Parametric interference effect in nonresonant pair photoproduction on a nucleus in the field of two pulsed light waves

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Nonresonant electron-positron pair photoproduction on a nucleus in the field of two pulsed light waves is studied theoretically. The process is considered in detail within the interference kinematic region, when stimulated absorption and emission of photons of external pulsed waves by an electron and a positron occurs in a correlated manner. Within this region, a correspondence between the emission angle and energy of the produced particles appears. The distribution of the obtained differential cross section over the pair kinetic energy is characterized by presence of oscillations, within the interference region. Each of the maxima corresponds to the definite partial process with emission and absorption of an equal number of photons of both waves. It was shown that the differential cross section within the interference region for certain values of the pair energy may exceed the cross section in other scattering kinematics in two orders of the magnitude. Obtained results may be experimentally verified, for example, by scientific facilities at sources of pulsed laser radiation (SLAC, FAIR, ELI, XCELS).

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I. INTRODUCTION

Photoproduction of a pair of an electron and a positron in the field of an atomic nucleus is of scientific interest due to the fact that it is one of the fundamental processes of interaction of γ photons with matter. The process of Coulomb photoproduction of pair (CPP) draws attention for quite a long time. For the first time, the differential cross section in the quantum electrodynamics (QED) approach was obtained by Bethe and Heitler in Ref. [1].

Nowadays, the relativistic-regime threshold (when the radiation intensity of the optical laser is higher than 10^{18} W cm^{-2}) has been already reached [2], and even exceeded in the world's leading scientific laboratories. Electron motion becomes highly nonlinear as a function of the laser electromagnetic field, under the relativistic regime, in laser-electron interaction. Thus, the laser system Vulcan at the Central Laser Facility (CLF) [3] in the United Kingdom and the Petawatt High-Energy Laser for heavy Ion eXperiments (PHELIX) [3,4] at the GSI Helmholtz Centre for Heavy Ion Research in Darmstadt (Germany) provide field powers of the order of 1 PW. Such intense electromagnetic fields allows to an electron to reach relativistic velocities already during the single laser-wave period. Ultrahigh intensity up to 10^{24} W cm^{-2} are envisaged in the laser facility of the Extreme Light Infrastructure (ELI) [3] project. The scientific interest to studying of processes of QED in laser fields of such intensity is caused by the possibility of testing of different aspects of fundamental physics for the first time. Experimental verification of QED effects in the laser field was carried out at the facility SLAC National Accelerator Laboratory (Stanford, USA) [5,6].

Nonlinear QED effects in force fields have been an object of scientific research for a long time already. QED processes of both the first and second order in the laser field were studied in Refs. [5–52]. The results have been summarized in monographs [7–10] and reviews [11–16].

Improvement of laser systems, generally, consists in production of increasingly short and intense laser pulses [2,3]. The amplitude of the intensity of the field of powerful ultrashort pulsed lasers changes greatly in space and time. New experimental conditions have required constant improvements in calculations and model development.

The theory of CPP process in the presence of a plane laser wave was developed in Refs. [19–28]. Borisov *et al.* in Ref. [19] studied resonant CPP on a nucleus in the particular case of ultrarelativistic energy of an electron and a positron, when an incident photon and a photon of a wave pump propagate towards each other. Photoproduction of electron-positron pair on a nucleus in the field of a pulsed wave was studied in detail in Refs. [24,25]. Nonresonant CPP in the field of two circularly polarized electromagnetic waves propagating in the same direction was considered in the general relativistic case and for arbitrary intensity of an external field in the Ref. [27].

The parametric interference quantum effect manifests when QED processes occur in the field of two laser waves. The specified kinematic region (called the interference region) appears in such a field, when scattering particles can forcedly absorb and emit photons of electromagnetic waves in a correlated manner, i.e., the equal number of photons of the first and second laser wave [42–44]. The parametric interference quantum effect in spontaneous bremsstrahlung (SB) of an electron scattered by a nucleus and in CPP on a nucleus in the field of two plane monochromatic waves was

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predicted theoretically [27,39–41]. Studying of effects caused by presence of a second pulsed laser wave is of interest due to the fact, that probability of stimulated processes in interference kinematics is generally greater than in the other one. The theory of QED processes in the field of two laser waves was developed in [27,28,39–51].

Hence, there is an interest in detailed study of CPP process in the field of two pulsed laser waves. In this work, we develop a theory of nonresonant production of electron-positron pair on a Coulomb center in presence of an external field of two pulsed electromagnetic waves. The main aim of the work is detailed analysis of the studied process within the region, where peculiar properties of stimulated absorption and emission of waves photons by an electron-positron pair (the parametric interference effect) appear.

The external pulsed field was chosen as a superposition of two plane nonmonochromatic waves, propagating in the same direction along the z axis, with the plane of polarization (xy). The four-potential of such a field has the form

$$A(\varphi_1,\varphi_2) = g_1\left(\frac{\varphi_1}{\omega_1\tau_1}\right)A_{1(\mathrm{mon})} + g_2\left(\frac{\varphi_2}{\omega_2\tau_2}\right)A_{2(\mathrm{mon})}, \quad (1)$$

$$A_{j(\mathrm{mon})} = \frac{F_{0j}}{\omega_j}(e_{jx}\cos\varphi_j + \delta_j e_{jy}\sin\varphi_j), \quad j = 1,2 \quad (2)$$

$$\varphi_j = (k_j x) = \omega_j \xi, \quad \xi = t - z/c. \quad (3)$$

Each of the summands in Eq. (1) corresponds to the field of the first and second pulsed laser wave (the index j labels the wave) and φ_j is the wave phase and τ_j is the pulse width. In Eqs. (1)–(3), c is the light velocity in vacuum; F_{0j} is the strength amplitude of the electric field in the pulse peak; ω_j is the laser-wave characteristic frequency; $k_j = (\omega_j,\mathbf{k}_j)$ is the wave four-vector; δ_j is the wave ellipticity parameter ($\delta_j = 0$ corresponds to linear polarization, $\delta_j = \pm 1$ corresponds to circular polarization); and $e_{jx} = (0,\mathbf{e}_{jx})$ and $e_{jy} = (0,\mathbf{e}_{jy})$ are four-vectors of wave polarization, meeting the conditions

$$e^2_{jx,jy} = -1, \quad (e_{jx,jy}k_j) = k_j^2 = 0. \quad (4)$$

Hereafter, the standard metric for four-vectors, $(ab) = a_0b_0 - \mathbf{ab}$, is used.

In what follows, we consider the case of circular polarization of external pulsed waves:

$$\delta_1 = +1, \quad \delta_2 = \mp 1. \quad (5)$$

It should be noted that in the case of waves' close frequency and same polarization, we have the case of a single wave [43]. Note also that description of a laser field by the potential (1)–(3) does not take into account the possible phase shift between laser waves and stipulates that laser pulses' maxima coincide.

Functions $g_j(\varphi_j/\omega_j\tau_j)$ in Eq. (1) are envelope functions of the four-potential of pulsed laser waves, that allows to take into account the pulsed character of a laser field [17]. The process is studied within the frame of the quasimonochromatic approximation, when a laser wave performs a lot of amplitude oscillation, i.e., the following condition is met:

$$\omega_j\tau_j \gg 1. \quad (6)$$

The condition (6) is satisfied for the majority of modern lasers [2,3].

Electron and positron interaction with a nucleus is considered in the frame of the Born approximation, that is, the case of rather fast particles is studied:

$$v_{\mp}/c \gg Z\alpha, \quad \alpha = e^2/\hbar c, \quad (7)$$

where v_- and v_+ are velocities of translation movement of an electron and a positron, respectively; Z is the nucleus charge number; $\hbar$ is the Planck constant; and α is the fine-structure constant.

