

# Transformation of the frequency-modulated continuous-wave field into a train of short pulses by resonant filters

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The resonant filtering method transforming the frequency-modulated radiation field into a train of short pulses is proposed to be applied in the optical domain. Effective frequency modulation can be achieved by using an electro-optic modulator. Due to frequency modulation, a narrow-spectrum cw radiation field is seen by the resonant filter as a comb of equidistant spectral components separated by the modulation frequency. Tuning a narrow-bandwidth filter in resonance with the  $n$ th spectral component of the comb transforms the radiation field into bunches of pulses, with  $n$  pulses in each bunch. The transformation is explained by the interference of the coherently scattered resonant component of the field with the whole comb. Constructive interference results in the formation of pulses, while destructive interference is seen as dark windows between pulses. It is indicated that the optimal thickness of the resonant filter is several orders of magnitude smaller than the necessary thickness of the dispersive filters used before in the optical domain to produce short pulses from the frequency-modulated field.

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## I. INTRODUCTION

Generating pedestal-free optical pulses with high peak power from a low-power laser is of great interest in optical communication [1]. Existing devices generally employ electro-optic amplitude modulators [2], acousto-optic modulators [3–5], frequency chirping followed by dispersive compensators [6–10], and dispersive modulators [11,12]. Application of the rapid  $\pi$ -phase-shift technique for a cw radiation field with subsequent filtering by the optically thick resonant absorber is also capable of producing short pulses in a controllable way [13–19]. However, this technique demands a very fast phase switch; otherwise, the amplitudes of the pulses reduce appreciably.

Recently, an original technique for producing short pulses was reported in [20,21]. This technique was experimentally tested with gamma photons having long coherence length (long duration of a single-photon wave packet), and the method of splitting of a single photon into pulses [20,21] was proposed to create time-bin qubits, whose concept was introduced before in [22,23] for optical photons. This technique could also be practical for protocols of memory-based quantum networks [24–27], which require narrow-band photons with optimal temporal wave forms to interact with atoms efficiently.

The method of single-photon shaping [20,21] is based on frequency modulation of the radiation field, which is also one of the basic elements of the frequency chirping, implemented in [7,8] by using an electro-optic modulator. However, in spite of following dispersion compensator, which is accomplished in [7,8] by using a near-resonance absorber containing alkaline vapor, subsequent absorption (removal) of a particular spectral component is used [20,21]. This removal method is much more flexible compared to the frequency chirping followed by a dispersive compensator [7,8] and allows fine control of the duration and repetition rate of the pulses.

In this paper, we analyze the capabilities of the removal method for application in the optical domain and compare it with the dispersion compensator method. The removal method

could be implemented with cold atoms, atomic vapors, and organic molecules doped in a polymer matrix. They could serve as filters to remove the single-frequency component of the comb spectrum produced by the electro-optical modulator from the cw radiation field. Cold atoms possess almost naturally broadened Lorentzian absorption lines, while atomic vapors at low pressure demonstrate the Doppler-broadened absorption lines. The effect of the wings of these lines on the shape of the pulses is discussed. The proposed method is capable of producing nanosecond or subnanosecond pulses, which could be directly resolved in time by modern detectors without use of a cross-correlation technique, based on Mach-Zehnder interferometer with a delay line in one of the arms.

There are also proposals and experimental implementations producing short pulses by strong laser excitation of atoms [28–32] and molecules [33–37]. The proposals [28–32] consider time-dependent perturbation of the excited atomic-energy level by the strong far off-resonance laser due to the Stark effect. The method [33–37] is based on the stimulated Raman scattering of a strong bichromatic laser radiation, which produces large molecular coherence employing an electromagnetically induced transparency scheme.

In this paper, the techniques based on using strong radiation fields are not considered, since the main focus of interest is the transformation of very weak fields into pulses with controllable wave forms. High efficiency and low losses of the transformation are quite important from the viewpoint of applications in telecommunication.

The paper is organized as follows. In Sec. II, a general formalism of the resonant filtering of a phase-modulated field is presented. In Sec. III, the resonant filtering through laser-cooled atoms is discussed. In Sec. IV, the atomic vapor of hot atoms is considered as a resonant filter. In Sec. V, the creation of femtosecond pulses by filtering high-order harmonics of the phase-modulated field through atomic vapor is proposed. In Sec. VI, comparison of the resonant filtering with dispersive filters is discussed.

## II. BASIC IDEA

The cw radiation field  $E(t) = E_0 \exp(-i\omega_r t + ikz)$  after passing through the electro-optic modulator acquires phase modulation,

$$E_{\text{EO}}(t) = E(t)e^{im \sin \Omega t}, \quad (1)$$

where  $\Omega$  and  $m$  are the frequency and index of the phase modulation. According to the Jacobi-Anger expansion,

$$E_{\text{EO}}(t) = E(t) \sum_{n=-\infty}^{+\infty} J_n(m) e^{in\Omega t}, \quad (2)$$

this field is transformed into an equidistant frequency comb with spectral components  $\omega_n = \omega_r - n\Omega$ , where  $J_n(m)$  is the Bessel function of the  $n$ th order. Fourier transform of the field is

$$E_{\text{EO}}(\omega) = E_0 \sum_{n=-\infty}^{+\infty} J_n(m) \delta(\omega - \omega_n), \quad (3)$$

where  $\delta(x)$  is the Dirac delta function. If the cw radiation field has finite spectral width, then the  $\delta$  function is to be substituted by  $f_r(\omega - \omega_n)$  describing the spectrum of the cw field.

We transmit the frequency comb through the resonant filter with a single absorption line  $F(\omega - \omega_f)$  centered at frequency  $\omega_f$ . We select the filter whose absorption linewidth  $\Gamma_f$  is much smaller than the distance  $\Omega$  between neighboring components of the frequency comb. Such a filter is capable of selectively removing one of the spectral components of the comb. Below we do not show explicit spatial dependence of the field amplitude hiding it into the parameters of the filtered field.

By changing the carrier frequency of the cw radiation field, we tune the  $n$ th component of the comb,  $\omega_n = \omega_r - n\Omega$ , close to resonance with the filter frequency  $\omega_f$ . Then the radiation field at the exit of the filter is transformed as

$$E_{f_n A}(\omega) = E_{\text{EO}}(\omega) + E_0 J_n(m) [T(\Delta_n) - 1] \delta(\omega - \omega_n), \quad (4)$$

where  $\Delta_n = \omega_n - \omega_f$  and  $T(\Delta_n)$  is a transmission function of the filter, which depends on its absorption coefficient and physical thickness (see Sec. III). In general, the transmission function  $T(\Delta_n)$  is a complex function, which takes into account attenuation of the field amplitude and phase shift due to the frequency-dependent refraction index (dispersion) after passing through the filter of length  $L$ .

For the optically thick filter at exact resonance,  $T(0) = T_0$  tends to zero. Indexes  $n$  and  $A$  in  $E_{f_n A}(\omega)$  mean that the  $n$ th component of the comb is in resonance with the filter and the solution is approximate since in Eq. (4) we disregard the interaction of nonresonant components with the filter. The small contribution from these components due to dispersion will be taken into account in Sec. III.

Equation (4) has a simple physical meaning. It is constructed such that the first term in square brackets just describes the amplitude of the attenuated spectral component, which is in resonance and proportional to  $J_n(m)T_0$ . The second term removes from the comb  $E_{\text{EO}}(\omega)$  the resonant component to avoid taking it into account twice. Within this approximation, other spectral components pass through the resonant filter with no change.

The inverse Fourier transformation of Eq. (4) results in

$$E_{f_n A}(t) = E_0 e^{-i\omega_r t} [e^{im \sin \Omega t} + (T_0 - 1)J_n(m)e^{in\Omega t}]. \quad (5)$$

The interpretation of this result is based on the interference of the incident and coherently scattered radiation fields [21]. The latter is described by the second term in the square brackets in Eq. (5). Therefore, for the frequency comb whose  $n$ th spectral component interacts with the filter, only this component is coherently scattered by the atoms in the filter. The scattered field interferes with the whole frequency comb at the exit of the filter. Therefore, the output radiation field reveals unusual properties.

