Partial polarization by quantum distinguishability

Mayukh Lahiri,[1,*] Armin Hochrainer,[1] Radek Lapkiewicz,[1,†] Gabriela Barreto Lemos,[1,2] and Anton Zeilinger[1,2]
[1]*Vienna Center for Quantum Science and Technology (VCQ), Faculty of Physics, Boltzmanngasse 5, University of Vienna, Vienna A-1090, Austria*
[2]*Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Boltzmanngasse 3, Vienna A-1090, Austria*

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We establish that a connection exists between wave-particle duality of photons and partial polarization of a light beam. We perform a two-path lowest-order (single photon) interference experiment and demonstrate both theoretically and experimentally that the degree of polarization of the light beam emerging from an output of the interferometer depends on path distinguishability. In our experiment, we are able to change the quantum state of the emerging photon from a pure state to a fully mixed state without any direct interaction with the photon. Although most lowest-order interference experiments can be explained by classical theory, our experiment has no genuine classical analog. Our results show that a case exists where the cause of partial polarization is beyond the scope of classical theory.

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I. INTRODUCTION

The interference effect displayed by a quantum system or entity when sent through an interferometer is a key feature of quantum mechanics [1,2]. If quantum entities of a particular kind are sent one at a time through a two-path interferometer and one can identify (even in principle) the path traversed by each of them, no interference occurs. The information that leads to the identification of the path is often called the which-path information. Because the common-sense understanding of a particle implies that the path traversed can always be identified, the particle behavior of the quantum entity is often interpreted as the complete availability of the which-path information, i.e., the complete distinguishability of the paths. On the other hand, when the which-path information is fully unavailable, i.e., when the paths are fully indistinguishable, perfect interference occurs—a characteristic of waves. The wave-particle duality of photons has been confirmed by numerous experiments [3]. Several theoretical studies have been made on the relationship between the path distinguishability and visibility of fringes in interference experiments [4–7].

Since fringe visibility is a measure of the ability of light to interfere, it has played an important role in the development of the field of statistical optics [8–10]. In the classical formulation of statistical optics, it is assumed that the electric field associated with light is not measurable and is considered to be a random quantity. The visibility of fringes in a lowest-order interference experiment (e.g., Young's double-slit experiment) then becomes a measure of the correlation between the interfering fields; in the simplest case, maximum visibility implies full correlation (coherent light) and zero visibility implies no correlation (incoherent light). Based on this observation a theoretical connection between path distinguishability and optical coherence was introduced by Mandel [5]. However, classical statistical optics describes lowest-order correlation effects so successfully that the difference between quantum mechanical interpretation and classical interpretation of such effects are often merely academic. It is, therefore, important to find a case where the quantum mechanical interpretation turns out to be the only possible interpretation. Such a case has been experimentally demonstrated [11,12] in the context of lowest-order interference, and the nonclassicality of the experimental results has been verified independently [13].

Studies in statistical optics have also revealed that partial polarization of a light beam is a manifestation of lowest-order correlation between two mutually orthogonal transverse field components [14–17]. It is therefore natural to ask whether a connection between partial polarization and path distinguishability can be established [18]. The aim of this paper is to show that the wave-particle duality of photons has an important implication in partial polarization of a light beam. We develop a two-path interferometer and show that the degree of polarization (DOP) of the light beam emerging from the output of the interferometer depends on the distinguishability of photon paths. We use two independent methods for introducing path distinguishability. While the distinguishability introduced in one of the methods can be erased by performing a suitable measurement on the photon emerging from the interferometer, the distinguishability introduced in the other method cannot be erased. Our key result is the dependence of the degree of polarization on the inerasable distinguishability, which cannot be explained by the classical theory of light.

In Sec. II, we provide a qualitative description of the phenomenon. In Sec. III, we describe the experimental setup and provide a theoretical analysis. Then in Sec. IV we present and discuss the experimental results. Finally, in Sec. VI we conclude by discussing the implications of our results.