It should be noted that there are several characteristic parameters in the problem of CPP process in the field of two pulsed laser waves. The first one is the classical relativistic-invariant parameter [11]

$$\eta_{0j} = \frac{eF_{0j}}{mc\omega_j}, \quad (8)$$

which numerically equals to the ratio of the work done by the field over an electron, on the wavelength, to the electron rest energy (here, e is an electron charge, m is an electron mass). The parameter (8) is one of the most important characteristics of the external electromagnetic field. In the classical consideration of laser-dressed electron motion, the parameter η_{0j} defines the characteristic velocity of electron oscillation in the case if $\eta_{0j} \ll 1$.

The multiplicity of multiphoton processes occurring in the Coulomb interaction between the particles in the laser field is also characterized by the Bunkin-Fedorov multiphoton quantum parameter [18]

$$\gamma'_{0j} = \eta_{0j}\frac{mc^2}{\hbar\omega_j}\frac{|\mathbf{p}|c}{E}. \quad (9)$$

It is equal to the ratio of the work done by the field at the distance, passed by a particle during the characteristic time of wave oscillation ω_j^{-1}, to the energy of an external-field photon. In Eq. (9), E and $\mathbf{p}$ are the particle energy and the particle momentum, respectively. Parameters γ'_{0j} determine stimulated processes of emission and absorption of photons of the first and second wave by an electron-positron pair, independently of each other.

The particular kinematic region, quite narrow and called the interference region, occurs in the field of two electromagnetic waves. The Bunkin-Fedorov quantum parameter is not manifested within this region. The following multiphoton parameters manifest within the interference region. The quantum interference parameter reveals in the field of two laser waves, propagating in the same direction and with different frequencies [43]. This parameter is the multiphoton major parameter out of the Bunkin-Fedorov kinematic region, and has the form

$$\alpha'_0 = \eta_{01}\eta_{02}\frac{|\mathbf{p}|c}{\hbar\omega_{\mathrm{com}}}\frac{m^2c^4}{E^2}, \quad \omega_{\mathrm{com}} \equiv \omega_1 - \delta_2\omega_2. \quad (10)$$

Parameters α'_0 [Eq. (10)] determine interference processes in correlated stimulated emission or absorption of photons of both waves by an electron-positron pair. It is easy to see from Eqs. (5) and (10) that parameter $\omega_{\mathrm{com}} = \omega_1 \pm \omega_2$ in the case of circular polarization.

The problem of CPP on a nucleus will be studied in the range of moderately strong fields, when outside the

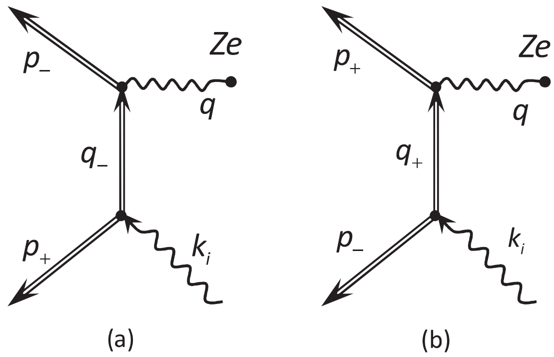

FIG. 1. Feynman diagram of CPP process in the field of two pulsed laser waves. Dual incoming and outgoing lines correspond to wave functions of an electron p_- and a positron p_+ in the field of two laser waves (1)–(3), the inner line corresponds to the Green function of an intermediate electron q_- and a positron q_+ in the field of two laser waves. Wavy lines correspond to an incident γ photon k_i and a "pseudophoton" q of a nucleus recoil.

interference region, the intensity meets the conditions

$$\eta_{0j} \ll 1, \quad \alpha_0' \gtrsim 1. \tag{11}$$

Under the conditions (11), $\gamma_{0j}' \gg 1$, however, within the interference region the influence of this parameter does not manifest by virtue of the special kinematics.

The relativistic system of units, $\hbar = c = 1$, will be used throughout this paper.

II. AMPLITUDE OF CPP IN TWO PULSED LASER WAVES

Let us consider production of a pair of an electron and a positron by a photon on a nucleus in the external field of two pulsed waves (1)–(3) in the first Born approximation on interaction of an electron and a positron with a nucleus Coulomb field. This process is of the second order in the fine-structure constant, and is described by two Feynman diagrams [53] (see Fig. 1).

In Fig. 1, $k_i = (\omega_i, \mathbf{k}_i)$ is the four-momentum of an incident γ photon; q is the four-momentum transferred to a nucleus; $p_- = (E_-, \mathbf{p}_-)$ and $p_+ = (E_+, \mathbf{p}_+)$ are four-momenta of an electron and a positron, respectively; and q_- and q_+ are four-momenta of particles in an intermediate state. We mark that a particle in an intermediate state is an electron for Fig. 1(a) and a positron for Fig. 1(b).

We emphasize that an electromagnetic field with the four-potential (1)–(3) represents a plane wave. Thereby Volkov functions [54], which are correct for a plane wave of arbitrary spectral composition, can be used for description of the state of an electron and a positron in the field of a quasimonochromatic wave.

The wave function of an incident photon is determined by the expression

$$A_i(x_{j'}, k_i) = \sqrt{\frac{2\pi}{\omega_i}}\,\varepsilon_i \exp(i k_i x_{j'}). \tag{12}$$

Here, ε_i is the polarization four-vector of an incident photon. The index $j' = 1,2$; it labels the integration variable in the first and second vertices of the diagram.

The field of a nucleus is described by the Coulomb potential in the form

$$A_0(|\mathbf{x}_{j'}|) = \frac{Ze}{|\mathbf{x}_{j'}|}. \tag{13}$$

We remind that CPP process is a crossed channel of bremsstrahlung due to electron scattering by a nucleus [29–40]. Nonresonant SB of an electron scattered by a nucleus in the field of two pulsed laser waves was studied in detail in [39,40]. In consideration of the known calculation procedure, we may obtain the required amplitude of nonresonant CPP on a nucleus by the following replacement and redesignation in expressions for laser-modified SB of an electron by a nucleus [41]:

$$p_f \to p_-, \quad p_i \to -p_+, \quad k' \to -k_i, \tag{14}$$

$$q_i \to q_-, \quad q_f \to -q_+. \tag{15}$$

Here, p_i and p_f are four-momenta of an electron in the initial and final states, k' is the four-momentum of an emitted photon, q_i and q_f are four-momenta of an electron in an intermediate state for SB process of an electron scattered by a nucleus.

We note that CPP in an external field may occur in a resonant manner, when a particle in an intermediate state becomes real [15,16]. The given paper presents studying of the nonresonant case, when the following condition is met:

$$q_\mp^2 - m^2 \gg \frac{(k_1 q_\mp)}{\omega_1 \tau_1}. \tag{16}$$

As it was shown in work [24], for CPP on a nucleus, the resonance condition is realized when the incident-photon energy exceeds the certain threshold value ($\omega_i \gtrsim m^2/\omega_{1,2}$); at that pairs are produced in the narrow cone relatively of a direction of incident-photon entrance. Therefore, the condition (16) includes a great kinematic region of nonrelativistic and relativistic energy and angles of emission of an electron and a positron.