The intensity of the field at the exit of the filter,  $I_{f_n A}(t) = |E_{f_n A}(t)|^2$ , is described by

$$I_{f_n A}(t) = I_0 [1 - 2S_n \cos \psi_n(t) + S_n^2], \quad (6)$$

where  $I_0 = |E_0|^2$ ,  $S_n = (1 - T_0)J_n(m)$ , and  $\psi_n(t) = n\Omega t - m \sin \Omega t$ . If the filter is opaque for the resonant component, the amplitude of the scattered field almost coincides with the amplitude of the resonant component and  $S_n \rightarrow J_n(m)$ . The phases of these fields are opposite; therefore, if  $\cos \psi_n(t)$  is positive, we have destructive interference seen as a drop of intensity  $I_{f_n A}(t)$ . If  $\cos \psi_n(t)$  is negative, the comb and the scattered resonant component interfere constructively producing pulses. The interference becomes pronounced if  $J_n(m)$  has a global maximum. For different spectral components  $n$ , this maximum is achieved at different values of the modulation index  $m$ . We denote these values as  $m_n$ , which are  $m_1 = 1.8$ ,  $m_2 = 3.1$ ,  $m_3 = 4.2, \dots$ . The phase difference of the comb  $E_{\text{EO}}(t)$  (whose phase evolves as  $m \sin \Omega t$ ) and the scattered field  $E_{\text{sc}}(t)$  (whose phase evolves as  $n\Omega t$ ) is  $\psi_n(t) + \pi$  if  $J_n(m) > 0$ . The evolution of  $\psi_n(t)$  in time fully describes the formation of the pulses and the dark windows between them.

The phase-modulated field  $E_{\text{EO}}(t)$  after passing through the resonant filter is transformed into bunches of pulses. The number of pulses in a bunch is equal to the number  $n$  of the filtered component of the frequency comb. The intensity of the pulses is equal to  $[1 + S_n(t)]^2 I_0$ . For example, the maximum intensities, predicted by Eq. (6) for optimal values of the modulation index  $m_n$  and opaque filter ( $T_0 = 0$ ), are  $I_{\text{max}} = 2.5I_0$  for  $n = 1$ ,  $I_{\text{max}} = 2.1I_0$  for  $n = 2$ , and  $I_{\text{max}} = 2.06I_0$  for  $n = 3$ . Thus, the intensity of the pulses exceeds almost two times the intensity of the radiation field if it would not interact with the filter. The radiation intensity between bunches is quite small because of destructive interference, which predicts  $I_{\text{min}} = [1 - S_n(t)]^2 I_0$ . For the same values of the parameters  $T_0$  and  $m_n$ , this intensity is  $I_{\text{min}} = 0.175I_0$  for  $n = 1$ ,  $I_{\text{min}} = 0.26I_0$  for  $n = 2$ , and  $I_{\text{min}} = 0.32I_0$  for  $n = 3$ , i.e., almost an order of magnitude smaller compared with the pulse maxima.

Within the period of the phase oscillation ( $T_{\text{EO}} = 2\pi/\Omega$ ), induced by the electro-optical modulator, one can distinguish two intervals. For half of the period  $T_{\text{EO}}/2$ , the phase  $\psi_n(t)$  evolves almost linearly as  $(n + m)\Omega t + C$ , where  $C$  is constant but different for different bunches. Within this half of the period, the relation  $\psi_n(t) = (2k + 1)\pi$ , where  $k$  is an integer, is satisfied  $n$  times, producing pulses. During the other half of the period  $T_{\text{EO}}/2$ , the evolution of phase  $\psi_n(t)$  almost stops

near the value  $2k\pi$ , resulting in destructive interference, seen as dark windows.

### III. FILTERING THROUGH COLD ATOMS

In this section, we consider the frequency-comb filtering through laser-cooled atoms with a modest optical depth. As an example, we take parameters of  $^{85}\text{Rb}$  atoms in a two-dimensional magneto-optical trap, described, for example, in [13]. The cw radiation field excites the  $^{85}\text{Rb}$   $D_1$ -line transition ( $\lambda = 795$  nm). Since the absorption line is almost Lorentzian, the transmission function in Eq. (4) can be described as [38]

$$T(\Delta_n) = \exp\left(-\frac{\alpha L \gamma / 2}{\gamma - i \Delta_n}\right), \quad (7)$$

where  $\alpha$  is the Beer's law absorption coefficient,  $L$  is the length of atomic cloud, and  $\gamma$  is a half width of the absorption line. If the  $n$ th component of the frequency comb is in exact resonance with  $D_1$  line, then the transmission function is  $T(0) = \exp(-\alpha L / 2)$ . An atomic cloud with a length of a few millimeters already demonstrates optical depth  $\alpha L = 5$ .

One can verify the approximate expression for the filtered field  $E_{fnA}(t)$  (5) comparing it with the exact expression, which could be obtained by the convolution of the  $E_{EO}(t)$  field,

$$E_{fn}(t) = \int_{-\infty}^{+\infty} E_{EO}(t - \tau) R(\tau) d\tau, \quad (8)$$

with the Green's function of the absorber of thickness  $L$  [16,17,38],

$$R(t) = \delta(t) - \Theta(t) e^{-i(\omega_f + \gamma)t} j_1(bt), \quad (9)$$

where  $\delta(t)$  is the Dirac  $\delta$  function,  $\Theta(t)$  is the Heaviside step function,  $j_1(bt) = \sqrt{b/t} J_1(2\sqrt{bt})$ ,  $J_1(x)$  is the Bessel function of the first order, and  $b = \alpha L \gamma / 2$ . For the infinitely lasting field, Eq. (8) is reduced to

$$E_{fn}(t) = E_{EO}(t) - E(t) \int_0^{+\infty} j_1(b\tau) e^{i(\Delta - \gamma)\tau + im \sin \Omega(t - \tau)} d\tau, \quad (10)$$

where  $\Delta = \omega_r - \omega_f$ . Index  $n$  in  $E_{fn}(t)$  implies that we consider the case when the  $n$ th spectral component of the comb is close to resonance with the filter, i.e.,  $\Delta = n\Omega + \Delta_n$ , and  $\Delta_n$  is close to zero.

Equation (10) takes into account the transformation of all spectral components of the comb, including those whose change is infinitely small since they are far from resonance with the filter. A comparison of the time dependencies of the approximate intensity  $I_{fnA}(t)$ , given by Eq. (6), and exact expression  $I_{fn}(t) = |E_{fn}(t)|^2$ , where  $\Delta = n\Omega$  and  $\Delta_n = 0$ , is shown in Fig. 1 for  $n = 1, 2$ , and 3. Optimal values of the modulation index  $m = m_n$  are adopted in each case. The following parameters for the cold-atom filter are taken:  $\gamma / 2\pi = 3$  MHz,  $\alpha L = 5$ , and  $b / 2\pi = 7.5$  MHz. We choose the modulation frequency of the field phase equal to 30 MHz, satisfying well the condition that  $\Omega$  is much larger than the spectral width of the absorption line of the filter  $\Gamma_f = 2\gamma$ .

