Tunable Landau-Zener transitions using continuous- and chirped-pulse-laser couplings

Farrokh Sarreshtedari[1,*] and Mehdi Hosseini[2]
[1]*Magnetic Resonance Research Laboratory, Department of Physics, University of Tehran, Tehran 143-9955961, Iran*
[2]*Department of Physics, Shiraz University of Technology, Shiraz 313-71555, Iran*
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The laser coupled Landau-Zener avoided crossing has been investigated with an aim towards obtaining the laser source parameters for precise controlling of the state dynamics in a two-level quantum system. The conventional Landau-Zener equation is modified for including the interaction of the system with a laser field during a bias energy sweep and the obtained Hamiltonian is numerically solved for the investigation of the two-state occupation probabilities. We have shown that in the Landau-Zener process, using an additional laser source with controlled amplitude, frequency, and phase, the system dynamics could be arbitrarily engineered. This is while, by synchronous frequency sweeping of a chirped-pulse laser, the system could be guided into a resonance condition, which again gives the remarkable possibility for precise tuning and controlling of the quantum system dynamics.

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I. INTRODUCTION

The Landau-Zener transition process in a two-level quantum system describes the dynamics of the states' occupation probabilities when the energy separation of the two states is linearly swept. Although the introduction of the Landau-Zener (LZ) model dates back to 1932 (Landau [1] and Zener [2]), it has many applications in today's atomic and molecular physics, quantum information science, quantum optics, and related fields [3–11]. Important experimental examples of using the Landau-Zener model include its application for preparation, manipulation, and reading out of quantum states in qubits and population inversion in many-body systems [5,9–14]. The Landau-Zener formalism is based on a time-dependent Hamiltonian with a constant variation rate of the energy difference. Assuming the system in a pure state in the infinite past, a general sweep rate of energy leads the system into nonadiabatic transitions and the Landau-Zener model analytically describes the dynamics of such process [1–3]. In fact, in the absence of diabatic coupling between the two states, the corresponding energy levels would have an exact crossing when the energy level is swept. However with a finite coupling value, the adiabatic energy levels E_1 and E_2 form an avoided crossing where the LZ transition takes place at such crossing points [3]. Significant utilization of the LZ model in experimental quantum systems inspired many works for considering new aspects and extensions to this classic problem. Among them controlling and tuning of the Landau-Zener transitions with different mechanisms has been investigated in different applications [15–20]. A tunable Landau-Zener mechanism could be categorized as a method in the quantum optimal control context whose goal is to precisely manipulate the dynamics of quantum processes [21]. References [15–17] are devoted to investigating the application of periodic modulations on the energy sweep of the Landau-Zener process for quantum manipulations. They have included a sinusoidal term into the diagonal energy which is superimposed to the conventional linear rate of energy change. The obtained Hamiltonian is solved there and the effect of periodic modulations on the tunneling process of the system is discussed. On the other hand, in the interesting work reported in Ref. [18], the effect of an oscillatory coupling energy as off-diagonal terms of the LZ Hamiltonian has been considered by applying an electromagnetic field to the quantum system. They have substituted the constant tunneling rate of the conventional LZ problem with an oscillatory coupling energy and investigated its effect on the LZ solution.

In this work, we have also investigated the effect of applying an electromagnetic field to the quantum system during a Landau-Zener transition process. Here, the off-diagonal coupling energy is considered as a superposition of the oscillatory electric dipole interaction term with the constant tunneling rate of the system to be used as a tunability mechanism. We have shown that in this problem, the consideration of the constant tunneling rate considerably affects the system dynamics. Two types of laser sources are considered in this work which include monochromatic and chirped-pulse-laser sources [22]. The interaction of the quantum system with the laser field is considered through conventional Rabi formalism in which the product of the atomic transition dipole moment and the illuminated light electric field give the measure of population fluctuations between the levels [23]. Here, investigating the mentioned quantum process, the dependence of the states' population to the monochromatic laser field parameters including intensity, frequency, and phase is obtained to be used for desired dynamics tuning. It is also shown that if a chirped-pulse laser is incorporated in LZ experiments where its frequency changes in synchrony to the sweeping rate of the bias energy, the resulting resonance between the states gives another interesting tool for precise controlling of the system dynamics.

