pi}{2}\right). \end{aligned} \tag{22}$$

IV. RESULTS AND DISCUSSIONS

In the following calculation, we fix the photon energy $E_p = 1.0\,\mathrm{eV}$; the frequencies of the bichromatic oscillating electric field are 2.42×10^{-5} a.u. and 4.84×10^{-5} a.u., respectively. Firstly, we study how the total time-dependent photodetachment cross section of the H^- ion varies with the strength of the bichromatic oscillating electric field F_0. Suppose the phase difference in the bichromatic oscillating electric field is $\phi = 0$. Figure 3 shows the detached electron's motion trajectory and the time-dependent photodetachment cross section in the bichromatic oscillating electric field with $F_0 = 10\,\mathrm{kV/cm}$. Under this condition, the influence of the bichromatic oscillating electric field on the movement of the detached electron is very small. Only the up closed orbit of the detached electron exists; the other three types of closed orbits are nonexistent in this condition. The reason can be interpreted as follows: As we know, the whole time-dependent electric field is given by a static electric field F_0 plus a bichromatic oscillating electric field $F_{bo}(t)$. The static electric field F_0 is always pointing along the $+z$ axis; the electric field force caused by the static electric field can return the electron when it is moving along the $+z$ axis, which is the up closed orbit for the detached electron. The other three types of closed orbits are induced by the oscillating electric field $F_{bo}(t)$. When the bichromatic oscillating electric field strength is relatively small, the electron cannot return to the origin to form a closed orbit. In Figs. 3(a) and 3(b), we plot the electron trajectory moving along the $+z$ axis and $-z$ axis, respectively. From these two figures, we can clearly see that when the electron is emitted along the $+z$ axis, the electric field force can return the electron to the origin to form a closed orbit; however,