In Fig. 1(a), the pulse duration from shoulder to shoulder for  $n = 1$  is slightly shorter than half of the period of the phase oscillation  $T_{EO}/2 = 16.7$  ns, while the duration of the dark

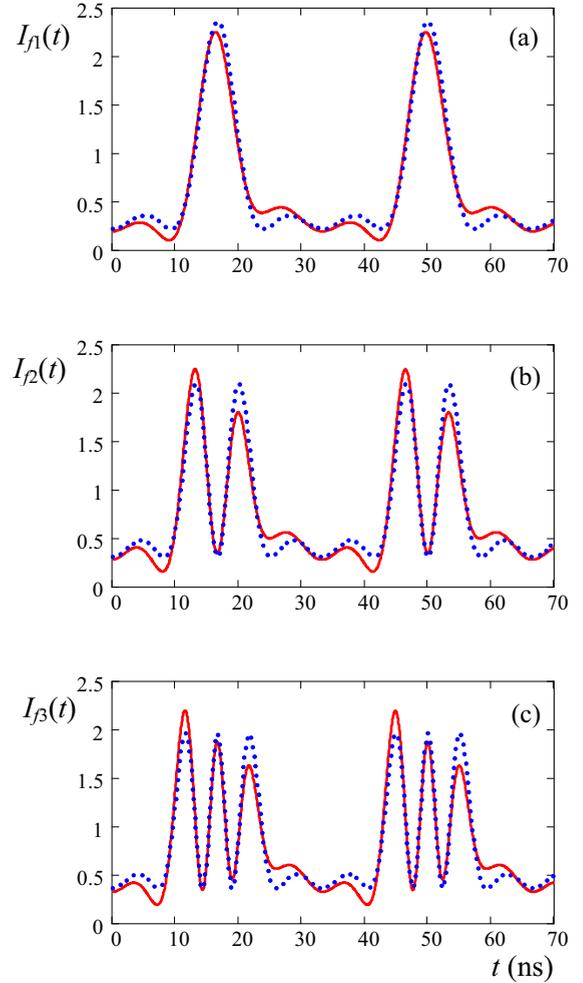


FIG. 1. Time dependence of the intensity of the filtered radiation field  $I_{fn}$ , where  $n$  means the number of the spectral component tuned in resonance with the filter. The solid red line corresponds to exact expression (10) and the dotted blue line represents the analytical approximation (6). Both are normalized to the intensity of the incident radiation field  $I_0$ . The modulation frequency  $\Omega / 2\pi = 30$  MHz is ten times larger than the half width of the absorption line of the filter  $\gamma / 2\pi = 3$  MHz; the optical depth of the filter is  $\alpha_0 L = 5$ . The number of the spectral component  $n$ , tuned in resonance ( $\Delta_n = 0$ ), and the optimal values of the modulation index are (a)  $n = 1, m_1 = 1.8$ , (b)  $n = 2, m_2 = 3.1$ , and (c)  $n = 3, m_3 = 4.2$ .

window is slightly longer than  $T_{EO}/2$ . The pulse width at half maximum is close to, but slightly shorter than,  $T_{EO}/4 = 8.3$  ns. If we have  $n > 1$  pulses in a bunch (see Fig. 1), their pulse width can be roughly estimated as  $T_{EO}/4n$ . Therefore, if, for example,  $n = 3$ , the pulse width is already close to 1 ns.

A small misfit between exact and approximate time dependencies is caused by the contribution of two nearest neighbors  $\omega_{n\pm 1} = \omega_r - (n \pm 1)\Omega$  of the resonant spectral component  $\omega_n$ . The corrected expression, which takes them into account, is easily found,

$$E_{fn}(t) \approx E_{fnA}(t) + E_{n+1}(t) + E_{n-1}(t), \quad (11)$$

where

$$E_{n\pm 1}(t) = E_0 e^{-i\omega_r t + i(n\pm 1)\Omega t} J_{n\pm 1}(m_n) \left( e^{\frac{-b}{\gamma - i(\Delta_n \mp \Omega)}} - 1 \right). \quad (12)$$

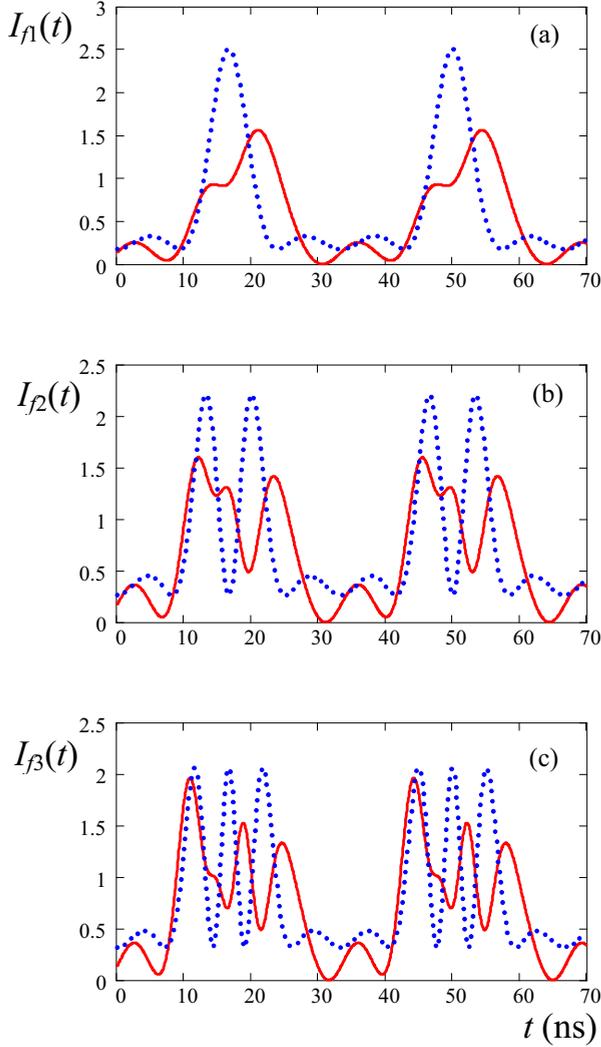


FIG. 2. Time dependence of the intensity of the filtered radiation field for an atomic cloud of length  $L = 1.5$  cm, which corresponds to the optical depth  $\alpha_0 L = 33$  [13]. Other parameters and notations are the same as in Fig. 1.

The field intensity, calculated with this correction, describes almost perfectly the exact time dependence of the filtered field intensity  $I_{fn}(t)$ . As it is seen in Fig. 1, the contribution of the sidebands introduces the asymmetry of the pulse intensities within a bunch and after-ringing, which appears in the beginning of dark windows.

Parameter  $b = \alpha L \gamma / 2$  plays a crucial role in the radiation filtering since the transmission function  $T(\Delta) = \exp[-b/(\gamma - i\Delta)]$  essentially broadens if  $b$  becomes larger than  $\gamma$ . In this case, many spectral components of the comb are modified by the filter and the interference of the comb with the scattered radiation field becomes messy. Figure 2 shows a comparison of the time dependencies of exact intensity  $I_{fn}(t)$  and approximate one  $I_{fnA}(t)$  for an atomic cloud with optical depth  $\alpha L = 33$ , which is achieved in [13] with cloud length  $L = 1.5$  cm. Instead of proper pulses, we see their appreciable distortion. This is not surprising since for this cloud the parameter  $b/2\pi = 49.5$  MHz is larger than the frequency-comb spacing  $\Omega/2\pi = 30$  MHz and many sidebands of the

resonant component contribute to the pulse generating. For these values of the parameters, it is necessary to take eight neighboring components into account, i.e., four red detuned from resonance and four blue detuned. Then, the approximate expression similar to Eq. (11) but with eight additional terms  $E_{n\pm k}(t)$ , where  $k = 1-4$ , gives the same result as the exact equation (10). Actually, the exact equation can be expressed as a result of filtering of all spectral components of the comb,

$$E_{fn}(t) = E_{fnA}(t) + \sum_{k=1}^{\infty} [E_{n+k}(t) + E_{n-k}(t)], \quad (13)$$

where

$$E_{n\pm k}(t) = E_0 e^{-i\omega_r t + i(n\pm k)\Omega t} J_{n\pm k}(m_n) \left( e^{\frac{-b}{\gamma - i(\Delta_n \mp k\Omega)}} - 1 \right). \quad (14)$$

Contrary to the example of the optically thick filter modifying many spectral components of the comb, we give another example when filtering becomes ideal. We take moderate optical depth  $\alpha L = 5$ , which corresponds to  $b/2\pi = 7.5$  MHz, and increase modulation frequency ten times to the value  $\Omega/2\pi = 300$  MHz. Then, the contribution of the sidebands becomes negligible and the intensity of the filtered radiation is well described by approximate equation (6) (see Fig. 3).

It is interesting to note that the phase modulation with frequency 300 MHz is capable of producing subnanosecond pulses. Tuning, for example, the fifth sideband of the frequency comb into resonance with the filter is capable of producing pulses with duration of 167 ps (see Fig. 4).

Cold  $^{85}\text{Rb}$  atoms are not the only example of the narrow-bandwidth filter. One can also use a cloud of cold potassium ( $^{39}\text{K}$ ) atoms generated in a vapor-cell magneto-optic trap whose excitation on the  $4S_{1/2}(F=1) \leftrightarrow 4P_{1/2}(F=2)$  transition (transition wavelength 770 nm) was studied in [39] for the observation of optical precursors.