II. QUALITATIVE DESCRIPTION OF THE PHENOMENON

Let us consider a two-path interferometer in which photon beams generated by two *identical* sources Q_1 and Q_2 are superposed by a lossless and balanced non-polarizing beam-splitter, BS (Fig. 1). Photons emerging from one of the outputs of BS are collected by a photodetector, D. Photons emitted by Q_1 can only travel via path 1; photons emitted by Q_2 travel via path 2 only. Suppose that the photons emitted by Q_1 and

*mayukh.lahiri@univie.ac.at
†Current address: Institute of Experimental Physics, Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland.

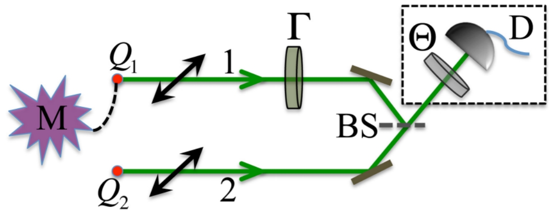

FIG. 1. Two identical sources, Q_1 and Q_2, emit identical linearly polarized photons. The photons from Q_1 are sent through a polarization rotator, Γ, and then superposed with the photons generated by Q_2 by a beam splitter, BS. The beam emerging from one of the outputs of BS is sent through a polarizer, Θ, and then detected by a photodetector, D. The device, M, attached to Q_1 determines with probability $1-\mathcal{M}^2$ whether Q_1 has emitted.

Q_2 are identical and linearly polarized in the same direction. On path 1, we place a polarization rotator (for example, a half-wave plate), Γ, by which we can rotate the direction of the incident linear polarization by an arbitrary angle γ, where $\cos\gamma \geqslant 0$. The superposed beam emerging from BS is sent through a polarizer, Θ, before arriving at D (Fig. 1). The light emerging from Θ is linearly polarized along a direction that makes an angle θ with the polarization direction of the light originally emitted by the sources. In this arrangement, Θ and D constitute the detection system that is used to measure the DOP of the beam before Θ.

Suppose Q_1 and Q_2 emit at the same rate and in such a way that only one photon exists in the system between an emission and a detection at D. Now, if path 2 is blocked, the probability amplitude of photodetection is given by $\alpha_1 = e^{i\phi_1}[\cos(\theta-\gamma)]/2$, where the phase ϕ_1 depends on path length. Similarly, if path 1 is blocked, the probability amplitude of photodetection will be $\alpha_2 = e^{i\phi_2}(\cos\theta)/2$. Suppose now that we attach (Fig. 1) to source Q_1 a device, M, that does *not* perform any measurement on the photons entering the interferometer but determines with a known probability whether Q_1 has emitted. Clearly, when both paths are open, there are three possible cases in which a photon can arrive at D after passing through Θ:

(I) Q_1 emits and M reports the emission

(II) Q_1 emits and M does not report the emission

(III) Q_2 emits.

Note that the possibilities II and III are indistinguishable. This is because photons coming from both arms have certain probabilities of passing through Θ given by $|\alpha_1|^2$ and $|\alpha_2|^2$; once they have passed through Θ, it is not possible to say in which source they were produced. On the other hand, when M reports an emission, it becomes known that the detected photon was emitted by Q_1. Hence, possibility I is fully distinguishable from possibilities II and III. For obtaining the total probability of detecting a photon at D, one therefore needs to add the probability associated with possibility I to the modulus square of the sum of *probability amplitudes* associated with II and III.