II. THEORETICAL MODEL

In this work, the developed Hamiltonian of the laser coupled LZ process is numerically solved for obtaining the evolution of energy levels as well as the transition probabilities between the states. Equation (1) shows the time-dependent Hamiltonian

*f.sarreshtedari@ut.ac.ir

of the conventional Landau-Zener model [24].

$$H_{LZ}(t) = H_{12}\sigma_x + \tfrac{1}{2}\alpha t\sigma_z, \quad (1)$$

where H_{12} is the tunneling rate between the states at $t = 0$, and α is the sweeping rate of the system bias energy. On the other hand the Hamiltonian of the laser-atom interaction is considered as

$$H_{\text{Int}}(t) = \hbar\Omega \cos{[2\pi f_L(t+\theta)]}\sigma_x, \quad (2)$$

where, f_L is the laser frequency and Ω is the Rabi frequency which is related to the fluctuation of population between the levels and is determined by the coupling strength between the laser field and the atomic transition dipole moment

$$\Omega = \frac{\vec{E}\cdot\vec{P}}{\hbar}, \quad (3)$$

where $\vec{E}$ is the laser electric field and $\vec{P}$ is the atomic transition dipole moment. Incorporating Eqs. (1) and (2), the total Hamiltonian would be obtained as Eq. (4) which is solved for obtaining the system dynamics.

$$H(t) = H_{LZ}(t) + H_{\text{Int}}(t) = \begin{Bmatrix} \alpha t/2 & H_{12} + \hbar\Omega\cos{[2\pi f(t+\theta)]} \\ H_{12}^* + \hbar\Omega\cos{[2\pi f(t+\theta)]} & -\alpha t/2 \end{Bmatrix}. \quad (4)$$

Considering Eq. (4), if the intensity of the applied field vanishes, the Hamiltonian reduces to the conventional LZ Hamiltonian. In this model, the existence of nonzero H_{12} prevents the coupling energy to vanish at zero field. However, for nonzero fields, the value of both the field intensity and H_{12} determine if the off-diagonal terms go to zero during the process.

It should be also noted that, here, for convenience the natural dimensionless unit system is used in which all the time variables are divided by time constant τ and the energy variables are multiplied by τ and divided by $\hbar$. In this natural system considering a specific value for τ, all the other parameters could be accordingly found using the mentioned units. In this regard, if we choose $\hat{t} = t/\tau$, and $\hat{\alpha} = \alpha\tau^2/\hbar$, $\hat{H}_{12} = \tau H_{12}/\hbar$, $\hat{\varepsilon} = \tau\varepsilon/\hbar$, $\hat{\Omega} = \Omega\tau$, $\hat{f} = f\tau$, the Schrödinger equation would be obtained as follows

$$\begin{pmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{pmatrix} = \begin{pmatrix} -i\frac{\hat{\alpha}}{2}\hat{t} & -i[\hat{\Omega}\cos(2\pi\hat{f}\cdot\hat{t}+\theta) + \hat{H}_{12}] \\ -[\hat{\Omega}\cos(2\pi\hat{f}\cdot\hat{t}+\theta) + \hat{H}_{12}^*] & i\frac{\hat{\alpha}}{2}\hat{t} \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad (5)$$

where $\phi_1(t)$ and $\phi_2(t)$ describe the states of the system, and their absolute squares [$P1(t)$ and $P2(t)$] represent the probability of finding the system in each state. It should be noted that for each value of τ, the solution result of Eq. (5) would be the same.

III. RESULTS AND DISCUSSION

We have numerically solved Eq. (5) for the investigation of the states' evolution and their eigenenergies. For comparison, the solution of the considered Landau-Zener problem with zero intensity of the laser field ($\hat{\Omega} = 0$) is depicted in Fig. 1. In this simulation the time intervals are chosen in such a way that the probabilities could reach stable values.

Figure 1 shows that in the LZ crossing, after the transition at $t = 0$, the probability of finding the system in each state is a damping oscillatory function about a determined value which can be described by the classical LZ equation. The frequency of these oscillations increases with time, which is related to the increase of energy in time with the rate of α. In our problem the end probability value of state 1 could be found using the conventional LZ formula of $P1 = \exp(-\pi H_{12}^2/8\alpha)$ [3].