#### IV. FILTERING THROUGH ATOMIC VAPOR

In this section, we consider the filtering of the frequency comb through a vapor of  $^{87}\text{Rb}$  atoms and take the parameters of the experiment [40] where spectral properties of the electromagnetically induced transparency were studied. Assume that the fundamental frequency of the comb is close to the  $S_{1/2}, F=1 \rightarrow P_{1/2}, F=2$  transition of the  $D_1$  line of natural Rb ( $\lambda = 795$  nm). The atoms are confined in a cell of length  $L = 5$  cm. We take two temperatures of Rb vapor (50 and 70 °C), which correspond to atomic densities  $N_1 = 6 \times 10^{10} \text{ cm}^{-3}$  and  $N_2 = 6 \times 10^{11} \text{ cm}^{-3}$ , respectively. Natural linewidth of the Rb  $D_1$  line is  $\Gamma_f/2\pi = \gamma/\pi = 5.4$  MHz and Doppler broadening is  $\Delta\omega_D/2\pi = 500$  MHz. We take the phase-modulation frequency  $\Omega/2\pi = 10$  GHz, which is 20 times larger than the Doppler width  $\Delta\omega_D/2\pi = 500$  MHz.

The transmission function for the atomic vapor is

$$T_D(\Delta_n) = \exp[-\alpha_{1,2} L F_D(\Delta_n)/2], \quad (15)$$

where  $\alpha_{1,2} = 3N_{1,2}\lambda^2/2\pi$  is the absorption coefficient of the naturally broadened line and  $F_D(\Delta_n)$  is a Doppler-broadened absorption line, which is

$$F_D(\Delta_n) = \sqrt{\frac{\ln 2}{\pi}} \frac{2\gamma}{\Delta\omega_D} \int_{-\infty}^{+\infty} \frac{\exp[-\ln 2(2x/\Delta\omega_D)^2]}{\gamma - i(\Delta_n + x)} dx. \quad (16)$$

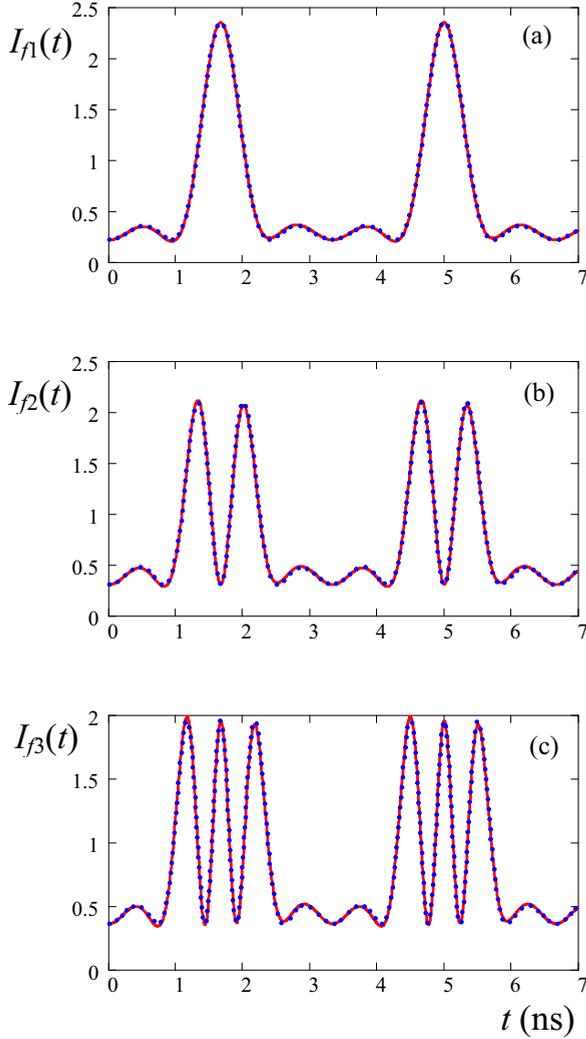


FIG. 3. Comparison of the exact time dependence of the intensity of the filtered radiation field  $I_{fn}(t)$  (solid red line) with the approximate one  $I_{fnA}(t)$  (blue dots) for an atomic cloud with optical depth  $\alpha_0 L = 5$ . The modulation frequency is  $\Omega/2\pi = 300$  MHz. Other parameters and notations are the same as in Fig. 1.

We have to emphasize that for both densities of Rb atoms, the cell is optically thick, i.e.,  $\alpha_1 L = 905$  and  $\alpha_2 L = 9053$ . It is easy to show (see, for example, [41]) that at exact resonance ( $\Delta_n = 0$ ), Eq. (16) can be approximated as

$$F_D(0) = \sqrt{\pi \ln 2} \frac{2\gamma}{\Delta\omega_D}. \quad (17)$$

Therefore, for  $\Delta_n = 0$ , the effective optical depth,  $\alpha_{1,2} L F_D(0)$ , is reduced almost one-hundred times since the Doppler width  $\Delta\omega_D$  is two orders of magnitude larger than the natural linewidth  $2\gamma$ .

If  $|\Delta_n| > 1.8\Delta\omega_D$ , then the absorption line reveals Lorentzian wings (see, for example, Fig. 1 in Ref. [41]), which can be approximated as

$$F_D(\Delta_n) = \frac{\gamma}{\gamma - i\Delta_n}. \quad (18)$$

Therefore, the contribution of far wings of the dispersion  $\chi'(\Delta_{n\pm k}) \sim \text{Im} F_D(\Delta_{n\pm k})$  in the filtering of nonresonant

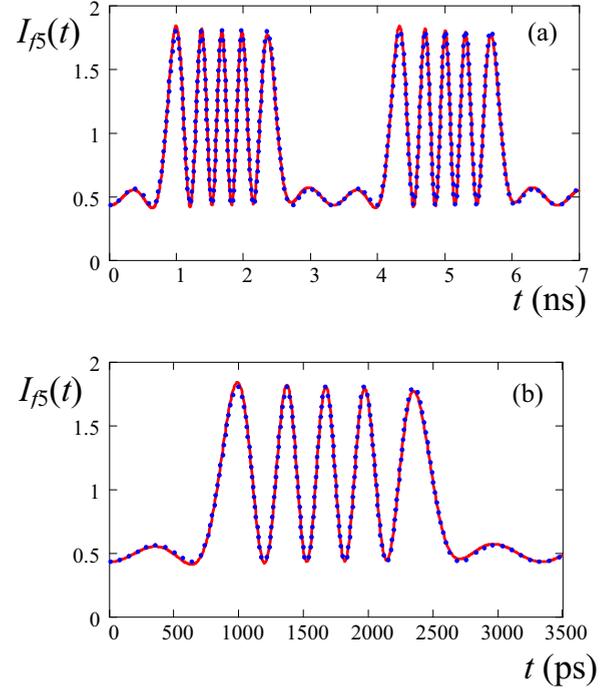


FIG. 4. (a) Time evolution of intensity of the filtered radiation field for an atomic cloud with optical depth  $\alpha_0 L = 5$ . The modulation frequency is  $\Omega/2\pi = 300$  MHz. The sideband  $n = 5$  is tuned in resonance with the filter. The value of the modulation index  $m_5 = 6.4$  is close to  $2\pi$ . (b) Zoom of the first bunch of pulses shown in (a). The solid red line shows  $I_{fn}(t)$  and the blue dots correspond to  $I_{fnA}(t)$ . Both are normalized to  $I_0$ .

components  $\omega_r - (n \pm k)\Omega$  of the frequency comb could be noticeable if  $\Delta_n = 0$ . For example, the contribution of the nearest sidebands ( $k = \pm 1$ ) of the resonant component is proportional to

$$E_{n\pm 1}(t) \sim J_{n\pm 1}(m_n)(e^{\mp i b_{1,2}/\Omega} - 1), \quad (19)$$

where  $b_{1,2} = \alpha_{1,2}\gamma/2$ . For the atomic density  $N_1$ , the ratio  $b_1/\Omega = 0.122$  is small since  $b_1/2\pi = 1.2$  GHz and modification of the sidebands due to filtering does not significantly influence the shape of the produced pulses. For the atomic density  $N_2$ , the ratio  $b_2/\Omega = 1.22$  is large since  $b_2/2\pi = 12$  GHz and the sidebands change their phases due to filtering. In this case, one can expect appreciable corruption of the produced pulses. To verify these expectations, we calculate the intensity of the filtered comb, taking into account the modification of many sidebands. The number of them can be limited by  $\pm k_{\max}$  if the contribution of the next sidebands  $\pm(k_{\max} + 1)$  does not change the signal.