Hence, the amplitude of nonresonant CPP on a nucleus in the field of two pulsed laser waves in the moderately strong field of two pulsed circularly polarized waves (1)–(3), (5), and (11) is presented in the form of a sum over partial components:

$$S_{fi} = \sum_{l_1, l_2 = -\infty}^{\infty} S_{l_1 l_2}. \tag{17}$$

Here, S_{l_s} is the partial amplitude of CPP with absorption ($l_1, l_2 < 0$) or emission ($l_1, l_2 > 0$) of l_1 photons of the first wave and l_2 photons of the second wave by an electron and a positron:

$$S_{l_1 l_2} = -i\frac{Ze^3\sqrt{\pi}}{\sqrt{2\omega_i E_- E_+}}\bar{u}_-\big[B_{l_1 l_2}^{(a)} + B_{l_1 l_2}^{(b)}\big]u_+. \tag{18}$$

Here, $\bar{u}_- = \bar{u}(p_-)$ and $u_+ = u(-p_+)$ are Dirac bispinors, functions $B_{l_1 l_2}^{(a)}$ and $B_{l_1 l_2}^{(b)}$ correspond to Figs. 1(a) and 1(b) of the considered process, respectively:

$$B^{(a)}_{l_1 l_2} = \sum_{s_1,s_2=-\infty}^{\infty} 4\pi \frac{\Delta^{(a)}_{l_{1,2}-s_{1,2},s_{1,2}}(q_-,q_0)}{q_x^2+q_y^2+(q_0-q_z)^2} \tilde{\gamma}^0 \frac{\hat{q}_-+m}{q_-^2-m^2} \hat{\varepsilon}_i, \quad (19)$$

$$B^{(b)}_{l_1 l_2} = \sum_{s_1,s_2=-\infty}^{\infty} 4\pi \frac{\Delta^{(b)}_{s_{1,2},l_{1,2}-s_{1,2}}(q_+,q_0)}{q_x^2+q_y^2+(q_0-q_z)^2} \hat{\varepsilon}_i \frac{-\hat{q}_++m}{q_+^2-m^2} \tilde{\gamma}^0. \quad (20)$$

The hat above the four-vector means the scalar product of the corresponding four-vector with Dirac matrices. Functions $\Delta^{(a)}_{l_{1,2}-s_{1,2},s_{1,2}}(q_-,q_0)$ and $\Delta^{(b)}_{s_{1,2},l_{1,2}-s_{1,2}}(q_+,q_0)$ are specified as

$$\begin{aligned}\Delta^{(a)}_{l_{1,2}-s_{1,2},s_{1,2}}(q_-,q_0) &= \tau_1 \int_{-\infty}^{\infty} d\phi \, \exp\{iq_0\tau_1\phi\} \\ &\times I_{l_1-s_1,l_2-s_2}(\chi_j,\gamma_j(p_-,q_-,\phi),\alpha(p_-,q_-,\phi)) \\ &\times I_{s_1,s_2}(\chi_j,\gamma_j(q_-,-p_+,\phi),\alpha(q_-,-p_+,\phi)), \quad (21)\end{aligned}$$

$$\begin{aligned}\Delta^{(b)}_{s_{1,2},l_{1,2}-s_{1,2}}(q_+,q_0) &= \tau_1 \int_{-\infty}^{\infty} d\phi \, \exp\{iq_0\tau_1\phi\} \\ &\times I_{s_1,s_2}(\chi_j,\gamma_j(p_-,-q_+,\phi),\alpha(p_-,-q_+,\phi)) \quad (22) \\ &\times I_{l_1-s_1,l_2-s_2}(\chi_j,\gamma_j(-q_+,-p_+,\phi),\alpha(-q_+,-p_+,\phi)),\end{aligned}$$

$$\phi \equiv \frac{\varphi_j}{\omega_j\tau_1} = \frac{\xi}{\tau_1}. \quad (23)$$

Here, ϕ is dimensionless variable of a laser field. Quantities s_1, (l_1-s_1) and s_2, (l_2-s_2) are numbers of photons, which are forcedly emitted or absorbed by an electron-positron pair from the first and second waves in the first (second) vertex. Therefore, parameters l_1 and l_2 are total number of external-field photons participated in the production process. They can be measured experimentally. Four-momenta q_- and q_+, q have the form

$$\begin{aligned} q_- &= k_i - p_+ - s_1k_1 - s_2k_2 \\ &= p_- - q + (l_1-s_1)k_1 + (l_2-s_2)k_2, \\ q_+ &= k_i - p_- - s_1k_1 - s_2k_2 \\ &= p_+ - q + (l_1-s_1)k_1 + (l_2-s_2)k_2, \\ q &= p_- + p_+ - k_i + l_1k_1 + l_2k_2. \end{aligned} \quad (24)$$

Integral functions (21) and (22) are smoothly dependent on the argument ϕ [Eq. (23)]. They will not be small only if the relation $q_0\tau_1 \lesssim 1$ is correct, which in fact represents the energy conservation law:

$$q_0 = E_- + E_+ - \omega_i + l_1\omega_1 + l_2\omega_2 \lesssim \tau_1^{-1} \ll \omega_1. \quad (25)$$

Special functions I_{n_1,n_2} in expressions (21) and (22) determine the probability of partial multiphoton processes in the field of two pulsed laser waves. Functions $I_{s_1,s_2}(\chi_j,\gamma_j(q_-,-p_+,\phi),\alpha(q_-,-p_+,\phi))$ and $I_{s_1,s_2}(\chi_j,\gamma_j(p_-,-q_+,\phi),\alpha(p_-,-q_+,\phi))$ correspond to the process of pair production by a photon k_i in the field of two pulsed laser waves [8]. Simultaneously, functions $I_{l_1-s_1,l_2-s_2}(\chi_j,\gamma_j(p_-,q_-,\phi),\alpha(p_-,q_-,\phi))$ and $I_{l_1-s_1,l_2-s_2}(\chi_j,\gamma_j(-q_+,-p_+,\phi),\alpha(-q_+,-p_+,\phi))$ correspond to scattering of an intermediate electron q_- and a positron q_+ by a nucleus in the field of two pulsed waves [45]. These special functions I_{n_1,n_2} are studied in detail in Ref. [55]. They can be represented in the form of expansion into series of integer-order Bessel functions. In the case of laser-wave circular polarization (5), functions I_{n_1,n_2} have the form

$$I_{n_1,n_2}(\chi_j,\gamma_j,\alpha) = \exp\{-i(n_1\chi_1+n_2\chi_{02})\} \times \sum_{r=-\infty}^{\infty} J_r(\alpha)J_{n_1-r}(\gamma_1)J_{n_2+\delta_2 r}(\gamma_2). \quad (26)$$

Arguments of functions (26) are determined as

$$\begin{aligned}\gamma_1(p,p',\phi) &= g_1(\phi)\gamma_{01}(p,p'), \\ \gamma_2(p,p',\phi) &= g_2(\phi\tau_1/\tau_2)\gamma_{02}(p,p'),\end{aligned} \quad (27)$$

$$\gamma_{0j}(p,p') = \eta_{0j}\frac{m}{\omega_j}\sqrt{-Q^2_{pp'}}, \quad Q_{pp'} = \frac{p}{(np)} - \frac{p'}{(np')}, \quad (28)$$

$$\tan\chi_1 = \frac{(e_{1y}Q_{pp'})}{(e_{1x}Q_{pp'})}, \quad \tan\chi_2 = \delta_2\frac{(e_{2y}Q_{pp'})}{(e_{2x}Q_{pp'})}, \quad (29)$$

$$\alpha(p,p',\phi) = \alpha_0(p,p')g_1(\phi)g_2(\phi\tau_1/\tau_2), \quad (30)$$

$$\alpha_0(p,p') = \eta_{01}\eta_{02}\frac{m^2}{\omega_{\text{com}}}\left[\frac{1}{(np)} - \frac{1}{(np')}\right]. \quad (31)$$

Here, $n \equiv (1,\mathbf{n}) = k_j/\omega_j$, $\mathbf{n}$ is a unit vector along the direction of propagation of laser waves. Using Eqs. (26)–(31), it is easy to obtain an explicit form of arguments of all the functions I_{n_1,n_2} by corresponding replacement. For example, for the function $I_{l_1-s_1,l_2-s_2}(\chi_j,\gamma_j(p_-,q_-,\phi),\alpha(p_-,q_-,\phi))$ indices assume values $n_1 = l_1 - s_1$ and $n_2 = l_2 - s_2$; quantities p and p' assume values p_- and q_-, respectively.