A. Coupling of monochromatic laser source

In the conventional Landau-Zener process, the adiabatic energy levels of E_1 and E_2 change in time because of the $\alpha t/2$ term in the Hamiltonian of the system. Although at $t = 0$ they reach their minimum energy gap of $2H_{12}$, they never cross each other. However, in the presence of an electromagnetic field, based on the values of H_{12} and Rabi frequency, the two energy levels could make the level crossing. In this regard, for low field intensities of $\hat{\Omega} < \hat{H}_{12}$, adiabatic energy levels of E_1 and E_2 never make an exact crossing. But for $\hat{\Omega} \geqslant \hat{H}_{12}$, based on the field phase of θ, crossing between the states is possible. Figure 2 shows the variation of energy levels for the four cases of conventional LZ with $\hat{\Omega} = 0$, low field intensity of $\hat{\Omega} = 1/2\hat{H}_{12}$, and high field intensity of $\hat{\Omega} = \hat{H}_{12}$ with $\theta = 0$ and $\theta = \cos^{-1}(-\hat{H}_{12}/\hat{\Omega})$. It is obvious that for the last case and for the mentioned phase the energy levels no longer exhibit an avoided crossing but rather an exact crossing. This emphasizes

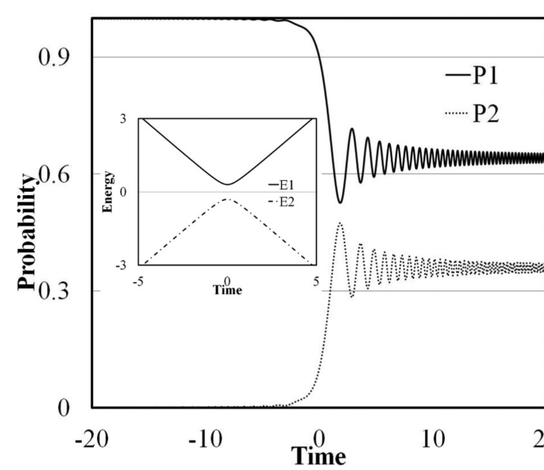

FIG. 1. The probability amplitudes of states 1 and 2 versus time through avoided crossing for $\hat{H}_{12} = 0.3$ and $\hat{\alpha} = 1.25$. The inset shows the eigenenergies of the quantum system. The time interval is $-\hat{t}_{\text{end}} \leqslant \hat{t} \leqslant \hat{t}_{\text{end}}$ where $\hat{t}_{\text{end}} = t_{\text{end}}\tau = 20$.

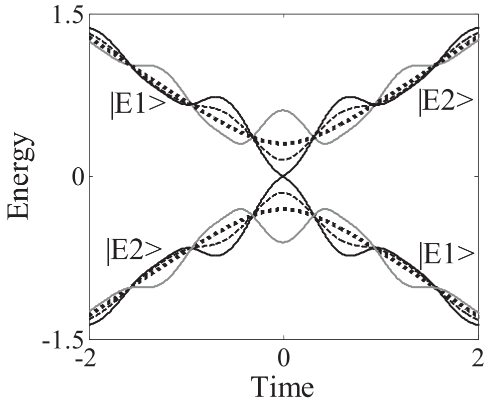

FIG. 2. The variation of adiabatic energy levels of E_1 and E_2 for cases with no field or $\hat{\Omega} = 0$ (dotted black line), low field intensity of $\hat{\Omega} = 1/2\hat{H}_{12}$ with $\theta = \pi$ (dashed black line), and high field intensity of $\hat{\Omega} = \hat{H}_{12}$ with $\theta = 0$ (gray solid line) and $\theta = \cos^{-1}(-\hat{H}_{12}/\hat{\Omega})$ (black solid line). The other parameters are $\hat{H}_{12} = 0.3$ and $\hat{\alpha} = 1.25$.

the effect of the field phase on the system dynamics which will be further considered in the following.

Figure 3 show the results of the mentioned LZ problem when the laser field is applied. In these figures, the probability amplitudes of the two states versus time are shown for two laser frequencies of $\hat{f} = 2.5$ and $\hat{f} = 0.75$ for different Rabi frequencies. It should be noted that for a physical amount of τ, these frequencies are in the range of near-optical wavelengths. In Fig. 3, after transition at $t = 0$, a specific interference pattern is obvious in the solution of the probability amplitudes which are due to the beat phenomenon between the intrinsic oscillation frequency of the system and the frequency of the applied laser field. The effect of laser field strength is also notable in these figures. Increasing the laser field strength proportionally increases the Rabi frequency where the probabilities of the states change correspondingly. It can be inferred that for the considered field frequencies, increasing the Rabi frequency in the specified range monolithically increases the transition probabilities. However, this is not the case for all the range of Rabi frequencies. For investigation of this dependence, the laser frequency and intensity is swept for different initial phases and the probability amplitudes are averaged in the 5% of the end time interval. For simplicity we name this value the averaged probability (AP) in the following.