Substituting the transmission function  $T_D(\Delta_n)$  into Eqs. (5) and (14), one obtains the modified Eq. (13), which describes the transformation of the frequency comb after passing through the atomic-vapor filter. The substitution changes the functions  $S_n$  and  $E_{n\pm k}(t)$  to

$$S_n = J_n(m_n)[1 - T_D(0)], \quad (20)$$

$$E_{n\pm k}(t) = E_0 e^{-i\omega_r t + i(n\pm k)\Omega t} J_{n\pm k}(m_n)[T_D(\mp k\Omega) - 1], \quad (21)$$

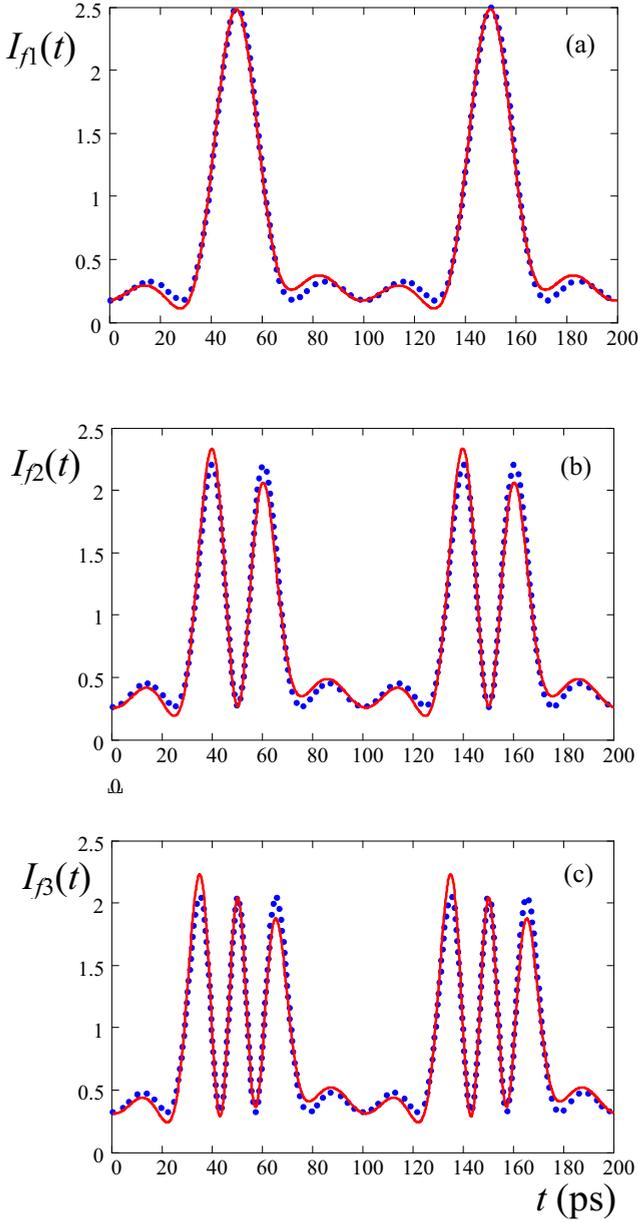


FIG. 5. Time dependence of the intensity of the filtered radiation field  $I_{fn}(t)$  (solid red line) for atomic vapor with optical depth  $\alpha_1 L = 905$ . The modulation frequency is  $\Omega/2\pi = 10$  GHz. The approximate time dependence of the intensity  $I_{fnA}(t)$  is shown by blue dots. Other parameters and notations are the same as in Fig. 1.

where  $\Delta_n = 0$ , which implies that the  $n$ th spectral component is in exact resonance with the filter.

For the Rb cell with atomic density  $N_1 = 6 \times 10^{10} \text{ cm}^{-3}$ , it is enough to take into account the contribution of two spectral components ( $n \pm 1$ ) neighboring the resonant component  $n$ . For this density, the effective optical depth of the cell at the absorption line center is  $\alpha_1 L F_D(0) = 14.4$ . The result of the comb filtering through the cell is shown in Fig. 5. If the  $n = 1, 2$ , or  $3$  spectral component is in resonance with the filter, the pulses with the width 25, 12.5, and 8.3 ps, respectively, are produced.

If atomic density is increased by the order of magnitude to  $N_2 = 6 \times 10^{11} \text{ cm}^{-3}$ , then eight sidebands of the resonant

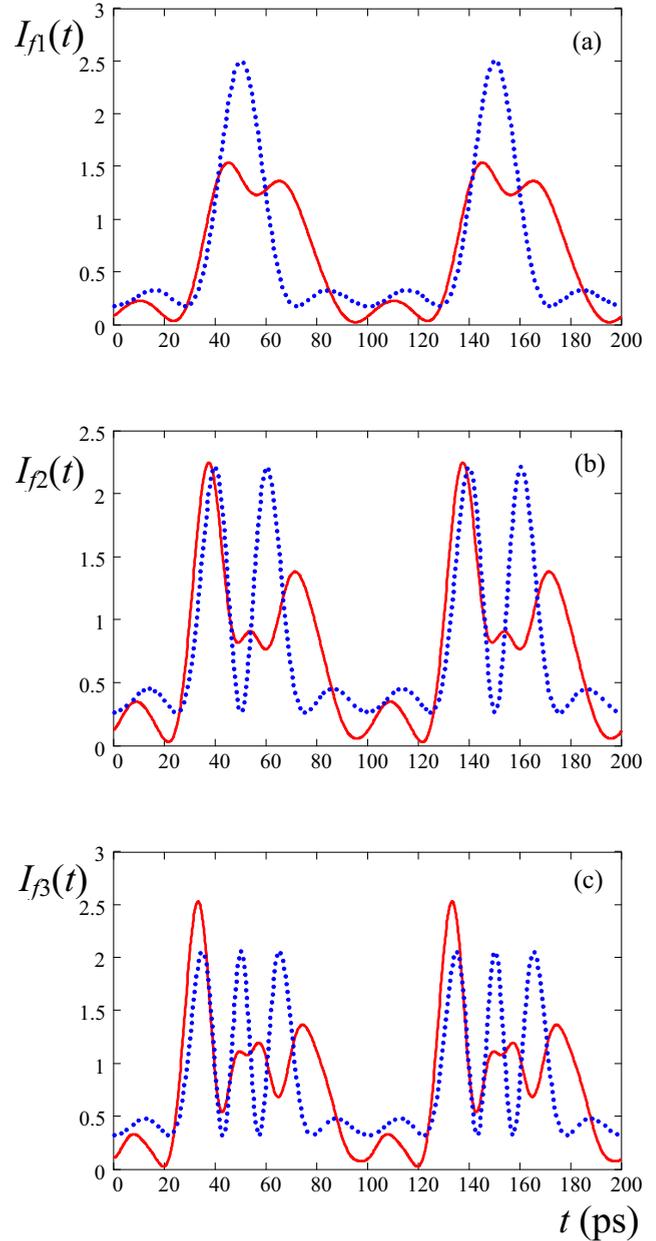


FIG. 6. Time dependence of intensity of the radiation field filtered through the atomic vapor with the optical depth  $\alpha_2 L = 9053$  (solid red line). Other parameters and notations are the same as in Fig. 5.

component  $n \pm 1, n \pm 2, n \pm 3$ , and  $n \pm 4$  give a noticeable contribution. Therefore, the intensity modulation of the filtered comb becomes messy (see Fig. 6). Actually, for  $n = 2$  and  $n = 3$ , it is necessary to take into account the contribution of 14 sidebands (up to  $n \pm 7$ ).