Let us assume that when Q_1 has emitted a photon, the probability of M *not* reporting the emission is equal to $\mathcal{M}^2$, where $0 \leqslant \mathcal{M} \leqslant 1$. The probability amplitudes associated with cases I, II, and III are then given by $\alpha_1\sqrt{1-\mathcal{M}^2}$, $\alpha_1\mathcal{M}$, and α_2, respectively. The probability of photodetection at D is thus given by

$$\begin{aligned}\Phi &= |\alpha_1\sqrt{1-\mathcal{M}^2}|^2 + |\alpha_1\mathcal{M}+\alpha_2|^2\\ &= \tfrac{1}{4}[\cos^2\theta + \cos^2(\theta-\gamma)\\ &\quad + 2\mathcal{M}\cos\theta\cos(\theta-\gamma)\cos(\phi_2-\phi_1)],\end{aligned} \tag{1}$$

which is directly proportional to the photon counting rate at the detector. We choose the path lengths such that $\phi_2-\phi_1$ is equal to a multiple of 2π, i.e., $\cos(\phi_2-\phi_1)=1$. In this case, it can be readily shown from Eq. (1) that when $\theta=\gamma/2$, the probability Φ attains its maximum value $\Phi_{\max} = (1+\mathcal{M})\cos^2(\gamma/2)/2$; and when $\theta = \gamma/2 \pm \pi/2$, it attains the minimum value $\Phi_{\min} = (1-\mathcal{M})\sin^2(\gamma/2)/2$. The degree of polarization (DOP) of the beam generated by superposition (after the beam splitter and before the polarizer) is given by [19]

$$P = \frac{\Phi_{\max}-\Phi_{\min}}{\Phi_{\max}+\Phi_{\min}} = \frac{\mathcal{M}+\cos\gamma}{1+\mathcal{M}\cos\gamma}. \tag{2}$$

We now discuss the role of path distinguishability in our interferometer. This should not be confused with the distinguishability of the possible cases mentioned above. We consider a photon right after the beam splitter before it has passed through the polarizer and ask under which conditions it is possible to know through which path (1 or 2) it traveled. Clearly, the path distinguishability can be introduced by two independent methods: (a) by the polarization rotator Γ and (b) by the device M.

The path distinguishability introduced by Γ is maximum when $\gamma=\pi/2$ and minimum when $\gamma=0$. A measure of the path distinguishability introduced by Γ is given by $\cos\gamma$.

The device M works in such a way that when $\mathcal{M}=0$, it determines with complete certainty whether Q_1 has emitted; in this case, the paths are fully distinguishable and no interference occurs irrespective of the orientation of Γ. On the other hand, when $\mathcal{M}=1$, the device M does not provide any information about the origin of the detected photon. A measure of the path distinguishability introduced by M is, therefore, given by $\mathcal{M}$.

It follows from Eq. (2) that *both* devices (Γ and M) must be used to introduce path distinguishability for generating a partially polarized ($0<P<1$) beam. If only one of the devices introduces path distinguishability, i.e., if either $\mathcal{M}=1$ or $\cos\gamma=1$, the beam is always fully polarized ($P=1$). On the other hand, the beam is unpolarized ($P=0$) if and only if *both* devices introduce maximum path distinguishability, i.e., if and only if $\mathcal{M}=0$ and $\cos\gamma=0$.

Note that the path distinguishability introduced by Γ can be erased by placing a polarizer after the beam splitter, whereas the path distinguishability introduced by M *cannot be erased* by any other device. The central feature of this thought experiment is the *dependence* of the DOP on the inerasable distinguishability ($\mathcal{M}$) for a fixed amount of the erasable distinguishability ($\cos\gamma \neq 1$).

III. EXPERIMENTAL AND QUANTITATIVE ANALYSIS

We now discuss an experiment (Fig. 2) in which the above-mentioned phenomenon is observed. This experiment is based on the concept of "induced coherence without induced emission" [11,12]. Two identical nonlinear crystals, NL1 and NL2, are pumped by two mutually coherent pump beams, P_1 and P_2, respectively. Each crystal converts a pump photon into