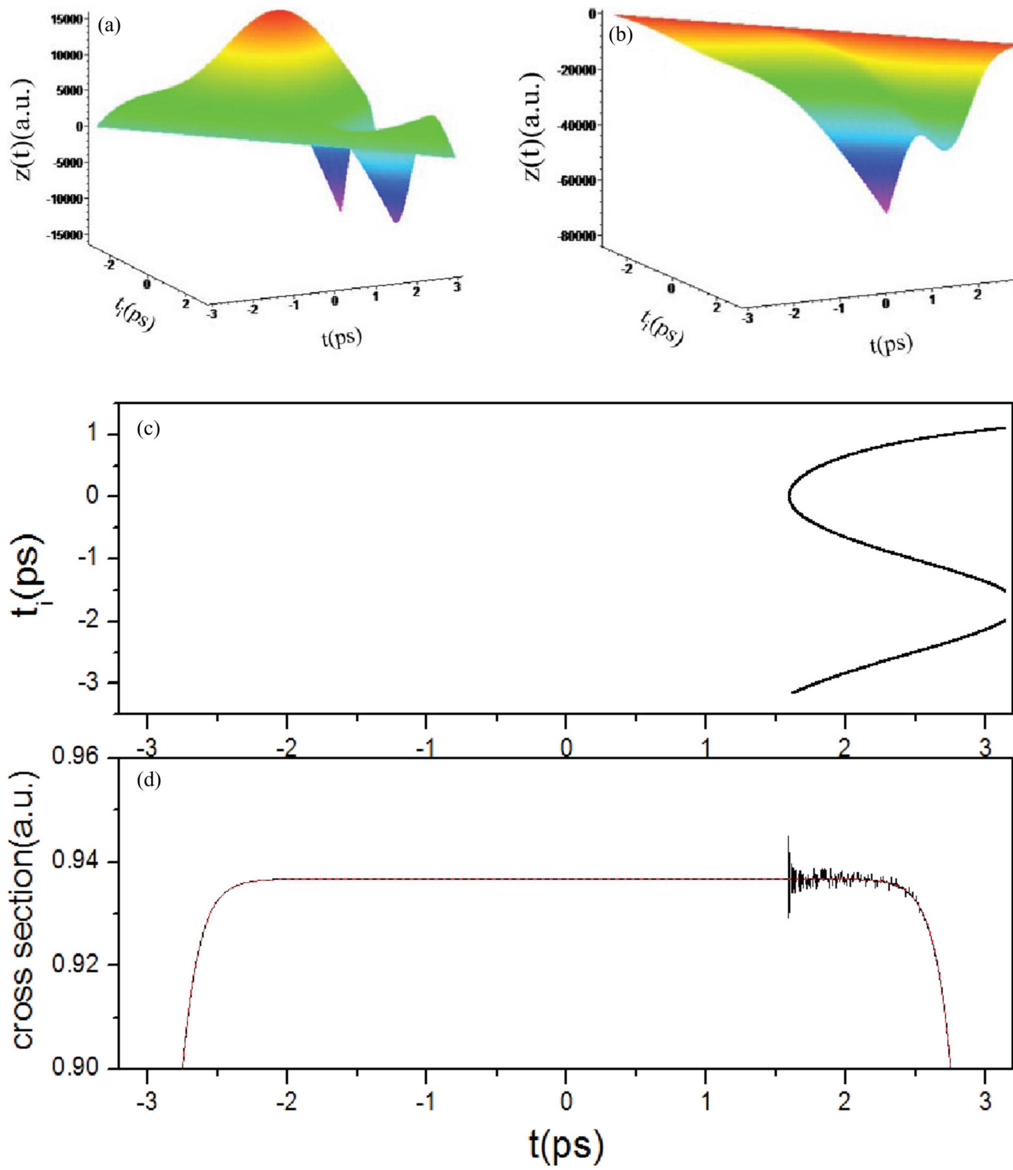

FIG. 3. (a) The detached electron's trajectory moving along the $+z$ axis in the bichromatic oscillating electric field. The photon energy is $E_p = 1.0\,\text{eV}$; the strength of the electric field is $F_0 = 10\,\text{kV/cm}$; the frequency of the bichromatic oscillating electric field is 2.42×10^{-5} and 4.84×10^{-5} a.u., respectively. (b) The detached electron's trajectory moving along the $-z$ axis in the same bichromatic oscillating electric field. (c) The $t - t_i$ curve for the detached electron's up closed orbit in the bichromatic oscillating electric field. (d) The solid line denotes the corresponding time-dependent photodetachment cross section in the bichromatic oscillating electric field, while the red dashed line is the case without the time-dependent electric field.

as the electron travels along the $-z$ axis, the electric force caused by the oscillating electron field is small enough so that it cannot return the electron to the origin to form a closed orbit. Figure 3(c) shows the $t - t_i$ curve for the up closed orbit of the detached electron. The corresponding time-dependent photodetachment cross section is shown in Fig. 3(d). The sold line is the photodetachment cross section in the bichromatic oscillating electric field, while the dashed line is the case without the time-dependent electric field. From this figure, we find oscillatory structures appear in the photodetachment cross section for the bichromatic oscillating electric field case, in contrast with the smooth curve without the time-dependent electric field. However, the oscillatory structure only appears in a small region $1.62 < t < 3.14$ps, which is caused by the interference between the returning electron wave traveling along the up closed orbit with the outgoing electron wave.

As we increase the strength of the bichromatic oscillating electric field, $F_0 = 25\,\text{kV/cm}$, the influence of the oscillating electric field on the detached electron's movement becomes strong. Except when the up orbit can be found, the up-down closed orbit for the detached electron begins to appear. However, the down closed orbit and the down-up closed orbit are still nonexistent. The $t - t_i$ plots for these two closed orbits are shown in Fig. 4(a). From this figure, we find that the returning time curve is mostly dominated by the up closed obit. The up-down closed orbit is only localized in a small interval $-0.956 < t < -0.007$ ps. Figure 4(b) shows the corresponding time-dependent photodetachment cross section.