To obtain proper and clean pulses, it is preferable to use a filter with a smaller optical depth. An example of filtering by a cell with optical depth  $\alpha L = 453$  is shown in Fig. 7. The filter is tuned in resonance with the spectral component  $n = 10$  of the frequency comb, produced by phase modulation with the optimal value of the modulation index  $m_{10} = 11.8$  (which is close to  $2\pi$ ). Such a filtering is capable of producing pulses as short as 2.5 ps, which are grouped in bunches consisting of 10 pulses. Thus, by using the phase-modulation

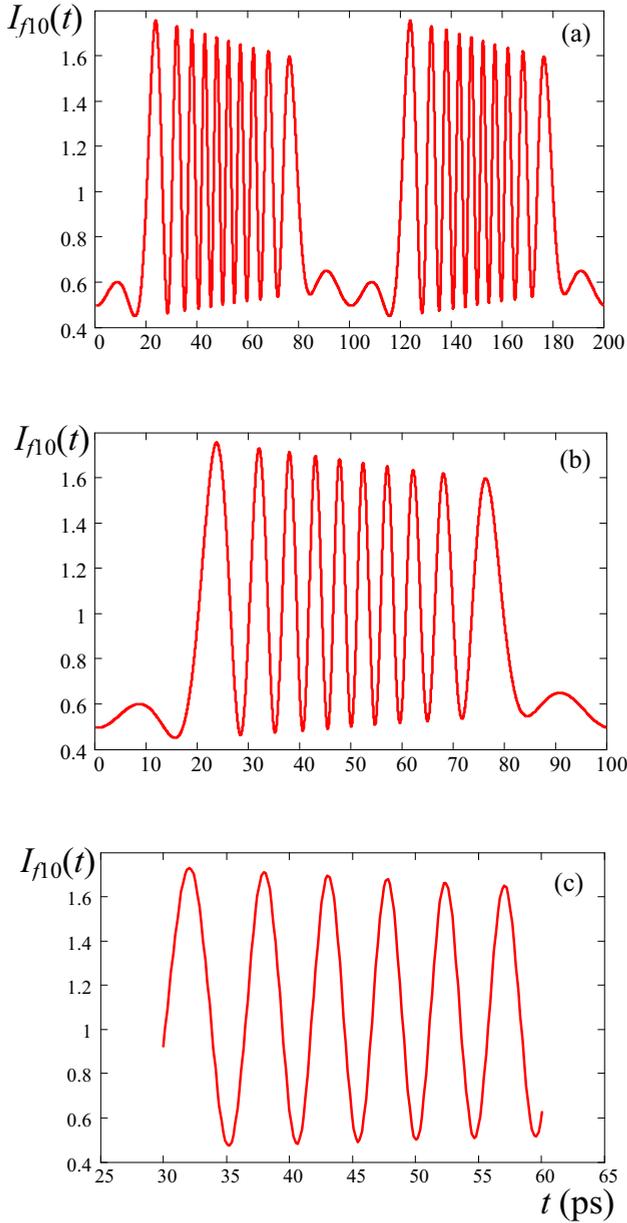


FIG. 7. (a) Time dependence of the intensity of the radiation field filtered through the atomic vapor with the optical depth  $\alpha_0 L = 453$ . The spectral component  $n = 10$  is tuned in resonance with the filter and the phase-modulation index is  $m_{10} = 11.8$ . (b) The content of a bunch of pulses. (c) Zoom of a central part of the bunch. Other parameters and notations are the same as in Fig. 5.

technique and subsequent filtering, it is possible to create pulses whose duration is 40 times shorter than the modulation period.

## V. FEMTOSECOND PULSES

It is also possible to create femtosecond pulses if the high-order harmonic of the frequency comb is removed. For simplicity, we consider an ensemble of two-level atoms, for example, a vapor of alkaline atoms. We take the modulation frequency of the radiation field equal to  $\Omega/2\pi = 10$  GHz.

If the modulation index is  $m_{100} = 104$ , the amplitude of the frequency component  $\omega_r - 100\Omega$  takes its first maximum value, which is proportional to  $J_{100}(104) = 0.144$ . Removal of this spectral component of the comb by atomic filter leads to high-frequency oscillations of the transmitted field intensity. Duration of the pulses is estimated as  $T_{EO}/400 = 250$  fs, where  $T_{EO}$  is the phase-modulation period. The necessary modulation index  $m_{100} = 104$  corresponds to a voltage, which is 33 times larger than the half-wave voltage of the electro-optical modulator. One can expect to reach this value by increasing the voltage and/or the physical length of the modulator by an order of magnitude. This is technically possible if one employs electro-optical modulators, based on the integrated-optical waveguides guiding the light along a determined path analog to optical fiber. Such modulators are fabricated in planar waveguiding substrates, which are 3 to 9 microns in width and depth (see, for example, the specifications of Jenoptik integrated-optical modulators). Therefore, half-wave voltage is quite low and the maximum applicable voltage, which is approximately  $\pm 30$  V, produces a phase shift  $20\pi$  for the red light. Extending three times the length of the electrodes, placed along the waveguide, and applying voltage  $\pm 15$  V, one can reach the desirable modulation index.

Below, the simplified picture of the filtering of high-order harmonics of the frequency comb is illustrated. To avoid complicated expressions, we approximate the coherently scattered component of the field in Eq. (6) as proportional to  $S_{100} = J_{100}(m_{100})$ , assuming that the resonant absorption is close to 100%, i.e.,  $T_D(0) \approx 0$ . We disregard the dispersive contribution of the neighboring spectral components. The time dependence of the intensity of the filtered radiation field is shown in Figs. 8(a) and 8(c). The contrast of the pulses is not as large as for the filtering of low-order harmonics. This is because the intensity of the maxima and minima of the pulses can be estimated as  $I_{\max} \approx [1 + 2J_{100}(104)]I_0$  and  $I_{\min} \approx [1 - 2J_{100}(104)]I_0$ . Since  $J_{100}(104) = 0.144$ , the maxima exceeds by 30% the level of the intensity of the incident radiation field, while minima are smaller than  $I_0$  on 30% of its value.

The amplitudes of high harmonics of the frequency comb decrease with increase of their number  $n$  since the first maxima of the Bessel functions  $J_n(m_n)$  decrease with increase of the order  $n$ . To achieve large contrast of the pulses, one can remove several spectral components of the comb neighboring  $n = 100$ , if they have the same phase at the time of the pulse formation. As an example, we consider the case where filtering of the comb component  $n = 100$  is accompanied by filtering of the components with the numbers  $n + 2k$ , where  $k$  takes values  $\pm 1$  and  $\pm 2$ . These spectral components have the same phase as a  $n = 100$  spectral component at times when the central pulses of the bunch are formed. The amplitude of the filtered radiation field with a cumulative contribution of the extra four spectral components of the comb due to their removal by separate filters is approximated as

$$E_{\text{cum}}(t) = E(t) \left[ e^{im \sin \Omega t} - \sum_{k=-2}^2 J_{n+2k}(m) e^{i(n+2k)\Omega t} \right], \quad (22)$$

where, for simplicity, it is supposed that  $T_D(0) = 0$ . The intensity of the filtered radiation field,  $I_{\text{cum}}(t) = |E_{\text{cum}}(t)|^2$ , is shown in Figs. 8(b) and 8(c). The central part of the