Notice that arguments $\gamma_{0j}(p,p')$ [Eq. (28)] and $\alpha_0(p,p')$ [Eq. (31)] are essentially quantum in the general case. It is evident that they can have a different order of the magnitude with respect to each other, depending on scattering kinematics. Arguments $\gamma_{0j}(p,p')$ [Eq. (28)] are Bunkin-Fedorov parameters, which determine the probability of stimulated processes in the field of each of wave, independently each wave from the other, in Coulomb interaction between particles. Parameters $\alpha_0(p,p')$ [Eq. (31)] are determined by the term that refers to interference of the first and second waves [42–44]. They determine the probability of correlated absorption-emission processes of photons of both waves by an electron and positron in CPP on a nucleus in the field of two pulsed laser waves.

Also, we underline that the obtained amplitudes (17)–(31) in the limit case $\omega_j\tau_j \to \infty$ coincide with the corresponding amplitude in the field of two monochromatic waves [27]. When one of the waves is "switched off" ($\eta_{02} = 0$), the amplitudes (17)–(31) coincide with the expression for the case of a single pulsed wave [20].

Interference region

Values of multiphoton parameters $\gamma_{0j}(p,p')$ [Eq. (28)] and $\alpha_0(p,p')$ [Eq. (31)] depend greatly on scattering kinematics. Such a kinematic region (the interference region) can be distinguished, where quantum parameters $\gamma_{0j}(p,p') \to 0$ and

$\alpha_0(p,p')$ become the major multiphoton parameters. At that, the parametric interference effect is most pronounced for circular polarization [42–44]. It was shown in Ref. [27] that the partial cross section of the studied process within the interference region can considerably exceed the corresponding partial cross section in any other geometry, for the case of monochromatic waves. Therefore, this article will consider the case of the interference region:

$$\gamma_{0j}(p,p') \approx 0. \tag{32}$$

The condition (32) and the explicit form (28) result to relativistic-invariant relations for amplitudes corresponding to Figs. 1(a) and 1(b):

$$Q^2_{p_-q_-} = Q^2_{q_-p_+} = Q^2_{p_-q_+} = Q^2_{q_+p_+} = 0. \tag{33}$$

Conditions (33) are satisfied when corresponding vectors $\mathbf{Q}_{pp'}$ are directed along or against the direction of propagation of waves $\mathbf{n}$, i.e., perpendicular to the wave polarization plane. Kinematics of CPP process in the field of two laser waves is identical for the amplitudes at the first and second diagram [27]. We also note that the following condition is met within the interference region for the azimuthal angles, along with the condition (33):

$$\varphi_+ = \varphi_- = \varphi_i. \tag{34}$$

Equation (33) allows to obtain the required relation of polar angles and energy of a pair with the angle of entrance of an incident photon:

$$a_\mp \equiv \frac{|\mathbf{p}_\mp|}{(np_\mp)}\sin\theta_\mp, \quad a_- = a_+ = \cot\frac{\theta_i}{2}, \tag{35}$$

$$(np_\mp) = E_\mp - |\mathbf{p}_\mp|\cos\theta_\mp. \tag{36}$$

Here, the angle $\theta_i = \angle(\mathbf{n},\mathbf{k}_i)$ denotes the polar angle of an incident photon, $\theta_\mp = \angle(\mathbf{n},\mathbf{p}_\mp)$ are the polar angles of an electron (θ_-) and a positron (θ_+).

From Eqs. (35) and (36), it is easy to determine the velocity (energy) of an electron and a positron produced by a photon within the interference region, depending on their polar angles

$$v_\mp = \frac{|\mathbf{p}_\mp|}{E_\mp} = \left[\cos\theta_\mp + \sin\theta_\mp \tan\frac{\theta_i}{2}\right]^{-1}. \tag{37}$$

Equation (37) can be easily presented in the form of an equation with respect to pair polar angles:

$$\tan\frac{\theta_\mp}{2} = \frac{v_\mp}{(1+v_\mp)\cos(\theta_i/2)}\left[\sin\frac{\theta_i}{2} \mp \sqrt{1 - \frac{\cos^2(\theta_i/2)}{v_\mp^2}}\right], \tag{38}$$

where the sign "$\mp$" in front of the square root refers both to an electron and to a positron and indicates symmetry of the interference condition (35) with respect to electron-positron replacement. It is important that in the frame of Eqs. (37) and (38), within the region, where the parametric interference effect appears, the correspondence between exit angles and energy of final particles appears. This dependence differs significantly production process within the interference region from any other geometry.

It can be seen from Eq. (38) that velocities of translational motion of an electron and a positron within the interference region are bounded below by a value, which is determined by the incidence angle of γ photon

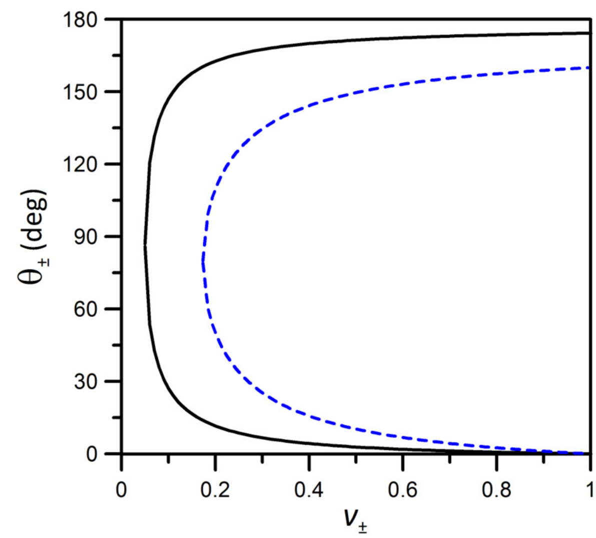

FIG. 2. Dependence of angles of emission of an electron and a positron on their energy within the interference region. The black curve corresponds to the angle between incident-photon entrance and the waves' propagation direction $\theta_i = 175^\circ$, the blue curve corresponds to the angle $\theta_i = 160^\circ$.

$$v_{\min} \equiv \cos(\theta_i/2). \tag{39}$$

In the case when an electron and a positron are produced with velocities close to the minimal value ($v_\mp \to v_{\min}$), we have

$$\theta_\mp \to \theta_{\lim} = \theta_i/2 \mp \sqrt{2(1 - v_{\min}/v_\mp)} \approx \theta_i/2. \tag{40}$$

That is, an electron-positron pair is emitted within a narrow cone along the bisector of the angle between the wave vector $\mathbf{k}_1$ and the incident-photon momentum $\mathbf{k}_i$. It is peculiar to the considered process that, with energy growing (far from the threshold), the direction of electron and positron exit recedes from the bisector of this angle (see [27]). It can be seen from Eq. (39) that by specifying angles θ_i and the photon energy, we can smoothly change the minimal energy of produced electron and positron.

If assume that the incident-photon momentum is nearly antiparallel to the wave vector $\mathbf{k}_1$:

$$\Delta\theta_i = (\pi - \theta_i) \ll 1, \quad v_{\min} \approx \Delta\theta_i/2 \ll 1. \tag{41}$$

In this case, the threshold energy for an incident photon has the form

$$\omega_i^{\text{th}} = 2\big(m + m v_{\min}^2/2\big) + l_1\omega_1 + l_2\omega_2. \tag{42}$$

The second term in the parenthetical expression on the right-hand side of Eq. (42) has the meaning of the minimal kinetic energy of an electron (positron). In the opposite limiting case when the incident-photon momentum is nearly parallel to the wave vector $\mathbf{k}_1$ ($\theta_i \ll 1$), we have the ultrarelativistic minimal energy of an electron and a positron within the interference region (35)–(38).

Dependence of angles of emission of an electron and a positron on their energy within the interference region is shown in Fig. 2. Curves differ by angles between incident-photon entrance and the waves' propagation direction. For example, let us fix both the exit angle and velocity of an electron; these values meet the selected point on the lower part of the curve. We obtain in this case that both the positron velocity (absorbed and emitted field energy is taken into account) and its emission angle correspond to the upper part of the curve. The law of energy conservation, restrictions with the minimal velocity (39), and the characteristic range of numbers of photons l_1 and l_2 define the section of the curve, where the positron can be produced.