Figure 4 shows the averaged final probability amplitudes of state 1 for different laser frequency and strength and for different initial phase angles. It is assumed that at the initial time, this state was in the ground-state or spin-up condition. In these figures, the dark region represents the complete spin transition from up to down and correspondingly the bright region represents the final up state. The other colors show the final mixed states.

Figure 4 provides a sample tool for selection of intensity, frequency, and phase of the laser source for controlling of the state dynamics and engineering the desired final spin state in the LZ experiment. It also reveals that the complete spin-flip region (dark) has an oscillatory behavior which, when increasing the laser frequency, makes it more frequent. This is while considering lines with constant laser amplitude,

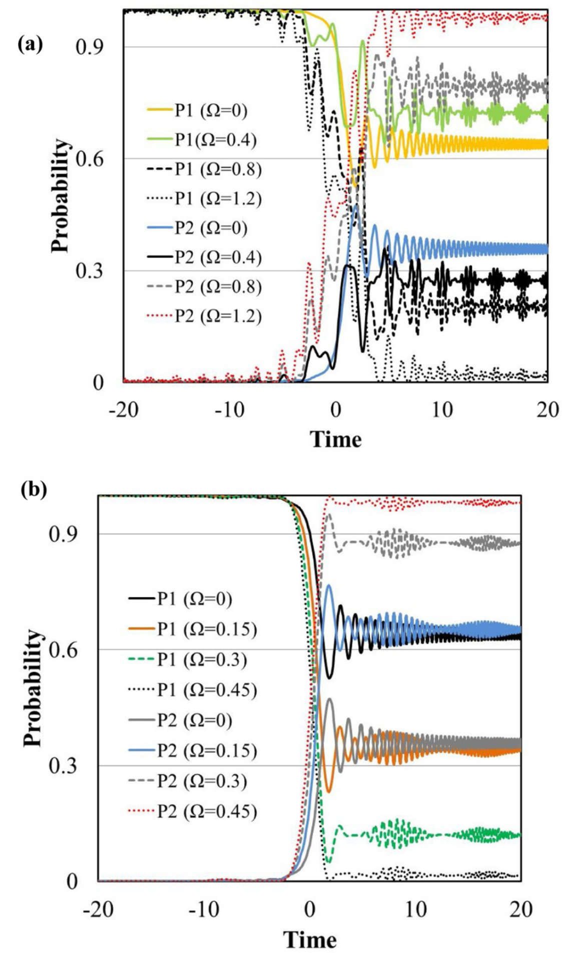

FIG. 3. The probability amplitudes of states 1 and 2 versus time through avoided crossing for different field and Rabi frequencies. (a) $\hat{f} = 2.5$; (b) $\hat{f} = 0.75$. The other parameters are $\hat{H}_{12} = 0.3$ and $\hat{\alpha} = 1.25$.

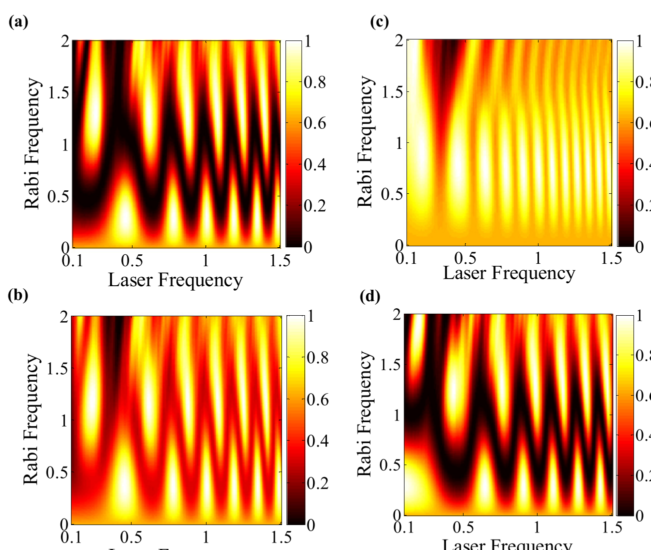

FIG. 4. The AP of state 1 for sweeping of the Rabi frequency and laser frequency with phase difference of (a) $\theta = 0^\circ$, (b) $\theta = 45^\circ$, (c) $\theta = 90^\circ$, and (d) $\theta = 180^\circ$. The other parameters are $\hat{H}_{12} = 0.3$ and $\hat{\alpha} = 1.25$.