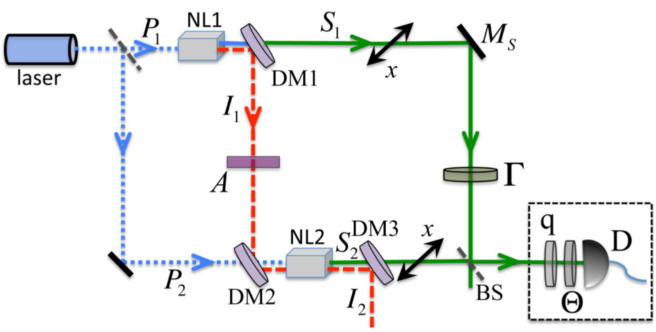

FIG. 2. Pump beams P_1 and P_2 (dotted lines) are generated by a CW laser source (532 nm). The crystals, NL1 and NL2, produce linearly polarized signal (S_1, S_2; solid line; 810 nm) and idler (I_1, I_2; dashed line; 1550 nm) photons. A neutral density filter, A, is placed on the path of I_1 between NL1 and NL2. Initially, S_1 and S_2 are polarized in the same direction x. S_1 is sent through a half-wave plate, Γ. S_1 and S_2 are superposed by a nonpolarizing beam splitter, BS. The detection-system (inside dotted box) used for polarization state tomography consists of a quarter-wave plate, q, a polarizer, Θ, and an avalanche photo detector, D; detection time 15 s. The mirror, M_S, is placed on a piezo-driven stage; DM1, DM2 and DM3 are dichroic mirrors.

a photon pair (signal and idler), each *linearly polarized*, by the process of spontaneous parametric down-conversion. We denote the signal and the idler generated in crystal NLj by S_j and I_j, respectively. The idler beam, I_1, is sent through NL2 and is aligned with I_2 (Fig. 2). The down-converted light is weak enough, so that it is highly improbable for photon pairs emitted by both crystals to be simultaneously present in the system. Under this condition the effect of stimulated emission at NL2 is negligible. An attenuator (neutral density filter), A, is placed on the path of I_1 between NL1 and NL2; the transmission coefficient of A can be varied. The signal beam S_1 is sent through a half-wave plate, Γ, such that its polarization direction can be rotated by a chosen angle γ. It is then superposed with S_2 by a nonpolarizing beam splitter, BS. The DOP of the superposed signal beam emerging from one of the outputs of BS is determined. We choose the path lengths appropriately [12] such that the beams S_1 and S_2 interfere when $\gamma = 0$ and A is absent.

The signal photons are generated as linearly polarized in the direction x, say; I_1 and I_2 are linearly polarized along the direction x' [20]. The quantum state of light (interaction picture [21]) generated by a crystal is given by the well-known formula (first-order approximation)

$$[\mathbb{1} + g_j \widehat{a}^{\dagger}_{S_j x} \widehat{a}^{\dagger}_{I_j x'} - g_j^* \widehat{a}_{S_j x} \widehat{a}_{I_j x'}]|\psi_{j0}\rangle \equiv \widehat{U}_j |\psi_{j0}\rangle, \quad (3)$$

where $\mathbb{1}$ is the identity operator, $j = 1,2$ labels the crystals, g_j provides a measure of the rate of parametric down conversion, $\widehat{a}^{\dagger}_{S_j x}$ and $\widehat{a}^{\dagger}_{I_j x'}$ are creation operators for S_j and I_j photons, respectively, and $|\psi_{j0}\rangle$ is the state of light before down-conversion (input state). The action of the attenuator, A, on the quantized field associated with I_1 photons is equivalent to that of a lossless beam splitter [11,12]; then one has

$$\widehat{a}_{I_2 x'} = [T \widehat{a}_{I_1 x'} + R' \widehat{a}_{0x'}] e^{i\phi_I}, \quad (4)$$

where T is the complex amplitude transmission coefficient of A, $|T|^2 + |R'|^2 = 1$, $\widehat{a}_{0x'}$ represents the vacuum field at the unused port of the beam splitter (the attenuator A), and ϕ_I is a phase factor due to propagation of I_1 from NL1 to NL2. It follows from Eqs. (3) and (4) that the quantum state of light in this system is given by $|\Psi\rangle = \widehat{U}_2 \widehat{U}_1 |\text{vac}\rangle$, i.e., by (neglecting the higher-order terms)