It follows from properties of the Bessel function and Eq. (26) that $n_2 = -\delta_2 n_1$ under the condition (32). Thus, functions I_{n_1,n_2}, which determine the amplitude of CPP in two laser waves, are reduced over one of indices and transform into Bessel functions within the interference region (33) for circular polarization:

$$I_{l_1-s_1,l_2-s_2}(\chi_j,\gamma_j,\alpha) \to e^{-i(l-s)\Delta} J_{l-s}(\alpha),$$
$$I_{s_1,s_2}(\chi_j,\gamma_j,\alpha) \to e^{-is\Delta} J_s(\alpha), \qquad (43)$$

$$l \equiv l_1 = -\delta_2 l_2, \quad s \equiv s_1 = -\delta_2 s_2. \qquad (44)$$

Here, $\Delta = \angle(\mathbf{e}_{1x},\mathbf{e}_{2x})$ is the angle between vectors of polarization of laser waves. Numbers of photons of both waves (44) are emitted and absorbed forcedly by an electron-positron pair within the interference region. Therefore, an electron-positron pair emits (absorbs) photons of both waves in correlated manner within the interference region. Formally, it looks like that an electron-positron pair emits (absorbs) forcedly as though an integer number of l photons of combination frequencies ω_{com} (10).

Nonresonant amplitudes (17)–(22) can be summed over the index s for moderately strong field (11). Given the properties of Bessel functions after some uncomplicated calculations, within the interference region, we obtain

$$S_{fi} = \sum_{l=-\infty}^{\infty} S_l, \quad S_l = -i\frac{Ze^3\sqrt{\pi}}{\sqrt{2\omega_i E_- E_+}}\bar{u}_- B_l u_+, \qquad (45)$$

$$B_l = \frac{4\pi e^{-il\Delta} H}{q_x^2 + q_y^2 + (q_0 - q_z)^2}\Delta_l(p_-,-p_+,q_0), \qquad (46)$$

$$H = \tilde{\gamma}^0\frac{\hat{q}_- + m}{q_-^2 - m^2}\hat{\varepsilon}_i + \hat{\varepsilon}_i\frac{-\hat{q}_+ + m}{q_+^2 - m^2}\tilde{\gamma}^0, \qquad (47)$$

$$\Delta_l(p_-,-p_+,q_0) = \tau_1\int_{-\infty}^{\infty} d\phi\, \exp\{iq_0\tau_1\phi\} J_l(\alpha_0(p_-,-p_+)g_1(\phi)g_2(\phi\tau_1/\tau_2)), \qquad (48)$$

$$\alpha_0(p_-,-p_+) = \frac{m^2\eta_{01}\eta_{02}}{\omega_{\text{com}}}\left[\frac{1}{(np_-)} + \frac{1}{(np_+)}\right]. \qquad (49)$$

Here, the values of velocities and polar angles of the pair are in accordance with Eqs. (37) and (38).

Therefore, expressions (45)–(48) determine the required nonresonant amplitude of CPP in the field of two pulsed moderately strong circularly polarized waves, within the interference region. The energy conservation law (25) within the interference region takes the form

$$q_0 = E_- + E_+ - \omega_i + l\omega_{\text{com}} \ll \omega_1. \qquad (50)$$

Emphasize also that the amplitudes (17)–(22) can be summed over the index s outside the interference region as well. In this case, the transition amplitude has the form of a double sum over numbers of photons of each of waves l_1,l_2, and is determined by special functions $I_{l_1,l_2}(\alpha_\pm(p_-,-p_+,\phi),\gamma_1(p_-,-p_+,\phi),\gamma_2(p_-,-p_+,\phi))$.

III. CROSS SECTION OF CPP PROCESS

Let us obtain the differential cross section for relativistic energies of an electron and a positron using the amplitudes (45)–(48) by standard mode [53]

$$d\sigma = \frac{|S_{fi}|^2}{T}\frac{d^3p_-}{(2\pi)^3}\frac{d^3p_+}{(2\pi)^3}. \qquad (51)$$

Here, the parameter T is some comparatively great time span. Let us take into account the correlation $d^3p_\mp = |\mathbf{p}_\mp|E_\mp dE_\mp d\Omega_\mp$. Thus, the differential cross section can be presented as a sum of partial components

$$d\sigma = \sum_{l=-\infty}^{\infty} d\sigma_l, \qquad (52)$$

where $d\sigma_l$ is the partial differential cross section of production of an electron into the energy range dE_- and the solid angle $d\Omega_-$ and a positron into the energy range dE_+ and the solid angle $d\Omega_+$ on a nucleus, with emission ($l > 0$) or absorption ($l < 0$) of $|l|$ photons of the combination frequency ω_{com} has the form

$$\frac{d\sigma_l}{dE_- dE_+ d\Omega_- d\Omega_+} = \frac{Z^2\alpha r_e^2}{(2\pi)^2 T}\frac{m^2|\mathbf{p}_+||\mathbf{p}_-|}{\omega_i \mathbf{q}^4} \times |\bar{u}_- H u_+|^2 |\Delta_l(p_-,-p_+,q_0)|^2. \qquad (53)$$

We choose envelopes of the potential of pulsed waves in the form of Gaussian functions:

$$g_1(\phi) = g_2(\phi\tau_1/\tau_2) = \exp\{-\phi^2\}, \quad \tau_1 = \tau_2. \qquad (54)$$

Let us not be interested in polarization effects. After appropriate averaging and summation over polarizations of an incidence photon and produced electron and positron, the expression for the partial section (53) assumes the form

$$d\sigma_l = d\sigma_l^* W_l. \qquad (55)$$

The quantity $d\sigma_l^*$ transforms into the cross section of differential cross section of a free-field CPP on a nucleus (Bethe-Heitler cross section $d\sigma_{\text{BH}}$) [1,53], when energy corrections can be negligible. The quantity W_l is the probability of stimulated emission and absorption of the equal number of photons of pulsed waves by an electron and a positron:

$$W_l(p_-,-p_+) = \sum_r \frac{1}{\rho}\int_0^\rho d\phi\, J_r^2(\alpha) J_{l-r}^2(\gamma_1) J_{l+\delta_2 r}^2(\gamma_2). \qquad (56)$$

Within the interference region, when particles are produced with energy and angles in accordance with conditions (38), then the probability of stimulated absorption and emission processes is simplified to the form

$$W_l = \frac{1}{\rho}\int_0^\rho J_l^2(\alpha_0(p_-,-p_+)\exp(-2\phi^2))d\phi. \qquad (57)$$

The quantity $\rho = T/\tau_1$ in Eqs. (56) and (57) defines the averaged interval over a laser pulse. Its value is determined by the concrete experiment conditions. Thus, if the external field represents itself the sequence of consecutive pulses, the parameter ρ assumes the sense of the ratio of the distance between adjacent pulses to the character pulse width.

Notice that the argument of the Bessel function (57) is the same order in the magnitude with the quantum interference parameter $\alpha_0(p_-,-p_+)$ [Eq. (49)]. Therefore, for nonresonant CPP on a nucleus, electron and positron relativistic energy and moderately strong pulsed waves, within the interference region, the probability of stimulated absorption and emission of equal number of photons of both waves is determined by the quantum interference parameter (49). We study the process when $\eta_{0j} \ll 1$, $\gamma_{0j} \ll 1$, and the main multiphoton parameter is $\alpha_0 \gg 1$.

Figure 3 presents partial probabilities of emission of an equal number ($l = 0; 1; 2$) of external-field photons (56), depending on the angle of positron emission θ_+, near the interference region (38) for circular polarization. An incident photon collides with a laser pulse at the angle 5° ($\theta_i = 175^\circ$). Figure 3(a) corresponds to pair relativistic energy, and Fig. 3(b) to pair nonrelativistic energy. The electron polar angle and its energy are fixed and meet the interference condition (38).