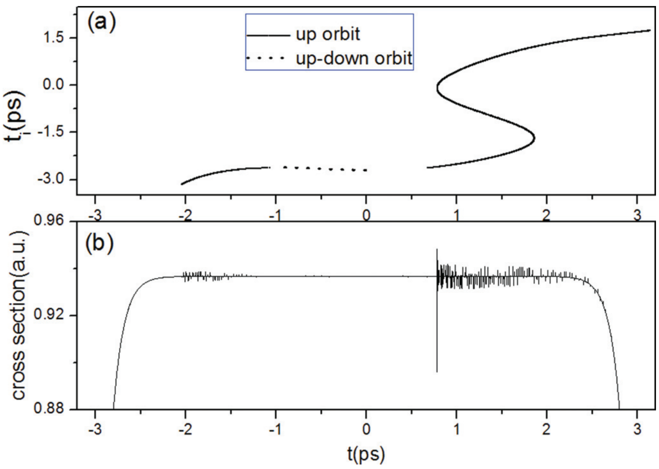

FIG. 4. The $t - t_i$ curve for the detached electron's up orbit and up-down orbits in the bichromatic oscillating electric field. The photon energy is $E_p = 1.0\,\text{eV}$; the strength of the electric field is $F_0 = 25\,\text{kV/cm}$; the frequency of the bichromatic oscillating electric field is 2.42×10^{-5} and 4.84×10^{-5} a.u., respectively. (b) The corresponding time-dependent photodetachment cross section in the bichromatic oscillating electric field.

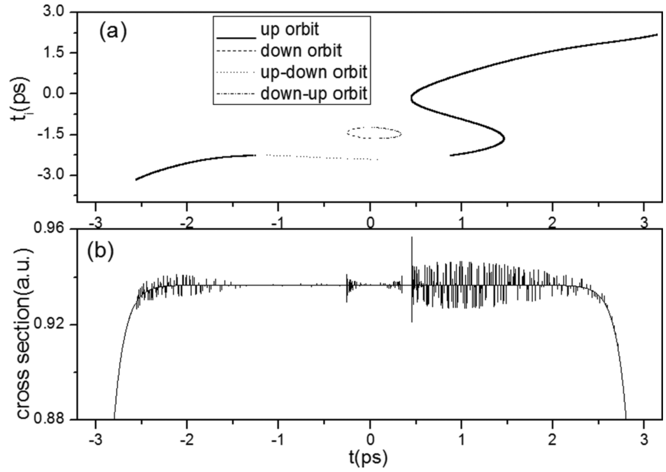

FIG. 5. (a) The $t - t_i$ curve for the detached electron's four different types of closed orbits in the bichromatic oscillating electric field. The photon energy is $E_p = 1.0\,\text{eV}$; the strength of the electric field is $F_0 = 50\,\text{kV/cm}$; the frequency of the bichromatic oscillating electric field is 2.42×10^{-5} and 4.84×10^{-5} a.u., respectively. (b) The corresponding time-dependent photodetachment cross section in the bichromatic oscillating electric field.

We find the oscillatory structure in the photodetachment cross section is mainly caused by the up closed orbit, and the contribution of the up-down closed orbit is small and can be neglected. Compared to Fig. 3(b), we find the oscillating region in the cross section is enlarged and the oscillatory structures in the photodetachment cross section become complex.

As we further increase the bichromatic oscillating electric field strength, $F_0 = 50\,\text{kV/cm}$, the electric field force acting on the detached electron caused by the oscillating electric field becomes stronger. Except for the up orbit and the up-down closed orbit, when the electron travels along the $-z$ axis, the electric field force induced in the oscillating electric field is strong enough to return the electron to the origin to form a closed orbit. Therefore, the down closed orbit and the down-up closed orbit are found under this condition. Figure 5(a) shows the $t - t_i$ plot for these four types of closed orbits. The corresponding time-dependent photodetachment cross section is given in Fig. 5(b). We find that except for the contribution of the up closed orbit to the photodetachment cross section, the other three types of closed orbit also make a certain contribution to the cross section, which makes the oscillatory structure in the total photodetachment cross section become much more complex.