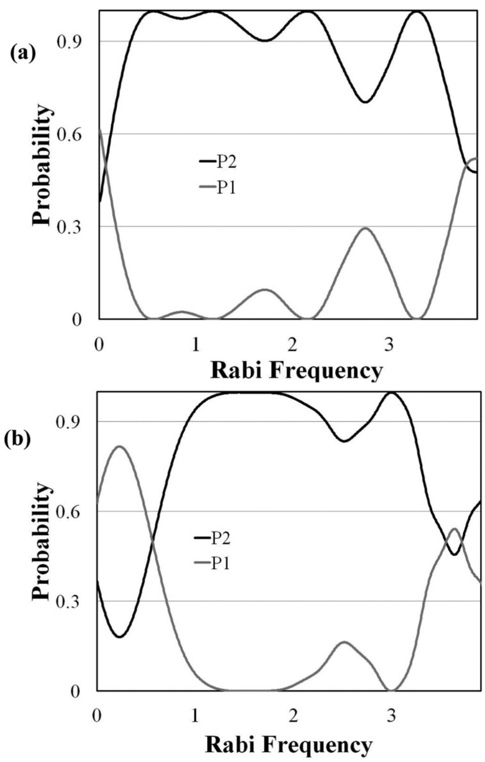

FIG. 5. The AP of states 1 and 2 versus Rabi frequency for the laser frequency of (a) $\hat{f} = 0.12$, and (b) $\hat{f} = 0.4$. The other parameters are $\hat{H}_{12} = 0.3$, $\theta = 0°$, and $\hat{\alpha} = 1.25$.

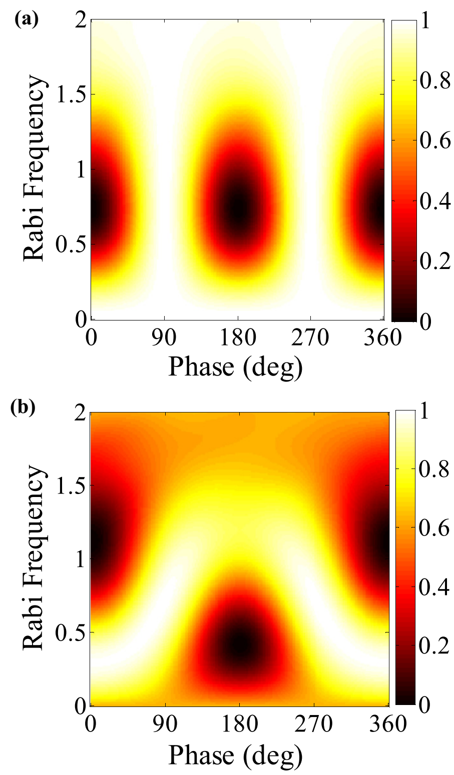

FIG. 7. The AP of state 1 for sweeping of the Rabi frequency and the laser field phase with (a) $\hat{H}_{12} = 0$, and (b) $\hat{H}_{12} = 0.3$. The other parameters are $\hat{f} = 1$, and $\hat{\alpha} = 1.25$.

the large varying of the final population reveals the impact of laser frequency on the system dynamics. From Fig. 4 it is also evident that for very low Rabi frequencies, the probability amplitudes are independent of the laser frequency and approach the classical LZ result to which no external field is applied. For better comparison, the obtained results for the field frequencies of $\hat{f} = 0.12$ and $\hat{f} = 0.4$ at $\theta = 0°$ are shown in Fig. 5.