It can be seen from Fig. 3 that distribution of partial probabilities over positron polar angle has an obviously pronounced peak. The provision of the distribution maximum in Fig. 3 is determined by values of positron polar angle; these values meet the interference condition (38), (a) $\theta_+ = 167.55^\circ$ and (b) $\theta_+ = 147.15^\circ$. The top of the peak may be described by Eq. (57). At that, the value of the multiphoton parameter (49) for Fig. 3(a) $\alpha_0(p_-,-p_+) = 134$ is slightly greater, then for Fig. 3(b) $\alpha_0(p_-,-p_+) = 123$. This situation explains the slight difference in the height of maxima in Figs. 3(a) and 3(b). When the number of photons increases, the height of the peak decreases, and its position slightly shifts relative to the interference angle of positron exit. From Fig. 3 one can conclude that partial probabilities within the interference region may exceed the corresponding probabilities in other scattering kinematics in two orders of the magnitude. Note also that the parameter $\alpha'_0 \sim 10$ [see Eq. (10)] in the presented calculations.

Notice that for nonrelativistic energy of an electron and a positron [Fig. 3(b)], the range of polar angles within the interference region is wider in comparison with the relativistic energy case [Fig. 3(a)]. Also, within the range of angles presented at Figs. 3(a) and 3(b), the distribution of partial probabilities is practically symmetrical relatively to the interference angle of positron emission.

All reasonings and laws mentioned above are valid when considering the distribution over the angle of electron emission due to the symmetry of main expressions with respect to the replacement of an electron by a positron.

In the case of electron and positron relativistic energy, corrections, related to stimulated emission and absorption of external-field photons, can be neglected in the energy conservation law, in comparison with electron and positron energy. Consequently, the partial cross section of nonresonant CPP process is factorized into the cross section of free-

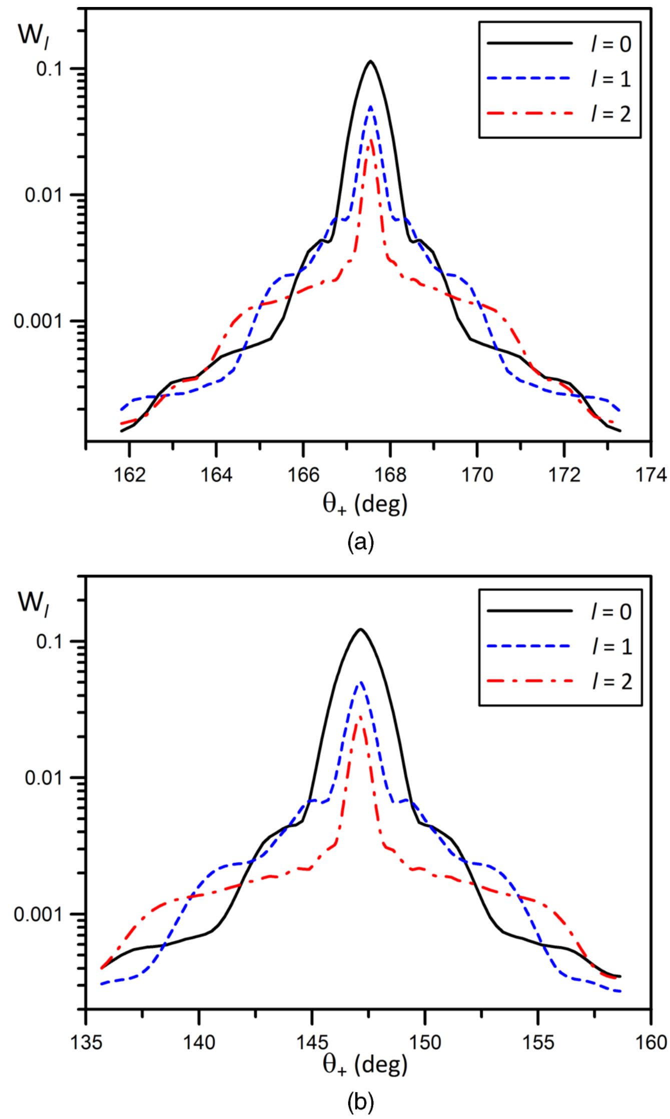

FIG. 3. Dependence of partial process probabilities on the polar angle of positron near the interference region at emission of an equal number of external-field photons (56) and other fixed parameters. Field parameters: $\eta_{01} = \eta_{02} = 0.02$, ($I_{01} = 3.74 \times 10^{15}$ W cm^{-2}, $I_{02} = 0.68 \times 10^{15}$ W cm^{-2}), $\omega_1 = 2.35$ eV, $\omega_2 = 1$ eV, $\delta_1 = +1$, $\delta_2 = -1$, $\rho = \sqrt{3}$. The polar angle of incident photon $\theta_i = 175^\circ$. (a) The incident-photon energy $\omega_i = 1.066$ MeV ($T_i = 0.09$), electron and positron velocities $v_- = 0.3$ and $v_+ = 0.3$, the electron polar angle $\theta_- = 6.73^\circ$. (b) $\omega_i = 1.025$ MeV ($T_i = 0.01$), $v_- = 0.1$, $v_+ = 0.1$, $\theta_- = 27.12^\circ$.

field CPP on a nucleus ($d\sigma_l^* \approx d\sigma_{\rm BH}$), and the probability of stimulated emission (absorption) of a certain number of external-field photons. In other words, photoproduction of an electron-positron pair and stimulated emission (absorption) of external-field photons occur independently of each other. Notice, for the relativistic energy such a factorization is realized outside the interference region as well.

The differential cross sections (52), (55), and (56), for the case of electron and positron relativistic energy, can be summed over all possible processes of stimulated emission and absorption of laser-field photons. After uncomplicated computations, we obtain

$$d\sigma = \sum_{l=-\infty}^{\infty} d\sigma_l = d\sigma_l^* \sum_{l=-\infty}^{\infty} W_l \approx d\sigma_{\rm BH}. \tag{58}$$

Thus, for electron and positron relativistic energy, within the range of moderately strong fields, all essentially quantum contributions are compensated after summation of partial cross sections over all processes of stimulated emission and absorption of external-field photons. Therefore, the differential cross section (52) coincides with the differential cross section of free-field CPP on a nucleus.

We note that, in the limit case of two plane monochromatic waves, the relation (56) transforms into

$$W_l = J_l^2(\alpha_0(p_-,-p_+)), \tag{59}$$

and Eqs. (52), (55), and (59) represent the differential cross section of photoproduction of a relativistic electron-positron pair on a nucleus in the field of two monochromatic waves, within the interference region [27].

It is of interest to consider the nonrelativistic case, when the contribution of field photons into the energy law cannot be neglected in comparison with electron and positron energy.