Figure 6(a) shows the total time-dependent photodetachment cross section with the strength of the bichromatic oscillating electric field $F_0 = 200\,\text{kV/cm}$. There are still four types of closed orbits for the detached electron, as we can see from the $t - t_i$ curve shown in Fig. 1. The oscillatory structure in the cross section gets more complicated. In order to see the contribution of the detached electron's closed orbit to the time-dependent photodetachment cross section, we remove the background term $\sigma_0(E,t)$ in the total cross section shown in Fig. 6(a) and only calculate the oscillating part $\sigma_{\text{osc}}(E,t)$ in the photodetachment cross section. From Eq. (21), we can see that the oscillating part $\sigma_{\text{osc}}(E,t)$ is a sum of many terms, and each term is related to a sinusoidal function; therefore the value of the oscillating cross section can be negative or positive. Figure 6(b) shows the oscillating cross section $\sigma_{\text{osc}}(E,t)$ caused by the four types of closed orbit of the detached electron. Figures 6(c)–6(f) show the contribution of each type of closed orbit to the oscillating cross section, respectively. This figure suggests that the contribution of the up closed orbit to the cross section plays a significant role, followed by the contribution of the down closed orbit. The contributions of the up-down and down-up closed orbits to the cross section are relatively small and can be omitted. The reason can be analyzed with aid of the $t - t_i$ curve for each closed orbit shown in Fig. 1. From Fig. 1, we find the up closed orbit dominates a large part of the whole $t - t_i$ curve, while the other three types of closed orbits dominate a small region in the $t - t_i$ curve; thus the contribution of the up closed orbit to the cross section is larger than the other orbits. The up closed orbit is localized in two intervals, $-2.98 < t < -1.51$ ps and $1.00 < t < 3.14$ ps. The oscillating frequency in the oscillating cross section given in Fig. 6(c) is larger than the oscillating electric field frequency ω except at the region $1.00 > t > 0.36$ ps. For the down closed orbit, we find it is localized in the region $-1.06 < t < -0.01$ ps. In the region $-0.01 > t > -0.77$ ps, the oscillating frequency in the oscillating cross section corresponding to the down orbit [Fig. 6(d)] is smaller than the oscillating electric field frequency ω, while in the other region, the oscillating frequency is larger than oscillating electric field frequency ω. The oscillating frequencies in the oscillating cross section caused by the up-down and down-up closed orbits are much smaller than the oscillating electric field frequency ω; correspondingly, their contribution to the cross section is small.

Next, we fix the electric field strength $F_0 = 300\,\text{kV/cm}$ and discuss the influence of the phase difference ϕ in the bichromatic oscillating electric field on the time-dependent photodetachment cross section. The result is shown in Fig. 7. Figure 7(a) shows the photodetachment cross section with the