$$|\Psi\rangle \approx |\text{vac}\rangle + \left(g_1 |x\rangle_{S_1} + g_2 e^{-i\phi_I} T^* |x\rangle_{S_2}\right) |x'\rangle_{I_1} + g_2 e^{-i\phi_I} R'^* |x\rangle_{S_2} |x'\rangle_0, \quad (5)$$

where $|\text{vac}\rangle$ is the vacuum state, $|x\rangle_{S_j} \equiv \widehat{a}^{\dagger}_{S_j x}|\text{vac}\rangle$ represents an x-polarized signal photon, $|x'\rangle_{I_j} \equiv \widehat{a}^{\dagger}_{I_j x'}|\text{vac}\rangle$, $|x'\rangle_0 \equiv \widehat{a}^{\dagger}_{0x'}|\text{vac}\rangle$, and ${}_0\langle x'|x'\rangle_0 = 1$.

The quantized field components (positive-frequency part) at one of the outputs of the beam splitter are given by

$$\widehat{E}^{(+)}_{Sx} = e^{i\phi_{S1}} \left(\cos\gamma \widehat{a}_{S_1 x} + \sin\gamma \widehat{a}_{S_1 y}\right) + i e^{i\phi_{S2}} \widehat{a}_{S_2 x}, \quad (6a)$$

$$\widehat{E}^{(+)}_{Sy} = e^{i\phi_{S1}} \left(\sin\gamma \widehat{a}_{S_1 x} - \cos\gamma \widehat{a}_{S_1 y}\right) + i e^{i\phi_{S2}} \widehat{a}_{S_2 y}, \quad (6b)$$

where y is the Cartesian direction orthogonal to x, ϕ_{S1} and ϕ_{S2} are the phase changes associated with the propagation from NL1 to BS and from NL2 to BS, respectively, and $\gamma/2$ is the angle of the half-wave plate Γ. The DOP is determined by using the formula [16,22]

$$P = \sqrt{1 - \frac{4 \det \mathbf{G}^{(1)}}{[\text{tr}\mathbf{G}^{(1)}]^2}}, \quad (7)$$

where det and tr represent the determinant and the trace of a matrix, respectively, and the matrix $\mathbf{G}^{(1)}$ is given by the elements $G^{(1)}_{pq} = \langle\Psi|\widehat{E}^{(-)}_{Sp}\widehat{E}^{(+)}_{Sq}|\Psi\rangle$ [23]; $p = x,y, q = x,y$, and $\widehat{E}^{(-)}_{Sp}(\mathbf{r},t) = \{\widehat{E}^{(+)}_{Sp}(\mathbf{r},t)\}^{\dagger}$.

If the crystals emit at the same rate ($|g_1| = |g_2|$), it follows from Eqs. (5)–(7) that

$$P = \{\cos^2\gamma + |T|^2(\sin^2\gamma + \cos^2\gamma\cos^2\beta) + 2|T|\cos\gamma\cos\beta\}^{\frac{1}{2}} / \{1 + |T|\cos\gamma\cos\beta\}, \quad (8)$$

where $\beta = \phi_{S2} - \phi_{S1} - \phi_I - \arg(T) + \arg(g_2) - \arg(g_1) + \pi/2$. If we set $\cos\beta = 1$, Eq. (8) reduces to the form

$$P = \frac{|T| + \cos\gamma}{1 + |T|\cos\gamma}. \quad (9)$$

Note that replacing $\mathcal{M}$ with $|T|$ in Eq. (2) yields Eq. (9). This is due to the following reason: When the I_1 beam passes through A, its intensity drops by a factor of $|T|^2$. Since a photon cannot be broken into further fractions, an idler photon can either be fully transmitted or fully blocked by A. The probability of an idler photon being transmitted through A is therefore equal to $|T|^2$. If the idler photon is blocked, the paths of the signal photon are fully distinguishable for any orientation of Γ; this is because the full which-path information can, in principle, be extracted by performing a coincidence measurement on a signal photon emerging from BS and an idler photon after NL2 [11]. On the other hand, if the idler photon is transmitted, the paths of the signal photon become