Cross section of CPP for nonrelativistic energy

In this section, we consider the case of nonrelativistic energy of produced electron and positron

$$v_\mp \ll 1. \tag{60}$$

We remind that the classical parameter η_{0j} (8) makes sense the velocity of electron (positron) oscillation in the external laser field of moderately strong intensities. We will study nonresonant photoproduction of nonrelativistic electron-positron pair on a nucleus in the moderately strong field of two pulsed waves (11), when the velocity of electron and positron oscillations in a laser pulse is less than or the same order with the velocities of their translational movement

$$\eta_{01,02} \lesssim v_\mp. \tag{61}$$

Inasmuch as we consider the process within the Born approximation, then the value of the energy of an incident photon has to meet the following condition:

$$Z\alpha \ll \sqrt{\frac{\omega_i - 2m}{m}} \ll 1. \tag{62}$$

The energy conservation law (25) in this case assumes the form

$$\frac{mv_-^2}{2} + \frac{mv_+^2}{2} - mT_i + l_1\omega_1 + l_1\omega_1 \ll \omega_1. \tag{63}$$

Here, T_i is the dimensionless parameter, which is determined by the energy of incident γ photon:

$$T_i = \frac{\omega_i - 2m}{m}. \tag{64}$$

The differential cross section of CPP process for nonrelativistic energy has the form of sum over partial components in general case of scattering kinematics:

$$d\sigma = \sum_{l_1,l_2=-\infty}^{\infty} d\sigma_{l_1 l_2}^{(v_\mp \ll 1)}. \tag{65}$$

The cross section $d\sigma_{l_1 l_2}^{(v_\mp \ll 1)}$ is the partial differential cross section of photoproduction of an electron into the nonrelativistic energy range dE_- and the solid angle $d\Omega_-$ and a positron into the range dE_+ and the solid angle $d\Omega_+$ on a nucleus, with emission $(l_1,l_2 > 0)$ or absorption $(l_1,l_2 < 0)$ of a certain number of laser-field photons:

$$\frac{d\sigma_{l_1 l_2}^{(v_\mp \ll 1)}}{dE_- dE_+ d\Omega_- d\Omega_+} = \frac{d\sigma_{l_1 l_2}^{*(v_\mp \ll 1)} W_{l_1 l_2}}{dE_- dE_+ d\Omega_- d\Omega_+} f_\delta(q_0). \tag{66}$$

Here, the function $f_\delta(q_0)$ is the wide Dirac delta function. The cross section $d\sigma_{l_1 l_2}^{*(v_\mp \ll 1)}$ depends on the number of stimulated photons and transforms into the cross section of a free-field process $d\sigma_{\rm BH}$ under $l_1 = l_2 = 0$:

$$\frac{d\sigma_{l_1 l_2}^{*(v_\mp \ll 1)}}{dE_- dE_+ d\Omega_- d\Omega_+} = \frac{Z^2\alpha r_e^2}{64\pi^2}\frac{v_+ v_-}{m} \times (v_+^2 \sin^2\theta_+ + v_-^2 \sin^2\theta_-). \tag{67}$$

The function $W_{l_1 l_2}$ defines the probability of stimulated emission and absorption of laser-field photons by an electron and a positron:

$$W_{l_1 l_2} = \sum_r \frac{1}{\rho}\int_0^\rho d\phi\, J_r^2(\alpha_0 \exp(-2\phi^2)) \times J_{l_1-r}^2(\gamma_{01}\exp(-\phi^2)) J_{l_2+\delta_2 r}^2(\gamma_{02}\exp(-\phi^2)). \tag{68}$$

Here, Bessel function arguments in the nonrelativistic case are specified as

$$\gamma_{0j} = \eta_{0j}\frac{m}{\omega_j}\sqrt{a_-^2 + a_+^2 - 2a_- a_+ \cos(\varphi_- - \varphi_+)}, \tag{69}$$

$$a_- = \frac{v_- \sin\theta_-}{1 - v_- \cos\theta_-}, \quad a_+ = \frac{v_+ \sin\theta_+}{1 - v_+ \cos\theta_+}, \tag{70}$$

$$\alpha_0 = \eta_{01}\eta_{02}\frac{m}{\omega_{\rm com}}(2 + v_- \cos\theta_- + v_+ \cos\theta_+). \tag{71}$$

Note that the parameter (71) for CPP process in one order of magnitude greater than corresponding parameter for the process of spontaneous bremsstrahlung of an electron on a nucleus in two laser waves [41].

With considering of Eq. (71), we estimate the contribution of field corrections into the conservation law (63):

$$\frac{|l_{1,2}|\omega_{1,2}}{mv_\mp^2} \lesssim \alpha_0 \frac{\omega_{1,2}}{mv_\mp^2} \sim \frac{\eta_{01}\eta_{02}}{v_\mp^2} \lesssim 1. \tag{72}$$

Thus, for waves intensities (61), the energy corrections on the external field cannot be neglected in the energy conservation law (63), in contrast to the previously considered case of electron and positron relativistic energies. Note that the function $W_{l_1 l_2}$ [Eq. (68)], strictly speaking, does not make sense of the probability of stimulated emission or absorption of laser-field photons, due to dependence of its arguments on the photon number l_1,l_2.

Expressions (65)–(68) determine the differential cross section of nonresonant photoproduction of a pair of a nonrelativistic electron and a positron on a nucleus, in a moderately strong field of two pulsed laser waves. Accordingly, the partial cross section of nonresonant CPP does not factorize into the cross section of free-field CPP on a nucleus, and the probability of emission (absorption) of a certain number of laser-field photons, when the velocity of electron and positron oscillations in a laser pulse is less than or the same order with the velocities of their translational movement (61). Such a factorization takes place for quite weak fields $\eta_{01,02} \ll v_{\mp}$ only [8,27].

We pose the problem to study the distribution of the differential cross section over electron and positron energy with the fixed geometry of production process.

In the case of a pulsed laser field, the energy conservation law is not rigorously fulfilled, however, the essential region of integration narrows sharply [Eq. (63)] and $f_\delta(q_0) \to \delta(q_0)$, due to the quasimonochromatic conditions (6). We emphasize that electron and positron energy in the final state are determined by the energy of an incident photon ω_i and a number of photons of the first l_1 and second l_2 wave. The conservation law establishes correspondences between the five values. Experimentally, the number of external-field photons, which participated in CPP process, is hardly determined directly. Therefore, it is convenient to use the conservation law for the convolution of one of sums, for example, over l_2, in Eq. (65).

Then, for electron fixed energy, it is possible to obtain the distribution of the differential cross section over the positron energy in the form

$$\frac{d\sigma}{dE_- d\Omega_- d\Omega_+} = \frac{Z^2\alpha r_e^2}{64\pi^2} \sum_{l_1=-\infty}^{\infty} \frac{v_+(l_1)v_-}{m} \times [v_+^2(l_1)\sin^2\theta_+ + v_-^2\sin^2\theta_-] W_{l_1 l_2} \frac{dE_+}{\omega_2}. \tag{73}$$

Here, it should be the integer number instead of the index l_2 in the function $W_{l_1 l_2}$ [Eq. (68)]:

$$l_2 \Rightarrow \left[-l_1 \frac{\omega_1}{\omega_2} + (1 - \varepsilon_{\text{kin}}) \frac{mT_i}{\omega_2} \right]. \tag{74}$$

In Eq. (74), the quantity ε_{kin} is specified as

$$\varepsilon_{\text{kin}} \equiv \frac{v_+^2 + v_-^2}{2T_i} = \frac{mv_+^2/2 + mv_-^2/2}{\omega_i - 2m}. \tag{75}$$

The quantity ε_{kin} makes sense of the ratio of the pair kinetic energy (the sum of kinetic energy of electron and positron) to the difference between the energy an incident photon and the pair rest energy. This dimensionless parameter is useful in the further study of the differential cross section: $dE_+ = mT_i d\varepsilon_{\text{kin}}$. Underline that the value of the parameter $\varepsilon_{\text{kin}} = 1$ corresponds to partial processes with the photons number $l_1 = l_2 = 0$ and velocity values for free-field CPP process.