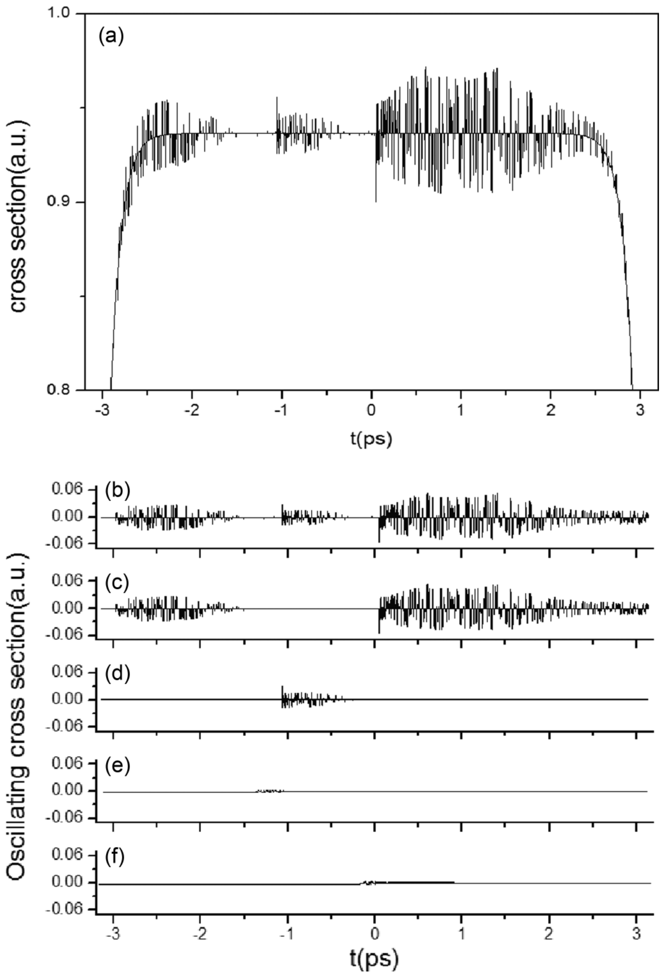

FIG. 6. (a) The total time-dependent photodetachment cross section of the H^- ion in the bichromatic oscillating electric field. The photon energy is $E_p = 1.0\,\mathrm{eV}$; the strength of the electric field is $F_0 = 200\,\mathrm{kV/cm}$; the frequency of the bichromatic oscillating electric field is 2.42×10^{-5} and 4.84×10^{-5} a.u., respectively. (b) The total oscillating cross section $\sigma_{\mathrm{osc}}(E,t)$ in the bichromatic oscillating electric field. (c–f) The oscillating cross section induced by the up closed orbit, down closed orbit, up-down closed orbit, and down-up closed orbit, respectively.

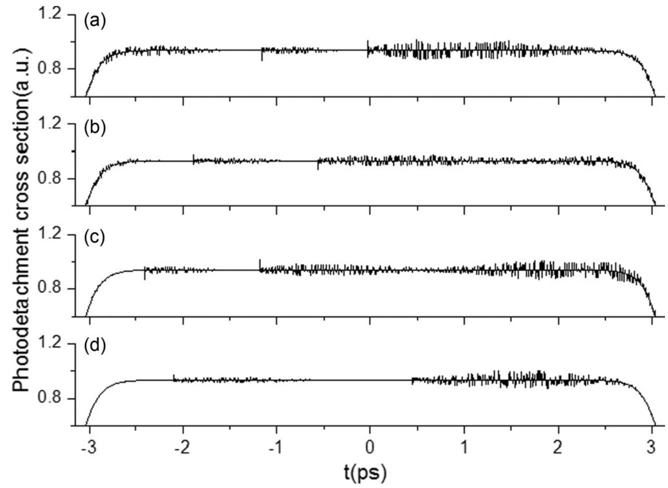

FIG. 7. The influence of the phase difference in the bichromatic oscillating electric field on the time-dependent photodetachment cross section of the H^- ion. The photon energy is $E_p = 1.0\,\mathrm{eV}$; the strength of the electric field is $F_0 = 300\,\mathrm{kV/cm}$; the frequency of the bichromatic oscillating electric field is 2.42×10^{-5} and 4.84×10^{-5} a.u. The phase difference in the bichromatic oscillating electric field is as follows: (a) $\phi = 0$; (b) $\phi = \pi/2$; (c) $\phi = \pi$; (d) $\phi = 3\pi/2$.