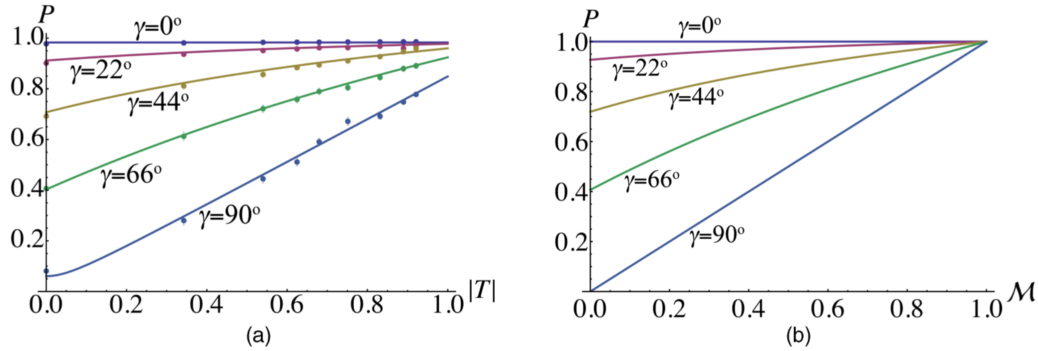

FIG. 3. Dependence of the degree of polarization P on inerasable path distinguishability: (a) Experimentally observed dependence of P on $|T|$ for various values of γ, and computed curves considering experimental imperfections (solid lines). Data points are represented by filled circles with error bars including both systematic and statistical errors. (b) Dependence of P on $\mathcal{M}$ for various values of γ predicted by the thought experiment [Eq. (2)]. The difference with the experimentally obtained results is because BS is slightly polarizing and I_1 beam suffers losses at the various optical components.

indistinguishable in absence of Γ (or $\cos\gamma = 1$), because there remains no way to extract the path information even in principle. Since the effect of stimulated emission at NL2 is negligible (see Sec. V for further discussion), the insertion of A does not change the intensities of the signal beams. The attenuator, A, of Fig. 2 thus introduces path distinguishability for the signal photons without interacting with them and plays the role of the device, M, of Fig. 1.

IV. EXPERIMENTAL RESULTS

In the experiment (Fig. 2), the value of $|T|$ is changed by using neutral density filters (A) of different values of transmittance. For each choice of T and γ, the interferometric phase is set equal to a multiple of 2π by varying the path length of S_1 with the mirror M_S. For each such setting, polarization state tomography is performed on the superposed signal beam in the following way: photon counting rates at the detector (D) are measured (detection time 15 s) in three mutually unbiased polarization bases that are set by the quarter-wave plate (q) and the polarizer (Θ) placed in front of D (Fig. 2); the matrix $\mathbf{G}^{(1)}$ is determined from the background-corrected photon counting rates by the maximum likelihood technique for a single-qubit system [24–26]. The DOP is then calculated by using Eq. (7). The experimentally observed dependence of the DOP on $|T|$ is shown in Fig. 3(a). The data points are represented by filled circles with error bars that contain both systematic and statistical errors. The solid lines are computed by using the theory discussed in Sec. III and considering the two following experimental imperfections: BS is slightly polarizing; and I_1 beam suffers losses at the various optical components.

In Fig. 3(b) we illustrate the results predicted by the thought experiment (Fig. 1). A comparison of Figs. 3(a) and 3(b) shows that the predictions of the thought experiment (Fig. 1) have been practically realized in the actual experiment (Fig. 2).