For quantitative analysis, we consider the ratio of the obtained differential cross section to the cross section in the absence of an external field. We derive

$$R = \int d\varepsilon_{\text{kin}} \frac{d\sigma(\varepsilon_{\text{kin}})}{d\sigma_{\text{BH}}}, \tag{76}$$

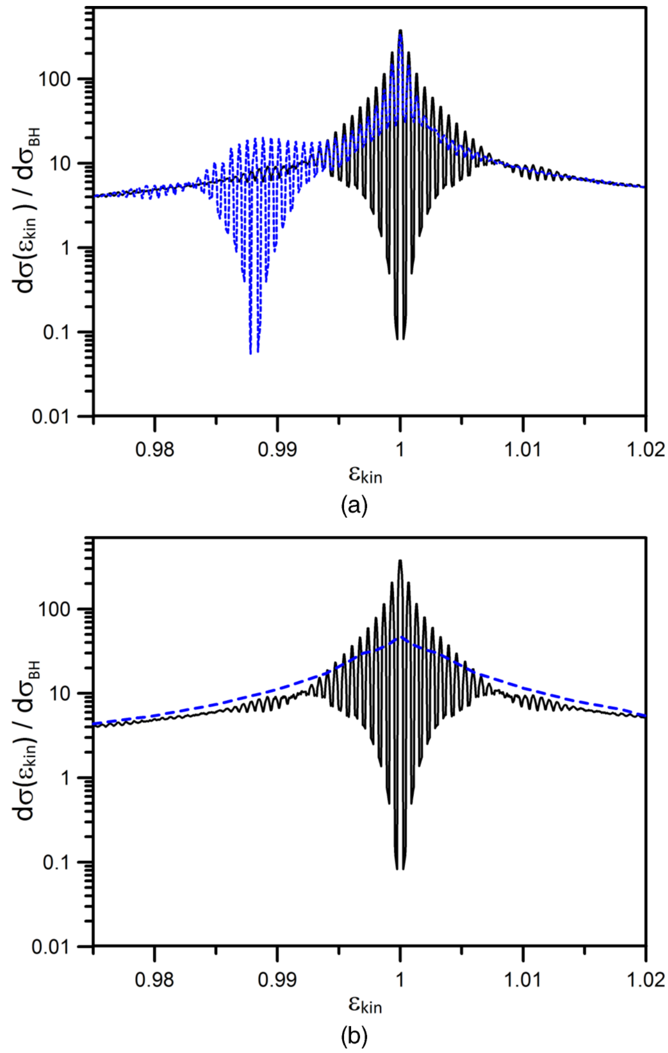

FIG. 4. The distribution of the differential cross section of CPP on a nucleus in the field of two pulsed laser waves (77) depending on the pair kinetic energy at the fixed geometry of production. An incident photon with the energy $\omega_i = 1.025$ MeV ($T_i = 0.01$) collides with laser pulse at the angle 5° ($\theta_i = 175°$). Field parameters: $\eta_{01} = \eta_{02} = 0.02$, ($I_{01} = 3.74 \times 10^{15}$ W cm^{-2}, $I_{02} = 0.68 \times 10^{15}$ W cm^{-2}), $\omega_1 = 2.35$ eV, $\omega_2 = 1$ eV, $\delta_1 = +1$, $\delta_2 = -1$, $\rho = \sqrt{3}$. The velocity and polar angle of an electron, $v_- = 0.1$ and $\theta_- = 27.12°$, meet the interference condition (38). The angle of positron emission for the solid curve in (a) and (b) $\theta_+ = 147.15°$, for the dashed curve in (a) $\theta_+ = 146.74°$ and (b) $\theta_+ = 158.61°$.

$$\frac{d\sigma(\varepsilon_{\text{kin}})}{d\sigma_{\text{BH}}} = \frac{mT_i}{\omega_2}\sqrt{\frac{2T_i\varepsilon_{\text{kin}} - v_-^2}{2T_i - v_-^2}} \times \frac{(2T_i\varepsilon_{\text{kin}} - v_-^2)\sin^2\theta_+ + v_-^2\sin^2\theta_-}{(2T_i - v_-^2)\sin^2\theta_+ + v_-^2\sin^2\theta_-} \times \sum_{l_1,r=-\infty}^{\infty} \frac{1}{\rho}\int_0^\rho d\phi\, J_r^2(\alpha) J_{l_1-r}^2(\gamma_1) J_{l_2+\delta_2 r}^2(\gamma_2). \tag{77}$$

Figure 4 presents the distribution of the differential cross section (77) depending on the pair kinetic energy. Figure 4(a) represents comparison for different angles of positron emission, within the interference region. Figure 4(b) represents comparison of distribution over the pair energy for cases of the interference region and the Bunkin-Fedorov region. For the case, which is described by the solid curve, the geometry is chosen in accordance with the interference condition (38) [$\alpha_0(p_-,-p_+) = 123$] and for the small values of photon number.

It is obvious from Fig. 4 the distribution over the pair kinetic energy is characterized by presence of oscillations, within the interference region. Each of the maxima corresponds to the definite partial process with emission and absorption of an equal number of photons of the both waves [see Fig. 3(b)]. Wherein the energy of an electron-positron pair changes, therefore each of the peaks is possible to separately be observed in the distribution in Fig. 4. The height of the corresponding peak decreases with increasing of the photon number. When a polar angle of a positron changes slightly [see dashed curve in Fig. 4(a)], the interference region is shifting in accordance with the condition (38). In this case, the maxima heights within the interference region are less, and are determined by positron polar angle, as well as value of photon number.

When a pair is produced within the Bunkin-Fedorov region [see the dashed curve in Fig. 4(b)], the distribution of the differential cross section (77) over the pair energy changes smoothly and has a maximum. This maximum corresponds to partial processes with a number of photons $l_1 = l_2 = 0$. Wherein, the differential cross section can be greater or less than the cross section within the interference region, depending on the pair energy. But, as Fig. 4 concludes, the differential cross section within the interference region for certain values of the pair energy may exceed the cross section in other scattering kinematics in two orders of magnitude.

It should be emphasized that the parametric interference effect is manifested in the specified kinematic region. Consequently, the experimental verification of the obtained results is only possible when measuring the differential characteristics of the CPP cross section in the field of two pulsed waves. For this purpose, the pair production process should be necessarily considered in the plane defined by an initial photon momentum and the wave vectors of a laser field. An electron and a positron should be detected at the polar angle (38). Peculiarities of cross-section distribution within the interference region can be observed experimentally by using detectors with high resolution.

Note that the researched process may be considered for an alternative statement of the problem. For example, the conditions of pair production can be realized in a collision of an ultrarelativistic ion beam and x-ray photons. In this case, of course, the changes of particle's energy and angle have to be taken into account according to the Lorentz transformations.

A quantitative analysis was carried out at characteristics of radiation for the laser system Phelix (FAIR project, GSI, Germany). This facility can generate laser pulse at two different photon energies with the values which were used in the numeric calculations. The scientific program of the FAIR project involves the study of the interaction of high-energy ions with different particles and the study of pair production in collisions of heavy ions. Thus, the experimental verification of the theoretical results related to a laser-modified photoproduction of pairs on a nucleus is proposed within the framework of the FAIR project since all the necessary conditions may be implemented.

IV. CONCLUSIONS

(1) CPP process in the field of two pulsed laser waves is characterized by presence of the selected kinematic region (the interference region), where correlated emission and absorption of photons of both waves dominate. Within this region, a correspondence between the polar angle and energy of the produced particles appears. The particle minimal energy is determined by the angle of an incident-photon entrance.

(2) Partial probabilities of stimulated emission and absorption of an equal number of photons of the both waves within the interference region may exceed the corresponding probabilities in other scattering kinematics in two orders of the magnitude. At that, for fixed electron and positron energy, the maximum of distribution of the probability over the angle of emission of each of particles corresponds to the interference polar angle.

(3) The distribution of the obtained differential cross section on the pair kinetic energy is characterized by presence of oscillations, within the interference region. Each of the maxima corresponds to the definite partial process with emission and absorption of an equal number of photons of both waves. When a polar angle of a positron changes, the interference region is shifting in distribution over the energy. This case corresponds to the obtained conformity between the polar angle and energy of produced particles. At that, the differential cross section within the interference region for certain values of the pair energy may exceed the cross section in other scattering kinematics in two orders of magnitude.

The obtained results may be experimentally verified, for example, by the scientific facilities at sources of pulsed laser radiation (FAIR, SLAC, XFEL, ELI, XCELS).

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