Figure 6 shows the dependence of the averaged final probability amplitude of state 1 to both laser frequency and field phase. This figure again shows the oscillatory behavior of the transition probability with changing the laser frequency,

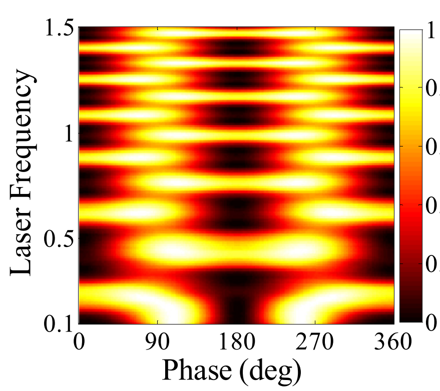

FIG. 6. The AP of state 1 for sweeping of the laser frequency and the laser field phase. The other parameters are $\hat{H}_{12} = 0.3$, $\hat{\alpha} = 1.25$ and $\hat{\Omega} = 1$.

especially for high frequencies. It is worth mentioning that robustness is an important issue in selection of the quantum control strategy [21]. From an experimental point of view, Figs. 4–6 suggest that in choosing the laser parameters, it is desirable to select them in regions where the variation of the probability amplitudes is minimum. The reason is that the minimum variation results in the minimum sensitivity of the system dynamics to the laser stability or the experimental uncertainties. In contrast, in regions with large variations, small changes in frequency, intensity, or phase of the laser source would considerably change the final value of the states. For example, in selecting the frequency of the laser source, Fig. 6 shows that selecting the lowest possible frequency for the desired dynamics engineering gives the benefit of minimum required laser frequency stability.

Figure 7 shows the dependence of the state 1 probability amplitude to both the Rabi frequency and the laser field phase at a constant frequency of $\hat{\omega} = 1$. In this figure, part (a) shows the result for zero constant tunneling rate of $\hat{H}_{12} = 0$ and part (b) shows the obtained result for $\hat{H}_{12} = 0.3$. Wubs *et al.* have analytically derived the final transition probability of the states for $\hat{H}_{12} = 0$ [17]. They have shown that the transition probability equation has a factor of $\cos^2(\theta)$ and also the maximum of the transition probability takes place when the relation between the Rabi frequency and the energy sweeping rate satisfy $\hat{\Omega}/\sqrt{\hat{\alpha}} = [2\ln(2)/\pi]^{1/2} \approx 0.664$. Figure 7(a) exactly shows the same result which for $\hat{\alpha} = 1.25$ at $\theta = 0°$; the maximum transition probability takes place at $\hat{\Omega} = 0.75$. This is while the transition probability has a $\cos^2(\theta)$

dependence on the changes of laser field angle. It is also worth mentioning that Fig. 7(a) shows a symmetric behavior for the positive and negative sign of the applied field, in which if we increase the field angle by an amount of 180°, nothing would be changed. In comparison, Fig. 7(b) shows that the existence of a nonzero constant tunneling rate of $\hat{H}_{12} = 0.3$ has a considerable effect on the pattern of the probability amplitudes.

It also shows that the mentioned symmetry for reversing the sign of the field would be broken in this case and hence the maximums of the transition probability for $\theta = 0^\circ$ and $\theta = 180^\circ$ take place at different Rabi frequencies of $\hat{\Omega} = 1.13$ and $\hat{\Omega} = 0.43$, respectively.

B. Coupling of chirped-pulse-laser source

The eigenenergies of Eq. (4) in the absence of the laser field can be obtained as Eq. (6).

$$\hat{\varepsilon}_g(\hat{t}) = 2\pi \hat{f}_g(\hat{t}) = \pm\sqrt{\hat{H}_{12}^2 + (\hat{\alpha}\hat{t}/2)^2}. \tag{6}$$

Furthermore, for small values of energy gap and far enough from $t = 0$, the eigenenergies could be approximated by a linear function of time $\hat{\varepsilon}_g(t) \approx \pm\hat{\alpha}\hat{t}/2$. Considering these energy levels, if in a specific period of time, the energy of the laser photons follows the gap energy, the resonance happens between the two states of the system. Accordingly if the frequency of a tunable chirped laser follows the energy gap of the eigenenergies, we can guide the system into the resonance condition. Figure 8 shows the obtained result of the probability amplitudes with consideration of the LZ Hamiltonian for two cases of on-resonance and 5% off-resonance laser frequency. Comparison of these figures confirms that ε_g is the resonance frequency. The frequency of the oscillations in Fig. 8(a) is determined by the Rabi frequency which can be chosen by adjusting the laser field intensity. Because of the mentioned resonance phenomenon, changing the field intensity does not change the behavior of system dynamics, except the frequency of the mentioned oscillations.