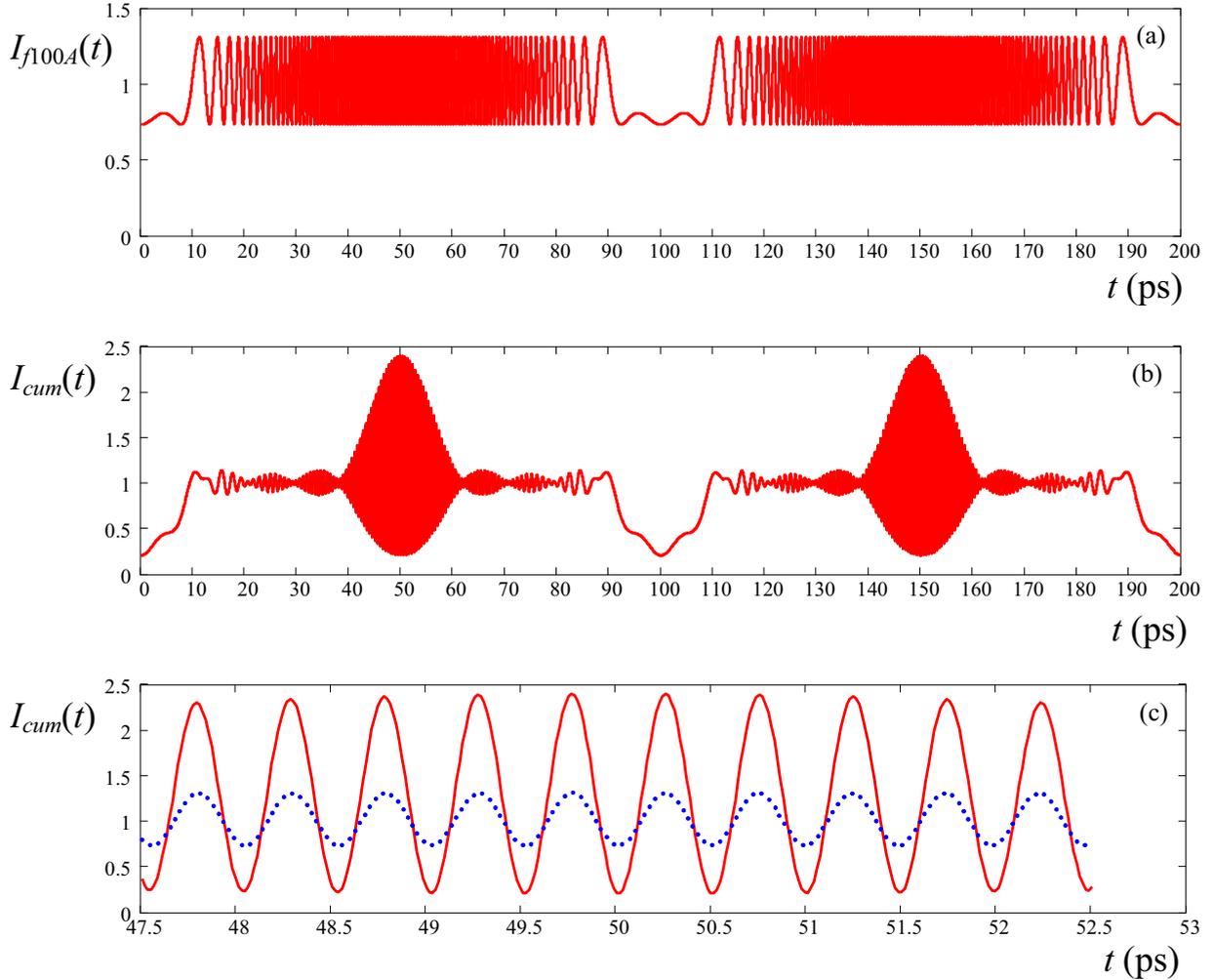


FIG. 8. (a) Time dependence of the intensity of the radiation field whose  $n = 100$  spectral component is removed by the filter. The modulation frequency is  $\Omega/2\pi = 10$  GHz. (b) Accumulative result of the filtering of the additional four spectral components of the frequency comb (see the text for details). (c) Zoom of a central part of the bunch. Accumulative effect is shown by the solid red line, while the dotted blue line demonstrates the result of the filtering of only one spectral component with  $n = 100$ . The full width at half maximum of the pulses is close to 250 fs as is expected from simple estimations.

pulse bunches demonstrates good contrast. The maximum accumulative effect of five spectral components is achieved if  $m = 103$ .

Cumulative filtering is possible if we have narrow-bandwidth filters properly adjusted for the chosen spectral components of the frequency comb. By a set of cells with atomic vapors, it is hard to construct such a specific multifrequency filter having a large spacing between the absorptive frequency components of the order of 20 GHz. However, for example, cesium atoms have a large hyperfine splitting of the ground state seen as a spectral doublet with a spacing equal to 9.2 GHz. If we reduce the modulation frequency of the field phase down to  $\Omega/2\pi = 4.6$  GHz, and properly populate ground-state levels by pumping Cs atoms such that both components of the doublet are strongly absorptive, then we can remove two spectral components of the frequency comb. An example of cumulative filtering of two spectral components  $100\Omega$  and  $98\Omega$  by cesium atoms is shown in Figs. 9(b) and 9(c). The increase of the intensity of the pulses

is clearly seen. Since the modulation frequency is decreased by a factor of two, the duration of pulses increases by the same factor to 500 fs.

Cumulative filtering of two spectral components by atoms with the spectral doublet produces pulses with modest contrast. The ratio of the maxima of the pulse intensities to their minima is close to three. To make the pulse minima close to zero, one can use destructive interference of the filtered radiation field  $E_{cum}(t)$  with the field  $E_{EO}(t)$  from the same electro-optic modulator but with the opposite phase and reduced amplitude, i.e.,

$$E_{rdc}(t) = E_{cum}(t) - RE_{EO}(t), \quad (23)$$

where  $R$  is the amplitude reduction factor. For the case of the filtering with the doublet, the reduction factor is  $R = 1 - J_{100}(103) - J_{98}(103)$ . Then, due the interference, the pulse minima at the bunch center become zero, while the amplitude of the pulse maxima reduces to the value  $2[J_{100}(103) + J_{98}(103)]E_0 = 0.55E_0$ . The result of this