V. DISCUSSION

As mentioned above, the attenuator (A) introduces path distinguishability for the signal photons *without interacting with them*. This path distinguishability (quantified by $|T|$) cannot be erased by introducing any device that interacts with the signal photons (in this context, see Ref. [27]). It can be readily checked that if $\cos\beta = 1$ and $\gamma = \pi/2$ (compare the curves labeled by $\gamma = 90^\circ$ in Fig. 3), the density operator representing a signal photon, emerging from BS, takes the form

$$\hat{\rho}_S = \tfrac{1}{2}(|x\rangle\langle x| + |y\rangle\langle y| + |T||x\rangle\langle y| + |T||y\rangle\langle x|). \qquad (10)$$

Clearly, when $|T| = 0$, the state is fully mixed (unpolarized light) and when $|T| = 1$, the state is pure (polarized light). We are therefore able to change the "intrinsic" correlation between the transverse field components using the attenuator, A, without any direct interaction with the beam; this phenomenon cannot be explained classically. Classical theory does not provide any space for the particle behavior of light. When a classical field is sent through a transmission object (attenuator), it gets attenuated without any change in its "intrinsic" statistical properties. On the other hand when a photon is incident on a transmission object, it either fully passes or gets fully blocked with a certain probability. The effect of an attenuator on a photon is therefore not deterministic. This is why an appropriately placed attenuator (A) affects the inherent statistical properties of the output light in our experiment and changes the DOP.

Our method of controlling the DOP of the light beam is based on a procedure that was introduced in Refs. [11,12]; this procedure allows one to generate mutual coherence between the signal beams S_1 and S_2 without any induced (stimulated) emission at NL2 (Q_2). One can argue that there is always a possibility of stimulated emission since the I_1 beam is sent through NL2. However, if both crystals are weakly pumped, the low down-conversion rates of the crystals assure that the effect of stimulated emission on the lowest-order interference of the two signal beams is negligible (see, for example, Refs. [11,12,28–30]). The absence of induced emission at NL2 can be quantitatively established by the dependence of the mutual correlation between S_1 and S_2 on $|T|$. It was shown in Refs. [11,12] that in this case the modulus of the degree of mutual coherence between the two beams (the visibility characterizing their interference) is linearly proportional to $|T|$. This fact was also verified independently in another publication [13]. In our experiment, the equivalent result is the linear dependence of the DOP on $|T|$ when $\gamma = \pi/2$ and $\cos\beta = 1$; this shows that contributions of photons generated by stimulated emission are negligible compared to the ones generated by spontaneous parametric down-conversion. In absence of the stimulated emission, it cannot be argued that the detected light (signal) interacted with the attenuator. The appearance of $|T|$ in the expression of the DOP is thus beyond the scope of classical theory where interaction of the field with an object is mandatory for any such effect.

We now compare our experiment with a purely classical situation that may mistakenly appear to reproduce some of our results. Suppose that two mutually orthogonal and mutually correlated field components are superposed by a two-arm interferometer. Since all optical fields have finite coherence time, the correlation between the superposed field components depends on the difference between optical distances traversed by the beams. Therefore, the DOP of the output beam can be changed by varying the path difference. However, if the output beam is filtered to a much narrower frequency width, the DOP approaches its maximum possible value that is independent of the path difference [31]. Therefore, the change of the DOP on the optical path difference is not due to "intrinsic" change of field correlation. In contrary, the DOP of the beam generated in our experiment cannot be altered by enhancing the coherence time of the beam.

VI. CONCLUSIONS

Coincidence or heralded detection plays a key role in establishing the failure of classical electromagnetic theory to explain certain higher-order correlation effects, e.g., photon antibunching [32], Hong-Ou-Mandel effect [33], etc. However, coincidence detection is not at all required for observing lowest-order correlation effects, and not enough attempts have been made to demonstrate the limitations of classical theory in explaining such effects. It is a common perception that the phenomenon of partial polarization (a lowest-order correlation effect) is a classical effect. We have demonstrated that partial polarization of a light beam may not always be explained by the classical theory, and we have not performed any coincidence measurement for this purpose. Our results show that it is possible to develop a source that generates a low-intensity light beam with a controllable degree of polarization. Furthermore, the inability of classical theory to explain our experiment opens the door for further systematic investigations into the lowest-order correlation effects from the perspective of quantum mechanics. We hope this will lead to the discovery of other novel phenomena with significant applications.

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