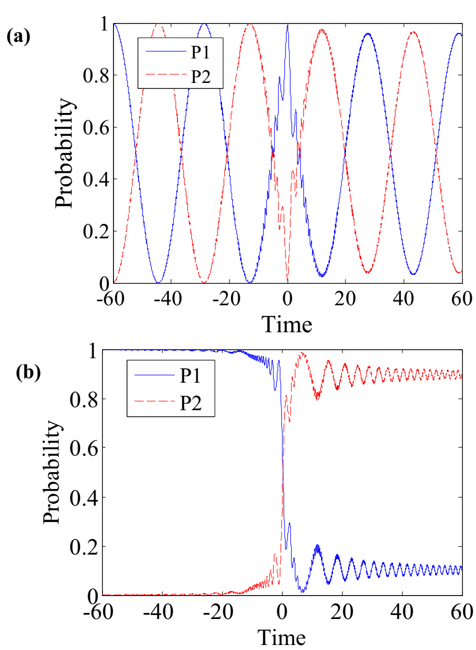

FIG. 8. The probability amplitudes of the states ($P1$ is the solid line and $P2$ is the dashed line) in the (a) resonance, and (b) 5% off-resonance cases. The considered parameters are $\hat{H}_{12} = 0.3$, $\hat{\alpha} = 1.25$, and $\hat{\Omega} = 0.2$.

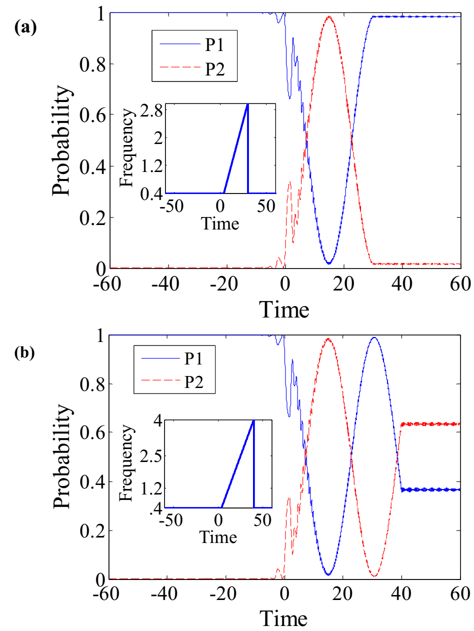

FIG. 9. The probability amplitudes of the states ($P1$ is the solid line and $P2$ is the dashed line) in the resonance case for a limited time interval (a) $\hat{t} = [0–40]$, and (b) $\hat{t} = [0–30]$. The insets show the changes of the laser frequency for each case. The considered parameters are $\hat{H}_{12} = 0.3$, $\hat{\alpha} = 1.25$, and $\hat{\Omega} = 0.2$.

Furthermore if instead of applying the chirped laser source in the whole LZ experiment period, a chirped-pulse laser interacts with the system in a specific time interval, it is possible to precisely control and tune the population of the states. Figure 9 shows the probability amplitudes for the case that the chirped-pulse laser follows the resonance frequency condition in two specific intervals of time.

IV. CONCLUSION

We have investigated the Landau-Zener process when an electromagnetic source interacts with the two-level quantum system. By developing and solving the system Hamiltonian, the effect of the monochromatic laser parameters including intensity, frequency, and phase on the dynamics of the system has been studied. It is shown that consideration of the constant tunneling rate (H_{12}), in addition to effect of the field interaction, considerably changes the probability amplitudes. While the probability amplitudes have a $\cos^2(\theta)$ dependence on the field phase for $H_{12} = 0$, for nonzero H_{12} the system has completely different behavior. The obtained results show that because of the specific behavior of the states in response to the laser parameters, robust experimental control of the system dynamics needs special considerations in external source parameter selection. For example, it is shown that in selecting the frequency of the laser in the considered range, choosing the lowest possible frequency gives the maximum robustness against experimental uncertainties. In this work we have also studied the effect of a special chirped-laser source for controlling the LZ process. It is shown that if the chirped-laser frequency follows the LZ energy gap, the system would be guided into resonance which can be used for arbitrary final-state engineering. It should be mentioned that, in comparison of the two approaches for tuning of the state populations in the LZ experiments, the chirped-pulse method has appreciably lower sensitivity to the laser field intensity. In fact, in the chirped-pulse-laser approach, because of the resonance condition, by decreasing the laser field intensity and increasing the interaction time period, the same transitions could be obtained.

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