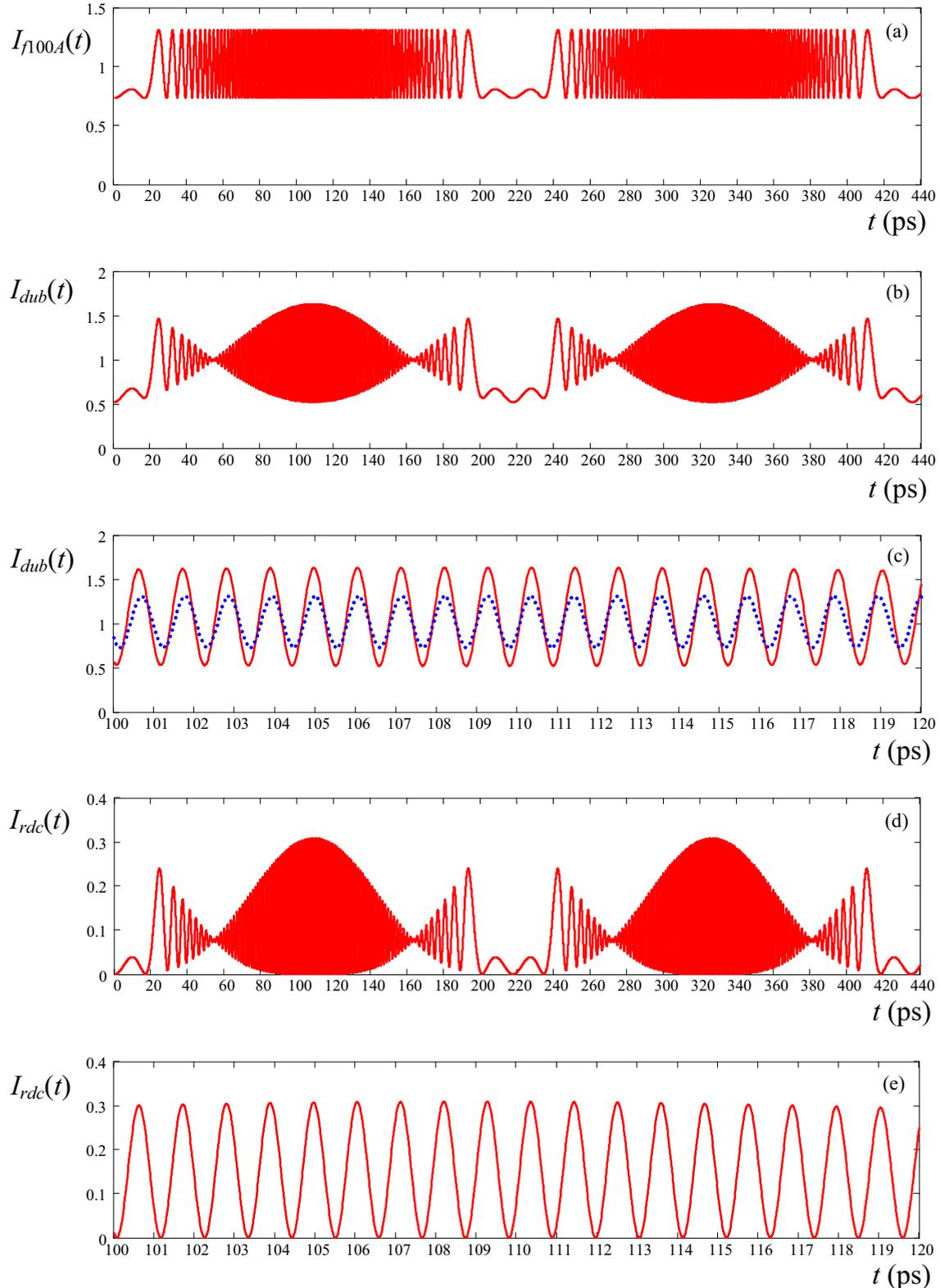


FIG. 9. (a) Time dependence of the intensity of the radiation field whose  $n = 100$  spectral component is removed by the filter. The modulation frequency is  $\Omega/2\pi = 4.6$  GHz and modulation index is  $m_{100} = 104$ . (b) Accumulative result of the filtering of the additional spectral component  $n = 98$  of the frequency comb by the doublet (see the text for details). The modulation index is  $m = 103$ . (c) Zoom of a central part of the bunch. The accumulative effect is shown by the solid red line, while the dotted blue line demonstrates the result of the filtering of only one spectral component with  $n = 100$ . The full width at half maximum of the pulses is close to 500 fs as expected from simple estimations. (d) The result of the filtering with the doublet, followed by reduction of the field amplitude due to the destructive interference with the reduced field from the electro-optic modulator whose phase is shifted by  $\pi$ . (e) Zoom of the central part of the pulse bunch, shown in (d).

interference is shown in Figs. 9(d) and 9(e) for the field intensity  $I_{\text{rdc}} = |E_{\text{rdc}}(t)|^2$ .

Cumulative filtering is better to implement using organic molecules doped in a polymer matrix, which undergo persistent spectral hole burning at liquid-helium temperature. In such a filter, the frequency resolution is limited by the width of the homogeneous zero-phonon lines of the chromophore molecules. For example, a waveguide narrow-band optical filter, which consists of a planar waveguide with a thin polymer film containing molecules, which undergo persistent spectral hole burning at liquid-helium temperature, demonstrates a transmission bandwidth of less than 1 GHz [42,43]. Saturated holes are burned in waveguide geometry by illumination in the transverse direction with low absorption, whereas the probing is carried out in longitudinal waveguiding directions with high absorption. The waveguide with spectral hole burning can act as an integrated subgigahertz narrow-band filter, which is proposed to observe the slow-light phenomenon [44,45].

Comb structures of arbitrary shapes in transmission spectra were created experimentally in organic molecules doped in a polymer [46,47]. Therefore, one can expect that it is experimentally possible to create a broad hole with, say, five absorptive peaks in it. For cumulative filtering, it is enough to create a broad hole with the width  $\sim 10 \text{ cm}^{-1}$  and five absorptive peaks in it with 20 GHz spacing and widths less than 1 GHz.

## VI. DISCUSSION

Resonant filtering by cold atoms is capable of producing nanosecond and picosecond pulses in a controllable way. The nanosecond range could be achieved since cold-atom absorption lines are homogeneously broadened and hence very narrow. This filtering could be applied to create time-bin qubits from single-photon wave packets with a long coherence length. Several examples of such a qubit formation are experimentally demonstrated in Refs. [20,21], where gamma photons with coherence time 141 ns are transformed into a train of short pulses.

The absorption lines of the atomic vapors are Doppler broadened and they are relatively broad. Since the frequency spacing of the filtered combs must be larger than the absorption linewidth  $\Delta\omega_D$ , the vapor filtering is capable of producing picosecond or shorter pulses. Nanosecond pulses cannot be produced by the vapor filters because of large absorption linewidth.

Both cold- and hot-atomic filters are quite flexible in producing a large variety of pulse trains. By changing the modulation index or frequency, one can change the pulse duration and repetition time in a wide range. These filters should not be very thick and can have a fixed density, which is not constrained by a desired pulse duration.

This is very different from dispersive filters [6–10], which involve frequency modulation followed by a dispersive compensator for producing short optical pulses. The idea of dispersive filtering has been based on the concept of chirp radar [48–50]. In the chirp radar system, the transmitted pulse is of relatively long duration  $\Delta t$  during which time the instantaneous frequency is swept over the range  $\omega \rightarrow$

$\omega + \Delta_{\text{chirp}}$  satisfying the condition  $\Delta_{\text{chirp}}\Delta t \gg 1$ . The return pulse is passed through a dispersive network providing a differential delay  $\Delta t$  over the frequency range  $\Delta_{\text{chirp}}$ . As a result, the energy at the beginning of the pulse is delayed so as to reach the end of the network at the same time as the energy at the end of the pulse. The duration of the compressed pulse produced this way is of the order of  $\delta t \approx 1/\Delta_{\text{chirp}} \ll \Delta t$ .

In the optical domain, electro-optical modulation of the radiation frequency [7] spreads the field spectrum over the range  $\omega_r - m\Omega \rightarrow \omega_r + m\Omega$ . In this scheme, dispersive properties of atoms are used to create frequency-dispersive group velocities of the spectral components produced by modulation. Therefore, the dispersive filtering scheme requires resonant media with a very large optical thickness and which employs large offset  $\Delta = \omega_r - \omega_f$  between the radiation frequency  $\omega_r$  and the absorption line center  $\omega_f$  ( $|\Delta| > m\Omega + \Delta\omega_D$ ) to avoid the absorptive losses. These constraints force one to work with relatively small frequencies  $\Omega$  and large modulation index  $m$ . Then, many spectral components of the comb acquire appreciable dispersive phase shifts with a large difference between the blue and red borders of the comb spectrum. The disadvantage of the dispersive filter is a requirement of a fixed, certain value of the vapor density, which is necessary for pulse compression for given values of the modulation index  $m$  and frequency  $\Omega$ , and offset between the radiation frequency and the absorption line center  $\Delta$ . Any deviation from the predefined, fixed value of the vapor density essentially reduces pulse production efficiency.

## VII. CONCLUSION

The resonant method of converting phase modulation into amplitude modulation of the radiation field in the optical domain is discussed. Phase modulation could be implemented by using an electro-optic modulator, which converts a single-line radiation field into a frequency comb consisting of fundamental frequency  $\omega_r$  and sidebands spaced apart at distances that are multiples of the modulation frequency, i.e.,  $\omega_r \pm n\Omega$  ( $n = 1, 2, \dots$ ). The intensity of the spectral components of the comb is proportional to  $J_n^2(m)$ , where  $m$  is the phase-modulation index. If the  $n$ th spectral component of the comb is tuned in resonance with atoms in the filter whose resonant absorption line is narrower than the frequency spacing  $\Omega$  of the comb, then the filtered field is transformed into pulses demonstrating a conversion of the phase modulation into intensity modulation. The effect is explained by the interference of the coherently scattered resonant component in the forward direction with the whole comb. Constructive interference of the fields results in formation of pulses. Their destructive interference is seen as dark windows. Three examples of the resonant filter are analyzed. They are an ensemble of cold atoms, atomic vapor of alkaline atoms, and organic molecules doped in polymer matrix. It is shown that it is preferable to work with the filters with moderate optical depth. If the optical depth is large, then due to dispersive wings of the filter many spectral components of the comb are modified, producing pulse corruption. The creation of nanosecond, subnanosecond, and femtosecond pulses is analyzed. The number of pulses and their spacing is well

controlled by the modulation frequency and modulation index. The proposed method could also be applied for photon shaping. Single photons of spectral widths ranging from several MHz up to 1 GHz could be modified to encode the information into time-bin qubits.

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