phase difference $\phi = 0$. Under this condition, four different types of the detached electron's closed orbits are found and they all make a certain contribution to the photodetachment cross section. Figure 7(b) is the case with the phase difference $\phi = \pi/2$. We find the oscillating amplitude in the photodetachment cross section gets decreased. Figure 7(c) shows the photodetachment cross section with the phase difference $\phi = \pi$. We find the oscillatory structure in the photodetachment cross section has nearly disappeared in the region $-3.14 \leqslant t \leqslant -2.39$ ps. As we further change the phase difference $\phi = 3\pi/2$, only the up closed orbit for the detached electron makes a contribution to the photodetachment cross section. The contributions of the other three types of closed orbits to the photodetachment cross section have disappeared, as we can clearly see from Fig. 7(d). The reason is as follows: When we choose the phase difference $\phi = 3\pi/2$, the time-dependent electric field becomes $F(t) = F_0[1 + \sin(\omega t) - \cos(2\omega t)]$, which is mostly larger than 0 as we change the time t, which means the electric field is pointing along the $+z$ axis most of the time. When the electron is emitting along the $+z$ axis, the electric field force is along the $-z$ axis; then after a period of time, it will return to the origin to form an up closed orbit. However, when the electron is traveling along the $-z$ axis, the electric field force is still along the $-z$ axis, which can accelerate the movement of the electron; thus it cannot return to the origin to form a closed orbit. Therefore, when the phase difference in the bichromatic oscillating electric field is $\phi = 3\pi/2$, only the up closed orbit exists and makes a contribution to the photodetachment cross section.

Finally, we compare the time-dependent photodetachment cross section for the H^- ion in the bichromatic oscillating electric field with the case of the single oscillating electric field. Suppose the electric field strength $F_0 = 500\,\mathrm{kV/cm}$. The frequency of the single oscillating electric field is $\omega_0 = 2.42 \times 10^{-5}$ a.u.. The results are shown in Fig. 8. The $t - t_i$ plot for the detached electron's closed orbit in the single oscillating electric field is shown in Fig. 8(a). We find only the up closed orbit exists. Figure 8(b) shows the corresponding photodetachment cross section. Figure 8(c) shows the $t - t_i$ curve for the detached electron's closed orbit in the bichromatic oscillating electric field. We find the number of closed orbits for the detached electron has increased. Four types of the detached electron's closed orbits are found, which makes the oscillatory structures in the photodetachment cross section become much more complicated, as we can see from Fig. 8(d). The reason can be analyzed as follows: According to the semiclassical closed orbit theory, the interference between the returning electron wave traveling along the closed orbit with the initial outgoing electron wave leads to the oscillatory structure in the photodetachment cross section. The influence of the bichromatic

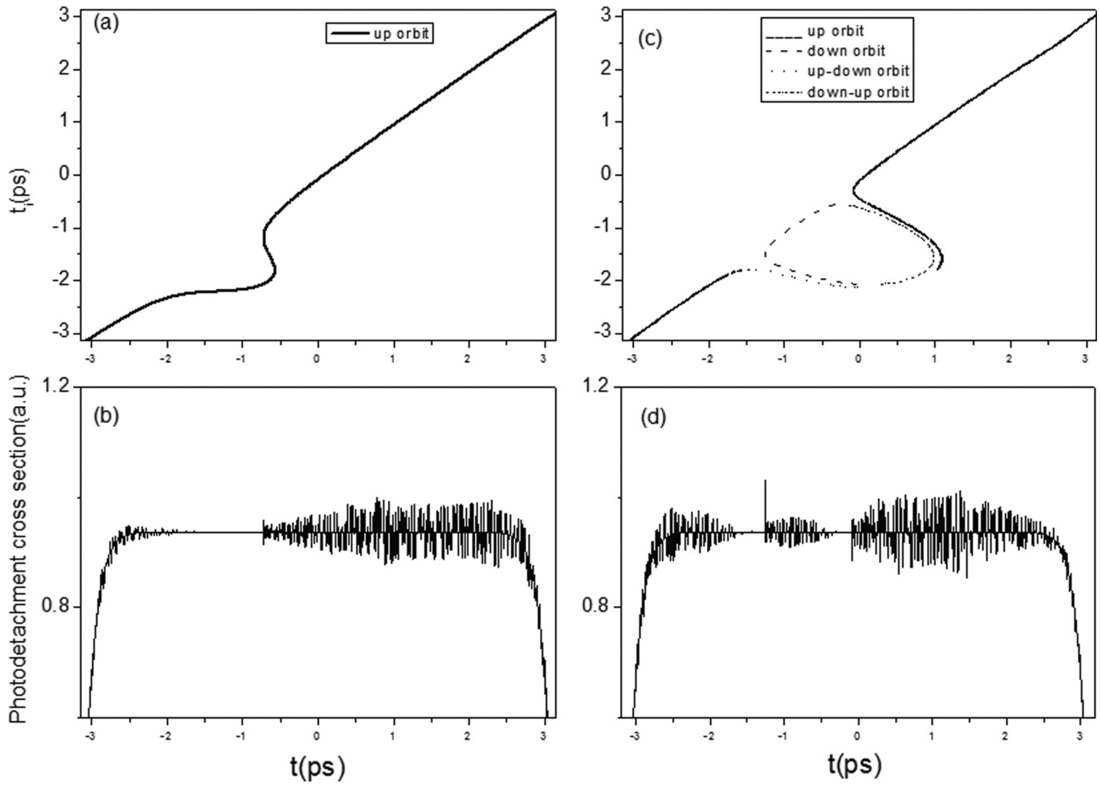

FIG. 8. Comparison of the time-dependent photodetachment cross section for the H^-ion in the bichromatic oscillating electric field with the case of the single oscillating electric field. The photon energy is $E_p = 1.0\,\text{eV}$; the strength of the electric field is $F_0 = 500\,\text{kV/cm}$. The frequency of the single oscillating electric field is 2.42×10^{-5} a.u. and the frequency in the bichromatic oscillating electric field is 2.42×10^{-5} and 4.84×10^{-5} a.u., respectively. (a) The $t - t_i$ curve for the detached electron's closed orbit in the single oscillating electric field. (b) The photodetachment cross section in the single oscillating electric field. (c) The $t - t_i$ curves for the detached electron's closed orbits in the bichromatic oscillating electric field. (d) The photodetachment cross section in the bichromatic oscillating electric field.

oscillating electron field on the detached electron's movement becomes stronger than the single oscillating electric field. In addition to the up closed orbit for the detached electron in the single oscillating electric field, the other three types of the electron's closed orbits also exist. The greater the number of the closed orbits, the more returning waves are returned by the oscillating electric field to the origin; therefore, their contribution to the photodetachment cross section becomes strong.

V. CONCLUSION

As a summary, the time-dependent closed orbit theory provides a clear physical description for investigating the photodetachment of the H^- ion in the bichromatic oscillating electric field. In particular, the time-dependent photodetachment cross sections in the bichromatic oscillating electric field for different electric field strength and various phase difference are calculated. It is found as the electric field strength is relatively weak, its influence on the photodetachment cross section is very small. However, if the electric field strength gets strong enough, the photodetachment cross section oscillates in a complicated manner. In addition, the phase difference in the bichromatic oscillating electric field can also affect the detached electron's movement and have some influence on the photodetachment cross section. Compared to the single oscillating electric field, the number of the closed orbit for the detached electron is increased, which makes the oscillatory structure in the photodetachment cross section become much more complex. In this work, we only consider the case that the strength of the bichromatic oscillating electric field is equal to the static electric field. More interesting work can be expected when the oscillating electric field strength is larger than the static field strength. In addition, we only calculated the photodetachment cross section with a 1:2 frequency ratio in the bichromatic oscillating electric field, since the 1:2 frequency ratio is simple and makes it easy to discuss the electron motion clearly. For other frequency ratios, such as 1:3, 2:3, etc., the time-dependent closed orbit theory we used in this work and the formula we put forward for calculating the time-dependent photodetachment cross section are still applicable; however, the movement of the detached electron and the oscillatory structure in the photodetachment cross section will become different. In future work, we will discuss the case for different frequency ratios in the bichromatic oscillating electric field and compare with the results given in this work. Our study provides a general framework for the understanding of the photodetachment of a negative ion in the presence of an oscillating electric field. We hope that our study can provide some reference for future experimental research of the photodetachment processes of negative ions in the oscillating electric field.

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